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Engineering Science Series

ENGINEERING SURVEYS:
ELEMENTARY AND APPLIED

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ENGINEERING SURVEYS:
ELEMENTARY AND APPLIED

By HARRY RUBEY, B.S., C.E.
Chairman, Department of Civil Engineering
University of Missouri

GEORGE EDWARD LOMMEL, B.S., C.E. *Professor of Topographical Engineering • Purdue University*

MARION WESLEY TODD, M.S.C.E. *Late Associate Professor of Topographical Engineering • Purdue University*

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PREFACE TO THE REVISED EDITION

The changes in this Second Edition result in large part from uggestions made by those who have used the text in the classoom. Other changes and additions have been made in order o conform as closely as possible to present-day surveying ractice.

The major differences between this edition and the earlier ne will be found in:

1. Chapter XII on Land Surveys. Most of these changes were made necessary by the 1947 revision of the *Manual of nstruction for the Survey of the Public Lands of the United States* sued by the United States Bureau of Land Management.

2. Chapter XXI on Engineering Astronomy. The 1949 phemeris data are used in computing latitudes and azimuths rom sun and star observations. Guide outlines of procedure or showing relations between places and celestial bodies are added for the benefit of the student.

3. Chapter XXIV on Photogrammetry. This has been nlarged, revised, and brought up to date by Professor H. O. arp. Practical applications of the use of photogrammetric inciples are stressed for such problems as highway location.

4. Additions to and deletions from the lists of references rinted on the last page of each chapter. These are helpful o the practicing engineer or student when more details of a articular subject are needed. These references have been arefully selected.

As in the previous edition, the cooperation of associates in eaching and practice, of students, of manufacturers, and of overnmental agencies is gratefully acknowledged. The uthors will be most appreciative of further desirable changes at may be brought to their attention by those who use the ext.

THE AUTHORS

ebruary, 1950

PREFACE TO THE FIRST EDITION

This combined volume of Engineering Surveys has been prepared as a text and reference book for the first (Elementary) and the second (Applied or Higher) courses in surveying. The elementary part is identical with *Engineering Surveys: Elementary,* which has passed through several corrected printings and editions, and which continues to be published separately for those preferring a shorter text. The same assignments can be made from both books.

The present volume aims to present the entire subject as it is taught throughout the United States, and as specified in the following minimum course content adopted by the Second National Surveying Teachers' Conference of the Society for the Promotion of Engineering Education held at Camp Case, Ohio, in August 1940:

> "*Elementary Engineering Surveys:* History and purposes of surveying; note keeping and theory of errors; distance measurement; leveling; horizontal and vertical angle measurement; methods of traversing; meridian determination; location of details; traverse computations with special emphasis on coordinates; plotting; methods and sources of information of land description, including United States Public Land System, by metes and bounds, and by named subdivision; responsibilities and limitations of land surveyors; adjustment of instruments; and appropriate field practice.
>
> "*Applied Engineering Surveys:* Field practice in adjustments and practical astronomy together with theory dealing with the application of errors and precision; topography and mapping with horizontal and vertical control by transit and stadia, plane table, and the elements of photogrammetry unless the latter is covered in a separate course; hydrographic surveying, including stream gaging where possible; and a general knowledge of other forms of surveying such as underground and mine surveying, use of state plane coordinates, and legal aspects of land surveying."

The Conference also suggested that ". . . teachers of surveying 'streamline' their surveying courses by eliminating nonessentials and adding new material made necessary by future widespread use of precision surveying for control purposes, state-wide coordinates, and photogrammetry; and that they create a new 'esprit de corps' . . . and an aroused interest in the subject, with general recognition of recent important developments and future potentialities."

More than the minimum content for these subjects is included and additional subjects useful to most engineers have been added, such as the chapters on city planning surveys and construction surveys.

Many phases of surveying are advancing so rapidly that they must be followed in technical periodicals and in engineering practice rather than in textbooks. For this reason a sound fundamental treatment is given in this text and adequate annotated references occur at the ends of the "Applied" chapters and elsewhere. Were this volume doubled in size by including the ever changing details covered in current publications, its purpose of serving as an easily comprehended and practical text would be defeated. Regarding current developments, the reader must consult the Engineering Index and the Industrial Arts Index which list periodical articles describing latest procedures. It is essential that the engineer form the habit of thus familiarizing himself with contemporary practice.

For the few engineers who engage in the most precise and specialized types of surveying, as for example first order control of the United States Coast and Geodetic Survey and photogrammetry, it will be necessary to study further these subjects which are treated only briefly in this volume, either in more advanced collegiate courses or in actual practice with governmental or private organizations engaged in this work.

The comprehensive Tables include five-place logarithms and trigonometric functions for use in ordinary surveying, and seven-place natural functions of sines and cosines for use in the more precise surveys, assuming that the natural functions generally will be used on the electric computing machine which has replaced logarithms for extensive office computations. The previously published student note forms have been improved and extended to include more difficult types in Table X. Answers to the first problem of most sets of problems are given to aid the student, and the further unsolved problems are provided for testing his knowledge. Different sets of numerical data permit the problems to be used for several semesters.

Special effort has been made to facilitate ease of reading and reference through large type, clear figures, a comprehensive index, and the numbering of tables, figures, and formulas so that they may be located by means of the section and chapter numbers carried at the head of each page. It is hoped that joint authorship of five men as well as the suggestions which have been received from many associates in teaching and in engineering practice will insure an authoritative treatment. The cooperation of these associates as well as the assistance of manufacturers of equipment and of governmental agencies in providing illustrative material are gratefully acknowledged. Valued suggestions were received from the editor, Dean H. P. Hammond of The Pennsylvania State College.

Chapter XXIV, Photogrammetry, was written by Professor H. O. Sharp of Rensselaer Polytechnic Institute; and Chapter XXVI, Hydrographic Surveys, by Professor W. I. Freel of Purdue University. Advice and criticism were given by Professor A. H. Holt of Worcester Polytechnic Institute for Chapter XXVII, Legal Principles of Boundary Surveys; and by Professor C. R. Forbes of the Missouri School of Mines and Metallurgy for Chapter XXIX, Underground Surveys.

Tables I, II, and III are those of the Macmillan Company; Tables V and VI are from the *Standard Field Tables of the United States Land Office;* Table VII is that of the *Paulin System Manual of Altimetry;* and Table XI is from Ives' *Seven-place Tables.*

It has been the purpose to prepare a practical, up-to-date text and reference book covering material most needed by practicing engineers, adapted to modern trends in surveying instruction for all departments of engineering; and suitable for either campus classes or surveying camp.

This volume, together with Rubey's *Route Surveys,* completes a series of coordinated texts covering all of the undergraduate surveying courses ordinarily required. It is offered with the hope of improving instruction in this basic field of engineering study and practice.

THE AUTHORS

January 1, 1942

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ENGINEERING SURVEYS: ELEMENTARY AND APPLIED

ENGINEERING SURVEYS: ELEMENTARY

CHAPTER I

INTRODUCTION

1. Why Study Engineering Surveys? Briefly, the broader answers to this question may be summarized as follows:

1. It is a required subject in engineering curricula throughout the country and is the basis for a large part of most engineering work.
2. Large numbers of engineers engage in surveying, or work closely associated therewith, as their first position after leaving college.
3. Other engineers are so engaged in subsequent practice either directly, indirectly, or in a supervisory capacity.
4. Surveying serves to introduce the engineer favorably to influential persons who are undertaking expensive projects. Desirable professional and business connections often grow out of such introductions.
5. A considerable portion of those engineers who became top engineering and management officers of corporations and governmental enterprises began their careers in surveying.
6. It is commonly thought that engineers are usually hampered in their advancement by lack of friendly extroverted public contacts and that successful careers in engineering and management are better attained by men possessing these characteristics. Surveying, and the contacts associated with it, tend to develop such characteristics.

Other viewpoints and benefits resulting from surveying may be found in and inferred from the following sections of this introductory chapter.

2. What Engineering Surveys Are. *To survey* is defined by Webster's Dictionary as follows: "To determine and delineate the form, extent, position, etc., of, as a tract of land, a coast, harbor, or the like, by taking linear and angular measurements, and by applying the principles of geometry and trigonometry."

All surveys have the same basic characteristic — that of delineating a portion of the earth's surface or sub-surface or of establishing the position or boundaries of some object upon it; but engineering surveys usually serve the additional purpose of furnishing information upon which an engineering project is planned, laid out on the ground, and constructed.

Not all surveys are of such nature or extent as properly to be called engineering surveys; some, for example, are merely simple surveys of land boundaries that may be conducted by following well-established routine methods. Some land surveyors of the old school, who learned to make surveys of this kind by a sort of apprenticeship, are not engineers and are not classed as such when they are licensed by law, since they are not qualified to execute engineering projects. Engineers, however, must have a working knowledge of surveying because engineering projects so often involve surveys.

Surveys cover a wide range in scope and complexity, from the staking out of simple structures or the surveying of small parcels of land to the extensive and difficult surveys incident to great construction projects, such as a railway or an aqueduct, or the surveys of relatively large portions of the earth's surface. The latter often require precise scientific methods and a very high degree of skill, for which education, training, and experience of a high order are necessary.

In recent years, in increasing proportion, many surveys are made by engineers, even though the survey may not form part of an engineering construction project.

Surveys are divided into two general classes, *plane* surveys and *geodetic* surveys. Plane surveys are those made upon the assumption that the earth is flat. Most engineering surveys are of this nature. The error involved in this assumption as to an area as large as a township (six miles square) will not ordinarily be excessive.

Geodetic surveys are those in which the surveyor takes into account the curvature of the earth, generally considering it as a sphere, but in some instances allowing for its true spheroidal shape. Geodetic surveys are usually made by the United

States Coast and Geodetic Survey; they are partly of a directly practical nature and partly in the nature of purely scientific work. *Engineering Surveys: Elementary* treats only of plane surveys, such as are the basis of most engineering projects of which surveys are a part. Some consideration is given to geodetic surveying in *Engineering Surveys: Applied.*

3. Uses of Surveys. All engineering and construction projects extending over a considerable area or distance are based upon elaborate and complete surveys. Such projects include railways, highways, transmission lines, canals, pipe lines, sewers, streets, buildings, industrial plants, wharfs, piers, dams, reservoirs, and the like. Real estate holdings must be surveyed for transfer, mortgaging, and determination of boundaries on the ground. Extensive maps, based upon surveys, are made by the Federal Government. The use of maps and surveys in military operations is well known. Most constructions involve a succession of surveys.

Regarding the importance of surveys in their respective fields, it is probable that the various branches of engineering rank in somewhat the following order: civil, mining, agricultural, electrical, mechanical, and chemical.

The more scientific the basis upon which a corporation in a given field conducts its operations, the more surveys are made to aid the management in reaching its decisions. As a consequence, the modern tendency to replace the small business project under individual ownership by the large corporation, results in the making of more and more surveys.

Because of the extent to which surveys and maps are the basis of all extensive engineering projects and most industrial operations, executives find knowledge of the subject valuable, particularly a knowledge of land surveys. Officers of all branches of the army are required to be familiar with surveying and mapping, and certain branches of the service use it extensively. Many corporation officials have had early experience in surveying.

4. Training Provided by Surveying. Surveying provides training of many kinds, in addition to inculcating knowledge of surveys themselves. The subject provides an excellent review and application of the principles of geometry and trigonometry, those branches of mathematics that are so widely used in engineering. It provides training in the systematic and neat arrangement of notes taken in the field, a training that is equally applicable to the factory and to the laboratory; and it provides drill in long and systematic calculations which cannot be carried out haphazardly, in the use of logarithms, in mapping, and in drawing.

The field work gives practice in planning the operations of groups of skilled men; it gives close contact with fellow workers and with the inhabitants of the country in which surveys are made; and it develops in the chief of the survey party the qualities of leadership needed in the negotiations and the carefully planned scientific operations that are required in modern industry.

All of the office training in surveying will be found helpful in other types of engineering office work.

Possibly one of the greatest benefits to the individual grows out of the healthfulness of the work. Largely conducted out-of-doors, with sufficient office work to occupy times of inclement weather, it combines the healthful benefits of hunting or fishing with the satisfaction that one is engaged in constructive and remunerative work rather than in non-productive play.

It is thus not difficult to understand why some of our great leaders practiced surveying at some time in their lives. George Washington* continued his surveying as opportunity permitted up to the time of his death. Lincoln and Hoover were surveyors for a time. Ex-president Rubio of Mexico states that he has carried a transit over most of that country. A number of the presidents and chairmen of the boards of directors of our railways, steel and oil companies, and the like, began their careers in work involving surveys. Good health, the breadth and balance of mind resulting from outdoor work, the confidence developed by seeing scientifically planned surveys made and checked with precision and accuracy, the projects for the benefit of mankind that are reared upon these surveys, all tend towards making a full and well-balanced life for those so engaged.

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* In his definitive biography, *Washington*, 1948, Freeman says, "Washington's maturity, his energy, his high code of behaviour and his skill in dealing with most of those in authority were rendered more effective by his strong sense of order. Every task was performed as if it were a land survey — step by step, with closest possible approach to absolute precision. As his memory was not especially strong, Washington relied on memoranda, the military equivalent of his surveyor's field notes. Whatever he undertook to do, he did thoroughly and methodically."

The late Judge E. H. Gary, for many years Chairman of the Board of Directors of the United States Steel Corporation, said: "Of highest importance and consequence . . . a man should be absolutely honest. Akin to honesty is accuracy. . . ." Both honesty and accuracy are fostered by surveying. One must be absolutely honest and accurate in his survey work, not only to his associates but to himself. No mental evasion or play upon the weakness of humanity will cover up errors or make poor work check properly. Careful, painstaking effort resulting in definite material benefit cannot but make for the development of character.

Unfortunately, the remuneration of the older surveyor, both financial and in prestige, is not great. For this reason most of the technically educated engineers turn their attention to other types of activity, such as engineering and administration, after some months or years spent in surveys. Through the contacts with engineers, managers, and owners, surveying provides an excellent opening into these activities. Memories of that portion of the younger life so spent, and the occasional opportunities for surveying in later life, will be recognized as having contributed toward substantial success in the worthwhile things of life.

It is hoped that the engineer, especially the student engineer, will approach his survey work with these broad views in mind.

5. Factors Controlling Surveys. The factors that control methods used in surveys are determined by the need —

1. To secure the needed information
2. With adequate accuracy or precision
3. At a minimum cost, and
4. Frequently in a minimum time

Neither time nor money may be spent indefinitely in attempting to secure a perfect survey; in other words, the law of diminishing returns operates here as elsewhere.

As in most human affairs, the theory involved in ordinary surveying is not particularly difficult to understand. Thought, judgment, skill, experience, care, and honesty are necessary in the execution of surveys and survey computations so that they may conform to the criteria given above. The greatest care must be exercised to avoid mistakes and small errors, and a procedure must be adopted which will permit of continual checking, since only by such checking is correct work assured.

Good surveys are based on painstaking and systematic work which has been executed with good judgment and good human relations. Surveying is thus partly a science and partly an art.

CHAPTER II

MEASUREMENT OF DISTANCES

6. Introduction. Rough estimates of distances may be made by eye or scaled from maps or aerial photographs. Occasionally distances are secured by triangulation, as described in Chap. XI. The more common field methods are described in the following sections. The necessary *precision* (degree of accuracy) of measurement should be decided upon; then that method should be used by means of which the measurement will be made most conveniently.

7. Pacing. The length of the human pace varies with the individual, few people having a natural pace as long as 3 feet. For pacing short distances, such as the dimensions of a room or building, good results may be obtained with an artificial pace of 3 feet, but few can maintain such a pace for long distances without fatigue. For these longer distances the natural length of pace is determined by pacing over a course of known length, a quarter of a mile or more, and thus standardizing the pace of the individual.

The length of pace will vary with fatigue, with the weight of shoes and clothing, with the roughness of the ground, with the slope of the country, and with the speed of pacing. In general, the length of the pace decreases as any of these factors increases, except the speed.

Considerable difficulty is encountered in keeping count of the paces. This is facilitated by counting *strides*, each stride consisting of two paces and being between 5 and 6 feet in length. A sort of rhythm may be developed in counting 5- or 10-stride units instead of single-stride units. Each 10 or 100 strides may be tallied by closing one of the fingers of the open hand; or a notched stick may be provided upon which to move the thumb forward, each notch representing 10 or 100 strides. A mechanical *tally* which is operated by the thumb may be used similarly. A *pedometer* when strapped to the pacer will automatically record in miles the distance covered, after it has been adjusted to the pace of the wearer. A similar instrument called a *passometer* automatically records the number of paces.

A good pacer will attain results within 1 per cent of the true distances; and an inexperienced man, after standardizing his pace, will seldom be more than 2 or 3 per cent in error, unless some mistake has been made in counting.

8. Speedometers. The speedometer of an automobile will give better results than pacing, provided the route is reasonably smooth, as along a road. The nearest tenth of a mile may be read on the ordinary speedometer and the nearest hundredth of a mile may be estimated. By drawing proper lines upon the instrument, the nearest hundredth of a mile may be read directly. The precision of this method, when used for distances of over a mile, is of the order of 1/2 per cent, or 1/200. The car should be tested over a known distance, since the size of the wheel and tire and the tire pressure affect the results. In both pacing and speedometer measurements, it must be remembered that the distances obtained are along the slope, while the distances required in surveying are horizontal, and due allowance should be made for this fact. However, for a slope of 8° 07′, which is a 14 per cent grade, the horizontal distance is only 1 per cent less than the slope distance.

The speed in miles per hour or feet per minute of a pedestrian, a saddle horse, or an automobile were formerly used and may prove useful upon occasion, although they are not precise.

9. Stadia. A more precise method of measuring distances by using stadia is described in Chap. IX. It is more expensive than the foregoing procedures, but gives results with an error of the order of 1/500, or better, if unusually careful work is done. Stadia errors tend to *compensate* or balance one another, so that the result of a series of stadia measurements is more precise than 1/500.

10. Rules. Short wooden or steel folding rules from 2 to 6 feet long are commonly used by mechanics and occasionally by surveyors. Since mechanics generally measure in feet and inches and surveyors in feet and decimals of feet, conversion is facilitated by having rules graduated in eighths of inches on one side and hundredths of feet on the other. One must carefully avoid using the wrong side of such a rule; perhaps the sides should be of different colors.

One-hundredth of a foot is closely equal to one-eighth of an inch (error 4 per cent). By the use of this relationship and of Table A, inches may be approximately converted mentally to decimals of a foot, and vice versa. For example, $0.72' = 9'' - \frac{3}{8}'' = 8\frac{5}{8}''$; or $0.70' = 8'' + \frac{3}{8}'' = 8\frac{3}{8}''$; or $7\frac{5}{8}'' = 0.50' + 0.08' + 0.05' = 0.63'$; or $7\frac{5}{8}'' = 0.67' - 0.03' = 0.64'$.

TABLE A

Feet	Inches
0.0104+	1/8
0 0417−	1/2
0.0833+	1
0.25	3
0.3333+	4
0.50	6
0.6667−	8
0.75	9

A wider range of more precise equivalents may be found in **Table VIII.**

11. Chains. The *engineer's chain* was divided into 100 parts each 1 foot in length and consequently was 100 feet long. The *surveyor's* or *Gunter's chain* was only 66 feet (called 1 *chain*) long and was composed of 100 *links* each 0.66 of a foot, or 7.92 inches, long. Chains were used until the latter part of the nineteenth century, and were then abandoned in favor of the steel tape described in the following section. They were heavy and awkward and they were not precise.

While the chain has been abandoned, the word *chaining* persists as applied to the measurement of distances. In references to distances measured with a tape, it is preferable, however, to use the words *taping* and *tapeman* instead of *chaining* and *chainman.*

The word *chain,* meaning a distance of 66 feet, is still properly used in referring to United States public land survey distances, that is, to surveys of sections of land. The United States General Land Office, now known as The Bureau of Land Management, still records and measures its distances in *66-foot chains, 80 chains being one mile* and *ten square chains being an acre.* However, the agency uses a modern steel tape, usually five or eight chains long, instead of the old-fashioned wire-link chain.

In old surveys the *rod, pole, or perch* was frequently used, *1 chain being equal to 4 rods or 4 poles or 4 perches.*

12. Tapes. Manufacturers of surveying equipment offer measuring tapes in various lengths, widths, and weights. Each type is designed to serve a particular need. Those made of high-grade tape steel may be had in lengths of from 5 to 500 feet and in widths of from 1/8 inch to 1/2 inch. Tape surfaces may be the ordinary bright steel; steel processed to a dark color; or a white, flexible, baked enamel surface with black markings.

Up to 50-foot lengths, tapes are supplied in leather cases. Longer tapes may be purchased with or without winding reels.

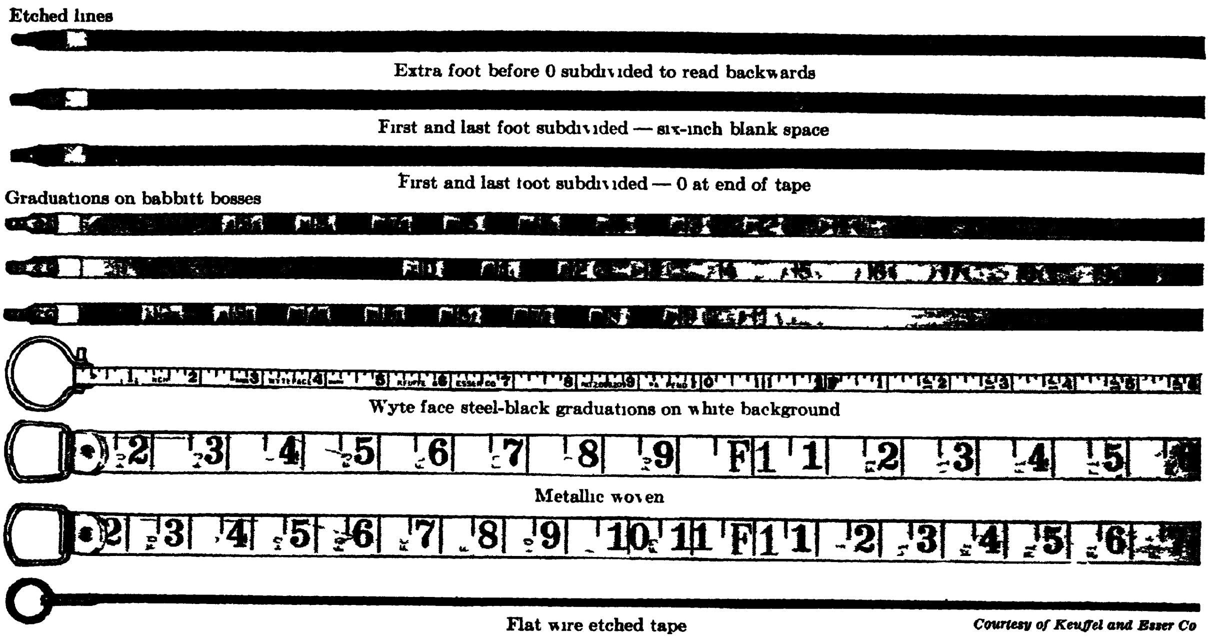
Graduation marks and numbers are either etched on the steel surface or stamped on babbitt-metal bosses. These graduation marks are in feet and decimals; in feet, inches, and fractions; or in metric units.

In surveying practice, the standard tape is 100 feet long with graduation marks every foot. Decimals of a foot are provided at one or both ends of the tape. The types which seem to be most popular are 1/4-inch or 5/16-inch wide, 0.015 to 0.020 inches thick, and with the 0 mark and the 100-foot mark placed about 6 inches in from the ends of the steel ribbon. Plated steel or "stainless" steel is desirable.

Structural engineers, architects, mechanical engineers, and mechanics prefer tapes graduated to inches and fractional parts of an inch.

For surveying operations in which a high degree of precision is unnecessary, a "metallic" tape is used. This is a waterproofed linen tape into which fine wire strands have been woven. Metallic tapes are usually 50 feet long with graduation marks at every 0.05-foot point. Each tenth is numbered for the entire tape. Some have inch and half-inch graduations.

When the linear measurements of a survey must be made with a high degree of precision, Invar or other nickel-steel alloy tapes are used. These tapes are of soft metal, kink easily, and cost much more than ordinary steel tapes. The combination of nickel and steel used in their manufacture results in a product


Etched lines
Extra foot before 0 subdivided to read backwards
First and last foot subdivided — six-inch blank space
First and last foot subdivided — 0 at end of tape
Graduations on babbitt bosses
Wyte face steel-black graduations on white background
Metallic woven
Flat wire etched tape

Courtesy of Keuffel and Esser Co

FIG 12 TAPES

which has an extremely low coefficient of thermal expansion. Invar tapes should never be wound on a reel less than 18 inches in diameter, nor "thrown" into circular form from the "figure eight." In the field they should be carried from point to point and not dragged along the ground.

Reels are provided by the manufacturers and are used quite generally for 300-foot and 500-foot tapes. They are seldom used for 100-foot tapes because of the ease with which these may be carried when done up in a figure eight or in a circle. When a reel is not used, the tape is first done up in 5-foot loops, keeping the same side of the tape upward, thus causing it to form a figure eight. This figure eight may be "thrown" into a smaller circle.

The process is reversed in undoing the tape. Care must be exercised in straightening the tape from the figure-eight form or it will become badly tangled and probably kinked. Once a tape has been kinked, it breaks easily at that point. By taking off the loops successively, beginning at the outside or top, this difficulty will be avoided.

13. Horizontal Distances Required in Taping. In surveying practice, all the distances, angles, and areas are either measured in horizontal and vertical planes or, if measured in inclined planes, are reduced by computation to horizontal or vertical components. In the determination of linear distances, the surveyor may decide to measure the horizontal component directly by holding the tape level, or he may choose to measure the slope distance and, with either the difference in elevation between the ends of the line or the slope angle, to compute the horizontal distance.

If he chooses the second method, the following approximations give sufficiently accurate results for the average conditions encountered in the field. From Fig. 13 approximate practical formulas are derived, as follows:

$$s^2 - h^2 = V^2,$$

$$(s - h)(s + h) = V^2,$$

$$s - h = \frac{V^2}{s + h}.$$

(13) Correction (to be subtracted from s)

$$= \frac{V^2}{2s} \qquad \text{(approximately).}$$

For example:

$$s = 100 \text{ feet}, V = 17 \text{ feet}, s - h = \frac{V^2}{2s} = \frac{289}{200} = 1.445 \text{ feet},$$

whereas the true value of $s - h$ is 1.456 feet, an error of about 0.01 foot.

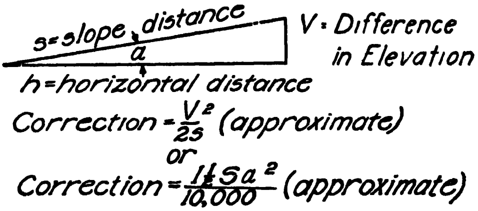

s = slope distance
a
V = Difference in Elevation
h = horizontal distance
Correction = $\frac{V^2}{2s}$ (approximate)
or
Correction = $\frac{1\frac{1}{2}sa^2}{10,000}$ (approximate)

FIG 13. CORRECTION TO HORIZONTAL.

If the small acute angle α (in degrees) is known rather than the height V, then by using the arc value of the angle, we find

$$V = 0.0175s\alpha \qquad \text{(approximately)} \tag{13a}$$

and formula 13 becomes

$$s - h = \frac{V^2}{2s} = \frac{(0.0175s\alpha)^2}{2s}$$

$$= 0.00015s\alpha^2$$

$$= \frac{1\frac{1}{2}s\alpha^2}{10,000} \qquad \text{(approximately).} \tag{13b}$$

For example:

$$s = 100 \text{ feet}, \quad \alpha = 5°, \quad s - h = \frac{(1\frac{1}{2})(100)(25)}{10,000} = 0.375 \text{ foot},$$

whereas the true value $s - h$ is 0.381 foot.

The approximation is less exact for large values of s or α, but it is frequently satisfactory. The computation is facilitated, although the result is less precise, by taking α to the nearest degree and s to the nearest 100 feet, one larger and one smaller than the true values.

In Fig. 13a, the versine is used to secure the horizontal distance by means of a correction, because the tabular versine has fewer digits and consequently is more easily used as a multiplier than is the cosine. Frequently the multiplication may be made mentally. If a table of versines is not available, the versine may be obtained from a table of cosines, since the versine = 1 − cosine. See also **Table IV.**

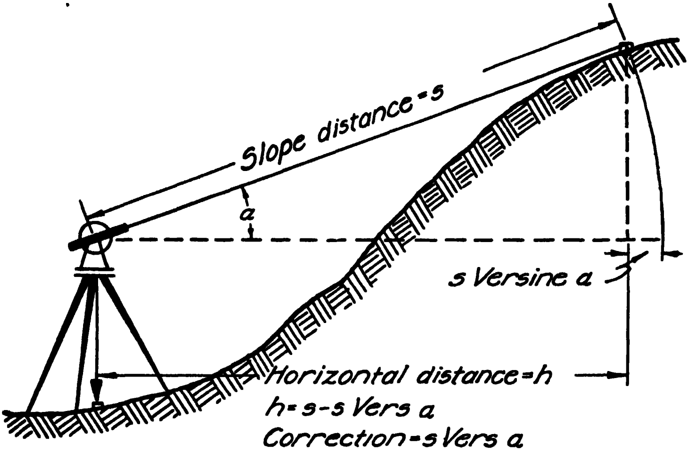
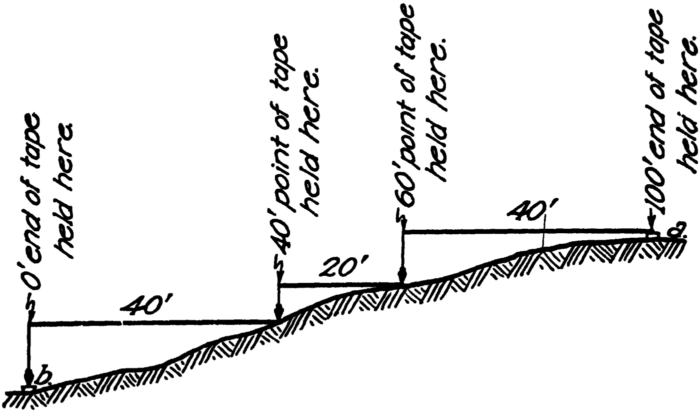


Fig. 13a. Correction to Horizontal.

It is preferable to measure with the tape lying supported on some level surface such as a railroad, a pavement, or a sidewalk, but such convenient aids are seldom available. In order to keep the tape level and to raise it above irregularities of the ground, brush, and weeds, it is customary for the head or rear tapeman, or both, to hold the end or ends of the tape above the ground and to use a plumb bob to transfer the position of the end of the tape to the ground. The head tapeman allows the plumb bob to hang an inch or so above the ground, and drops it to mark the point after smoothing the ground with his foot.

It is desirable for either the head tapeman or the rear tapeman to hold his end on the ground or stake, if possible. Distances are taped down hill more effectively than up hill, provided the rear tapeman can hold his end on the ground or stake and so keep the tape steady.

14. Breaking Tape. It frequently happens that the ground slopes so steeply that the full 100-foot length cannot be held level. In such a case (Fig. 14), the rear tapeman holds the tape on the ground or as near it as possible, and the head tapeman proceeds ahead until, with the tape level, his point on the tape is about at the level of his head or as high as he can conveniently reach. He plumbs this point, say 40 feet from the rear end, to the ground. The rear tapeman proceeds to this point and holds the same point of the tape on or near the ground, while the head tapeman moves down the slope until he has again reached a point which requires him to hold his part of the tape as high as he can conveniently reach. This process is repeated for the entire tape length. The procedure is called *breaking tape* and permits measurements to be made on sloping ground with the tape held level. It is best first to pull the tape out its full length, the rear tapeman actually holding the same point on the tape that was held by the head tapeman, as shown in Fig. 14, rather than to measure a number of fractional tape lengths and then to add them arithmetically. The process is reversed in taping up hill.


0' end of tape held here
40' point of tape held here
60' point of tape held here
100' end of tape held here
40'
20'
40'
a
b

Pull tape out full 100 feet from a towards b before setting intermediate temporary points.

FIG. 14. BREAKING TAPE.

If taping pins (see page 17) are being used, the head tapeman sticks a pin at each fractional tape length to hold the point and the rear tapeman gives him a pin to keep the tally straight.

If a plumb bob is not available or requires more time than the required precision of measurement necessitates, a taping pin or stone may be dropped from the high end of the tape to the ground. Sometimes a flag pole may be plumbed or swung vertically at the end of the tape, and its lower end will mark the approximate point on the ground.

15. Lining and Stationing. For long lines, it is desirable that the head tapeman be *lined in* by the transitman. For distances under several hundred feet, however, it is entirely possible for the rear tapeman to line in the head tapeman, sighting over the point already set and the objective point. If the points are considerably different in elevation, or if close alignment is desired, the rear tapeman may hold a plumb bob over his point and sight past the string. Points thus set may be a few tenths of a foot out of line but this will not seriously affect ordinary surveys — in fact the errors in distance due to holding the tape out of level will far exceed those due to alignment. As an example, holding one end of a 100-foot tape 1.4 feet out of line, or out of level, causes an error of 0.01 foot in distance.

In most taping it is customary to record the distances in 100-foot units called *stations*, as carried forward from the beginning of the survey, which is customarily called *station zero or* $0 + 00$. If there is a horizontal angle at some point such as 1365.4 feet from the beginning of the survey, that point would therefore be marked on the stake and in the notes as $13 + 65^{\underline{4}}$. Because the decimal point may become obliterated in the notes or on the stakes, many engineers record the decimal of a foot above a short horizontal line instead of using a decimal point. The usual decimal form may be used in the office if desired.

After taping to the angle point, station $13 + 65^{\underline{4}}$, station 14 is set by the head tapeman, who holds the zero end of the tape. On flatter angles and on less precise surveys, the rear tapeman may hold the end of the tape on station 13 and allow the tape to bend around the angle point, thus avoiding possible errors due to holding the wrong graduation on the tape or to mistakes in subtraction.

16. Field Procedure. A steel tape, two plumb bobs, and a set of eleven taping pins on a carrying ring are necessary equipment for a taping party.

In the measurement of a distance, the head tapeman, *carrying the zero end of the tape,* moves out along the line. The rear tapeman holds the hundred-foot point of the tape on station 0 + 00, where a single taping pin is left. The other ten pins are in the possession of the head tapeman.

The tape is then pulled *taut* by the head tapeman, who, in the meantime, has been signaled into line by the rear tapeman. Both cooperate in seeing that the tape is straight and horizontal throughout its entire length. A taping pin is then set in the ground under the zero point of the tape. This pin marks station 1 + 00. The pins slope at right angles to the line and their intersections with the ground surface are the points used for measurement.

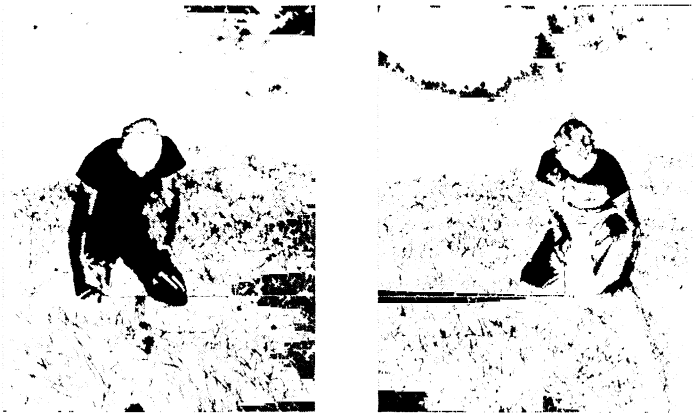


Fig. 16. Taping.

The tape is then dragged along the ground by the head tapeman, moving along the line being measured. The rear tapeman moves from station 0 + 00 to station 1 + 00 and carries with him the first taping pin. Upon his arrival at station 1 + 00, he calls out "tape," whereupon the head tapeman stops, is lined in, and a second hundred feet of distance is marked on the ground, setting station 2 + 00. The rear tapeman then picks up the pin at station 1 + 00 before moving on to station 2 + 00.

If permanent stakes are to be set on line, instead of taping pins, the same procedure is followed, points on the tops of the stakes being marked with a pencil or, if a transit is used to *line in,* tacks are driven. The stakes are labeled 1, 2 and so on. Careful checking of the station number by both head and rear tapemen is necessary in order to avoid costly mistakes.

Under no condition should the rear tapeman pull the taping pin at his point until the forward pin has been set and checked. If stakes are being set, he should take care that the position of his stake is not disturbed while the measurement is being taken. He should also call out the station number of his point either by counting the number of taping pins in his possession or by reading the number marked on his stake. With eleven taping pins to the set, a distance of one thousand feet can be measured before a transfer of pins from rear tapeman to head tapeman is necessary.

It is suggested that Fig. 16 be studied carefully. Note the positions assumed by both tapemen. They are thus enabled to exert a *strong, steady pull* on the tape and at the same time to stay off line.

17 Stakes. Survey stakes are either cut or split from available limbs of trees or from waste lumber, or are secured ready-made from a saw mill or lumber dealer. Various types are shown in Fig. 17. Ordinary lath make satisfactory stakes

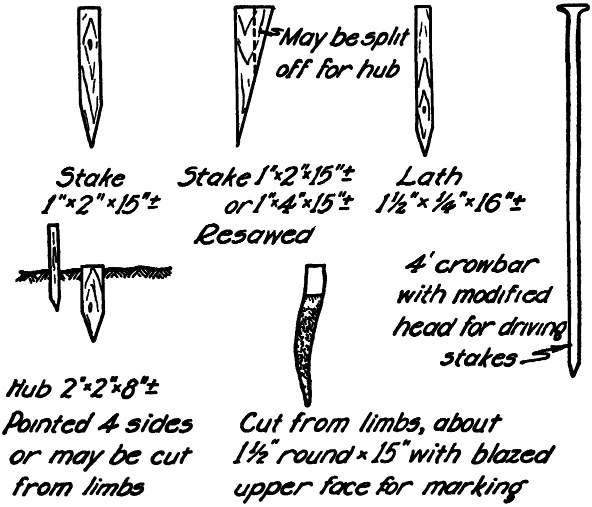

May be split off for hub
Stake 1"×2"×15"±
Stake 1"×2"×15"± or 1"×4"×15"± Resawed
Lath 1½"×¼"×16"±
4' crowbar with modified head for driving stakes
Hub 2"×2"×8"± Pointed 4 sides or may be cut from limbs
Cut from limbs, about 1½" round × 15" with blazed upper face for marking

FIG. 17. STAKES.

if cut in 16-inch lengths, although they cannot be driven in hard ground. They are cheap and easy to obtain, and the full 4-foot length is useful in setting up sights. When used as stakes, they break off readily at the ground, leaving a broken end in the ground, where it may be found. If a heavier stake is kicked or tramped or otherwise disturbed, it will be knocked entirely out of the ground and its position lost.

The kind of stakes used depends on the purpose and permanence of the survey and on relative availability and cost. While stakes are usually driven with a light ax or hand ax, the bar shown in Fig. 17 is useful where many stakes are to be driven, particularly in hard or frozen ground. The stake is inserted in a hole which has been made with a stroke of the bar, pointed end down. The bar is then reversed and a blow or two with the flat head drives the stake home.

Where stakes cannot be driven, as along a railroad, a paved highway, a sidewalk, or the like, the stations may be marked thereon with lumber crayon or traffic paint.

Rough field checks of taped distances may be had by stadia, by counting railroad rail lengths, or by pacing.

18. Measuring Fractional Tape Lengths. It is customary for the head tapeman to carry the zero end of the tape and for the rear tapeman to allow his end to drag along the ground between stations. Tapemen must carefully examine the ends of the tape to determine where the end graduations occur, since this is not the same on all tapes. Most survey tapes have only the foot marks graduated on them, except at one or both ends where the end foot is divided into tenths. Some tapes have an extra foot, divided into tenths, beyond the zero end. Few modern tapes are just 100 feet long over all, most of them having extra lengths beyond the graduated portion on both ends. *Unless the type of end graduation is carefully observed, annoying mistakes will occur.*

When it is necessary to measure a distance of less than 100 feet, say 38.7 feet, the head tapeman first holds the zero mark of the tape on the point and the rear tapeman notes that the taping pin is between 38 and 39. He also makes an estimate of the fraction of a foot. This enables him to "rough check" the exact measurement which is made by the head tapeman. The rear tapeman calls "38 plus," then holds 39 on the pin. The tape is pulled taut by the head tapeman and the fractional distance between the 0-foot mark on the tape and the point on the stake is read, in this case 0.3 foot. The head tapeman calls "cut three tenths," which subtracted from 39 gives 38.7 feet as the measured distance. Hundredths of a foot, if needed, are usually estimated.

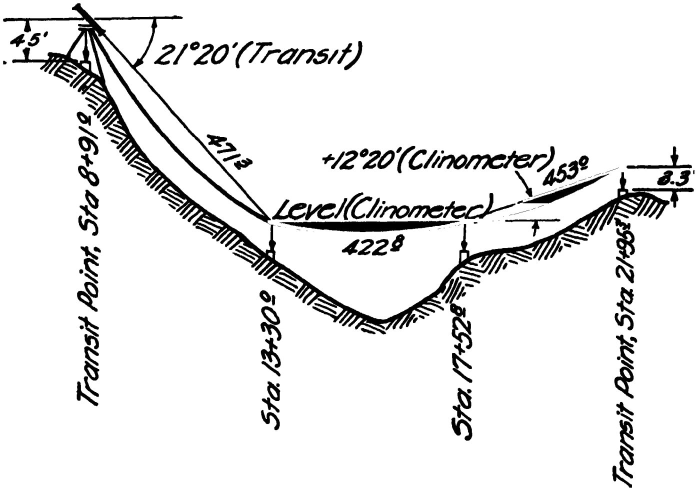
If there is an extra foot graduated to tenths on the zero end of the tape, the rear tapeman holds 38 feet and the head tapeman reads 0.7 foot on the extra foot beyond the zero end.

Some distinctive method of calling numbers to one another must be adopted by the tapemen, perhaps similar to that used by telephone operators, in order to avoid misunderstandings. Possibly the best way of calling 1218.06 would be *one two one eight point oh six.* This should be repeated back to the caller in some different manner, for example, *twelve plus eighteen point zero six.*

19. Slope Taping. Figure 19 shows procedure when taping on a slope. The rear end of the tape is held at the end of the horizontal axis of the transit when that instrument is being used to measure the vertical angle, while the head end may be held either above the stake or preferably on it. The 100-foot tape is seldom used on the slope but the 300-foot and 500-foot tapes are generally so used. The vertical angle along the tape or parallel to it is measured with a transit or with a *clinometer* (Fig. 68). One possible plan is to measure these *slope angles* only at alternate tape ends, one angle forward and one backward from the same point. In Fig. 19, for example, it would be unnecessary to have a clinometer reading at station 13 + 30, since the forward and backward readings of vertical angles could be taken at station 17 + 52.8.

Measured slope distances are reduced to their horizontal components by one of the methods of Figs. 13 and 13a, or by diagrams or tables prepared from these formulas. The methods of formulas 13 and 13b (page 13) give closely approximate results and can generally be accomplished by mental arithmetic or with a slide rule.

It occasionally happens that the relative elevations of the ends of the tape are known or may easily be obtained or estimated. No transit or clinometer is then necessary and no vertical angle is measured. The difference or correction between the slope length and the horizontal length is most easily computed, the height between ends being known, according to Fig. 13 (page 13).



From Engineering News-Record, *courtesy McGraw-Hill Co.*

FIG. 19. SLOPE TAPING.

The differences in elevation between the tape ends may be desired occasionally for vertical as well as for horizontal distance. If fairly precise elevations are required or if the vertical angles are large, the transit must be used so that more precise vertical angles are available. The slope distance is ordinarily multiplied by the sine of the vertical angle to get difference in elevation. Approximate differences in elevation may be taken from tables or diagrams, or they may be computed from formula 13a of page 13.

Slope taping and long tapes have been used in underground surveys almost exclusively for many years. They are now used to some extent for surface surveys, perhaps more in the West than in the East. Under many circumstances their use will materially increase the speed and precision of taping and minimize mistakes. In rough, open country, the speed and precision may be doubled. *The Manual of Instructions for the Survey of the Public Lands of the United States* says: "By a skillful use of the long steel tape on the slope, with correct determinations of the vertical angle, and proper reductions from the slope to the true horizontal distance, the surveyor obtains one of the most rapid and reliable methods of measurement." These methods are used also by the Forest Service, the Coast and Geodetic Survey, and the Geological Survey of the United States.

Long tapes cannot be used to advantage where a considerable amount of traffic would run over them. Further procedure is given in Engineering News-Record, December 24, 1925, page 1024.

A few long tapes have several extra feet of length graduated beyond the normal end graduation. A slope angle is marked on each of these extra graduations. For example, when taping on a 10° slope, the full tape length horizontally is obtained by measuring the greater length to the 10° slope mark. Thus no horizontal correction need be computed for full tape lengths.

A somewhat similar process is occasionally possible when a 100-foot tape with an extra foot graduated to tenths beyond the end is used on the slope to set stakes 100 feet apart horizontally.

20. Tape Standardization. A tape must be of known and definite length. As now manufactured, the better tapes are quite precisely graduated and are generally of correct length as nearly as ordinary field methods can detect. The 100-foot tapes are usually exactly* 100 feet long when supported (as on a floor) throughout their length under a 10-pound pull and at 68° F. The longer tapes are standardized under the same conditions, except that a 20-pound pull is used. The manufacturers sometimes state in their catalogs the conditions under which their tapes are precisely true, or they will furnish this information on request. For a small fee the United States Bureau of Standards in Washington will compare any good tape with their standard and certify its length, as shown in Fig. 20.

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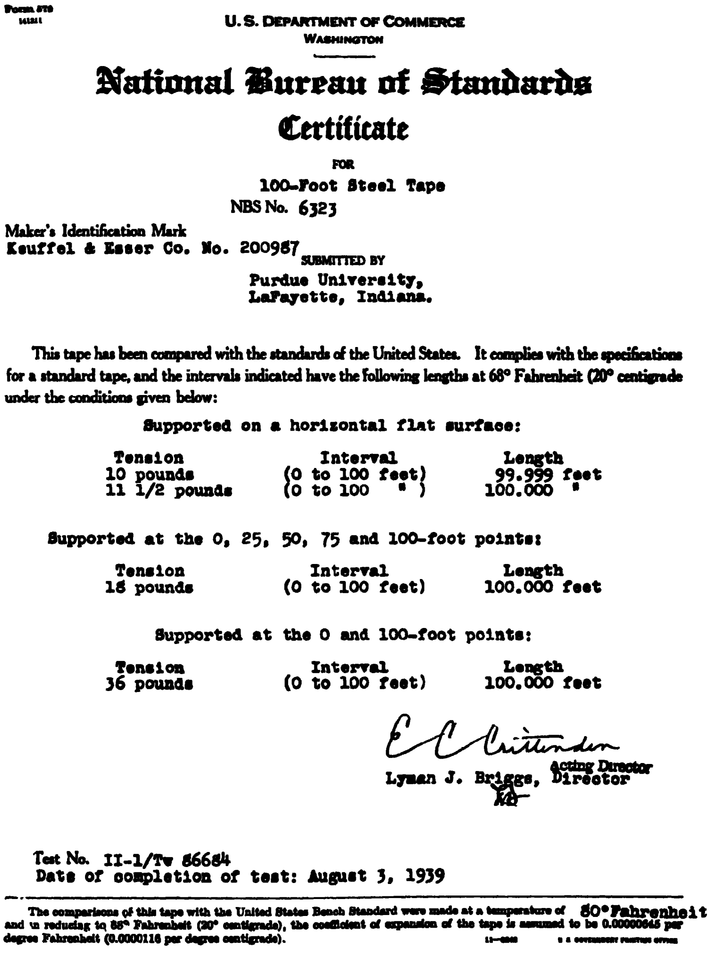
* Strictly speaking, no measuring device is exactly correct. The term as here used implies a very high degree of precision.



Form 570

U. S. DEPARTMENT OF COMMERCE
WASHINGTON

National Bureau of Standards
Certificate
FOR
100-Foot Steel Tape
NBS No. 6323

Maker's Identification Mark
Keuffel & Esser Co. No. 200987

SUBMITTED BY
Purdue University,
LaFayette, Indiana.

This tape has been compared with the standards of the United States. It complies with the specifications for a standard tape, and the intervals indicated have the following lengths at 68° Fahrenheit (20° centigrade under the conditions given below:

Supported on a horizontal flat surface:

Tension	Interval	Length
10 pounds	(0 to 100 feet)	99.999 feet
11 1/2 pounds	(0 to 100 ")	100.000 "

Supported at the 0, 25, 50, 75 and 100-foot points:

Tension	Interval	Length
18 pounds	(0 to 100 feet)	100.000 feet

Supported at the 0 and 100-foot points:

Tension	Interval	Length
36 pounds	(0 to 100 feet)	100.000 feet

E C Crittenden
Lyman J. ~~Briggs~~, ~~Director~~ Acting Director

Test No. II-1/Tw 86684
Date of completion of test: August 3, 1939

The comparisons of this tape with the United States Bench Standard were made at a temperature of 80° Fahrenheit and in reducing to 68° Fahrenheit (20° centigrade), the coefficient of expansion of the tape is assumed to be 0.00000645 per degree Fahrenheit (0.0000116 per degree centigrade).

FIG. 20. TAPE STANDARDIZATION CERTIFICATE.

Since a tape is seldom used in the field under the standard conditions mentioned above, it is necessary, for precise work, to make the corrections outlined in the following sections.

21. Temperature Correction. The temperature correction depends solely upon the coefficient of expansion of steel, usually taken as 0.000 006 45 foot per foot per degree F.

The following symbols will be used in the formulas of the next few sections:

t_s for the standard temperature at which the tape is true length, in degrees F,

t for the temperature of tape during measurement, in degrees F,

k for the coefficient of expansion of tape per degree F (0.000 006 45 for ordinary steel tapes),

p_s for the standard pull or tension at which the tape is true length, in pounds,

p for the pull or tension during measurement, in pounds,

E for the modulus of elasticity of tape, in pounds per sq. in. (usually taken as 28,000,000 for tape steel),

A for the cross-sectional area of tape, in square inches,

L for the length of tape between end graduations, in feet,

W for the total weight of tape between end graduations, in pounds.

$$\text{Temperature correction (in feet)} = Lk(t - t_s). \tag{21}$$

Example. A 100-foot steel tape is true length at 68° F. What is its length at 15° F?

$$\begin{aligned}\text{Temperature correction} &= (100)(0.000\,006\,45)(15 - 68)\\ &= -\,0.034 \text{ foot},\\ \text{Length at } 15^\circ &= 100.00 - 0.034 = 99.966 \text{ feet.}\end{aligned}$$

An approximation that is easily remembered and closely correct is that *each 15° F change in temperature changes the length of a 100-foot tape by 0.01 foot.* This may be applied mentally to all engineering cases involving the expansion of steel as well as to tapes and is consequently worth memorizing.

Second Solution.

$$\text{Temperature correction} = \left(\frac{15 - 68}{15}\right)(0.01) = -\,0.035 \text{ foot.}$$

It will be noted that neither the pull nor the cross-sectional area of the tape affects the temperature correction. It is a useful check to recall that heat expands and cold contracts the tape.

22. Special Tapes to Avoid Temperature Corrections. Invar and other nickel-steel alloy tapes have much lower coefficients of expansion* than do other steel tapes.

Figure 22 shows the 99.9-foot point of a rarely-used tape with several succeeding lines marked for different temperatures. If, for example, the tape temperature during measurement is 80°, then the true 100-foot length is found on the 80° line. Thus no computed correction is necessary.

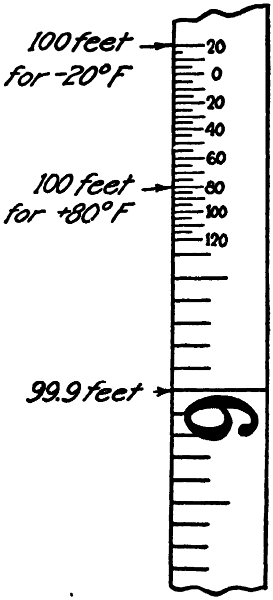


FIG. 22.

23. Pull or Tension Correction. The change in length due to pull or tension depends upon the modulus of elasticity of steel (usually taken for steel tapes at 28,000,000 pounds per square inch), upon the cross-sectional area of the tape, and upon the pull.

(23) Pull correction per tape length $= + \frac{L(p - p_s)}{EA}$.

EXAMPLE. A 100-foot tape is its true length when supported on a floor under a 10-pound pull. The cross-sectional area of the tape is 0.0047 square inch. How much does its length increase under a pull of 20 pounds?

$$\text{Pull correction} = \frac{(100)(20 - 10)}{(28{,}000{,}000)(0.0047)}$$
$$= + 0.0076 \text{ foot.}$$

It is difficult to measure the cross-sectional area directly, and consequently it is better to weigh the tape and compute its cross-section from the fact that steel weighs about 490 pounds per cubic foot or 0.28 pound per cubic inch. In the preceding example, the tape weighed 1.58 pounds per 100 feet, from which its cross-sectional area was computed to be 0.0047 square inch.

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* The coefficient varies with the quantity of nickel in the alloy. Approximately 36 per cent gives the lowest value of k, 0.0000001±.

If we follow this line of reasoning, substitute for A its equivalent $W/3.4L$ and use $E = 28{,}000{,}000$, formula (23) becomes

$$\text{(23a) Pull correction per tape length (in feet)} = +\frac{L^2(p - p_s)}{(8{,}200{,}000)W}.$$

On the ordinary 100-foot steel tape weighing a little under two pounds, a 15-pound additional pull stretches the tape 0.01 foot.

24. Sag Correction. The amount which the distance between the ends of the tape shortens when the tape is lifted from a floor and the middle sags toward the floor is called sag correction.

$$\text{(24) Sag correction per tape length (in feet)} = -\left(\frac{L}{24}\right)\left(\frac{W}{p}\right)^2.\text{*}$$

Example. A tape which is exactly 100 feet long when supported on a floor under a 10-pound pull is lifted from the floor and supported at the ends. The tape weighs 2 pounds. What is the corrected length between its ends when so supported?

$$\text{Sag correction (in feet)} = -\frac{100}{24}\left(\frac{2}{10}\right)^2$$

$$= -\,0.167 \text{ foot,}$$

Corrected distance = 100.000 − 0.167 = 99.833 feet.

In formula (24), L is the length of the tape between supports; for example, if a 100-foot tape be supported at the ends and in the middle, L would be 50 feet, W would be the weight of 50 feet of tape, and the total sag correction would be twice that computed with $L = 50$ feet.

The largest sag correction ordinarily encountered would occur with a heavy tape weighing, say, 10 pounds, 500 feet between supports and under a 20-pound pull. By substitution in formula (24) we find the sag correction to be 5.2 feet.

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* This closely approximate formula is developed by assuming that the curve of the sagged tape is a parabola. It is sufficiently precise for survey work and is simpler than the formula based on the catenary.

Another way to obtain this correction is to note that the sag correction varies as the cube of the length and inversely as the square of the pull. The result of the preceding example (− 0.167) would then be multiplied by $(500/100)^3(10/20)^2$. This checks the value of 5.2 feet.

Lighter or shorter tapes and heavier pulls will materially reduce these corrections. Support of the tape at one or more intermediate points will also reduce the correction.

The foregoing discussion applies when the ends of the tape are at the same elevation. When the tape is vertical, the sag correction becomes zero. The variation in sag correction for different end elevations is given in Mark's *Mechanical Engineers' Handbook.* The variation is ordinarily negligible unless the difference in end elevations is great.

A correction similar to the sag correction occurs when the wind blows the tape sidewise. It cannot be computed easily; hence efforts must be made to avoid such lateral curving of the tape.

25. Normal Tension. If sufficient pull is given the tape when it is suspended and sagged, the tape will be slightly stretched and the added pull will decrease the sag. At a certain pull, known as the normal tension, the suspended tape will measure its true length. The formula for normal tension is derived by equating formulas (23a) and (24). We may write

$$(25) \quad \text{Normal tension} = p = \sqrt{\frac{340{,}000\, W^3}{L(p - p_s)}}, \qquad \text{(approximately).}$$

It is simpler and more satisfactory to determine the normal tension by actually stretching the tape suspended above two points marked on a pavement at the ends of the tape when it was supported throughout its length under standard pull, and by noting the pull required to bring it to the true length, than it is to use formula (25). It is not practicable to exert a sufficient pull or tension, equal to the normal tension, on long or heavy tapes; consequently a computed or observed correction must be applied to the measurements.

The normal tension as above computed or observed only makes the tape its true length. It does not compensate all of the cumulative errors, and consequently a greater pull should be used when a line of many tape lengths is being measured, as explained in § 28.

26. Tape Standards. When precise surveying is in progress it is customary to establish two monuments exactly 100 feet apart. They may be set in a sheltered hallway, or if set out of doors, they must be deep enough in the ground to resist disturbance by frost. Points are established with a tape which has been standardized by the United States Bureau of Standards. The field tapes are stretched over this set of monuments and their lengths may be observed under measured conditions of temperature, pull, and support. In this way the computations of the several preceding sections of this text may be avoided.

For less extensive surveys, the field tapes are compared with a standardized tape. This tape is seldom or never used in the field, as such usage would be apt to break it or alter its length by kinking or bending.

27. Small Errors. Small errors in taping result from lack of skill and fallibility of the observer's senses. For example, it is difficult to judge by eye whether the tape is held level, particularly on sloping ground where the downhill end of the tape is generally held too low. Some tapemen use a hand level to avoid this error. Should one end of the 100-foot tape be 1.4 feet higher or lower than the other end, the tape forms the hypotenuse of a right triangle whose horizontal side is 0.01 foot shorter than the tape, or 99.99 feet long. A like error of 0.01 of a foot occurs when the end of the tape is held 1.4 feet to the right or left of its true position. A convenient formula for determining the amount of this error due to alignment, either horizontal or vertical, is given in Fig. 13.

If the head end or the rear end of the tape is not held exactly over the point on the ground, some slight error is introduced The setting of the pin or stake also produces a small error.

28. Cumulative Errors. Unless the conditions under which the work is being done change, small errors will accumulate and will not tend to compensate. They are also called *systematic*

Taping offers an example of a number of different kinds of cumulative errors. Not only do most of these accumulate with others of their kind, but also the various kinds tend to accumulate. If the tape is raised above the ground and no correction for shortening due to sag is applied, the distance as recorded in the notebook is too large. If an improper pull is applied to the tape, it is generally less than it should be, and the error makes the recorded distance too large. If the tape is not held horizontal, the resulting error makes the recorded distance too large. If the tape is held out of line, the recorded distance is again too large. Each of these cumulative errors, therefore, tends to accumulate with others of its kind, and the various kinds also tend to accumulate, so that the recorded distances are *too long.*

There is, fortunately, a field procedure which tends to offset these cumulative errors of taping. If a greater pull is applied to the tape than that which is theoretically necessary to make it exactly its true length, the tape is stretched and some of the sag correction is eliminated. Consequently a pull greater than that theoretically required tends to make the recorded distances *too short.* Competent head tapemen pull several pounds harder on the tape than is theoretically required.

In the use of tapes longer than 100 feet, it is frequently desirable or necessary to calculate the amount of the cumulative errors and to modify the observed distances accordingly. preferably before recording them. This is especially necessary for heavy tapes which have a large sag correction.

To summarize, the cumulative errors are eliminated either by

1. A pull greater than that theoretically required, or
2. Computing and applying a net correction to care for all cumulative errors.

Temperature corrections are usually cumulative but are not included in the foregoing discussion of this section since they may make the recorded distances either too long or too short, depending on whether the temperature during measurement is below or above that temperature at which the tape is true length.

In all engineering or surveying work, great care should be used to minimize these cumulative or systematic errors as they ordinarily exceed those of an accidental or compensating nature, such as those described in the following sections.

29. Compensating Errors. These are also called accidental errors and are those which are as likely to be positive as they are to be negative; that is, they tend to balance or to compensate one another. For example, in measuring many tape lengths, the tape may not be held directly over the stake or pin; one time the error will be added, another time subtracted; the errors ordinarily will not exactly balance one another but they will tend so to balance or compensate. Such errors are further treated in the next section.

30. Mathematical Theory of Probability and Least Squares. In treating so-called accidental or compensating errors, it is assumed that they are small and that they are as likely to be positive as negative. In an infinite number of observations (note that in surveying we will have only a small number of observations), the mean value would be the true one; that is, the accidental errors would exactly balance or compensate The smaller the number of observations, the less certainly do the errors balance.

The *probable error* is defined as an error of such magnitude that it is equally probable (we may say that it is an even bet) that the true error is greater or less than the probable error. For example, the mean value of several measurements of a line may be 967.23 ± 0.05 feet. Here the ± 0.05 foot is the probable error; that is, it is equally probable that the true error is greater than or less than 0.05 foot; and it is equally probable that the true value lies between 967.18 and 967.28, as it is that the true value lies outside of these values.

These theories, fully explained in mathematical works, are considerably used in surveying, engineering, and science. It is important that engineers understand their applications, two cases of which are here given in connection with taping. The computation and use of the probable error in ordinary engineering and surveying is primarily to give the engineer some rough idea of the precision of the work. Since only a few repeated measurements are usually available and the conditions of measurement are not rigidly controlled, the probable error as computed is only roughly approximate, not mathematically exact.

In the first case, assume that a line of 10 tape lengths is measured with a 100-foot tape. There is an accidental or compensating or approximate probable error of ± 0.02 foot each time the tape is laid down, due, let us say, to the tape's being held a little ahead or a little behind its proper position We do not know for any particular measurement whether it is ahead or behind. According to the theory of probability and least squares, the approximate probable error of the total 1000 feet is equal to the probable error of a single observation multiplied by the square root of the number of observations — in this case $0.02\sqrt{10} = \pm 0.06$ foot; and the result would be written 1000 ± 0.06 feet. Expressed in words, it is as probable that the result is between approximately 999.94 and 1000.06, as it is that it lies outside of these limits. Note that the actual error may be more than $(10)(0.02) = \pm 0.20$ (if we assume that the errors are large and / or cumulative). Or the actual error may be zero (provided all of the errors happened exactly to compensate). Or the actual error may be anywhere between these values. But the *probable error* is approximately ± 0.06 foot.

In the second case, assume that a line 100 feet in length is measured ten times with a 100-foot tape; that is, that the line is remeasured nine times. Again assume that the approximate probable error of each measurement is ± 0.02 foot. Then according to the theory of probability and least squares, the approximate probable error of the mean of the ten observations is the approximate probable error of a single observation divided by the square root of the number of observations — in this case $\pm (0.02/\sqrt{10}) = \pm 0.006$; and the result would be expressed 100 ± 0.006 feet. Or, expressed in words, it is as probable that the true value lies between approximately 99.994 and 100.006 as it is that it lies outside of these limits. Note that the actual error may be more than ± 0.02 foot (provided that the errors are large and / or cumulative). Or the actual error may be zero (assuming that all of the errors happened to compensate). Or the actual error may lie anywhere between these values. But the approximate probable error is ± 0.006 foot, which gives an idea of the precision attained.

31. Determination of Probable Error. In both of the preceding cases it is assumed that the probable error for a single measurement is 0.02 foot. Sometimes the surveyor or engineer knows from experience about what the probable error of a single measurement might be.

The procedure for determining mathematically an approximate probable error for a series of a small number of measurements, as ten measurements of a 100-foot length, follows. The residual V is the difference between the mean and any single measurement. Any measurement whose residual V is more than five times the probable error of a single measurement should be omitted from tabulations such as the following.

MEASUREMENT, IN FEET	RESIDUAL V, IN FEET	V^2
100.03	+0.025	0.000625
99.99	−0.015	0.000225
99.98	−0.025	0.000625
100.01	+0.005	0.000025
100.02	+0.015	0.000225
100.04	+0.035	0.001225
99.98	−0.025	0.000625
99.96	−0.045	0.002025
99.99	−0.015	0.000225
100.05	+0.045	0.002025
1000.05 Mean = 100.005±0.006	$\Sigma V = 0.000$	$\Sigma V^2 = 0.007850$

The probable error of a single measurement (from the theory of least squares) is slightly greater (varying in a complicated manner) than

$$(31)\quad \pm\, 0.6745\sqrt{\frac{\Sigma V^2}{n-1}}^{\,*} = \pm\, 0.6745\sqrt{\frac{0.00785}{9}} = 0.02 \text{ foot.}$$

---

* Formula (31) may be considered as almost exact for 30 or more observations but increasingly approximate for a smaller number of observations.

An approximation sufficiently precise for most engineering surveys is that the probable error of a single measurement is slightly greater than

$$(31a)\quad \frac{0.845\Sigma V}{\sqrt{n(n-1)}}.$$

The probable error of the mean of the series of measurements is the approximate probable error of a single measurement divided by the square root of the number of measurements; in this case, it is slightly greater than ± $0.02/\sqrt{10}$, or ± 0.006 foot.

It must be remembered that the preceding probable errors apply only to the accidental or compensating errors such as those due to plumbing or setting the pins. Moreover, many measurements are needed, since too few observations give materially larger probable errors. The cumulative errors due to lack of level, lack of alignment, and temperature are not considered in the preceding treatment.

32. Precision of Measurements and Computations. *Precision* will be emphasized throughout all survey studies and operations, as well as in most engineering and scientific observations and calculations. *Precision,* as used by the engineer, conveys the idea of degree of accuracy. It may be well to note here that it is impossible to make any measurement so that the observed value will be exactly correct. There will always be some error. A line whose true length is 728.74 feet may be measured by pacing and found to be 730 feet. Such a value should be sufficiently accurate for this method of measurement, that is, free from mistakes; but it would not be a precise value. The same line might be measured more carefully by stadia and found to be 728 feet. This would also be accurate, that is, free from mistakes, and more precise than the former value. Or it might be measured by ordinary taping methods and found to be 728.7 feet, which is still more precise. By means of careful taping methods, the value might be found to be 728.73 feet. No matter how carefully the value is determined, there would always be some error.

The degree of precision obtained in measurements or calculations is indicated, although not always exactly expressed, by the last digit of the number as written. When the preceding result is written 728.74, it means that the result is correct to the nearest 0.01 foot; that is, that the true value lies between 728.735 and 728.745. When the value is 730 feet, it means that it is correct to the nearest 10 feet; that is, that the true value lies between 725 and 735 feet. Likewise, the value 728 feet means that the correct figure lies between 727.5 and 728.5 feet. It will be noted that the preceding statements are not strictly exact mathematically, but they do represent what is generally accepted as good practice.

In intermediate steps of computations, values are expressed to one more digit than are the field data. Final computed results, however, should be expressed to the same degree of precision as that of the data on which they are based.

33. Precision Required in Taping. The essential requirement of all surveys is that they shall be sufficiently precise for the purpose of the survey. It is, of course, necessary that no error be permitted of sufficient magnitude to cause trouble in the engineering project or other ultimate purpose of the survey. This frequently requires that certain parts of a survey be made quite precisely, while other parts may be executed much less precisely.

Sometimes the money available for the survey is limited by the immediate value which the results will have. For example, where land is worth only a few dollars per acre, the owner hesitates to spend much on surveys, and some cheap method must be used. The young engineer should endeavor to avoid this type of work, since its poor quality will remain against his record long after the low price is forgotten.

It often happens that work of a little more precise character than is required for the immediate purpose of the survey can be executed without additional expense, and in such cases the better quality of work is advisable.

Most engineers prefer that precision of surveys should be beyond the requirements of the present or even the immediate future, but where added expense is involved many employers are unwilling to make this additional expenditure. Thus it frequently happens that land surveys are made with increasing precision as the value of the land continues to increase.

The Federal Government obtains expensive precision beyond immediate needs in most of its survey work, and doubtless this is a wise governmental policy.

At the beginning of a survey it is desirable by study to establish the required limits within which the errors of the survey should fall. The work can then be planned to accomplish these results with greatest convenience. Minimum cost is usually necessary and maximum speed is frequently desirable. Speed is best accomplished by planning in advance rather than by overworking the men, since the latter frequently results in errors and blunders, which delay, rather than speed, the work.

The so-called metallic tape (waterproofed cloth interwoven with wire) or the steel tape used hurriedly by marking the points on the ground with a scratch of the heel, give cheap results of low precision.

For ordinary engineering surveys with a 100-foot tape such as would be used for railways, highways, canals, pipe lines, transmission lines, and so on, and for the boundaries of moderately priced lands, the tape is held level as determined by eye; plumb bobs or flag poles, and stakes or taping pins are used; temperature and sag corrections are ignored or roughly estimated; the pull is made somewhat greater than theoretically required; and ordinary care is used without undue loss of time. Using these methods the errors of measurement will be of the order of:

1/5000 for gently rolling or level country without underbrush,

1/4000 for gently rolling or level country with underbrush,

1/1500 for abrupt slopes without underbrush,

1/1000 for abrupt slopes with underbrush.

Surveys for high-priced city land, bridge piers, tunnels, governmental purposes, and the like require precisions of between 1/5000 and 1/50,000 or even more. For these higher precisions, it is necessary to determine the tape temperature (with a thermometer attached to the tape), to use a spring balance at the end of the tape, to set stakes on grade or otherwise keep the tape level or on a known slope, and generally to exercise great care. Heavy stakes are used, upon the top of which are soft metal strips on which the position of the tape end is scratched. Frequently light stakes or lath with horizontal nails in them are set on line at the quarter or third points of the tape to minimize the sag correction, and to prevent side sag due to wind

By careful work in this manner, government engineers attain precision better than 1/1,000,000. Few engineers in private practice attempt or are capable of any such precision, and it is rarely necessary except in government work. When very precise measurements are needed, it is advisable to arrange for a specialized organization, such as the United States Coast and Geodetic Survey, to furnish personnel, equipment, and methods.

34. Importance of Good Taping. The importance of good taping cannot be overestimated. Any careful man can avoid blunders, but the elimination of the smaller errors requires some little knowledge and considerable skill. It is easier to do good instrument work, because of the excellence of the instruments, than it is to do good taping.

The best tapemen, particularly head tapemen, are frequently developed from active young laborers or handy men who make this their vocation. College men are apt to be promoted beyond this grade before they develop a good technique, although their understanding of the theory and purpose of the survey to some extent offsets this lack of manual skill. It is of greatest importance that the college man understand the theory and technique so that he can instruct his tapemen, since he usually has to break in green men for the work. The precision of a tape survey and the speed with which it progresses are dependent largely upon the head tapeman. He should be of the physically active type and capable of being held responsible for proper procedure.

Because of the many opportunities for error, the only way to insure correct taping is to develop as nearly a perfect system of checks as is possible. Such systems will vary according to circumstances. One example is the closing of traverses, as discussed in Chapter XVII.

35. Measurement of Angles with Tape. It is occasionally useful to measure or to lay out angles with a tape; this may be accomplished by the application of simple geometric and trigonometric principles. While the transit is far superior to the tape for measuring all ordinary angles, yet very small angles are more conveniently and precisely measured with the tape, as described in § 95.

In Fig. 35, a right triangle is formed if the sides are proportioned as 3 : 4 : 5. For example, 30 : 40 : 50 or 9 : 12 : 15.

In Fig. 35a, $AB = BC$ and radius AD = radius CD, which gives a right angle at B.

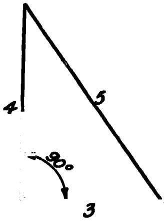

Fig. 35.

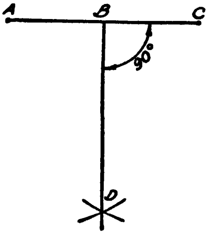

Fig. 35a.

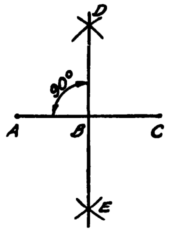

Fig. 35b.

In Fig. 35b, radius AD = radius CD and radius AE = radius CE, which gives a right angle at B. It is usually convenient to have all 4 radii equal.

In Fig. 35c, with a radius of 57.3 feet and an arc of 57.3 feet we have an angle of 57.3° or 1 radian at the center of the circle. Each foot of arc then evidently subtends 1° at the center of the circle and angles may be laid off or measured by the number

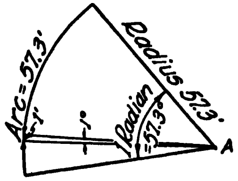

Fig. 35c.

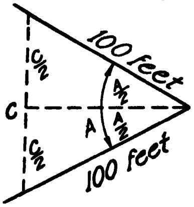

Fig. 35d.

of feet of arc. For example, 6.3 feet of arc laid out with a 57.3 foot radius subtends 6.3° or 6° 18′. While all other methods of laying off or measuring angles with a tape require trigonometric tables, this procedure eliminates their necessity. The arc is measured by setting pins or pegs and stretching the tape around them. A 5.7-inch radius is used on drawings.

Estimates of arcs may be made by this principle; for example, a 10° angle at the center of a circle of 600-foot radius subtends an arc of (10)(600/57.3) = 105 feet. Or we may estimate that a 10° angle will move a point 600 feet away, a distance of 105 feet laterally.

The quantity 5729.6 feet is used in curve work as explained in § 204. This value (more precisely 5729.578) divided by 100 gives the number of degrees in a radian. It is computed by dividing 360 by 2π. Because of its general use, it is worth memorizing, at least to the value 5730 or 57.3.

In Fig. 35d, to measure the angle A, lay off 100 feet on each line and measure the chord C; then we have

$$\sin\frac{A}{2} = \frac{C}{2} \div 100.$$

Find $C/2$ and move the decimal point two places to the left. This result is the natural sine of $A/2$.

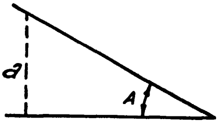

FIG. 35e.

In Fig. 35e, to lay off the angle at A, measure 100 feet and erect the perpendicular a. Look up the natural tangent of A and move the decimal point two places to the right. This gives a, which is then measured on the ground.

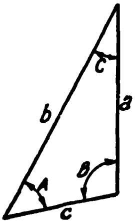

FIG. 35f.

In Fig. 35f, any oblique triangle may be solved when at least three of its parts (either angles or sides) are known. Formulas for the solution of such triangles are given in Table IX in the back of this book.

All of the methods of Figs. 35 to 35f inclusive may be used in drafting as well as in surveying.

PROBLEMS

1. (a) A line was measured with a steel tape and recorded as 1213.16 feet; the temperature reading was 98° F. Later the tape was found to be exactly 100 feet long at 68° F. What is the true length of the line? This type of question requires clear, not hazy, reasoning. It is frequently answered incorrectly. HINT. What was the true length of the tape at 98° and how many times was it laid down? *Ans.* 1213.40 feet

(b) The same as (a) except that the length was recorded as 847.63 feet at 8° F.

(c) The same as (a) except that the length was recorded as 1808.32 feet at 38° F.

2. (a) An old 50-foot metallic tape is used in setting corners for a building foundation. This tape is known to be 50.04 feet long. The foundation is to be 32 feet by 48 feet. If this tape is used, what measurements must we make? *Ans.* 31.97′ and 47.96′

(b) The same as (a) for a foundation 64 feet by 100 feet.

(c) The same as (a) for a foundation 45 feet by 108 feet.

3. (a) A 100-foot steel tape weighs 2.105 pounds. If steel weighs 490 pounds per cubic foot, what is the cross-sectional area of the tape in square inches? *Ans.* 0.0062 sq. in.

(b) The same as (a) except that the weight is 1.70 pounds.

(c) The same as (a) except that the weight is 1 pound 2 ounces.

4. (a) A Bureau of Standards Certificate states that a 2-pound steel tape is 100.000 feet long when supported throughout its length under a standard pull of 10 pounds. A line measured with this tape, but using a 15-pound pull, is recorded as 1832.72 feet. What is the true length of the line if the tape was supported during the measurement?

Ans. 1832.78 feet

(b) The same as (a) except for a 20-pound pull.

(c) The same as (a) except for a 25-pound pull.

5. (a) A line is measured with a 6-pound steel tape which is exactly 500 feet long when supported throughout its length under a 20-pound pull. During measurement the tape is swung off the ground and supported at the ends only with the same pull, and the distance is recorded as 3316.43 feet. What is the true length? *Ans.* 3304.05 feet

(b) The same as (a) except that a 10-pound tape is exactly 500 feet long when supported, and the recorded distance is 1963.71 feet.

(c) The same as (a) except that a 5-pound tape is exactly 500 feet long when supported and the recorded distance is 4317.86 feet.

6. (a) The following slope distances are measured with a 300-foot steel tape: 300 feet on a 7° slope; 286.3 feet on a 15° slope; 247.6 feet on a 2½° slope. What is the total horizontal distance? Use the approximate formula when it will give results to the nearest 0.1 foot. *Ans.* 821.7 feet

(b) The same as (a) except that a 100-foot tape is used: 100 feet on a 2° slope; 97.6 feet on a 10¼° slope; and 48.7 feet on a 20½° slope.

(c) The same as (a) except that a 500-foot tape is used: 500 feet on a 3° slope; 478.6 feet on a 10° slope; and 413.6 feet on a 20¼° slope.

7. The same as 6 except that the differences in elevation of the ends of the tape are given rather than the vertical angles:

(a) 30 feet; 100 feet; 6.5 feet. *Ans.* 814.3 feet

(b) 3 feet; 30 feet; 9.3 feet.

(c) 25 feet; 70 feet; 121 feet.

8. (a) The average rate of grade on U. S. 40 between Uniontown, Pa. and the Summit Hotel is 4.56 per cent. An automobile speedometer reading between the two places is 6.03 miles. What is the horizontal distance traveled? *Ans.* 6.02 miles

(b) The same as (a), but take the average grade to be 6.09 per cent.

(c) The same as (a), but take the average grade to be 2.32 per cent.

9. (a) A line is recorded as 1653.72 feet long. It is measured with a 1½-pound tape which is 100.013 feet long at 68° F under a 10-pound pull, supported throughout its length. During measurement the temperature is 38° F and the tape is suspended under a 15-pound pull. The line is measured on a 5 per cent grade (5-foot rise vertically for every 100 feet horizontally). What is the true horizontal distance? HINT. Compute the true horizontal distance secured each time the tape is laid down, considering all of the corrections. *Ans.* 1650.93 feet

(b) The same as (a) except that the length is recorded as 2286.43 feet and is measured with a 20-pound pull, tape suspended, temperature 8° F, grade 10 per cent.

(c) The same as (a) except that the length is recorded as 3179.84 feet, and is measured with a 17-pound pull, tape suspended, temperature 98° F, slope 5° (not per cent).

10. (a) The head end of a 100-foot tape is held $\frac{1}{2}$ foot off true line, first to the right and then to the left, and one foot out of level, first above and then below the rear end. The line is recorded as 2373.82 feet long. Is the error compensating or cumulative? What is the true length?

Ans. Cumulative; 2373.67 feet

(b) The same as (a) except 1 foot out of line and 2 feet out of level.

(c) The same as (a) except 2 feet out of line and 3 feet out of level.

11. (a) In a 1600-foot line, assume that an accidental or compensating probable error of ± 0.02 foot occurs at each 100-foot point due to swinging of the plumb bob. What is the total probable error from this source?

Ans. ± 0.08 foot

(b) The same as (a) except with a ± 0.03 foot probable error.

(c) The same as (a) except with a ± 0.04 foot probable error.

12. (a) The angle between two intersecting fences was determined with a tape. A point on each fence line was established 100 feet from the fence corner. The distance between these established points was found to be 127.62 feet. What was the angle between the two fence lines? If the points had been set at 57.3 feet radius instead of at 100 feet, what would have been the arc distance between them? *Ans.* 79° 18′, 79.3 feet

(b) The same as (a) except that the distance was found to be 167.35 feet.

(c) The same as (a) except that the distance was found to be 105.83 feet.

13. (a) A deed for a certain tract of land calls for a rectangular area 38 chains 13.3 links by 49 chains 86.7 links. How many acres are there in the tract? *Ans.* 190.16 Ac.

(b) The same as (a) except that the area is 28.75 chains by 53.25 chains.

(c) The same as (a) except that the area is 74 chains 28.0 links by 33 chains 54.5 links.

14. (a) Write a brief set of instructions for taping a railway location survey with stakes every 100 feet. The line runs across drainage, through second growth timber and underbrush, and a large part of it is on slopes up to 20 per cent. The permissible error is of the order of 1/5000.

(b) Write similar instructions for a transmission line location across open mountainous country with many draws and canyons. Stakes may be as much as 300 feet apart. The permissible error is of the order of 1/2000.

(c) Write similar instructions for city lot surveys, using taping pins. The permissible error is of the order of 1/10,000.

CHAPTER III

FIELD NOTES

36. Functions of Field Notes. Notes taken in the field are usually worked over in the office into some more advanced form of presentation, such as a map, a report, or a computation. The notes are then always kept for reference, since not all of the field information is ordinarily prepared on the more advanced form. In the event of litigation or of important reference, the original field notes are of primary importance, and are much more valuable than any copied or transferred data.

The notes must be completely and finally recorded in the notebook as the survey progresses, and not kept on scraps of paper and later copied into the book. In many engineering surveys the notes are used by an office man who is not familiar with conditions on the ground, hence *the essential qualities of field notes are clearness and completeness.* Even where the field man works up his own notes, it is possible that others will refer to them later. For these reasons, and for workmanlike appearance as well as for creating confidence in the work, notes must be neat and systematically arranged. They must be recorded in the conventional and generally used form, and not according to individual whim.

37. Technique of Note Keeping. A rather hard, sharply pointed pencil of about 4H grade is so used as slightly to indent the paper and to avoid smears. Should the notes be rubbed or wetted by rain or perspiration, they may generally be deciphered through the indentation of the paper. Ink is not used because it smears and may become obliterated from dampness. One exception is to be noted: entries made in the office are best written in red ink.

If changes are necessary in the notes, they are crossed out rather than erased, and the new material is inserted adjacent to the crossed-out material. This is a frank admission of error, while an erasure causes suspicion that dishonest alteration has occurred. It is sometimes desirable, particularly with a novice, to cross out and mark *abandoned* an entire page or more of notes and to refer on that page to the other page upon which the correct notes are to be found.

It is best not to crowd the notes, since paper is cheaper than labor or a possible misinterpretation. Frequently complete notes only require a few lines, and these few should be spread liberally over a large portion of the page rather than crowded towards the top or bottom.

The title of a survey is placed at the top of one or both pages. The names and duties of the party, the date, weather, and instrument used should be recorded on the upper part of the right-hand page. When a survey extends through several pages of notes or through several days, it is only necessary to record variations from these data on succeeding pages.

The notes must be complete, with nothing left to memory. It is better to include a few unnecessary measurements than to omit a needed one, since such omission involves delay and perhaps the expense of an entire party.

The notes should show the precision which was actually obtained in the field, neither more nor less. In recording distances to the nearest 0.01 foot, for example, the notes should read 734.00 feet rather than 734 feet. An angle of 18° 00′, correct to the nearest minute, should be so recorded rather than as 18°.

Such a number as 0.6 should be so recorded rather than .6, as the zero before the decimal point will serve to indicate its position even though the point is obliterated. Or the tenths and hundredths may be recorded over a line, as mentioned on page 15.

Two minor illogical exceptions are to be noted in current practice. Compass readings to the nearest 1/4° are usually recorded as N 68° 45′ E instead of N $68\frac{3}{4}^{\circ}$ E. Angles precise to the nearest 1/2′ are recorded 113° 28′ 30″ instead of 113° $28\frac{1}{2}'$. Perhaps the more logical N $68\frac{3}{4}^{\circ}$ E and 113° $28\frac{1}{2}'$ will be increasingly used in the future.

38. Types of Notes. Where the survey is of a repetitive nature, as for example in most leveling, it is best to tabulate the work. When made in the conventional form, such a tabulation will convey the required information in a simple and definite manner. In other types of surveys, the data may be of such varying character that they cannot be advantageously tabulated, but must be shown on a sketch or sketches. Such sketches are made large to avoid crowding of figures and are preferably drawn roughly to scale, not with a scale but proportioned by eye. Occasionally sketches are clearer if not made to scale, that is, if some important feature is exaggerated. The matter of *clearness should govern.* A small thin celluloid scale or triangle is useful in ruling lines. It is often desirable to make the sketch before the field measurements are taken. If some part of the sketch becomes crowded, it is best to repeat part or all of the sketch on a larger scale on the same page or on some other page.

Sketches are made on the right-hand page of the notebook or on one-tenth inch cross-section paper, larger than the usual field book. Such paper is available in pads, in books, or in single sheets and may be thumb-tacked to a sketching board. Notes are rarely recorded in narrative form, as in § 181.

The most common form of notes is that in which the tabulation occupies the left-hand page and a sketch is made on the right-hand page; this sketch adds data which do not readily lend themselves to tabulation and repeats as much of the tabulated data as may seem necessary. Examples are given as Figs. 96a, 99, and 204a in §§ 96, 99, and 204, respectively. Occasional explanatory notes may be necessary to supplement the tabulation and sketch.

The examples of notes given throughout this book have received general approval, but must be modified to meet special conditions.

Loose-leaf books are available but are not generally used, since the sheets are easily lost or improperly altered, and since a temptation exists to make poor field notes to be copied later, when the original notes are then destroyed. This is a bad practice. Loose-leaf note sheets should be numbered consecutively to avoid misplacement. Occasionally they are duplicated by carbon copy or by photostat to insure against loss and to make them available in the field and to one or more office men at the same time.

39. Index. The first few pages of the notebook must be filled in as an index as the work progresses (Fig. 39). It is sufficient to number each double page only at the upper left-hand corner of the left-hand page. Notes should be directly cross-referenced between pages, when possible. If many notebooks are kept, a book index or a card index should be prepared. It may be arranged according to the type of survey operation, as levels, triangulation, transit traverse, and so on; by geographical location of survey, as by section, county, or town; by the name of the man in charge of the party; by the name of the individual or company for whom the survey was made; or by some combination or cross reference of these

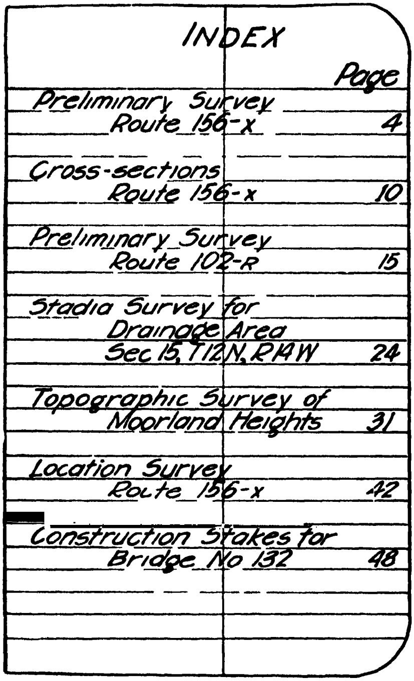

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FIG 39. FIELD NOTEBOOK INDEX.

Notebooks often contain information of considerable value and they should be protected against loss, if possible in fireproof vaults. It sometimes pays to copy the notes, preferably by photostat, both to insure against loss and to facilitate their use. Some engineers who use loose-leaf books make a carbon copy in the field, but the practice is not general.

40. Lettering and Symbols. It is the best practice to letter the notebooks in the prevailing style of rapid engineering lettering, usually the Reinhardt or a similar type. A good size for the capitals and figures is from 1/10″ to 3/20″ in height with the lower case letters in proportion. Slant letters are more easily and more rapidly made. Some individuality in lettering is permissible, providing clearness is not sacrificed.

Some common symbols and abbreviations are:

Transit point	⊙
Triangulation point	◬
Center line	℄
Section corner	$-\begin{array}{c\|c}4&3\\9&10\end{array}-$ or sec. cor.
One-quarter section corner	¼ sec. cor.
Stake	Stk.
Tack	Tk.
Crow-foot, a mark made thus	→ or c.f.
Bench mark	B.M.
Turning point	T.P.
Foresight	F.S. or −
Backsight	B.S. or +
Height of instrument	H.I. or $\overline{\wedge}$

41. Notekeeper. The notekeeper holds one of the more important positions of a survey party, usually ranking with that of instrument man. Before leaving the field, the notekeeper must carefully examine his notes for errors and omissions in the survey. Since the notekeeper has the sketch before him and has more time, it is proper that he should be responsible for errors and omissions. The instrument man has little time for notekeeping, since he is busy with instrumental manipulation and is responsible for the progress of the survey.

While it is undesirable to crowd the notes with useless information, the notekeeper should err on the side of too much, rather than too little, information. He should endeavor to criticize his notes from the viewpoint of the man who will use them, and to keep them accordingly.

The student should strive to improve each set of notes over the preceding one. He should avail himself of this opportunity to develop note-keeping technique, since it will greatly help him not only in surveying but also in other engineering work, such as recording experiments and tests as well as in sketching and in the preparation of memoranda used in executive duties. Poor habits acquired during the formative years in college are difficult to break in later practice. In the earlier years of engineering practice the *poor-habit man* spends considerable time and effort in correcting his faults while the competing *good-habit man* has his mind, time, and energy free for more productive work, which results in more rapid advancement.

Original field notes must not be altered to make them check, or for any other than honest and scientific reasons. *Erasures must not be made*, but faulty work should be crossed out and the correcting entries made directly adjacent. It is best practice to use red ink for any additions to the notes which are made in the office.

PROBLEM

Some technique in notekeeping should be developed before going into the field, where the attention and energy are occupied with the details of field work. Copy typical portions of field notes from this book or other sources, following the suggestions made in this chapter and endeavoring to improve on the originals, if possible.

CHAPTER IV

COMPASS SURVEYING

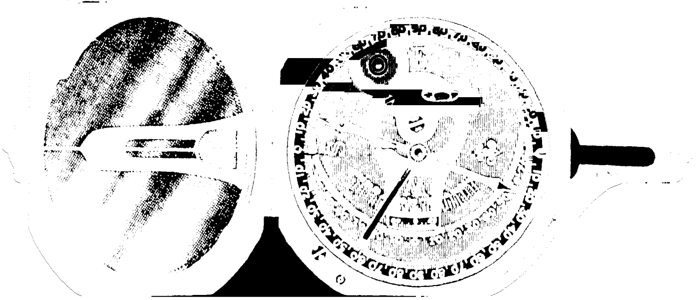
42. Types of Compasses. The somewhat elaborate surveyor's compass was used extensively before the introduction of the engineer's transit during the middle of the nineteenth century. Since the transit is now used for precise determination of direction, a less elaborate compass (the *Forest Service Compass*, Fig. 42) suffices for those relatively rare cases where a compass is necessary.

Figure 42 also shows a smaller and less precise compass which is used for rough determination of direction. The *Brunton Pocket Transit* is perhaps the best of the smaller instruments, and it may be used also in measuring approximately vertical angles. All of these types of compass have vertical slits for sighting instead of a telescope, such as is used in most modern surveying instruments. The compass most used by the engineer is that which forms a part of the engineer's transit, described later.

Figure 42a shows an approximate method of determining South with a watch.

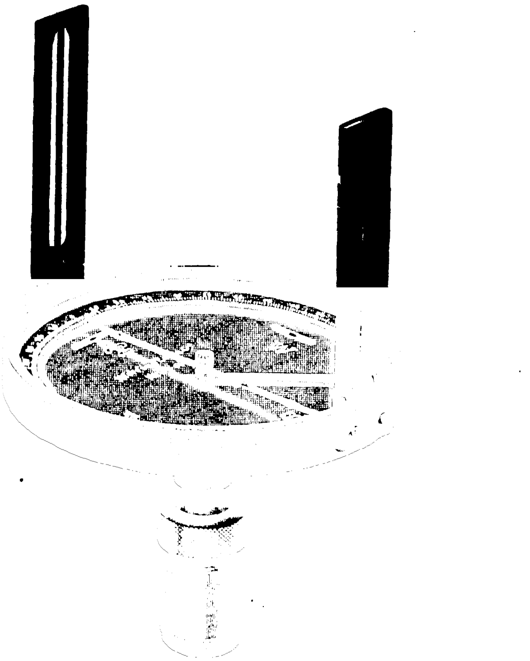
43. Magnetic Declination. The needle of the compass does not usually point true north. The angle between the needle and a true meridian (a true north and south line) at any particular locality is called the *magnetic declination;* east declination if the north end of the needle is east of true north, and west declination if the north end of the needle is west of true north.

The United States Coast and Geodetic Survey publishes a pamphlet and map of the United States (Fig. 43) and also separate pamphlets and maps for many of the states, giving the magnetic declination for various points on a particular date. By examining the original larger scale maps, especially the state maps, many minor changes become apparent which are not shown on the smaller scale map reproduced here. None of these maps can give the declination for all points as closely as it can be determined by comparing the needle with a true meridian which has been established astronomically.



Courtesy of Keuffel and Esser Co.

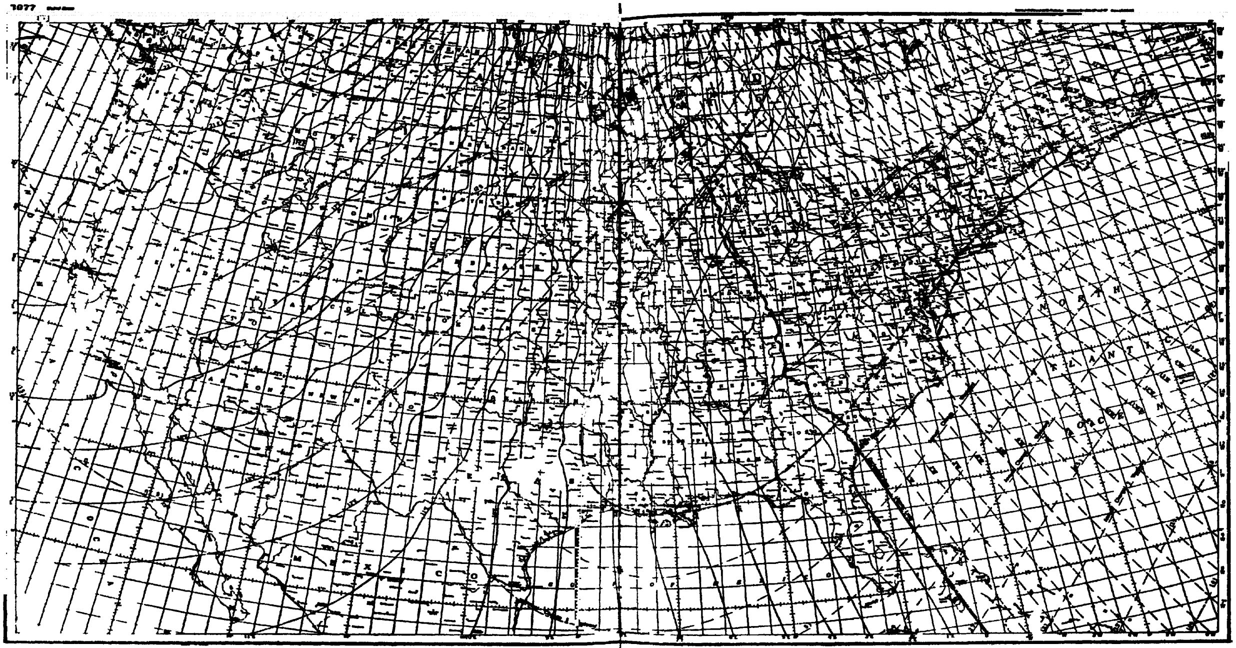
Brunton Compass.



Courtesy of Keuffel and Esser Co.

Forest Service Compass.

Fig. 42. Modern Compasses.



Courtesy of the United States Coast and Geodetic Survey

FIG 43 MAGNETIC DECLINATION IN THE UNITED STATES FOR 1945

The lines of equal magnetic declination (full lines) apply to January 1 1945 East of the line marked 0° (agonic line) the north end of the compass needle points west of north, west of that line it points east of north

The north end of the compass needle is moving to the westward for places north of the line of no change (the double dashed line) and to the eastward for places south of that line at an annual rate indicated by the lines of equal annual change (dashed lines)

The zero line on Fig. 43, called *the agonic line*, is that line along which at any point the needle points true north. East of this line the needle has a west declination and west of the line it has an east declination, of amounts shown by the various *isogonic* (equal declination) lines on the maps.

Charts similar to Fig. 43 are compiled from many observations made by the United States Coast and Geodetic Survey, in which a large and sensitive compass needle was compared with many astronomically established true meridians. Such charts may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C. A pamphlet entitled *Principal Facts of the Earth's Magnetism*, available from the same source, is a general treatment of the subject.

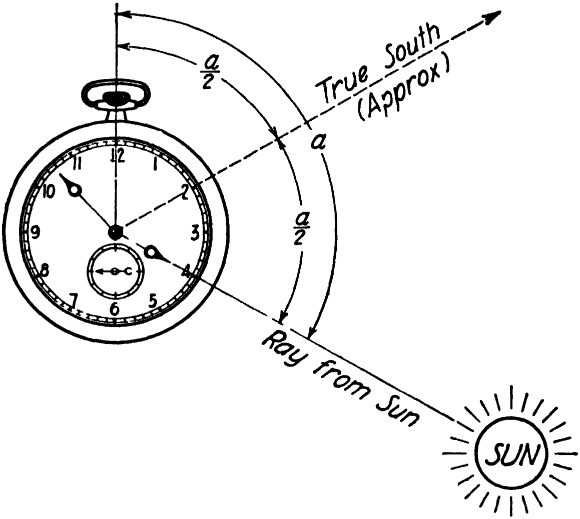


FIG. 42a.

The values of declination taken from the government charts are thought to be within 15 minutes of correct values for more than half of the territory which they cover. Occasionally, however, a difference of a degree or more may be found.

Closely correct values can be found only by comparing the needle with an astronomically established true meridian. Such a meridian may be established by the methods of Chapter VIII, or the closely correct bearing of some line may be secured from previous surveys which were astronomically controlled. The United States Geological Survey will furnish true bearings or azimuths of lines in the vicinity desired, and they may also be obtained from many local governmental surveys or from private corporations.

44. Variation of Declination. In a particular locality, the declination varies slightly from year to year (from 0 to 5 minutes annually in various parts of the United States). These annual changes are shown by the dashed lines which run easterly and westerly on isogonic maps. The exact character of this *secular* variation in declination is not definitely known but is thought to be *periodic*. It amounts to several degrees in 150 years. Because of this considerable variation over a period of years, it is necessary to know the declination when an old compass survey was made (if it was referred to the magnetic meridian), as well as to know the present declination.

The daily, or *diurnal*, variation varies several minutes (in the United States) from the mean throughout 24 hours and is *periodic* in character, the extreme eastern position of the needle occurring at about 8 A.M. and the extreme western pointing at about 1 P.M. This is not sufficient in the United States to be discernible on the compasses used in surveys.

Annual and *lunar* variations are less than one minute and may therefore be neglected.

There are also irregular compass variations which cannot be predicted. These occasionally amount to nearly a degree, especially in northern latitudes. They occur during magnetic storms, sun spots, and displays of Aurora Borealis.

Local attraction of the needle is discussed in § 51.

45. Setting Off the Declination. Many compasses, particularly those on transits, are so arranged that the circle on which the degrees are graduated may be rotated with reference to the line of sight through the amount of the magnetic declination for the locality in which the instrument is being used. In this manner it is possible to adjust the compass so that needle readings on the compass circle may be read directly from true north and south rather than from magnetic north and south.

In Fig. 45, the magnetic declination has not been set off. The line of sight is pointing true north. Since the magnetic declination in this locality is 8° 10′ E (the needle pointing east of north by that amount) the needle reads N 8° 10′ W on the plate. This is the magnetic bearing, or the bearing from magnetic north, of the line of sight.

In Fig. 45a, the magnetic declination of 8° 10′ has been set off by rotating the circle on which the graduations occur through an angle of 8° 10′ with reference to the line of sight. The sights are still pointing due north and the needle is still pointing 8° 10′ east of north but the needle now reads 0° 00′, which is the bearing of the line of sight reckoned from true north. It is this bearing, reckoned from true north, that is usually desired. Most modern compasses or transits are provided with a means of thus setting off the declination.

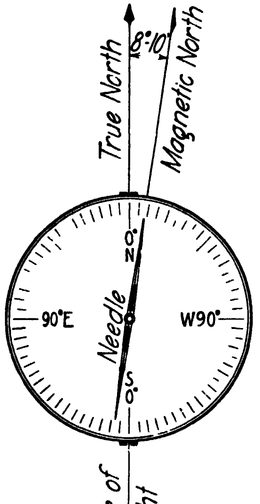


Fig. 45. Declination Not Set Off.

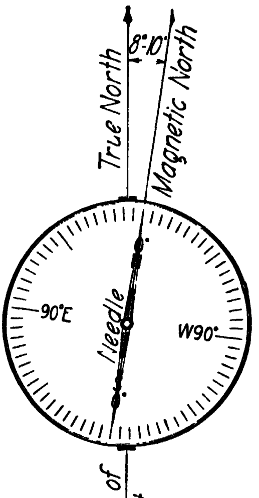


Fig. 45a. Declination Set Off.

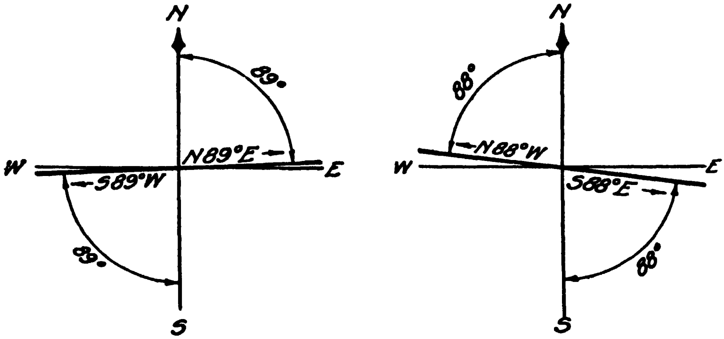
A needle reading, therefore, may refer to either true or magnetic north and south, depending upon whether the declination has been set off on the instrument. It is most desirable that the declination be set off on the compass or transit and that bearings from the true meridian be read and recorded. This should be done whenever the instrument is so constructed as to permit it.

A bearing which is read by means of a needle is properly called a *needle reading* or *observed bearing* to distinguish it from one which has been calculated from an astronomic meridian and measured angles and which is called a *calculated bearing.*

46. Precision of the Needle. Precise work should not be attempted with a compass, as it is not an instrument of precision. Under ordinary favorable conditions, a good compass needle should give bearings within 15 minutes of correct values, although such precision is not assured.

47. Dip of the Needle. In the northern hemisphere, the north end of the needle dips downward when freely supported at its center of gravity. A movable wire counterweight is slipped over the south end of the needle, and this also distinguishes the south end of the needle.

48. Bearings of Lines. The bearing or course of a line, a term used in navigation and surveying, is the angle which that line makes with a meridian. The bearing of any line cannot exceed ninety degrees. A line running almost due east might have a bearing of N 89° E, read "North 89 degrees East," as in Fig. 48. Looking in the opposite direction on the same line



The original line *The same line, rotated 3° clockwise*

FIG 48. BEARINGS.

the bearing would be S 89° W. If this same line were rotated three degrees in a clockwise direction, its bearing would become S 88° E, not N 92° E, and looking along the line in the opposite direction its bearing would be N 88° W.

Bearings may be measured and described either from *magnetic north and south*, that is from the direction in which the needle points, in which case they are called *magnetic bearings;* or they may be measured and described from a true astronomic meridian (*true north and south*), in which case they are called *true bearings.* The bearings of a given line as expressed under the two systems will differ by the amount of the magnetic declination of that date and locality.

49. Obtaining a Compass Bearing. The telescope or peep sight or vertical slit is sighted along the line whose bearing is to be determined and the position of the end of the needle on the graduated circle is read, as in Fig. 49. The compass must be level, so that the needle swings freely. The larger compasses have level bubbles for this purpose. The smaller compasses are held in the hand while the larger ones are supported on a light

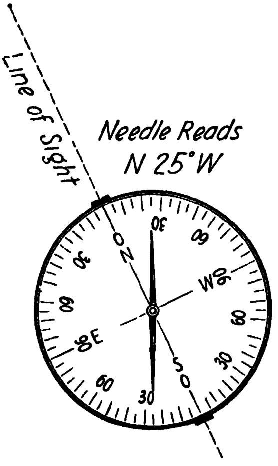


Fig. 49. Declination Not Set Off.

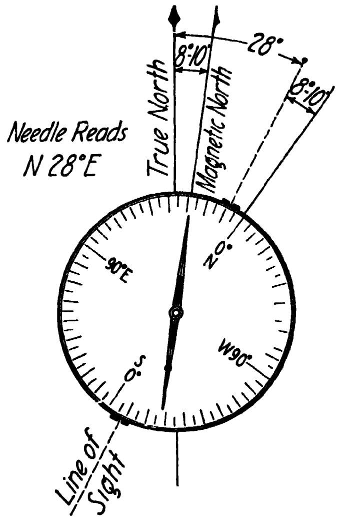


Fig. 49a. Declination Set Off.

tripod or on a single leg called a *Jacob's staff*, having some device such as a ball-and-socket joint provided for leveling.

The north end of the needle should be read in all cases where the instrument is used in normal position, that is, with the north end of the compass box, or graduated circle, towards the object and away from the observer. If for some reason a transit is sighted with the telescope upside down, or if any compass is sighted backwards, then the south end of the needle should be read; otherwise a reverse bearing is secured. The safest procedure is always *to point the north end of the compass box towards the object and to read the north end of the needle.*

Figure 49a shows the sights or telescope pointing along a line whose true bearing is N 28° E and the needle reads N 28° E on the compass circle, since the declination has been *set off*, that is, the line of sight has been turned with reference to the compass circle, through a declination of 8° 10′.

It will be noted that the letters E and W are interchanged on the plates from their true relative positions, so that the 28° which is read at the north end of the needle will lie between the N and E points on the plate. A careful consideration of the procedure of sighting the compass and reading the bearing will show the necessity of this interchange. It is well for the beginner, when reading bearings on the compass, to check this method of using the NESW letters on the compass plates, by roughly noting the position of the line of sight with reference to the needle. He will thus assure himself that he has read the bearing in its proper quadrant, NE, NW, SE, or SW.

50. Calculation of Angles from Bearings and Vice Versa.

GIVEN: Two lines with bearings as shown in Fig. 50.

REQUIRED: The angle between the lines. (Two angles are formed; the smaller is to be computed.)

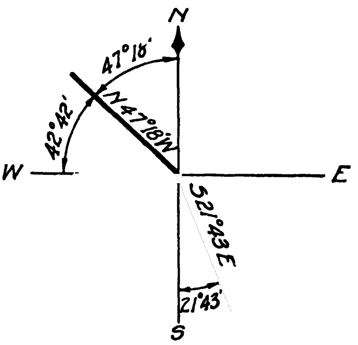


FIG. 50.

SOLUTION: Draw a sketch, as in Fig. 50. The angles 47° 18′ and 21° 43′ are given.

$$\begin{array}{r} 90^\circ\ 00' \\ \underline{47^\circ\ 18'} \\ 42^\circ\ 42' \end{array} \qquad \begin{array}{r} 42^\circ\ 42' \\ 90^\circ\ 00' \\ \underline{21^\circ\ 43'} \\ \text{Required angle} = 153^\circ\ 85' = 154^\circ\ 25'. \end{array}$$

GIVEN: A line bearing N 57° 43′ E making an angle (measured clockwise) of 92° 27′ with another line.

REQUIRED: The bearing of the second line.

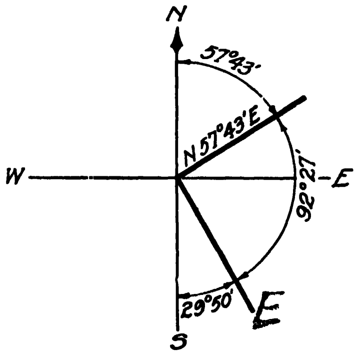


FIG. 50a.

SOLUTION: Draw a sketch, as in Fig. 50a. The angle 57° 43′ is given.

$$\begin{array}{r} 57^\circ\ 43' \\ \underline{92^\circ\ 27'} \\ 149^\circ\ 70' \end{array} \qquad \begin{array}{r} 180^\circ = 178^\circ\ 120' \\ \underline{149^\circ\ \ 70'} \\ 29^\circ\ \ 50' \end{array}$$

which gives the required bearing as S 29° 50′ E.

It is *essential* for the novice to make such simple sketches in calculations of this nature.

51. Local Attraction. The magnetic needle is attracted from its normal pointing towards magnetic north by local influences such as the proximity of steel, iron, magnetic ores, and direct (not alternating) electric current. While such influences seldom occur near enough to the compass to affect it, except in cities, yet they may occur at any time and may deflect the needle several degrees. The compassman must remove from his person, particularly the upper part of his body, any steel, such as knife, keys, etc.

If magnetic bearings are taken at two or more points along the same straight line, they will agree, provided there is no local attraction. For assurance that none exists, it is therefore necessary to take at least two magnetic bearings on the same straight line, usually at its ends. If they are in agreement there is no local attraction except in the unusual event that the same local attraction exists at both ends of the line.

In Fig. 51 there is no local attraction at *B* or *C* since the two needle readings for the line *BC* are the same. N 45° 10′ E and S 66° 40′ W are then correct needle readings since they are read from *C* and *B* respectively, at which points there is no local attraction.

By using the two needle readings taken at *D*, it is possible to compute the true angle at *D*, even though local attraction which pulls the needle from its true position may exist at this point. From the correct needle reading or magnetic bearing N 45° 10′ E of *CD* and the correct angle at *D*, it is possible to compute the correct magnetic bearing of *DE* (N 11° 30′ W), and so on.

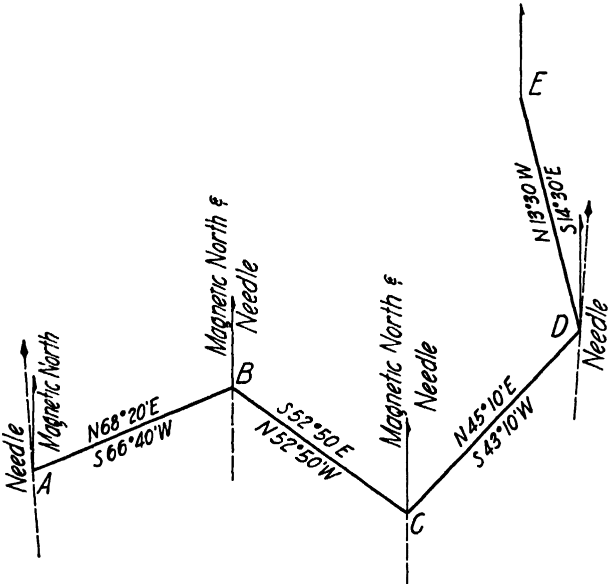


FIG. 51. LOCAL ATTRACTION.

It will be noticed that local attraction at a particular point may be constant, or may vary with varying magnetic influences. It is of course only constant local attraction which may be detected and eliminated as just explained. *There is no way to eliminate the effects of varying local attraction,* such as an intermittent direct-current electrical circuit, or such as an automobile which has been moved near a compass, except to avoid it. Fortunately, such intermittent effects are rarely encountered.

52. Common Errors. When the standard precaution of reading the bearing twice on each line is observed, usually at the ends of the line, there should be few errors. Of course the compass should be in adjustment as described in § 121. The needle sometimes sticks, especially near the point at which it is about to settle, and may be jarred loose by lightly tapping the glass cover with a pencil or finger, or by using the needle lifting screw. The glass cover, if rubbed, sometimes becomes slightly charged with electricity and attracts the needle. It may be discharged by touching with the moistened finger. After completing the compass readings at a particular set-up, and particularly before moving the instrument, the needle should be lifted off of the pivot to avoid wear on the pivot, which dulls the sensitivity of the compass.

The more sensitive the needle, the longer it will oscillate. If it comes to rest quickly, it either needs re-magnetizing or the pivot is not sharp. Wide oscillations may be checked by the lifting screw, but the needle must finally settle without interference, except perhaps for a light tapping on the glass cover to avoid sticking. To read the position of the needle precisely one must look along the direction of the needle and not obliquely, since this causes parallax and an incorrect reading.

53. Precision of Readings. On the larger compasses, the graduations of the circle are usually to half degrees. It is possible to estimate the position of the needle to about one-third of the one-half degree graduation, that is, to make needle readings to the nearest one-sixth of a degree, or ten minutes. Many instrument men read the needle to quarter degrees. If the needle is well magnetized and the pivot is sharp, it may be possible to estimate a bearing to the nearest five minutes, but it is not safe to expect to obtain a bearing closer than to the nearest ten or fifteen minutes. It must be remembered that the daily variation in declination of the needle amounts to several minutes and that other uncertainties may exist. On the small pocket compasses, somewhat less precision is obtainable, possibly to the nearest one or two degrees.

Note, however, that this lack of precision along a compass traverse is not generally cumulative but tends to compensate, and that while a single line may have an error of fifteen or twenty minutes or even more in its bearing, a series of lines of a survey is apt to check quite closely on the ground or on a map with the positions indicated by their true bearings. An example of this compensation is shown in Fig. 53.

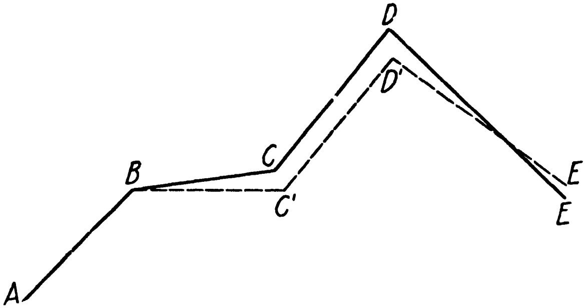


FIG. 53.

Correct bearings are shown solid while compensating observed bearings are shown dashed.

Because of this compensation of compass errors, it is possible with a compass to secure fairly satisfactory reconnaissance or preliminary surveys for railway, highway, transmission-line, military, and similar purposes.

However, the compass is not satisfactory for most of the usual engineering surveys of the present day, nor can it be used for laying out work with the precision demanded in modern engineering construction.

54. Uses of Compass. The compass is now used in surveying only, in general, for approximate work such as rough mapping, timber cruising, reconnaissance, and preliminary surveys. For most engineering work and even for land surveying, the compass has been replaced by the transit, which will give measured angles and calculated bearings precisely within a few minutes, or even more precisely. On the transit, the compass is most frequently used for checking the more precise angular work against large blunders. Its use in this manner is indicated on the field notes of Figs. 96a, 97a, and 99 in §§ 96, 97, and 99 respectively. For *compass traversing*, see § 198.

PROBLEMS

1. Find the smaller angle between each of the following pairs of lines. Use sketch.

(a) N 71°¼ E and N 13°½ E; (b) S 22°¾ E and S 48°½ W; (c) N 82 ¾ W and S 31°¾ E; (d) S 37°½ W and N 78°¼ E; (e) N 28°¾ W and S 89 W; (f) S 73°¾ E and N 37°½ W. *Ans.* (a) 57°¾

2. In the following tabulation, find the magnetic bearing that should be used at the present time in retracing each of the lines, each line being from an independent survey. Use a sketch and work from the true bearing, which always remains the same.

Old Surveys		Present Time Surveys	
Declination	Observed Magnetic Bearing	Declination	Required Magnetic Bearing
(a) 8 15′ W	N 13 45′ W	12 30′ W	*N 9 30′ W*
(b) 6 45′ E	N 82° 15′ E	10 00′ E	?
(c) 4 30′ W	S 16 45′ W	7 15′ E	?
(d) 12 00′ E	S 64° 30′ E	2 45 W	?
(e) 3 15′ W	S 47 30′ W	9 30′ E	?
(f) 5 45′ E	S 76 45′ E	1 15′ W	?

Given Line	Angle	Unknown Bearing
*(a) N 18 26 W	12° 16′	*N 6 10′ W*
(b) S 86 43′ E	63 52	?
(c) N 74 51′ W	113 46	?
(d) S 24 13 E	148 37′	?
(e) N 47 32′ E	208° 01′	?
(f) S 61 01′ E	307° 58′	?

3. In the accompanying tabulation there is given the true (astronomic) bearing of a line and the angle, measured clockwise, from this line to another line whose bearing is required. Use a sketch.

4. Change the given bearing of Problem 3 into azimuth (angle between 0 and 360°, clockwise from south). *Ans.* (a) 161 34′

5. In each of the several different closed traverses *ABCDEA* of the following tabulation, local attraction exists at one or more points. Compute the correct bearings of each of the lines in one of the traverses. Use a sketch. Check the total of interior angles of the figure. Compute each correct bearing from the preceding one and finally check on the bearing with which you started, thus obtaining an all-inclusive check.

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* Answers for (a) are shown in italics in tables.

	(a)	(b)	(c)	(d)	(e)	(f)
AB	S 4° 30′ W	S 15° 45′ W	S 2° 45 W	S 12° 45′ W	S 10° 30′ W	S 17° 00′ W
BA	N 3° 45′ E	N 17° 00′ E	N 4° 30′ E	N 15° 45′ E	N 4° 30′ E	N 15° 45′ E
BC	N 71° 30′ E	N 45° 00′ E	N 72° 15′ E	N 43° 45′ E	N 72° 15′ E	N 43° 45′ E
CB	S 72° 15′ W	S 43° 45′ W	S 73° 15′ W	S 42° 15′ W	S 72° 15′ W	S 43° 45′ W
CD	N 9° 15′ E	N 21° 30′ E	N 10° 15′ E	N 20° 00′ E	N 9° 15′ E	N 21° 30′ E
DC	S 9° 15′ W	S 21° 30′ W	S 9° 15′ W	S 21° 30′ W	S 12° 00′ W	S 23° 00′ W
DE	S 87° 00′ W	S 63° 30′ W	S 87° 00′ W	S 63° 30′ W	S 89° 45′ W	S 65° 00′ W
ED	N 89° 30′ E	N 61° 15′ E	N 87° 00′ E	N 63° 30′ E	N 87° 00′ E	N 63° 30′ E
EA	N 30° 45′ W	N 30° 45′ W	N 33° 15′ W	N 28° 30′ W	N 33° 15′ W	N 28° 30′ W
AE	S 33° 15′ E	S 28° 30′ E	S 35° 00′ E	S 31° 30′ E	S 27° 15′ E	S 27° 15′ E

6. From Fig 43, for your locality, find the magnetic declination for (a) present date, (b) 1910, (c) 1920, (d) 1930, (e) 1950, (f) 1960

7. Fill out this table using a declination of (a)* 4° 15′ E, (b) 3° 45′ W, (c) 8° 30′ E, (d) 11° 15′ W, (e) 19° 30′ E, (f) 21° 30′ W. Local attraction exists

Observed Magnetic Bearings		True Angle between Lines	Corrected Magnetic Bearings	True Bearings	True Azimuth 0° 00′ S
AB	N 80° E		*N 78° E*	*N 82° 15′ E*	*262° 15′*
BA	S 82° W	*122°* *	*S 78° W*	*S 82° 15′ W*	*82° 15′*
BC	S 40° E		*S 44° E*	*S 39° 45′ E*	*320° 15′*
CB	N 46° W	*64°*	*N 44° W*	*N 39° 45′ W*	*140° 15′*
CD	S 70° W		*S 72° W*	*S 76° 15′ W*	*76° 15′*
DC	N 72° E	*18°*	*N 72° E*	*N 76° 15′ E*	*256° 15′*
DE	N 90° E		*N 90° E*	*S 85° 45′ E*	*274° 15′*
ED	N 90° W	*130°*	*N 90° W*	*N 85° 45′ W*	*94° 15′*
EF	S 40° E		*S 40° E*	*S 35° 45′ E*	*324° 15′*
FE	N 36° W		*N 40° W*	*N 35° 45′ W*	*144° 15′*

*** Answers are shown in italics.**

CHAPTER V

LEVELING

55. Level Surface and Level Datums. A *level surface*, as the term is used in surveying, means an approximately spherical surface concentric with the surface of a body of still water; in other words, following the curvature of the earth. The tangent to such a surface at any point is at right angles to the direction of the force of gravity, or to a plumb-bob string. A level surface is therefore *not* a plane, but is *curved*, as shown in Fig. 55.

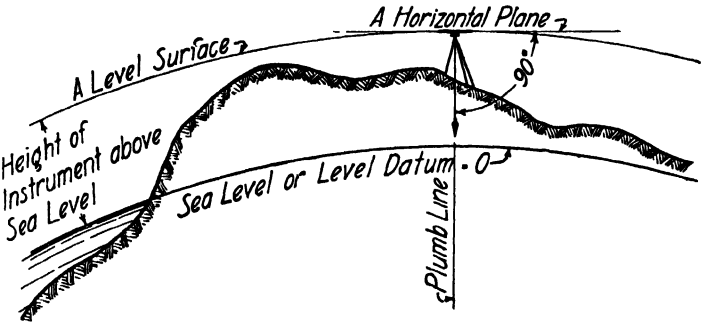


Fig. 55.

Elevations are generally referred to sea level. When we say that a certain point has an elevation of 722.84 feet, we mean, unless specified to the contrary, that the point is 722.84 feet above sea level. *Sea level* usually signifies the mean obtained by averaging high and low tides over at least a lunar month, and longer for precise results. However, in any particular locality there often may be several level surface *datums*, sometimes including different so-called sea-level datums, and since any one of these may have been used in leveling operations, it is necessary to specify the datum to which any particular elevation or set of elevations is referred.

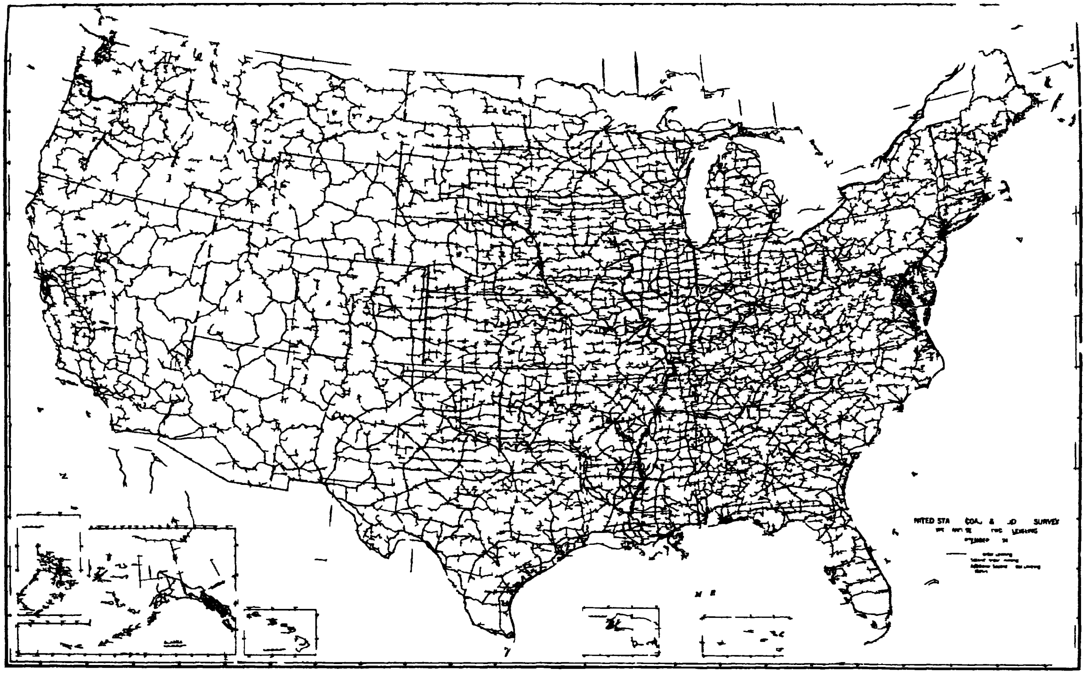
56. Assumed Level Datum. Where previously established bench marks are not available, or for small and inexpensive surveys, the surveyor may assume the elevation of some point and determine other elevations with reference thereto. In such cases the elevation of the initial point is assumed sufficiently above the assumed zero datum, so that no negative elevations will occur within the survey. The initial elevation of the survey is frequently chosen as 100.00 feet above the assumed datum, provided none of the elevations of the survey will be below the datum. This assumption of a datum is not a desirable practice since it sometimes leads to confusion. It should be used only for temporary work or in thoroughly isolated locations.

57. Available Elevation Data. The United States Coast and Geodetic Survey and the United States Geological Survey have run precise levels and established *bench marks* (B. Ms., or permanent points of known elevation) over the entire United States. Their mean sea-level datum is the datum in common use.

The network of published and unpublished precise leveling in the United States is shown in Fig. 57. This is the standard basic sea-level datum leveled inland from the oceans by the federal government and is quite precise, adjacent B.Ms. giving relative elevations within a few hundredths of a foot. The location and elevation of bench marks along these lines may be obtained from the United States Coast and Geodetic Survey in Washington, D. C.

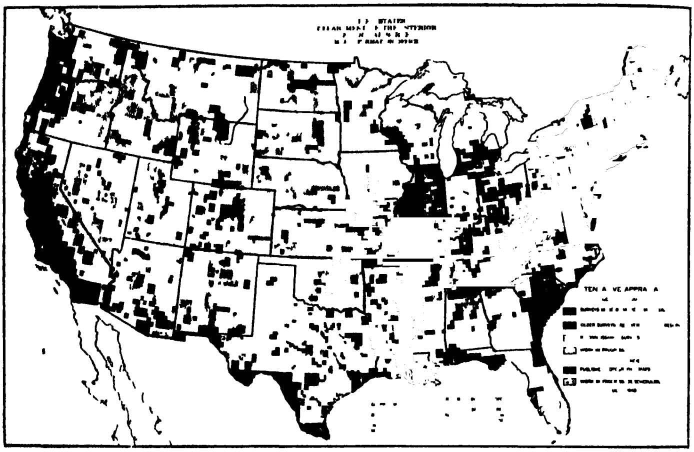
Many additional slightly less precise level data may be obtained from the United States Geological Survey in Washington, D. C. The work of this organization thoroughly covers the areas shown in Fig. 57a. Their published topographic sheets, widely used and generally available, cover this area and show the location and approximate elevation of many of their bench marks (permanent points of known elevation), as well as much additional information of approximate ground-surface elevations.

Other branches of the federal government; city, county, and state engineers; irrigation, drainage, and power projects; and railways and many corporations, have extensive level data and gladly furnish them gratis.



Courtesy of the United States Coast and Geodetic Survey

FIG 5



Courtesy of the U. S. Geological Survey

FIG. 57a. AREAS MAPPED BY THE U. S. GEOLOGICAL SURVEY

58. Determination of Approximate Differences in Elevation. The natural manner of obtaining differences in elevation is from the surface of a body of still water, but this is rarely available. Sighting across the water surfaces of a bent glass tube partly filled with water gives approximate relative elevations. A hose with glass tubes in the ends, filled with water, is sometimes used by mechanics and others. A triangular wooden frame with a plumb bob is sometimes used. Sighting along the top of a carpenter's or mechanic's level will give fair results if the level is a good one and is in adjustment. More frequently the mechanic's level is held on top of a straightedge so that the bottom of the straightedge is level. All of these methods are suitable for points relatively close together, say within 20 feet; except that a 100-foot hose is sometimes used.

59. Spirit Level. The mechanic or engineer avails himself of a more convenient and precise method. A slightly curved graduated glass tube filled with alcohol or ether, except for a small air bubble, is attached to some sort of a straightedge or telescope. The fluid drops to the lower part of the tube and the air bubble ascends to the highest point. A tangent to the top center of the bubble is a horizontal line at right angles to a plumb line. The instrument is adjusted so that the straightedge, or the line of sight of the telescope, is horizontal when the bubble is at the center of the graduations on the tube.

The sensitiveness of the bubble tube is dependent upon the viscosity of the liquid, the smoothness of the tube and the radius of curvature to which it is bent or ground. In the cheapest carpenters' or mechanics' levels the tube is bent to a short radius and filled with alcohol, but in the better grades of such levels and in all engineers' surveying instruments, the tubes are filled preferably with ether and smoothly ground to a long radius. On these better instruments, the radius of curvature determines the sensitiveness of the bubble tube, and this sensitiveness is expressed either as a radius (say 85 feet); or as the number of seconds of angle (say 20 seconds) subtended at the center of the circle of curvature by two radii from the ends of two adjacent bubble tube graduations, usually (but not always) 1/10 inch apart.

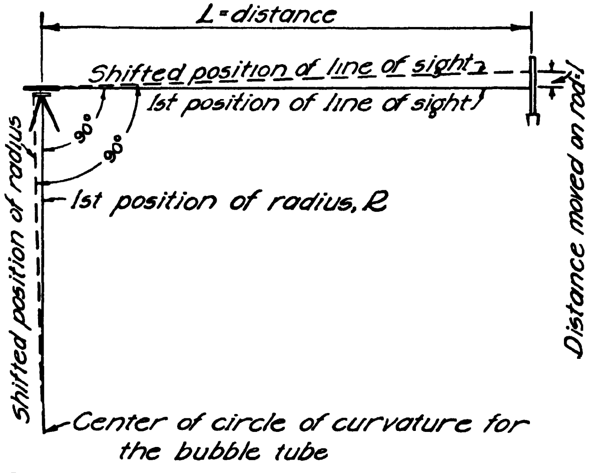

L = distance
Shifted position of line of sight
1st position of line of sight
90°
90°
1st position of radius, R
Shifted position of radius
Distance moved on rod
Center of circle of curvature for the bubble tube

FIG. 59. DETERMINING SENSITIVENESS OF BUBBLE TUBE.

The sensitiveness may be determined in the field as in Fig. 59. The bubble is centered and a rod reading taken on a point l feet from the instrument. The bubble is then shifted several divisions and another rod reading taken. The difference between the rod readings or distance which the line of sight has moved on the rod, may be called I. Call the distance which the bubble shifts D. Then, with all values in feet,

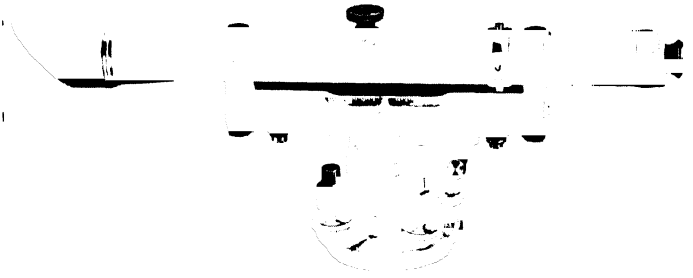
$$\frac{I}{L} = \frac{D}{R}, \qquad \text{or} \qquad R = \frac{LD}{I}. \tag{59}$$

The angular value between the first position and the shifted position of the line of sight is $\sin^{-1} (I/L)$. When this is divided by the number of bubble divisions shifted, we have the angular value of one bubble division.

The instrument maker may determine the sensitiveness more easily on a bubble testing apparatus in his shop.

60. Line of Sight. The line of sight of a telescope, such as is used on a level or other surveying instrument, is a straight line passing through the intersection of the cross-hairs and through the optical center of the objective lens (Fig. 62).

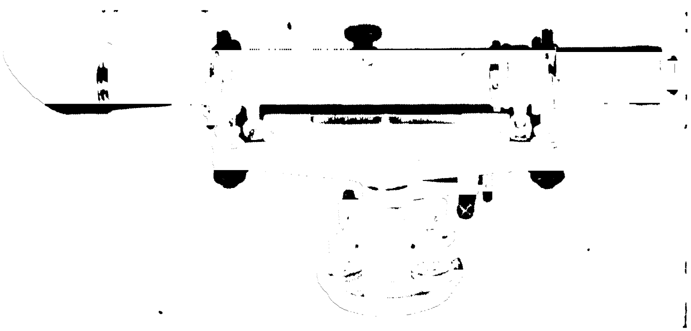
61. Engineers' Level. The engineers' level (Figs. 61 and 61a) consists essentially of a telescope defining a line of sight, and an attached sensitive bubble tube. If the level is in adjustment, every time the bubble is centered in the tube, the line of sight is horizontal. The standards holding the telescope, the foot-screws, the tripod, and so on, are added only for practical convenience in holding the instrument steady.



Courtesy of Keuffel and Esser Co.

FIG. 61. ENGINEERS' DUMPY LEVEL.

62. Optical Properties. The optical properties of the transit or level telescope should correspond with the type of work to



Courtesy of Keuffel and Esser Co.

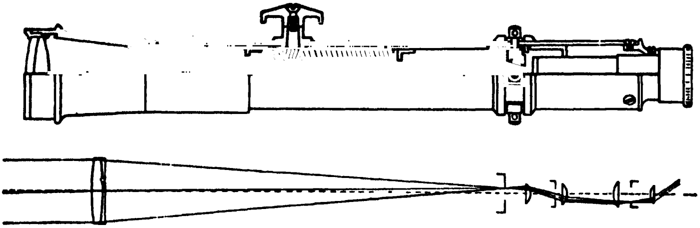
FIG. 61a. ENGINEERS' WYE LEVEL.

be done. The usual instrument serves the wide range of ordinary conditions encountered in engineering practice. Special instruments are made for special work, such, for example, as underground surveys. Some optical properties are:

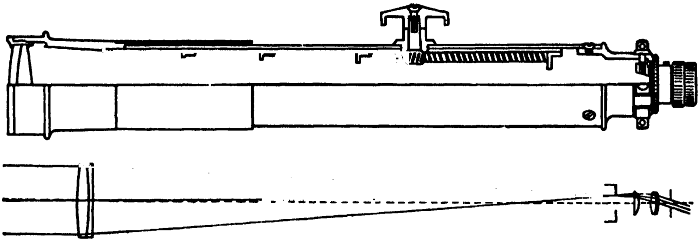
1. *Illumination*, which refers to how well lighted the image appears.
2. *Definition*, which has to do with sharpness of detail.
3. *Width of field of view*, expressed as an angle, which tells how much of the object is visible at one time, and
4. *Magnifying power*, which is the ratio between the apparent lineal dimension of the object and of its image.

In general, excellence in certain of these properties is obtained at the expense of others. Aside from optical properties, the type of level bubbles as well as the general workmanship on a level limit the precision of the work that can be done with a particular instrument. It is therefore desirable that on any level all of these factors be appropriately balanced with respect to the others; and it is especially desirable that the kind of work to be done with the instrument be kept in mind. On a good level, for example, the slightest perceptible motion of the bubble should show a like motion of the line of sight on the rod.

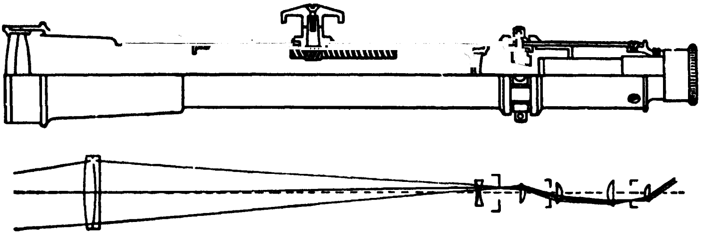
63. Optical Systems. In Fig. 62 the erecting eyepiece consists of four lenses which both magnify and erect the image, so that it is observed right side up. The image formed on the plane of the cross-hairs by the objective lens is always upside down.



Erecting Eyepiece. Conventional Focusing.



Inverting Eyepiece. Conventional Focusing.



Erecting Eyepiece. Internal Focusing.

Courtesy of C. L. Berger and Sons

FIG. 62. TELESCOPE OPTICAL SYSTEMS.

Another type, consisting of only two lenses, called the inverting eyepiece, shown in Fig. 62, only magnifies, but does not erect the image. Consequently the observer sees the image upside down. While this is something of a disadvantage to the inexperienced man, yet the image is clearer and better illuminated than with the four-lens eyepiece.

The inverting eyepiece is short, and the erecting eyepiece is relatively long. They may be distinguished by observing the distance of the cross-hair reticule ring, in Figs. 61, 61a and 62, from the outside end of the eyepiece, or by noting whether the image is inverted when the telescope is used.

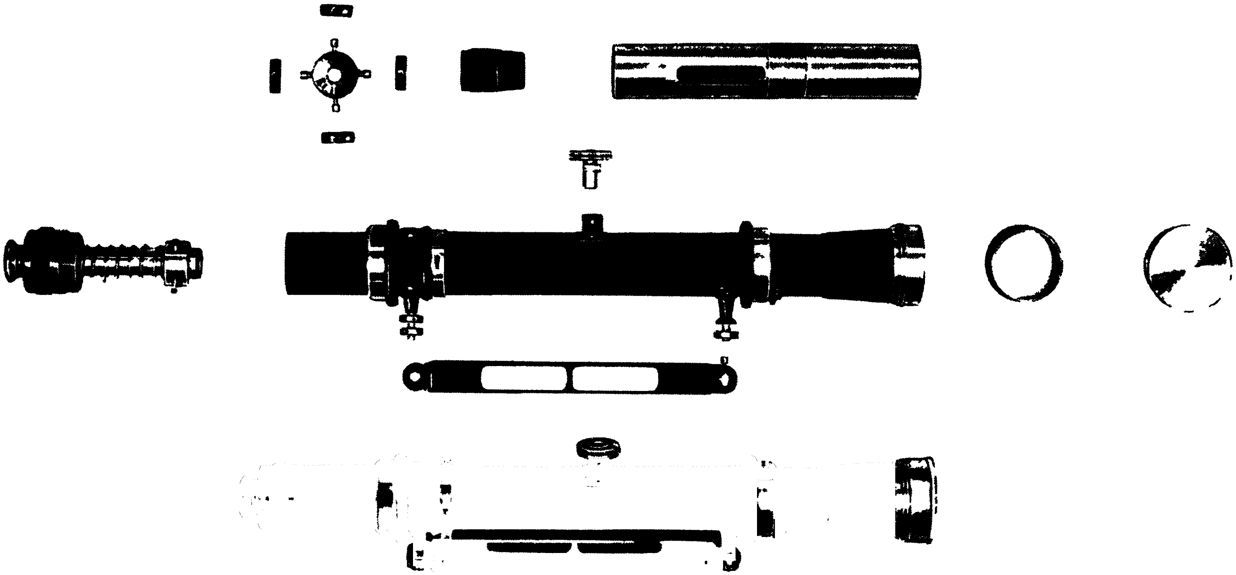


FIG. 63 INTERNAL FOCUSING TELESCOPE — DISASSEMBLED

Courtesy of Keuffel and Esser Co.

Present practice in the United States favors the erecting eyepiece for the usual surveying instruments and the inverting eyepiece for the more precise instruments.

Internal focusing telescopes (Figs. 62 and 63) are now standard on new instruments.

64. Parallax. When the eye is moved slightly while looking through the telescope, the intersection of the cross-hairs may appear to move. This is called *parallax*.

The object sighted at is optically projected by the outer, or objective lens, onto the plane of the cross-hairs as indicated on Fig. 62, but on this plane the image is very small. The small figure is magnified by the lenses of the eyepiece, which acts as a microscope. It is necessary, therefore, to focus the eyepiece on the cross-hairs. This is roughly accomplished by pointing the telescope toward some light-colored surface a few hundred feet distant, and moving the eyepiece in or out until the cross-hairs appear clearest to the eye. The eyepiece is moved either by a small thumb screw or by rotating the eyepiece so that a spiral groove, engaging upon a small projection, will move the eyepiece in or out.

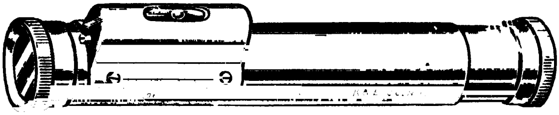
The cross-hairs then appear plainly to the eye and parallax is largely eliminated. Should some parallax still be noticeable, eliminate it as explained in § 115. Parallax *must* be eliminated from the level telescope before precise work is possible.

65. Dumpy Level. This is the simpler type of engineers' level and is increasingly used (Fig. 61). The telescope and supports are permanently fastened together. The inverting eyepiece is usually found in the older dumpy levels, while modern design favors the erecting eyepiece.

66. Wye Level. This type (Fig. 61a) has been used more in the past than the dumpy level, but it has been largely replaced by the dumpy type. It is called a *wye* or "*Y*" level because the telescope rests in *Y*-shaped supports and may be rotated about its axis, or removed from the *Y* supports and turned end for end. The *wye* level has a larger number of adjustable parts than the dumpy level. These added parts facilitate adjustment, but, on the other hand, more frequently require adjustment.

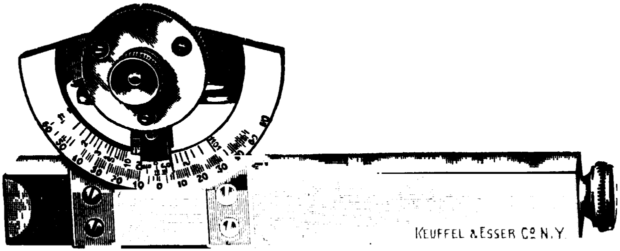
67. Other Types. European levels are rarely used in this country, since they are slightly more complex than the American types. Precise levels, used chiefly by the Federal Government, are quite similar to the engineers' dumpy level described above.

68. Hand Level and Clinometer. These two instruments, shown in Fig. 68, are of similar construction except that the clinometer has an added vertical circle upon which vertical angles may be read. The bubble on top of these instruments is reflected by a 45-degree mirror or prism inside the tube, so that the bubble and part of the landscape are visible at the same time. The bubbles are poor and there is no magnification of the landscape. The instruments are of relatively low precision and are used for approximate work only.



Courtesy of Keuffel and Esser Co

Hand Level.

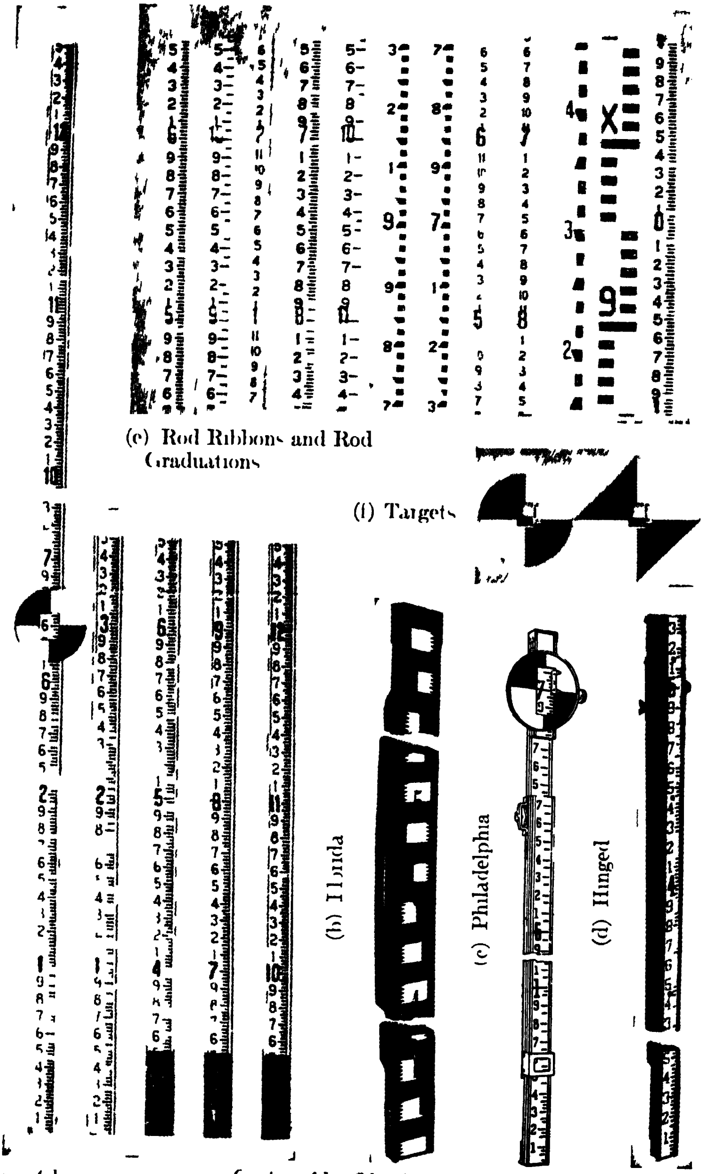


Courtesy of Keuffel and Esser Co.

Clinometer.

FIG. 68.

69. Leveling Rods. Several types are shown in Fig. 69. The single piece or Florida rod (b) is best for surveys in which little transportation is involved. The tenths of a foot on the Florida rod are alternately red and black. Hence these tenths may be read farther from the instrument than is possible with types (a), (c), and (d). However, the hundredths of a foot are not so distinct on the Florida rod as they are on other types.



(a) Chicago

(b) Florida

(c) Philadelphia

(d) Hinged

(e) Rod Ribbons and Rod Graduations

(f) Targets

Courtesy of Keuffel and Esser Co and Chicago Steel Tape Co

FIG. 69. LEVEL RODS.

The slip joint or Chicago rod (a), the sliding joint or Philadelphia rod (c), and the hinged joint rod (d), are types that are easily transported but must be used carefully to avoid small errors at the joints, particularly those due to the sliding down of the upper part of the Philadelphia rod.

Type (c), the Philadelphia rod, is generally used, but the more recent type (a), the Chicago rod, offers certain advantages, including less danger of error at the joint, wider figures which may be read at a longer distance, and a shorter length when collapsed for transportation. It is slightly more difficult to set a target above the seven-foot graduation on the Chicago rod, but this is rarely necessary.

Rod ribbons of heavy painted canvas (Fig. 69e) may be tacked to a long strip of wood. If glued or thoroughly thumb-tacked to the wood after comparison with a steel tape, they are sufficiently precise for ordinary leveling.

The rod is held vertically by the rodman, who, in calm weather, plumbs the rod between his fingers. In windy weather he must judge by eye whether the rod is vertical, perhaps comparing it with the vertical corners or lines on buildings and the like. Rod levels are available for holding the rod plumb but are seldom used.

For more precise work, the rod is waved towards and away from the level and the minimum rod reading, which occurs when the rod is vertical, is read from the level. The rod is waved in the same vertical plane with the instrument as directed by the levelman, who directs the rodman to lean the rod right or left until it appears parallel to the vertical cross-hair. This method should not be used with readings less than four feet, since the error due to low readings on an inclined rod is small, and since the raising of the rod on its rear edge introduces error. Neither should it be used unless the precision of the work so demands, since it slows progress.

70. Targets. Targets (Fig. 69f) were once generally used, but in modern practice the rods are read directly by the instrumentman and targets are used only where poor atmospheric conditions or extremely long sights make direct reading by the instrumentman difficult or impossible, where a single rod reading is used repeatedly, as for example in setting a number of construction stakes at the same elevation, or in other special circumstances.

The target is moved up or down under the direction of the levelman and is clamped and read by the rodman. When possible, this reading is read directly and checked by the levelman. With the Philadelphia rod, target settings above seven feet are made by clamping the target at exactly the vernier reading of the rear vernier, and sliding the rear section of the rod upward until the target is on the line of sight. The rod reading is then found on the back of the rod, *not* at the target.

Target verniers (Fig. 70) reading to thousandths of a foot or, rarely, to thirty-seconds of an inch, are furnished on most targets but are seldom used, since readings to hundredths of a foot, or eighths of an inch, are usually sufficiently precise. Furthermore, experience has demonstrated that instrumental and manipulative errors involved in leveling are so large that the smaller vernier readings are seldom justified. Also, the smaller errors of reading tend to compensate.

On the left of Fig. 70 is shown an ordinary level rod with a target to which is attached a vernier whose zero is on the same horizontal line as is the target center. Feet, tenths, and hundredths are read on the rod graduations opposite the zero of the vernier, and thousandths of a foot may be estimated by eye.

More precisely, the thousandths of a foot are read on the vernier by noting which line of the vernier coincides with (is a horizontal continuation of) a rod graduation. Thus in the enlarged vernier of the figure, it is noted that the zero of the vernier reads slightly above 5.32 on the rod. Thousandths are read on the vernier by noting that the 2 on the vernier coincides with a rod graduation, and the precise reading is therefore 5.322. Were the target and the vernier raised one-thousandth of a foot, the 3 on the vernier would coincide with a rod graduation and the precise reading would be 5.323.

To obtain the *least count* of a vernier, that is, the smallest fraction of a foot which it will read, it is necessary to divide the value of the smallest division on the rod scale by the number of divisions on the vernier. In this case the least count is 0.01 foot divided by 10, or 0.001 foot.

Another method of determining the least count of this vernier is to note that ten divisions on the vernier cover exactly nine divisions, or 0.09 foot, on the rod. Therefore, each vernier division is 0.009 foot and each rod division is 0.010 foot. The difference of 0.001 foot between these widths is the least count of the vernier.

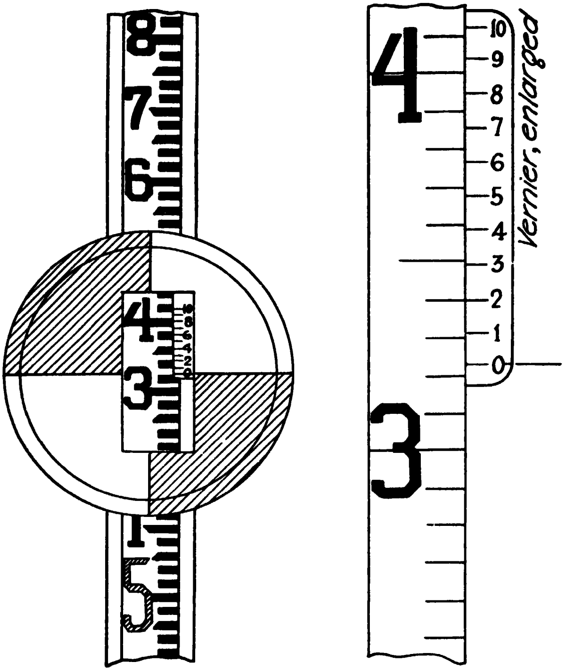


Fig. 70. Target Verniers.

This same principle is applied to many precise instruments. The transit verniers of § 91 are similarly constructed.

71. Setting Up the Level. The level is set up with the legs thrust firmly into the ground and with the top plate of the tripod approximately level. The wing nuts at the tripod head are kept sufficiently tight so that a tripod leg, when raised, will barely fall of its own weight. When the tripod points are on a smooth surface, the wing nuts are further tightened to prevent slippage and probable damage to the level if it were to fall. If possible, the tripod points should be set in indentations of the smooth surface. On hillsides, it is usually convenient to set two tripod legs downhill and one uphill.

With the telescope over one pair of foot screws, the bubble is brought approximately to the center of the tube by turning each of the footscrews of a pair in opposite directions at the same time by means of the thumbs and forefingers. The bubble moves in the same direction as the left thumb. The telescope is then turned 90° over the other set of footscrews and the bubble brought approximately to center. This procedure is repeated a time or two with increasing precision until the bubble is centered with the telescope over either set of footscrews. If the level is in adjustment the bubble will remain centered throughout a complete revolution of the telescope.

The telescope is focused on the rod by turning the focusing screw which is in the middle third of the telescope. The objective lens is moved outward for near sights and inward for distant sights, except for internal focusing.

An instrument should not be left set up unguarded where traffic or construction work will endanger it or where live stock are pastured. Painted tripod legs minimize danger in heavy traffic. It should not be leaned against a wall or in a corner. Indoors, or in timber, the instrument should be carried under the arm with the tripod to the rear, rather than on the shoulder, in order to avoid striking it against door jambs or trees. The footscrews and clamps should be lightly tightened or clamped sufficiently only to prevent movement. The lens should never be touched with anything but a camel's-hair brush or, at worst, a light touch with a soft silk handkerchief. Repairs to instruments are expensive.

72. Bench Marks and Turning Points. A *bench mark*, or B.M., is a definite permanent point of known elevation. Some attention must be given to the permanence of the objects on which the B.Ms. are established, particularly so that there shall be no change in elevation such as might result from freezing, settlement, or other disturbance. See page 61 for location of available bench marks.

A *turning point*, or T.P., is a temporary point of known elevation. While removable metal turning points may occaionally prove useful, the rodman ordinarily finds a hand ax, pikes, and wooden pegs sufficient. Turning points may, of ourse, be established on any solid object. Inexperienced rodmen err in holding the rod on the ground for a turning point instead of using something solid. This may cause an error of several hundredths of a foot in the elevation of all subsequent points. If held on the ground, the rod tends to settle downward, and the errors on a series of such unsatisfactory turning points tend to be cumulative.

Practice varies as to whether to use fairly permanent turning points and to describe them in the notes, or to omit the description and abandon the turning points as soon as they have been used. The purpose of the survey must determine this policy.

If the B.Ms. are to be used over a long period, it is possible, of course, that they may be disturbed or obliterated. To facilitate their re-establishment, or for assurance that they have not been disturbed, it is desirable that fairly permanent T.Ps. be established on each side of the B.M. and that these T.Ps. be thoroughly described in the notes, or that the B.Ms. should be established in pairs.

Independent foresights should not be taken on B.Ms. but the B.Ms. should be included in the regular level circuit, so that the circuit when finally checked will definitely check all intermediate B.Ms.

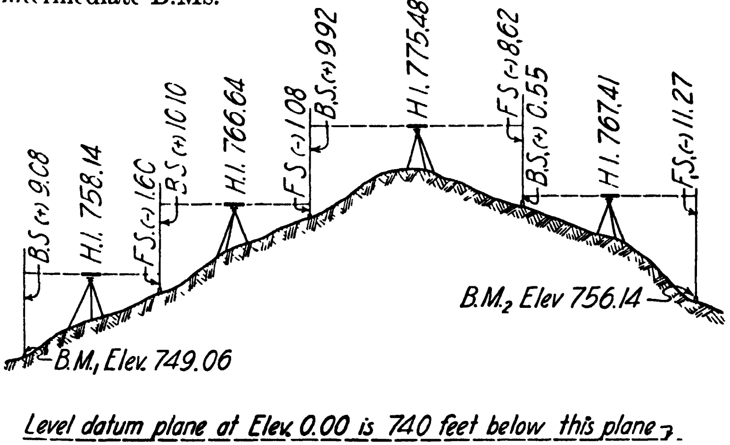
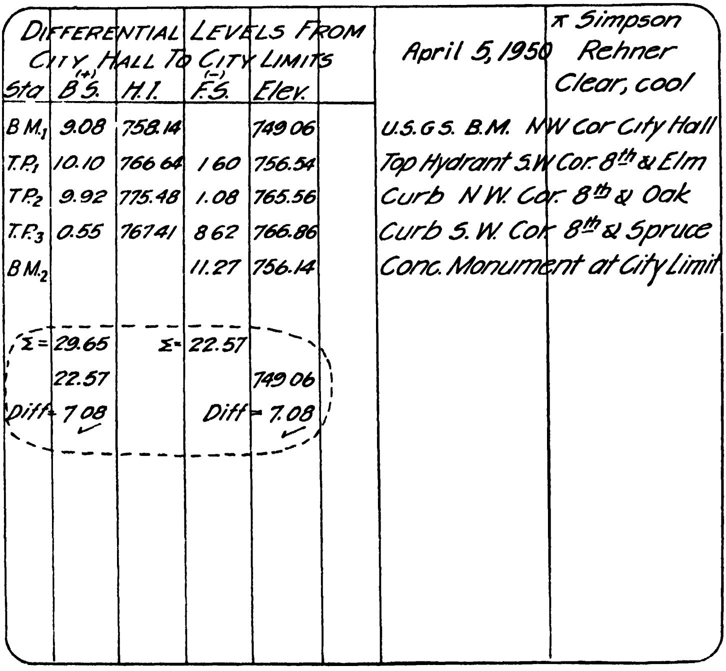


FIG. 73. DIFFERENTIAL LEVELING.

73. Differential Leveling. The process of determining the difference in elevation between two points some distance apart is indicated in Fig. 73, which illustrates the standard field notes of Fig. 73a.



Differential Levels From City Hall To City Limits

April 5, 1950

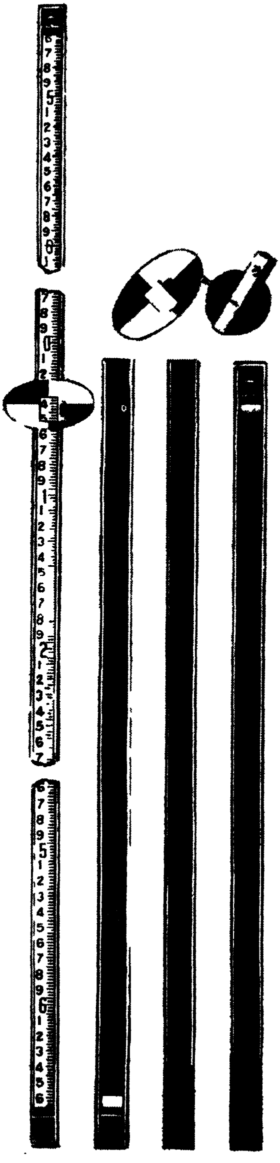
⊼ Simpson
Rehner
Clear, cool

Sta	(+) B.S.	H.I.	(−) F.S.	Elev.			
B.M.₁	9.08	758.14		749.06		U.S.G.S. B.M. N	W Cor City Hall
T.P.₁	10.10	766.64	1.60	756.54		Top Hydrant S.W	Cor. 8th & Elm
T.P.₂	9.92	775.48	1.08	765.56		Curb N.W. Co	r. 8th & Oak
T.P.₃	0.55	767.41	8.62	766.86		Curb S.W. Co	r. 8th & Spruce
B.M.₂			11.27	756.14		Conc. Monum	ent at City Limit
Σ =	29.65	Σ =	22.57				
	22.57			749.06			
Diff	7.08 ✓		Diff =	7.08 ✓			

FIG. 73a. STANDARD FIELD NOTES FOR DIFFERENTIAL LEVELING.

The rod reading is taken on a point of known elevation (*a backsight or + sight*) and then on a point of unknown elevation (*a foresight or − sight*). The instrument is moved to its next location and the process repeated until the last point is reached. The words *backsight* and *foresight* are not used in the sense of backward and forward but rather to designate rod readings taken on points of known elevation (a backsight or B.S.), or on points of unknown elevation (a foresight or F.S.).

In keeping the notes, the point or station is entered in the first column of the field notes. On the same horizontal line are entered the backsight and foresight taken *on* this point or station. From these the elevation of the point and also the



Courtesy of Chicago Steel Tape Co.

FIG. 74. TAPE ROD.

elevation of the height of instrument obtained *from* the point are determined by addition or subtraction, as shown by the notes of Fig. 73a. All of the data mentioned in this paragraph are entered on the same horizontal line.

The additions and subtractions are made horizontally across the page immediately after each rod reading is taken. The backsight or plus rod reading is added to the known elevation, which gives the *height of instrument*, or *H.I.* The foresight or minus rod reading, subtracted from the height of instrument, gives the unknown elevation. They are then checked at the bottom of the notes as shown in Fig. 73a, by obtaining the difference of the sums of backsights and foresights and by noting that this difference is identical with the difference between the original and final elevations.

It is evident that the work will be speeded-up, in leveling uphill, by taking backsights near the top of the rod and foresights near the bottom of the rod; and by the reverse of this in leveling downhill. The exercise of this judgment by the rodman in selecting T.Ps. and by the levelman in choosing instrument set-ups, develops with experience. The levelman should watch the rodman as he selects the T.P. and should tell him to take a higher or lower point, as is apparent through the level

74. Tape Rod. The tape rod of Fig. 74 is seldom used but offers advantages in some work; for example, where many elevations must be determined from one set-up of the level. The rod graduations, increasing downwards, are enameled on a movable endless metal tape about twenty feet long. If the elevation of the B.M. is 743.21, then the tape is moved until 3.21 is read on the horizontal cross-hair of the level when the rod is held on the B.M. The tape remains clamped in this position so long as the H.I. remains the same. When the rod is held on a new point of unknown elevation, a rod reading of, say, 1.13 means an elevation of 731.13, 741.13, or 751.13. A little study and use of the rod will indicate when it is necessary to add or subtract ten feet. This rod eliminates the need of additions and subtractions in the notes.

75. Signals. *Plumb the Rod.* Extend the arm vertically above the head and sway the arm slowly in the direction in which the rod is to be plumbed.

Up. Extend the right arm horizontally outward, palm up, from the shoulder and raise upward, an abrupt and large wave when a large movement is required, and a slower and smaller wave when a small movement is desired. Drop the arm after making the signal.

Down. The same as above except that the movement is downward, palm down, from the shoulder.

All Right. Extend both arms horizontally from the shoulder and wave up and down.

Establish Turning Point or Bench Mark. Extend one arm vertically above the head and wave in circles around a vertical axis.

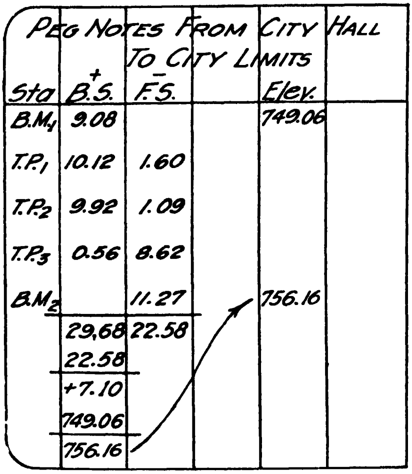
Numbers. Numbers may be transmitted a considerable distance by a system of signals, but they are seldom used.

Methods of increasing the visibility of signals on long sights are given in § 101.

76. Closing Level Circuits. In order to be certain that leveling has been accurately accomplished, it is customary to start from a bench mark of known elevation and to return to, or *close on*, the same point, noting whether the finally determined elevation corresponds with the original elevation. An equally satisfactory check may be made by closing on another bench mark of known elevation. Not infrequently, thousands of dollars of loss have resulted from erroneous levels. Occasionally corporations have been thrown into receiverships from this cause.

Since there are so many possibilities of error in leveling, and since the closing check is frequently not available until considerable leveling has been completed, the following sections will describe the usual blunders and small errors which may prevent a satisfactory closure. It should *not* be considered that any leveling technique is a satisfactory substitute for a closure as described in the preceding paragraph.

77. Common Mistakes.* Large mistakes in leveling can occur only through failure to center the bubble at the moment of observation, slippage at a rod joint, the use of a wrong or indefinite B.M. or T.P., a misreading of the rod, recording wrong readings in a field notebook, or arithmetical mistakes. Experienced levelers and rodmen make few such mistakes, but if it is desired to minimize them further, one of the following check procedures is followed. Such procedures are particularly desirable where the levels are not finally closed or checked on the original B.M. or on some other B.M.



Peg Notes From City Hall
To City Limits

Sta	+ B.S.	− F.S.		Elev.	
B.M.₁	9.08			749.06	
T.P.₁	10.12	1.60			
T.P.₂	9.92	1.09			
T.P.₃	0.56	8.62			
B.M.₂		11.27		756.16	
	29.68	22.58			
	22.58				
	+7.10				
	749.06				
	756.16				

FIG. 77. PEG NOTES.

The rodman may keep an extra set of notes in a *peg book* (Fig. 77), which corresponds to the instrumentman's notes in Fig. 73a. He sets the target, pencil, or card on the rod under direction of the levelman and reads and records the rod readings on all B.Ms. and T.Ps. He omits all extra foresights, as, for example, those taken in profile leveling. It should be noted that this is an entirely independent set of readings and notes, in addition to the regular data and notes kept by the levelman, who reads the rod directly (without target) from the level. The two sets of B.M. elevations should of course check within prescribed limits, since the same B.Ms. and T.Ps. are used.

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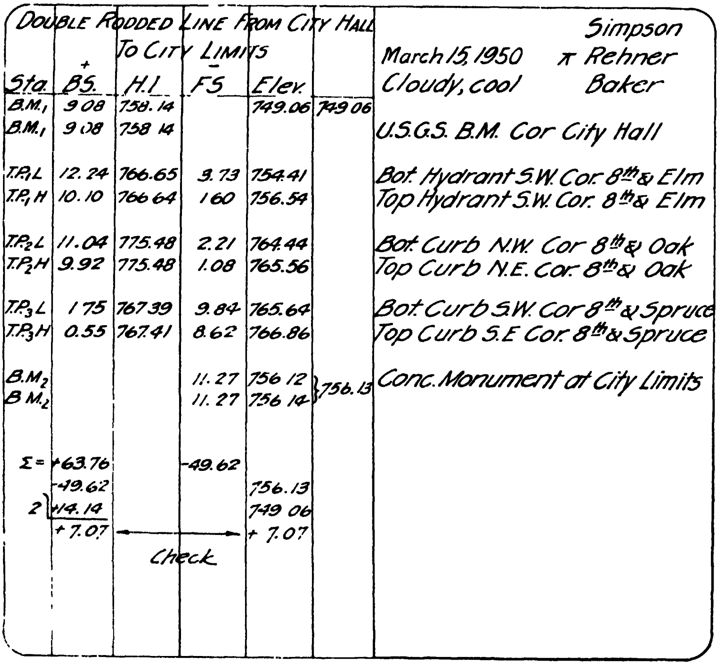
* *Mistakes* result from carelessness and can be detected and removed from final results. They should not be confused with the unavoidable small *errors* inherent in all measurements.



DOUBLE RODDED LINE FROM CITY HALL TO CITY LIMITS

Simpson
March 15, 1950 π Rehner
Cloudy, cool Baker

Sta.	+ BS.	H.I	− FS	Elev.		
B.M.₁	9 08	758. 14		749.06	749 06	
B.M.₁	9 08	758 14				U.S.G.S. B.M. Cor City Hall
T.P.₁ L	12. 24	766.65	9. 73	754.41		Bot. Hydrant S.W. Cor. 8th & Elm
T.P.₁ H	10. 10	766 64	1 60	756.54		Top Hydrant S.W. Cor. 8th & Elm
T.P.₂ L	11.04	775.48	2.21	764.44		Bot. Curb N.W. Cor 8th & Oak
T.P.₂ H	9.92	775.48	1.08	765.56		Top Curb N.E. Cor. 8th & Oak
T.P.₃ L	1 75	767 39	9.84	765.64		Bot. Curb S.W. Cor 8th & Spruce
T.P.₃ H	0.55	767.41	8.62	766.86		Top Curb S.E Cor. 8th & Spruce
B.M.₂			11. 27	756 12	756.13	Conc. Monument at City Limits
B M.₂			11. 27	756 14		
Σ =	+63.76		−49.62			
	−49.62			756.13		
2	+14.14			749 06		
	+ 7.07	←	→	+ 7.07		
		Check				

FIG. 77a. DOUBLE RODDED LINE.

In a procedure that is rarely used, one or two rodmen may be used to run a *double-rodded line*, in which two sets of readings are taken on different T.Ps. by the levelman who records and figures two independent sets of T.Ps. and H.Is. as in Fig. 77a, which covers the same territory as Fig. 73. It will be noted that one set of readings (marked *L* and read first) is on T.Ps. about a foot lower than the other set (marked *H* and read last). Some such precaution is necessary to prevent confusion. Since the two sets of T.Ps. are close together, a check between resulting H.Is. does not eliminate errors due to lack of adjustment and curvature and refraction, and if the required precision demands their elimination, it must be accomplished by equalizing the lengths of backsight and foresight.

78. Small Errors. Errors in adjustment of the level will cause some (usually small) error in results. Most of these, as well as errors of curvature and refraction, are balanced or eliminated if the lengths of backsight and foresight are made equal; that is, if the rod is held the same distance from the instrument on both the backsight and the foresight. In practice, every effort is made to do this, although it is not always easy of accomplishment. Under such circumstances, for example, if it has been necessary to take a foresight 300 feet long and a backsight 100 feet long, — at some future set-up, preferably the next one, the backsight length should exceed the foresight length by 200 feet; that is, if the backsight is 250 feet, the foresight should be reduced to 50 feet. The general rule or procedure, between any two points whose differences in elevation is desired, is that the *sum of the lengths of the foresights should be equal to the sum of the lengths of the backsights.* The theory underlying this elimination of errors of adjustment is explained in § 117 under the two-peg test in the chapter on adjustments. Errors due to differences in length of backsight and foresight are apt to accumulate when the leveling is predominantly uphill or predominantly downhill, since downhill sights are long and uphill sights are short.

If the rod is not held vertically, or on solid T.Ps., other errors are introduced. If a jointed rod is used, care must be exercised to see that there is no overlapping or shortage at the joint of the rod; that is, the distance from the foot mark below the joint to the foot mark above it, must be exactly one foot.

It is necessary for the instrumentman to eliminate parallax (see § 115) and to assure himself that the bubble is in the center of the tube at the instant at which he reads the rod. Such assurance is had by carefully leveling the instrument, taking the rod reading, and finally again looking at the bubble

By standing between the tripod legs, the instrumentman can observe the rod and see the bubble without moving his feet Moving the feet on soft ground may throw the bubble out of center. When the level is set up on soft ground or on frozen ground which is beginning to thaw, the bubble may move from the center due to settlement of the tripod. It is not essential that the bubble be exactly in the center when the level is turned at right angles to the line of sight, but it is customary and convenient to have it near the center or even exactly centered. *It is essential that the bubble be in the center when the sight is taken.*

Errors due to expansion or contraction of the rod (usually less than 0.01 foot), to settlement of the level between the instants of backsighting and foresighting, and to like causes, are seldom of noticeable importance in ordinary leveling. Special procedures are adopted in precise leveling, or in long lines of levels, to minimize these effects.

79. Profile Leveling. The *profile* may be defined as a vertical section of the ground surface along the line of some survey.

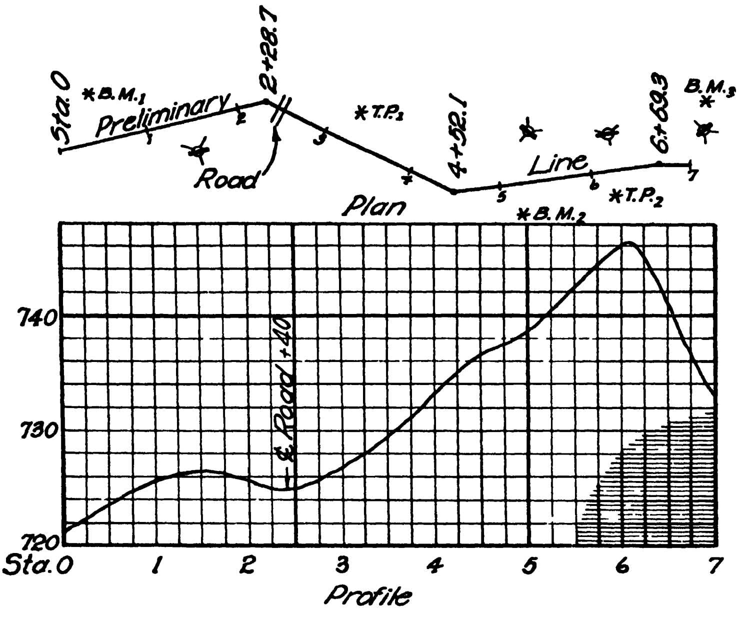
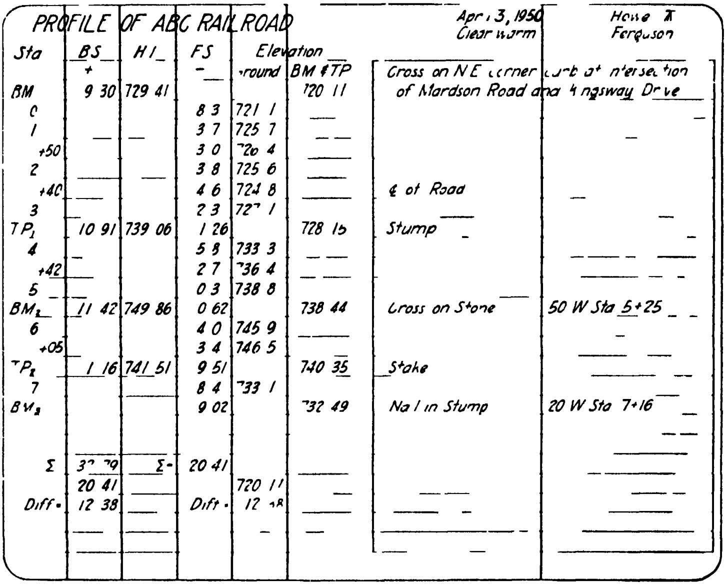


FIG. 79. PROFILE.

The profile shows the true length of the line; that is, it follows around angles or curves and is *not* a true side view. A side view, from any particular direction, would not show the true length of all portions of a survey having angles or curves. The vertical scale is much exaggerated.

The modern practice is to show a plan map, or at least an alignment sketch on the same sheet with the profile, and the profile than becomes the most important drawing in connection with many surveys. In the construction of railways, highways, sewers, pipe lines, canals, transmission lines, flumes, and the like, the construction engineers and contractors are primarily interested in the profile rather than in the map or plan, since the plan location is nearly always staked on the ground, whereas the stakes give little or no data regarding elevations.



PROFILE OF ABC RAILROAD

Apr 13, 1950
Clear warm

Howe ⊼
Ferguson

Sta	BS +	HI	FS −	Elevation Ground	Elevation BM & TP		
BM	9 30	729 41			720 11	Cross on N E corner curb at intersection of Mardson Road and Kingsway Drive	
0			8 3	721 1			
1			3 7	725 7			
+50			3 0	726 4			
2			3 8	725 6			
+40			4 6	724 8		℄ of Road	
3			2 3	727 1			
TP1	10 91	739 06	1 26		728 15	Stump	
4			5 8	733 3			
+42			2 7	736 4			
5			0 3	738 8			
BM2	11 42	749 86	0 62		738 44	Cross on Stone	50 W Sta 5+25
6			4 0	745 9			
+05			3 4	746 5			
TP2	1 16	741 51	9 51		740 35	Stake	
7			8 4	733 1			
BM3			9 02		732 49	Nail in Stump	20 W Sta 7+16
Σ	32 79	Σ−	20 41				
	20 41			720 11			
Diff =	12 38		Diff =	12 38			

FIG 79a PROFILE NOTES.

Field notes and procedure are shown in Figs. 79 and 79a. Profile leveling is similar to differential leveling, except that several additional foresights are read from a single H.I. at various points on the surface of the ground (not on top of the stakes) where ground elevations are needed. The *ground readings* are taken to tenths of a foot while B.M. and T.P. readings are taken to hundredths of a foot. The elevations are computed with similar precision; that is, tenths of feet on ground elevations and hundredths of feet on T.P. and B.M. elevations.

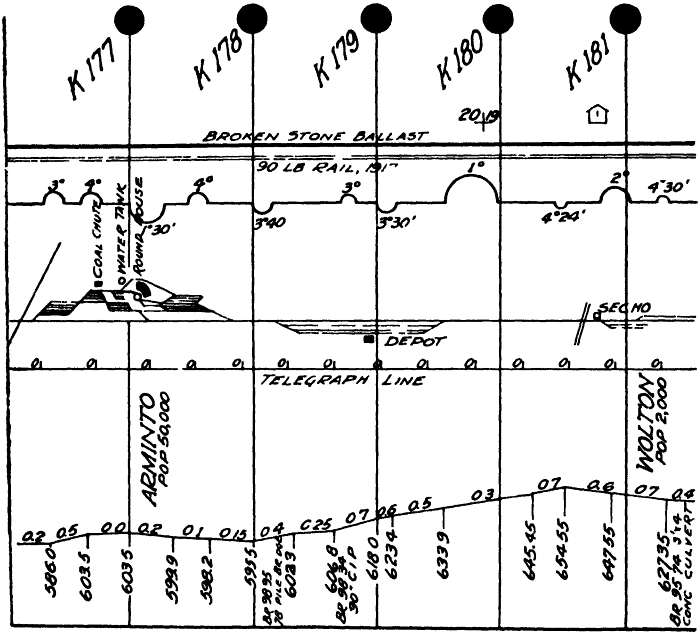


FIG. 79b. CONDENSED OR OPERATING PROFILE.

The profiles in this book are on Plate A profile paper. Plate B has a different vertical spacing.

Condensed or operating profiles are drawn to small scale and are used for planning operation rather than construction. Figure 79b shows such a condensed profile used by railway operating officials in determining locomotive loading, speed, and other operating factors.

80. Refraction. With the correctly adjusted instrument set up and the bubble centered, we would expect the line of sight to be represented by the top line A_1B_1 in Fig. 80. Actually the line of sight is *refracted*, or bent vertically, as it passes through air strata of different densities, as indicated by the middle line A_2B_2. The curvature of the earth, or of a level surface, is shown by the lower lines. The refraction A_3A_2 and B_3B_2 averages about 1/7 of the total error due to curvature and refraction, A_3A_1 and B_3B_1. Refraction is greatest where the line of sight passes near the ground or near a body of water, since the adjacent air strata here have maximum differences in temperature and consequently maximum differences in density. Refraction changes rapidly with changes in temperature; hence it is best to do precise leveling on cloudy days or in the middle of the afternoon on sunny days when the ground and air are uniformly warm, particularly if long sights are taken. On rare occasions, refraction may reach a maximum of 0.10 of a foot in a 200-foot sight. Heat waves or *boiling* air indicates rapidly changing refraction.

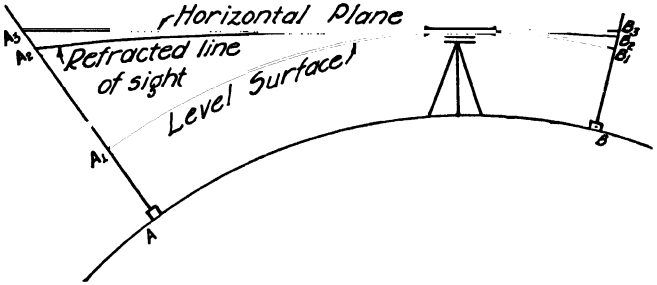


FIG. 80. REFRACTION AND CURVATURE.

81. Curvature. This is the correction (A_2A_1 and B_2B_1 of Fig. 80) introduced by the departure of the approximately spherical level surface from the surface generated by the refracted line of sight of the level. The amount of the correction, including average refraction mentioned in the preceding section, is slightly greater than:

$$\text{(81)}\quad \text{Curvature and refraction correction} = (0.0002)\,(D)^2,$$

with D in 100-foot units and the correction in feet. Stated in words, the correction varies with the square of the distance. For a distance of 300 feet, the usual maximum length of sight, the correction is only 0.002 foot, and if the lengths of backsight and foresight are equal, even this small error is eliminated. Such small corrections may be neglected for ordinary leveling.

A study of Fig. 80 will show that for equal lengths of backsight and foresight on a particular set-up of the level, the curvature and refraction corrections balance and need not be considered unless unusual differences of refraction occur. The corrections or errors are not exactly balanced, however, when only the total lengths of the backsights and foresights are equal, since the individual errors are proportional to the square of the distance. (See Problem 7, page 98.)

Neither curvature nor refraction need be considered in most engineering leveling, except for the long sights used in reciprocal leveling, as described on page 89.

82. Lengths of Sight. Maximum sights up to 300 feet are used in most engineering leveling, the longer sights speeding up the work. Beyond this distance it is difficult to read hundredths of a foot on the rod, the errors of curvature and refraction become appreciable, and the general precision for which the level is designed and manufactured tends to become insufficient. The less precise grades of leveling may frequently be accomplished more quickly by using much longer sights and target rods, and by using a sensitive level of high magnifying power, if it is available.

Where a target is used on long sights, it should be set three or more times and the average reading taken after casting out those readings which are evidently erroneous. The bisection target is particularly useful on long sights where the cross-hair would cover the line between colors on the ordinary target It consists of a horizontal white line of varying width on a black background. The diamond-shaped target (Fig. 69) with points at the sides is also useful in taking long sights.

When refraction is large and when fairly precise results are desired, it is best not to take sights exceeding 100 feet if the line of sight passes within two feet of the ground or water.

83. Probable Error in Setting Level Target. It is occasionally desirable, in reciprocal leveling, for example, to set a target on a rod several times, and to use the mean value of the several readings, after casting out any reading which is evidently erroneous.

If ± 0.03 foot is the approximate probable error of a single reading, and if nine readings are taken, then the approximate probable error of the mean of the average value is approximately $\pm 0.03/\sqrt{9}$ or ± 0.01 foot. While the theory of probability, as explained on page 30, is not entirely applicable to such a small number of observations, yet this example will indicate the improved precision obtained from the repeated readings. This repetition, of course, also tends to eliminate mistakes.

84. Reciprocal Leveling with One Level. *Reciprocal leveling* is the process of determining the difference in elevation between two points more than several hundred feet apart where it is not possible to set the level approximately midway between the points. It is commonly used in leveling across bodies of water or across deep gullies, canyons, or ravines.

In Fig. 84, the level is first set up near A and readings AA_1 and BB_2 taken, and their difference computed. The level is then set up near B and readings BB_1 and AA_2 taken, and their difference computed. The mean of the two differences gives the true difference in elevation between A and B, if we assume that the refraction has remained constant. All errors of curvature and of adjustment of the instrument are thus eliminated, and there remains only the error due to inequality of B_2B_3 and A_2A_3 if refraction has changed.

It will be noted that this is the *two peg* procedure described in § 117. The same CAUTION there mentioned must be observed, but here the combined error of adjustment, curvature, and refraction should be less than the differences in elevation of the stakes or pegs.

Carefully conducted reciprocal leveling is as precise as ordinary differential leveling, and in rough country it may be accomplished more quickly.

Refraction will be nearly constant if the temperature of the air remains nearly constant. It is desirable, therefore, when only one level is used, as described in this section, to choose constant temperature conditions such as are available on a cloudy day or in the middle of a sunny afternoon. Refraction across and near to water is apt to be large and variable, while across a ravine it is apt to be small and nearly constant.

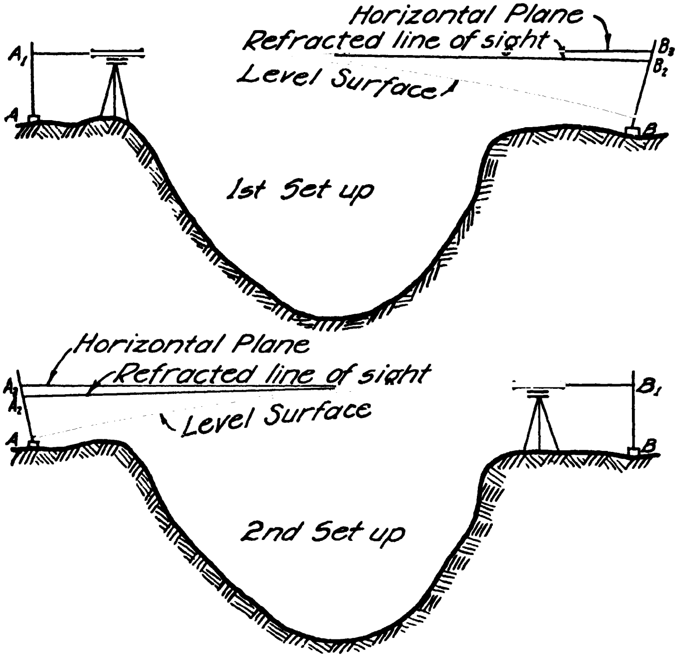


Fig. 84. Reciprocal Leveling.

85. Reciprocal Leveling with Two Instruments. If it is expected that refraction will change considerably between the times of two set-ups and if precise results are required, a more elaborate procedure is needed.

Two levels are required and simultaneous readings are taken with one level near each point. The levels, preferably of about the same order of sensitivity, are then interchanged and another set of simultaneous readings is taken. Two differences in elevation are figured, as in the preceding section, one difference for each instrument as though that instrument were the only one being used. The mean of the two differences is free from error due to change in refraction and of course free from all other errors mentioned under the foregoing section where only one level is used.

86. Customary Limits of Error. The formula most often used to express the maximum error permissible in engineering leveling is

(86) Permissible error $= 0.05\sqrt{D}$,

where the error is in feet and D is in miles. Expressed in words, the permissible error is equal to 0.05 foot times the square root of the length of the line of levels in miles. Such results may be obtained by any competent levelman in smooth or rolling country where 200- to 300-foot sights are possible. The engineers' level and the self-reading rod are used with the usual precautions of solid turning points, rod readings to hundredths of a foot, roughly equalized sums of backsight and foresight distances, and so on.

While the preceding precision is not difficult to attain, yet it is greater than is necessary for many types of engineering work. A slightly more rapid or cheaper type of leveling is increasingly permitted, especially in rougher country, allowing

(86a) Permissible error $= 0.10\sqrt{D}$.

Such levels would be precise enough for many types of engineering construction such as highway work.

For precise levels, usually governmental work, a common specification is

(86b) Permissible error $= 0.02\sqrt{D}$.

These more precise results usually require a precise level and rod and the many precautions mentioned in works on higher surveying. It is seldom that such precision is required for engineering purposes. It is best accomplished by men experienced in precise leveling.

If mistakes are eliminated, most of the errors in leveling are accidental and therefore tend to compensate. As explained on page 32, the theory of least squares shows that they will not exactly compensate or balance, but that uncompensated errors will tend to remain equal in number to the square root of the number of opportunities for error. In ordinary leveling, there will be an opportunity for error on each backsight and on each foresight. If there are 12 set-ups per mile there will be 24 opportunities for error and there would probably remain uncompensated about $\sqrt{24}$ times the value of a single error (say 0.01 foot), or about 0.05 foot per mile, as shown in (86) above. Results of actual leveling bear out this theory.

Formulas 86, 86a, and 86b are applicable where sights averaging between 200 and 300 feet may be obtained. In hilly country more set-ups per mile are necessary, and the added opportunities for error require a correspondingly more liberal allowance of error. Wherever possible, levels are therefore run along railways, highways, canals, valleys, and so on, where the work can be done more easily and more precisely.

It must be remembered that the foregoing formulas give *maximum* permissible errors and that the results usually obtained will be somewhat more accurate.

87. Adjustment of Level Circuits. Where levels are run between bench marks of precisely known elevation, or where the work closes back on the B.M. at the beginning of the survey, there will normally be an error of closure of the level circuit. The first and final elevations are correct and the intermediate elevations, which will be in error to greater or less degree, should be corrected in *direct proportion* to the length of levels run up to the respective intermediate points (not in proportion to the square root of the distance run).

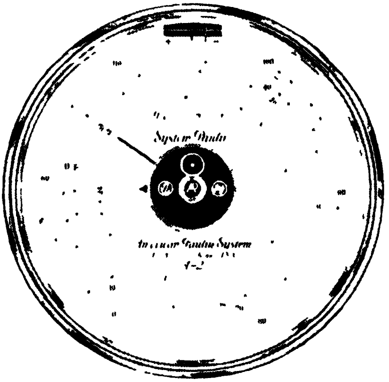
When levels are run forward and backward between two B.Ms., the mean difference is obviously the correct one to use.

More complicated cases of adjustment, where several lines of levels or level circuits meet on common points, are treated in books on higher surveying.

88. Trigonometric Leveling. As described in § 105, the transit may be used for a fairly precise determination of vertical angles which, together with the slope or horizontal distance, constitute the data from which the difference in elevation between two points may be computed. The distances may be obtained with a tape, by stadia, or by triangulation. This is further explained in § 157.

89. Leveling with Barometer. Approximate differences in elevation, correct to within 10 to 25 feet, may be quickly and cheaply obtained with a barometer, an instrument which measures atmospheric pressure. Such pressure is greater at low elevations than at high ones.

A mercury barometer similar to those used in physics laboratories cannot be used because it is awkward to handle and to carry. The *aneroid barometer* of Fig. 89 is used in surveying. Changes in pressure are registered by a hand traveling around a dial. The dial is graduated to feet of elevation, or inches of mercury (corresponding to a mercury barometer); preferably, it should have both graduations. The word *compensated* occurring on the face of most aneroids means that temperature changes within the instrument will not affect its results. However, correction must be made for differing air temperatures outside of the instrument because the temperature of the air affects its weight and pressure.



Courtesy of the American Paulin System

FIG. 89. ANEROID BAROMETER.

The derivation of formulas and the procedure for securing the most precise results possible with the aneroid are given in books on higher surveying.*

If approximate elevations with errors on the order of 10 to 25 feet will suffice, a relatively simple method may be used. The instrument is held at a point whose elevation is known or assumed and the pointer is read, or the dial rotated until he known or assumed reading of the elevation is under the pointer. The latter procedure is not theoretically correct since the spaces between elevation graduations are not equal, but it is sufficiently precise for most work, especially if the elevations are within a few hundred feet of one another. The aneroid is then taken to the points of unknown elevation and the pointer is read. The difference in readings (or the actual dial reading providing the dial was shifted to correspond with the elevation at the initial point) corrected for temperature and pressure changes as explained below, will give the desired elevations.

---

* See Rubey, *Route Surveys*, Macmillan, 1938, p. 11.

The atmospheric pressure at a given point ordinarily varies periodically throughout 24 hours, with two low and two high pressures that differ by the equivalent of some 50 feet of elevation. Changing weather conditions introduce other differences of pressure equivalent to several hundred feet of elevation, occasionally within a few hours. Both of these changes in pressure may be eliminated from the results by reading every hour or so a second barometer at a constant elevation in the vicinity of the moving barometer. A chart is drawn of the changes in pressure at the fixed station and the readings of the moving barometer are corrected accordingly. A good check is had when the moving barometer is read at different times on the same point or is read on points of known elevation. The barograph is a 7-day automatic recording aneroid, and is excellent for the fixed station instrument. Many of these are available since they are used for weather prediction. It is best not to attempt barometric work when the barometric pressures are varying rapidly.

For all but the least precise work, the air temperatures must be taken at each reading, and corresponding corrections taken from Table VII, or they may be computed by the formula:

$$\text{(89)} \quad \text{Temperature correction} = \frac{t_1 + t_2 - 100^\circ}{1000} \text{ approximately}$$

(in % of the difference of aneroid readings)

where all temperatures are Fahrenheit. The aneroid is a delicate instrument and most makes are subject to sudden partial derangement. It should be kept in the case and not exposed to the sun or to rapid temperature changes. It should be held horizontally while it is being read, and after a station is reached should be lightly tapped with a pencil and allowed to adjust itself for a few minutes before it is read.

Barometric elevations are only approximate, but they are quickly and easily taken. They are used chiefly in rough country when atmospheric conditions are relatively stable, and for work where elevations precise to within 10 feet to 25 feet are sufficient. Such large errors limit or prohibit their use in flat country.

PROBLEMS

1. (a) After the instrument has been leveled, a reading of 5.67 feet is made on a rod 300 feet from the instrument. The bubble is then moved out of center through 5 divisions of the bubble tube, and the rod reading is observed to be 5.82 feet. What is the curvature of the bubble tube? This curvature should be expressed in two ways: in seconds of angle subtended at the center of the circle by one division of the tube, and as a radius, 1 division being 0.1 inch. *Ans.* 20″; 83 feet

(b) The same as (a) except that the rod readings are 5.93 and 5.64 with a bubble movement of 3 divisions.

(c) The same as (a) except that the rod readings are 4.23 and 4.17 with a bubble movement of 4 divisions.

(d) The same as (a) except that the rod readings are 6.13 and 6.20 with a bubble movement of 5 divisions.

(e) The same as (a) except that the rod readings are 11.19 and 11.44 with a bubble movement of 5 divisions.

(f) The same as (a) except that the rod readings are 7.41 and 7.23 with a bubble movement of 6 divisions.

2. (a) Fill in the elevations and H.Is. in the following set of field notes. Check B.M.$_2$ by adding algebraically to B.M.$_1$, the difference between the sum of the backsights and the sum of the foresights. Answers are in italic type.

STATION	B.S. OR +	H.I. OR ⊼	F.S. OR −	ELEVATION
B.M.$_1$	0.13	*743.41*		743.28
T.P.$_1$	4.62	*736.55*	11.48	*731.93*
T.P.$_2$	1.18	*728.27*	9.46	*727.09*
T.P.$_3$	3.67	*719.53*	12.41	*715.86*
T.P.$_4$	2.76	*712.10*	10.19	*709.34*
B.M.$_2$			6.55	*705.55*
	12.36		*50.09*	*743.28*
			12.36	*705.55*
		Check	*37.73*	*37.73*

(b) B.M.$_1$, 343.72. Backsights: 0.22, 4.31, 7 93, 2.44, and 6.13. Foresights: 12.28, 9.46, 11.32, 7.78, and 8.37.

(c) B.M.$_1$, 843.38. Backsights: 11.18, 9.63, 12.14, 10.06, and 8.72 Foresights: 0.04, 1.22, 0.87, 2.35, and 0.89.

(d) B.M.$_1$, 113.64. Backsights: 1.18, 0 73, 1.65, 0.13, and 0.93. Foresights: 9.87, 12.42, 11.37, 12.16, and 10 17.

3. Referring to the following tabulated notes, let us suppose that the level is first set up midway between *A* and *B*, which are 300 feet apart It is afterward set up so that the eyepiece is over *B*. With the bubbles centered, is the line of sight inclined upward or downward, and how much vertical error, in feet, exists in a horizontal distance of 300 feet? What should be the reading on *A*, during the second set-up, to make the line of sight horizontal? This is the first "two-peg test" of item 7, § 117. It will be necessary either to read about the two-peg test in the aforesaid section, or to postpone working this problem until this section has been studied. Each horizontal line of the tabulation is a separate problem and only one is to be solved.

	Level Midway between *A* and *B*		Level near *B*	
	Rod Readings on		Rod Readings on	
	A	*B*	*A*	*B*
(a)	6.28	4 13	5.92	3.85
(b)	6.64	5.11	5.80	4.18
(c)	4.92	4.17	5 21	4.54
(d)	6.60	5 33	6.98	5.66
(e)	5.70	3.96	5.47	3.58
(f)	3.75	3 38	5.82	5.41

Ans. 3. (a) Downward 0.08 foot Rod reading = 6.00

4. In Problem 3, assume that the level is first set up near *A* instead of at the midpoint, and answer the same questions. This refers to the second "two-peg test" of item 7, § 117. Here again it will be necessary to study the two-peg test or to postpone solution of this problem. Observe the "Caution" which is given for this second method.

5. (a) Fill in all of the elevations and H.Is. for the following profile notes. Check elevation of B.M.$_2$ by algebraic summation of backsights and foresights, using only the foresights taken on T.Ps. and B.Ms. Answers are shown in italic type. Note that the rod readings which are taken on the ground are only read and recorded to tenths of feet.

Station	B.S. or +	H I. or ⊼	F.S. or −	Elevation	
				Ground	T.P. & B.M.
B M.$_1$	0.91	*747.73*			746.82
T P.$_1$	1.13	*738.47*	10.39		*737.34*
1			6.4	*732.1*	
2			8.2	*730.8*	
3			7.6	*730.9*	
$+28^{3}$			5.1	*733.4*	
4			10.3	*728.2*	
$+82^{1}$			10.4	*728.1*	
T P.$_2$	2.37	*729.16*	11.68		*726 79*
5			1.3	*727.9*	
6			3.4	*725.8*	
7			5 8	*723.4*	
$+48^{7}$			6.4	*722.8*	
T P.$_3$	3.64	*725.51*	7.29		*721.87*
B M.$_2$			5 91		*719.60*
	8.05		*35.27* *8.05*		*746.82* *719.60*
			27.22	*Check*	*27.22*

(b), (c), (d), (e), and (f) are the same as (a) except that the elevation of the initial B.M.$_1$ is changed as follows: (b) 100.00; (c) 843.71; (d) 138.49; (e) 467.43; and (f) 694.63.

6. Reciprocal levels are taken with one level as below:

Distance between *A* and *B*	Instrument at *A* Average Reading on		Instrument at *B* Average Reading on	
	A	*B*	*B*	*A*
(a) 2000′	4.79	6.87	5.43	3.15
(b) 2500′	3.41	5.28	6.13	4.33
(c) 2800′	5.82	7.96	4.84	2.82
(d) 2000′	6.71	9.01	5.23	3.10
(e) 2500′	4.03	6.01	5.24	3.34
(f) 2800′	6.28	8.38	4.97	3.07

If we assume constant refraction, what is the true difference of elevation? the level is out of adjustment, does the line of sight point downward or ward, and how much? *Ans.* 2.18 feet, downward 0.1 foot

If we assume that two levels are used and that the refraction varies, find e true difference in elevation. Use (a) and (d) for one complete set. e (b) and (e) for another complete set. Use (c) and (f) for another comte set. *Ans.* 2.197 feet

7. (a) At a certain level set-up, a 200-foot backsight of 6.327 and a 600-foot foresight of 4.181 are taken on two stakes. What is the true difference in elevation of the two stakes, allowing for curvature and the usual refraction? *Ans.* 2.152 feet

(b) The same, except 100-foot B.S. of 8.194 and 1000-foot F.S. of 7.428.

(c) The same, except 300-foot B.S. of 2.893 and 800-foot F.S. of 6.167.

(d) The same, except 800-foot B.S. of 11.321 and 200-foot F.S. of 5.131.

(e) The same, except 600-foot B.S. of 9.815 and 100-foot F.S. of 2.182.

(f) The same, except 1000-foot B.S. of 3.472 and 50-foot F.S. of 12.435.

8. Given three level rods graduated as follows: rod A to quarter inches, rod B to eighth inches, rod C to hundredths of a foot.

Design target verniers for these three rods:

(a) With least count of 1/32 inch for rod A.

Ans. 8 spaces on vernier, each space 7/32″ wide.

(b) With least count of 1/64 inch for rod A.

(c) With least count of 1/32 inch for rod B.

(d) With least count of 1/64 inch for rod B.

(e) With least count of 0.002 foot for rod C.

(f) With least count of 0.0005 foot for rod C.

9. (a) In 16,200 feet of levels, with the average length of sight 200 feet, the probable error of the readings is ± 0.005 foot each. What is the total probable error? *Ans.* (a) ± 0.045 foot

(b) The same, except 20,000 feet and ± 0.01 foot.

(c) The same, except 12,800 feet and ± 0.003 foot.

(d) The same, except 9800 feet and ± 0.004 foot.

(e) The same, except 5000 feet and ± 0.015 foot.

(f) The same, except 3200 feet and ± 0.02 foot.

CHAPTER VI

THE TRANSIT

90. General Uses. The engineers' transit (Figs. 90 and 90a) is used to measure both vertical and horizontal angles. Distances may be measured by means of the stadia hairs, described in Chapter IX. The transit has a magnetic needle and may be used as a compass. The results obtained in leveling with a transit may be nearly as precise as those obtained with the engineers' level, although the transit is not so well adapted to this purpose.

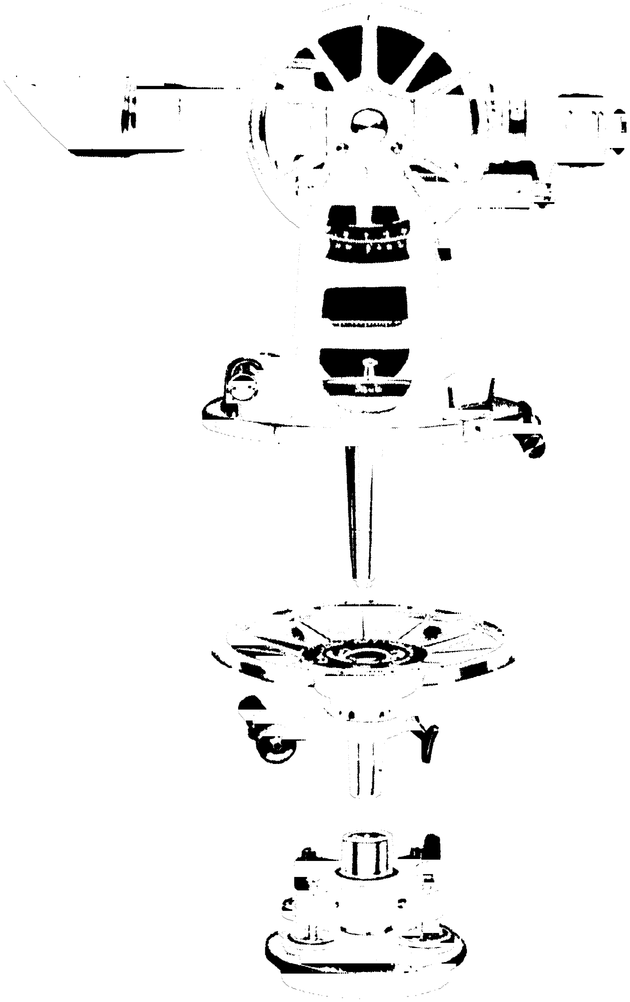
The transit may therefore be termed the universal surveying instrument, since those uses mentioned comprise almost the entire range of surveying. It is used for mapping, for astronomic observation, for laying out new work and for setting construction stakes. It is the one essential surveying instrument for modern practice, the other instruments serving only special purposes. The young engineer, surveyor, or engineering firm should first acquire a transit; then other special instruments may be added as occasion requires.

A very few European theodolites and transits have been recently used in this country, but it is probable that their extended use will develop slowly, if at all. They offer various refinements and a more complex construction. These are not likely to appeal to the American engineer, who usually engages only temporarily in survey work and consequently prefers the simpler instruments.

91. Transit Circle Graduations and Verniers. The horizontal and vertical circles used in angular measurement, as described later in this chapter, are similar in construction. Only the horizontal circle (*the horizontal limb*) will be described here Figure 91 shows the usual graduations on the horizontal circle of a transit reading to (least count of) one minute of arc. It will be noted that the graduations run from 0° to 360° both clockwise and counter-clockwise.

Courtesy of Keuffel and Esser Co

FIG. 90. ENGINEERS' TRANSIT



Courtesy of Keuffel and Esser Co.

FIG. 90a. ENGINEERS' TRANSIT DISASSEMBLED.

The more common double-direct verniers are shown in Fig. 91. The theory of these verniers is similar to that of the target verniers described on page 74. The upper vernier is by far the more commonly used on transits now in service. The main scale is divided into half-degrees, and 30 divisions on the vernier correspond to 29 divisions on the main scale; consequently the least count of the vernier is 1/30 of 1/2 degree or 1 minute. In other words, this vernier reads to the nearest minute of arc. The setting shown in the upper figure reads 17° + 25′ = 17° 25′ from left to right, and 342° 30′ + 05′ = 342° 35′ from right to left.

The lower vernier is used for slightly more precise work and, in accordance with increasing survey precision, is now standard on newly purchased transits. Here the main scale is divided into third-degrees, and 40 divisions on the vernier correspond to 39 divisions of the main scale; consequently the least count of the vernier is 1/40 of 1/3 degree or 1/2 minute. In other words, this vernier reads to the nearest half minute of angle. The figure reads 130° + 9′ 30″ = 130° 09′ 30″ from left to right, and 49° 40′ + 10′ 30″ = 49° 50′ 30″ from right to left.

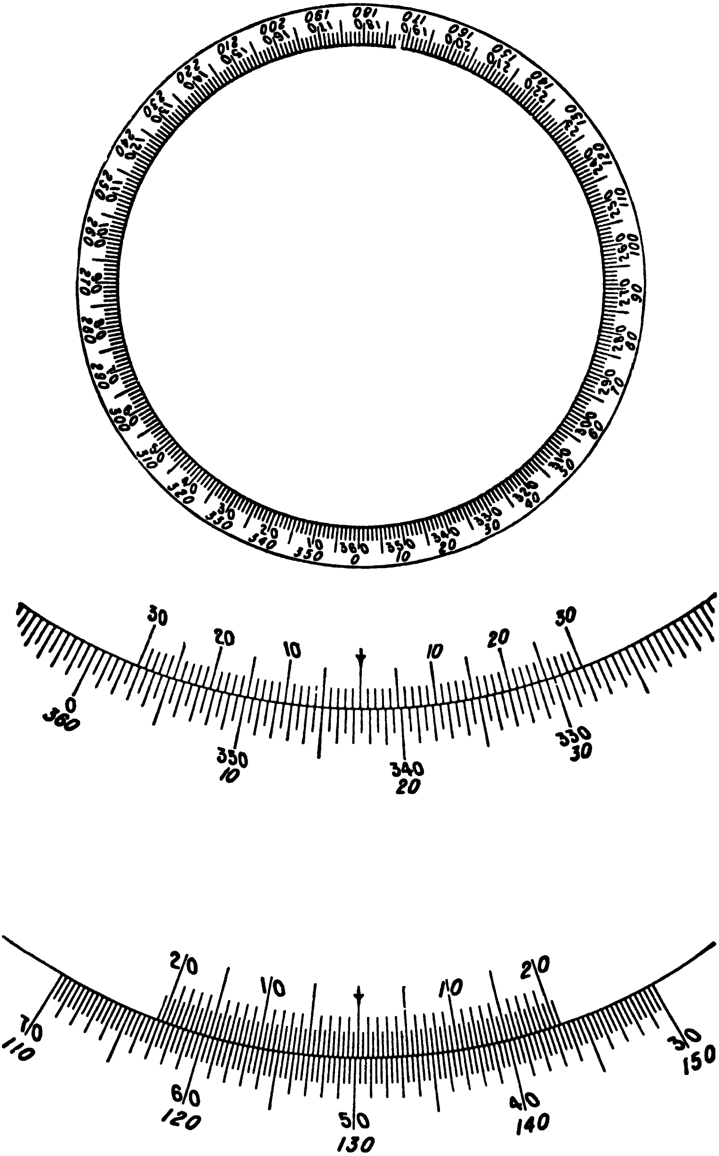
In rare instances, not frequent enough to warrant description here, one may encounter *single verniers*, *folded verniers*, and more precise verniers.

Vertical circles are generally graduated in quadrants from 0° to 90°, and are equipped with verniers reading to the nearest minute.

It is best first to read the zero line of the vernier along the main scale, as though no vernier existed, and to estimate the angle to within about 5 minutes. Then the coincidence of lines on the main scale and the vernier is closely read and compared with the angle previously estimated. This minimizes such blunders as reading the vernier on the wrong side of the zero, or omitting to add the fractional part of a degree which may occur on the main scale.

Always read along the main circle, in the direction in which the degrees are increasing, to the zero of the vernier; thence continue in the same direction along the vernier to the coincidence of the lines.

The check mentioned in the previous paragraph will also prevent a mistake in this respect. In deciding which pair of



Courtesy of Keuffel and Esser Co

FIG. 91. CIRCLE AND VERNIER GRADUATIONS.

lines coincide, the pairs of lines adjacent on either side should appear to fail to coincide by equal amounts in opposite directions. The eye should be in a vertical plane with the coinciding lines in order to prevent parallax, which results if the plates do not lie in exactly the same plane. Should two sets of lines appear to coincide equally well, then the reading lies half way between the two sets. A *reading glass* (small magnifying glass) is frequently used to determine precise coincidence of vernier lines. It both enables more precise reading and minimizes eye strain. Vernier *A* is the one nearer the eyepiece when the telescope is in normal position (bubble underneath). Vernier *B* is 180° around the horizontal circle.

92. Setting up the Transit. The transit is set up similarly to the level except that the two small plate bubbles make it unnecessary to rotate the transit about the vertical axis in order to get one bubble over opposite sets of foot screws.

The transit is first set up with the plumb bob approximately over the stake, tack, or point, and with the tripod head approximately level. The entire tripod is then lifted, without changing the relative position of the legs, and moved until the plumb bob hangs an inch or so above and within about 1/4 inch horizontally of the tack or point. The legs are then pressed firmly into the ground. Pressing a leg more deeply into the ground or moving it radially shifts the plumb bob in the direction of that leg without seriously changing the plate levels. A circular movement of a tripod leg alters the plate levels without greatly moving the plumb bob. Some of the suggestions regarding setting up the level (page 76) also apply to the transit.

The plumb bob is then lowered within about 1/8 inch of the point and exactly centered by loosening one footscrew of each pair and shifting the head of the transit with reference to the tripod. The previously loosened footscrews are then brought to a light uniform pressure or bearing on the plates.

The transit is turned about its vertical axis until the bubbles are over respective pairs of footscrews. First one bubble is brought approximately to the center, then the other bubble; and this is repeated with closer approximations for a time or two until finally both bubbles are centered.

93. Measuring Horizontal Angles. 1. Set up the transit, hing the plumb bob over the point, and level the instrument with the plate bubbles.

2. Set the vernier on 0° 00′ by using the *upper motion clamp* and the upper *slow motion or tangent screw.* The vernier and the horizontal circle will then rotate together on the lower motion.

3. Point the telescope along one line of the angle. The *lower motion clamp* and *slow motion or tangent screw* are used. The instrument is now pointing along one line of the angle and vernier *A* reads 0° 00′. Check vernier *A* after sighting and clamping along first line, to detect errors caused by turning wrong tangent screw.

4. Loosen the upper motion clamp and rotate the telescope horizontally, carrying the vernier with it, until the vertical cross-hair is approximately on the point defining the second line of the angle. The cross-hair is brought exactly on the point by clamping the upper motion and using the upper slow motion or tangent screw. Vernier *A* has now moved from its initial reading of 0° 00′ to a position reading the value of the angle in degrees and minutes. Check by reading vernier *B* independently.

The transit is leveled only at the beginning of the measurement and the foot screws are not touched during the measurement, even though the bubbles leave the central position.

If only one vernier is used, it is convenient to use the vernier under the eyepiece, usually marked vernier *A*, as the angle may be read without stepping around the instrument. Frequently both verniers are read to eliminate errors of eccentricity and incorrect placing of verniers (mentioned on page 142), and as a check. Should the readings disagree, either the plate graduations are incorrect, the verniers are incorrectly placed (not exactly 180° 00′ apart), eccentricity exists, or a mistake has been made.

Incorrect graduations of more than a few seconds are rare on the better grade of transits.

If the verniers are not exactly 180° 00′ apart, there will be a constant difference in vernier readings, seldom over a minute or two, regardless of the position of the verniers around the main circle. Correct angles are obtained by reading the same vernier, or by using the means of both vernier readings.

If eccentricity exists in a transit, the verniers will be 180° 00′ apart for certain vernier readings and perhaps a few minutes from 180° 00′ apart for other positions, the maximum discrepancy occurring when the verniers are at right angles to the position in which they agree. Correct angles are obtained by averaging the vernier readings.

To avoid *lost motion*, the tangent or slow motion screws should always be brought to their final position with a small clockwise turning, so that they are acting against compression in the opposing springs. The tangent screws should not be operated near the ends of their runs, where they tend to bind.

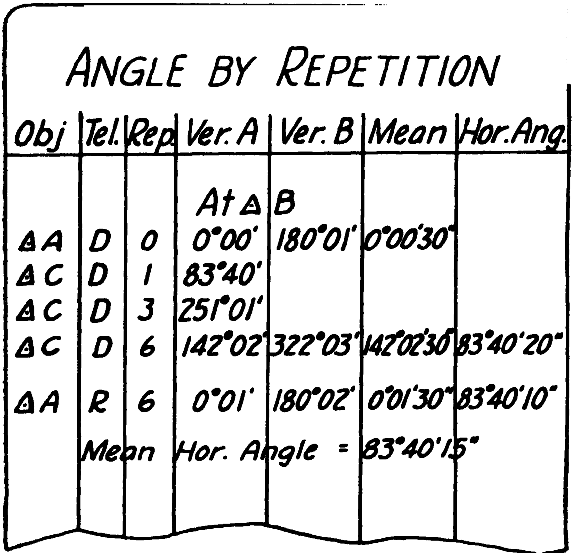
94. Measuring Horizontal Angles by Repetition. When the above process is repeated, that is, doubled without returning the plates to zero but starting the second measurement with the plates clamped on one times the value of the angle; then at the end of the second measurement the plates will read twice the value of the angle. This reading, divided by two, will yield a more precise value of the angle with less chance of a mistake, providing half of the second reading is compared with the first reading. Such *doubling* is customary for all important angles. To eliminate adjustment errors, the telescope should be *reversed in altitude* between measurements; that is, one measurement should be made with the bubble up and the other with the bubble down.

Greater precision results from a set of four to twelve repetitions. Half of the set, two to six repetitions, will be made with the bubble down (*telescope normal*). The telescope is then *inverted* (bubble up), the angle reading recorded, the plate readings unchanged, and the other half of the repetitions made measuring the *explement* of the angle (360° − the angle). The plates should then read the initial setting, usually zero degrees.

Still greater precision is obtained by averaging several sets, such as are described above, and observing certain added minor precautions which are mentioned in books on higher surveying and in publications of the United States Coast and Geodetic Survey.

Inverting the telescope in altitude eliminates all errors of adjustment except that of the plate bubbles.

Repetitions are usually made clockwise except that some deflection angles (page 109) which are doubled must be measured counter-clockwise. Both *A* and *B* verniers are customarily read. The plate bubbles must be in adjustment and the plates should not be releveled during a set. Field notes for a repeated angle are shown in Fig. 94.



ANGLE BY REPETITION

Obj	Tel.	Rep	Ver. A	Ver. B	Mean	Hor. Ang.
			At △	B		
△A	D	0	0°00'	180°01'	0°00'30"	
△C	D	1	83°40'			
△C	D	3	251°01'			
△C	D	6	142°02'	322°03'	142°02'30"	83°40'20"
△A	R	6	0°01'	180°02'	0°01'30"	83°40'10"
	Mean		Hor. Angle	=	83°40'15"	

FIG. 94.

It is seldom necessary, in customary engineering practice, to repeat an angle except that nearly all instrumentmen double important angles (once with bubble up and once with bubble down) to avoid blunders, to secure slightly greater precision, and to eliminate errors of adjustment, as mentioned on pages 136 to 138, inclusive.

95. Laying Off an Angle by Repetition. It is not possible directly to lay off an angle by repetition. The somewhat indirect method used is illustrated by the following example.

To lay off an angle of 18° 30′ 20″ with a transit reading to the nearest minute, first lay off an angle of 18° 30′ by a single setting and establish a temporary tacked stake. Continue to repeat this angle as precisely as required. Assume that repetition determines 18° 29′ 40″, as the value of the angle to the temporary stake, precise to perhaps 10″. A new tacked stake must then be set a short distance from the temporary stake, by computing and measuring an offset, so as to increase the angle 40″. If the temporary stake is 600 feet from the transit it would be necessary to set the final stake 600 times the sine, tangent, or arc of 40″ or 0.12 ft. from the temporary stake.

96. Direct Angles or Angles to the Right. Direct angles are measured directly from one line to another, that is, what we would ordinarily describe as the angle between two lines. These angles may lie anywhere between 0° and 360°. In order to establish uniformity of practice and to avoid misunderstanding as to which of two possible angles was measured, it is customary to sight first on the backsight with the plates at 0° 00′ and then to measure the angle clockwise to the foresight, as in Fig. 96.

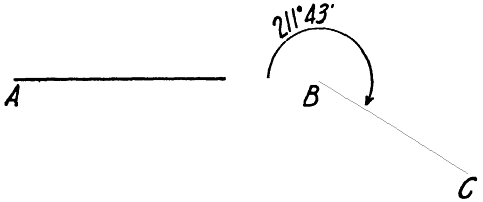
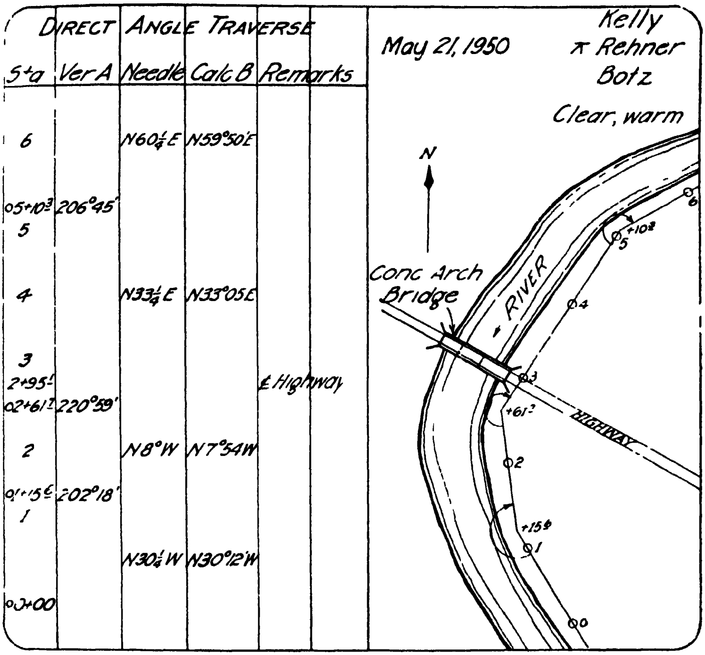


FIG. 96. DIRECT ANGLE, OR ANGLE TO RIGHT.

Direct angles are usually doubled by repetition, once with bubble down and once with bubble up, to give greater precision and to guard against errors of reading, slippage, and adjustment. Both verniers are ordinarily read, particularly when the angles are not doubled. Direct angles are commonly used in city and mine surveys. Examples of the field notes are given in Fig. 96a. The interior angles are frequently used on small closed traverses and in simple surveys where there is little opportunity for misinterpretation; for instance, it is the interior angle that is read where it is recorded in its proper place on a sketch rather than in a tabulation of data.



DIRECT	ANGLE	TRAVERSE		
Sta	Ver A	Needle	Calc B	Remarks
6		N60¼E	N59°50'E	
⊙ $5+10^{3}$	206°45'			
5				
4		N33¼E	N33°05'E	
3				
$2+95^{5}$				℄ Highway
⊙ $2+61^{7}$	220°59'			
2		N8°W	N7°54'W	
⊙ $1+15^{6}$	202°18'			
1				
		N30¼W	N30°12'W	
⊙ 0+00				

May 21, 1950

Kelly
⊼ Rehner
Botz

Clear, warm

N
Conc Arch Bridge
RIVER
HIGHWAY
6
+10³
5
4
3
+61⁷
2
+15⁶
1
0

FIG. 96a.

97. Deflection Angles. This is the older method of measuring horizontal angles, popular with railway and highway engineers and used in most traversing where only one angle is read at a point. The angle measured is that between the backsight produced and the line ahead, as shown in Fig. 97. Deflection angles vary from 0° to 180°, never exceeding 180°, and consequently must be designated as right (*R*) or left (*L*).

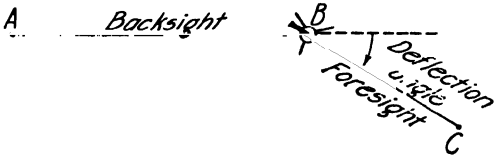
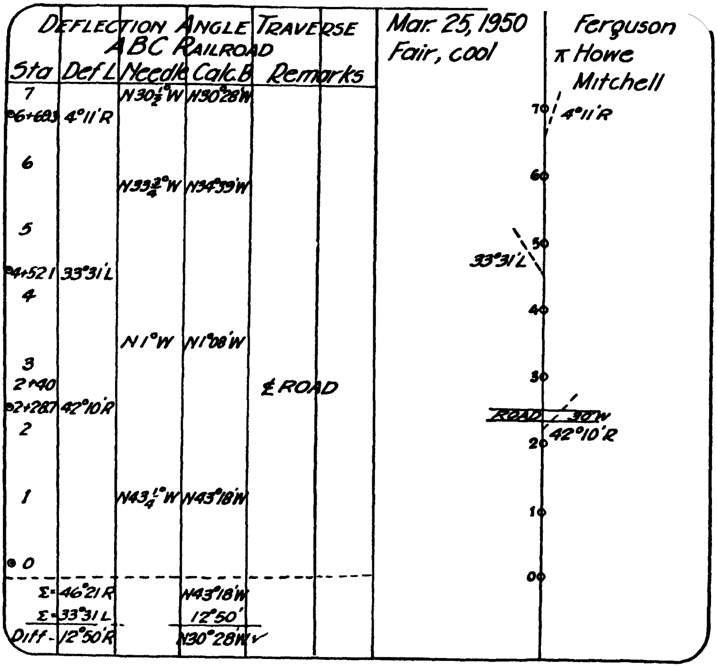


FIG. 97. DEFLECTION ANGLE.

The measurement of deflection angles with the transit is almost invariably repeated or *doubled* by the following process.



DEFLECTION ANGLE TRAVERSE
ABC RAILROAD

Sta	Def L	Needle	Calc. B	Remarks
7		N30½°W	N30°28'W	
⊙6+68.3	4°11'R			
6				
		N33¾°W	N34°39'W	
5				
⊙4+52.1	33°31'L			
4				
		N1°W	N1°08'W	
3				
2+40				℄ ROAD
⊙2+28.7	42°10'R			
2				
1		N43¼°W	N43°18'W	
⊙ 0				
Σ =	46°21'R		N43°18'W	
Σ =	33°31'L		12°50'	
Diff =	12°50'R		N30°28'W ✓	

Mar. 25, 1950
Fair, cool

Ferguson
⊼ Howe
Mitchell

7
4°11'R
6
5
33°31'L
4
3
ROAD N 30°W
2
42°10'R
1
0

FIG. 97a.

The transit is backsighted at A, with the bubble underneath the telescope and the vernier at 0° 00′; the telescope is plunged vertically and sighted at C by loosening and turning the upper motion; and the value of the deflection angle is read. The lower motion is then loosened, the telescope revolved horizontally and again backsighted at A, with the bubble now on top of the telescope and the plates reading one times the value of the angle instead of 0° 00′ as in the first backsight. The telescope is again plunged vertically and sighted at C with double the value of the angle appearing on the plates, and with the telescope in normal position, so that the line may be conveniently carried ahead.

One-half of the doubled value of the angle should agree with the previous single value within the least count of the instrument. This process gives greater precision, eliminates possible mistakes, and also eliminates all errors of adjustment mentioned on pages 136 to 138 except of plate bubbles. Field notes are shown in Fig. 97a.

In ordinary practice as outlined above, only one vernier is read. The other vernier may be read as a check but is not recorded. A few engineers, for example those in the United States Coast and Geodetic Survey, read and record both verniers on both the single and doubled angles.

As a deflection angle survey proceeds, the bearing of each line is computed from the preceding line and is entered in the notes as a calculated bearing. This is checked by the needle reading, which is also recorded. The same check is applied somewhat less frequently to direct angle surveys.

98. Azimuth Angles. Azimuth angles are those measured clockwise to the line under consideration from some given reference direction. They may lie anywhere between 0° and 360°. For important surveys the reference direction is true north or true south. For example, using true south as 0°, a line *AB* having a true bearing of N 32° 14′ E would have an azimuth of 212° 14′, and a line having a true bearing of N 63° 57′ E would have an azimuth of 243° 57′, as shown in Fig. 98. It should be noted also that the azimuth of *BA* (32° 14′) differs from that of *AB* by exactly 180° 00′.

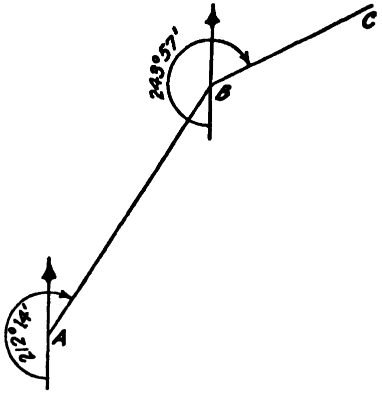


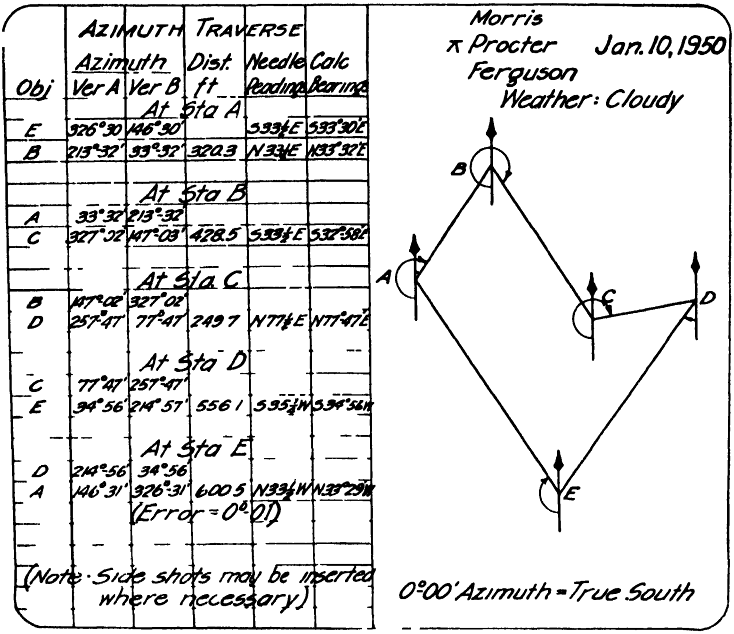
FIG. 98. AZIMUTH.

For less important surveys, magnetic north or magnetic south are convenient reference directions for the measurement of azimuth angles. The major direction of a survey or construction is rarely (and perhaps improperly) used, for example, along the major axis of a stadium. A note of the reference direction *must* be entered in the field notes in something of this form: 0° 00′ azimuth is true south.

The measurement of azimuth angles is perhaps the quickest and generally the most satisfactory method where several or more angles or directions are to be determined at one set-up of the transit.

Both verniers are read on important azimuths, since it is impossible to double azimuths and thus to avoid blunders, or otherwise to avoid small errors due to eccentricity.

99. Traversing by Azimuth. In Fig. 99, the transit is set up at *A* and sighted at *B* with the plates reading the true azimuth of *AB* (or any other known line and azimuth from *A*) All azimuths needed from *A* are then read on the plates. When the transit is moved to *B*, the plates are changed 180° to read the azimuth of *BA* and a backsight is taken on *A* with the lower motion. When the transit is turned horizontally to sight at *C*, using the upper motion, the azimuth of *BC* is read on the plates. This procedure is continued around the traverse. A check is possible whenever a plate azimuth can be read on a



AZIMUTH TRAVERSE

Obj	Azimuth Ver A	Azimuth Ver B	Dist. ft	Needle Reading	Calc Bearing
		At Sta A			
E	326°30'	146°30'		S33½E	S33°30'E
B	213°32'	33°32'	320.3	N33½E	N33°32'E
		At Sta B			
A	33°32'	213°32'			
C	327°02'	147°03'	428.5	S33½E	S32°58'E
		At Sta C			
B	147°02'	327°02'			
D	257°47'	77°47'	249.7	N77½E	N77°47'E
		At Sta D			
C	77°47'	257°47'			
E	34°56'	214°57'	556.1	S35¼W	S34°56'W
		At Sta E			
D	214°56'	34°56'			
A	146°31'	326°31'	600.5	N33½W	N33°29'W
		(Error = 0°01')			

(Note · Side shots may be inserted where necessary)

Morris
π Procter Jan. 10, 1950
Ferguson
Weather: Cloudy

0°00' Azimuth = True South

Fig. 99

line whose azimuth is already known, for example at E, providing the azimuth AE was previously read from A. The telescope is always in normal position (bubble down).

A less used method of traversing, when the surveyor is moving to B, is to leave the azimuth of AB clamped on the plates Set up at B, backsight at A with the telescope inverted (bubble up) and the azimuth AB on the plates. Plunge the instrument vertically and it will now be pointing along AB produced (bubble down). Turn to C, read the azimuth BC on the plates, and so on around the traverse to a final check as in the preceding method. In this second method, the backsight azimuths are omitted from the field notes (except that AE is recorded for final checking).

It is desirable to read and record both vernier readings and the needle reading on each important azimuth, since otherwise there is no check other than the final one previously mentioned. In the second method, the final check is no assurance that intermediate azimuths are read or recorded correctly.

Further discussion of transit traversing occurs in § 199.

100. Comparison of Methods of Designating and Measuring Angles with Transit. The table on the opposite page summarizes the methods of measuring angles with a transit.

101. Sight Points. If the point sighted at is near the transit and the stake can be seen, a pencil may be held on the tack, or a nail or taping pin may be stuck immediately behind the tack. These objects are more clearly seen if they are stuck through a card or a piece of white paper. If the stake is too low to be seen, a plumb bob may be held over the tack and the transit sighted on the string. A card or celluloid target may be attached to the string to increase its visibility.

For longer sights where these smaller objects would not be clearly visible, a pointed wooden or steel *flag pole* one inch or less in diameter and six to ten feet long is used. The alternate feet are painted red and white, since this increases visibility against different colors of background. Pieces of different colored cheese cloth, one light and one dark, as for example white and red, may be tacked to the top of the flag pole in order to facilitate locating it on long sights or in brush and

TABLE B

Azimuths	Deflection Angles	Direct Angles
	Advantages	
The azimuth or direction of lines is directly available on the transit plates, without computation. A rapid and simple method where more than one angle is to be measured at a point. Azimuths may be checked mentally against the needle reading.	The doubled angle prevents any mistakes, eliminates most of the errors of adjustment, and gives greater precision.	The doubled angle prevents any mistakes, eliminates most of the errors of adjustment, and gives greater precision.
	Disadvantages	
No check against mistakes in reading or recording the azimuths except the rough needle check, or reading both verniers. Not so precise as doubled angles. The second method is subject to the error of adjustment because the line of sight does not produce a straight line when revolved vertically.	Occasional mistakes are made in recording Right or Left after the angle. It is necessary to calculate the true bearings before they can be compared with the needle readings. Difficult to record where several angles are measured at one point.	Necessary to calculate true bearings before they can be compared with the needle readings. Not so simple to record where several angles are measured at a point.
	Present and Future Usage	
Widely and increasingly used, especially in stadia surveys.	Widely but decreasingly used. Commonly used in railway, highway, and other line surveys.	Perhaps their usage is decreasing in ordinary work. The method is commonly used for precise results and in city and mine surveys.

umber. Waving the flag pole will enable the transitman first to locate it, after which it is held stationary in a vertical position, *or plumbed,* during the sight. The flag pole is also called *color pole* or *range pole*. The pole should not be injured through use as a crowbar, spear, javelin, or vaulting pole.

A lath or branch of a tree may be driven near the stake to be sighted at and a triangular piece of paper or card set on line in its split top. It is then unnecessary for a man to remain at the point to give a sight. By employing the plumb bob, the tip of the triangular paper or card may be set directly over the tack.

If the transit is set over the stake, a lath or branch of a tree with a triangular white card at the top may be placed on line several feet from the transit, the transitman alternately sighting at the card and moving it onto line. When the transitman moves ahead to the next stake his backsight is already set, and no flagman has been necessary.

A flag pole or stick is sometimes set in an inclined position over the stake, so that the plumb bob may be left hanging directly over the tack.

If the transit is kept sighted along a line for some time, it is helpful to raise and lower the telescope slightly, until some point, such as an easily recognized spot on a tree or building, is found exactly on the vertical cross-hair. This point will then serve to check the alignment of the transit and it may not be necessary to establish a sight as described above or to keep a man, called a *back flagman,* on the sight point.

It is desirable in all survey work that the light or sunshine illuminate the face of the flag pole, rod, and other sight since if they are in shadow, they will be more difficult to see. Sometimes the face of the rod or sight may be turned slightly away from the instrument so that the light will fall on it and so that it may yet be visible from the instrument. In dark foggy weather, at night, or underground, sights are taken by illuminating both the sight and the cross-hairs, as described in § 127. In any method of giving sights, it is desirable that the apparent thickness of the cross-hair should not entirely cover the sight.

It is difficult to observe a sight satisfactorily if the sun is shining into the outer or objective end of the telescope. In such a case, a hat or other object so held as to shade the objective lens will increase the visibility. The tubular sunshade should always be on the instrument when in use. It serves the foregoing purpose of shading the objective lens, prevents rain and dust from falling on the lens, and better balances the telescope as it rotates about the horizontal axis.

102. Signals. Some set of definite signaling to and from the transitman must be adopted, since much of the work involves long sights where calling to one another is impractical. The following are commonly used, supplemented by such special signals as may be needed:

Give Line. The flagman holds the flag horizontal over his head, across the line of sight, then brings it to a vertical position in front of him. For exact line, the top of the flag is waved back and forth across the line of sight.

Pick up Transit. The party chief extends both arms outward below his shoulders and raises them suddenly.

Move Point to Right or Left. The transitman's appropriate arm is extended level with the shoulder. A quick motion demands a considerable movement, a slow movement only a slight movement of the flag. For short sights, the palm of the hand may be turned in or out.

Plumb the Flag. The transitman extends one arm vertically and inclines it in the direction towards which the flag should be plumbed, as shown by the vertical cross-hair.

All Right. Both hands are extended horizontally and moved up and down.

103. Prolonging a Straight Line. Several methods of prolonging a straight line are shown in Fig. 103.

Single Sighting. The transit at B is backsighted at A and plunged vertically. If the line of sight is not perpendicular to the horizontal axis (see page 137) the point C_1 as lined in by the transit is not on a true straight line produced. When the transit is moved ahead and set up at C_1 and similarly manipulated, the error accumulates at D_1, and so on. This error may be minimized by backsighting with the bubble down on the first set-up, then backsighting with the bubble up at the second set-up, and so on alternately.

Double Sighting. This is the method in general use. At the first set-up at B the transit is backsighted on A with the bubble down, plunged vertically, and C_1 is set. The transit is then revolved horizontally and backsighted on A, plunged vertically, and C_2 is set (bubble down). C is set halfway between C_1 and C_2, truly on the straight line produced. This procedure is repeated at each new set-up. In addition to eliminating the adjustment error of single sighting, the second plunging assures the instrument man that no mistake has been made, such as the plates slipping during the plunging, or the tripod settling or being disturbed.

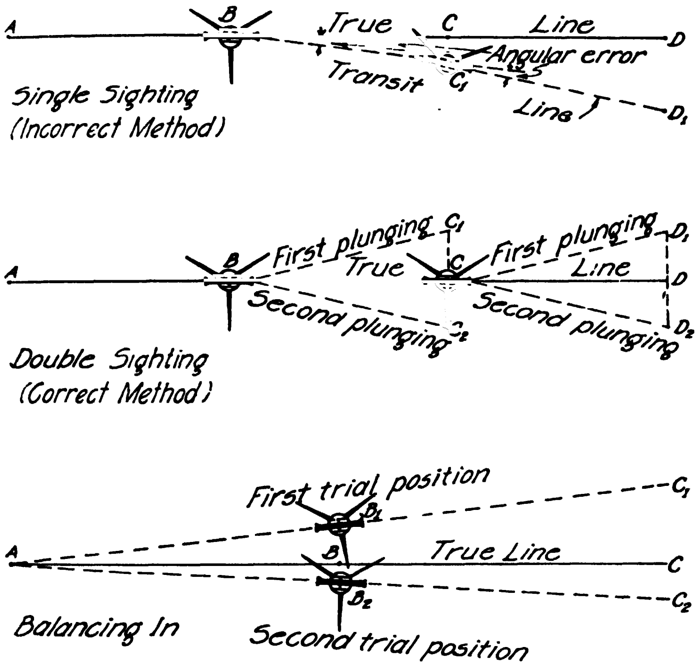


Fig. 103. Prolonging a Straight Line.

Balancing In. This is used where A and C are not intervisible and it is desired to establish B on the straight line between them. After single sighting with the transit approximately on line at B_1, the distance C_1C is estimated and the transit moved proportionately to B_2, which is nearer to, but not exactly on, the true straight line. The estimated error C_2C is less than C_1C. This process is repeated until the transit is sufficiently close to B and is then finally checked by double sighting. The final transit movement may be made by loosening the footscrews and shifting the head of the transit.

104. Passing Obstacles. The transit is generally available for passing obstacles but the same procedure may be less rapidly and less precisely accomplished with the tape only. The compass offers a less precise but more rapid method of passing obstacles by running the line to the obstacle and then setting up beyond the obstacle, without the use of a backsight.

Several transit methods of carrying alignment and distance around obstructions are shown in Fig. 104. In all cases the line is being run from left to right along the solid lines.

Equilateral Triangle. No computation of distance is necessary since the obstructed line BD is equal to the measured lines BC and CD. Sixty-degree transit angles are turned at B, C, and D.

Simple Triangulation (as for example in a river crossing). Angles are measured at A and A_1 (sometimes also at B as a check) and the distance AA_1 measured. AB is then computed.

Auxiliary Line. The angle α is a small one. AC is computed.

Offset Line. Transit set-ups are made at A, B, B_1, C_1, C, and D. No angle is turned at B_1, a straight line being produced to C_1 and D_1 after backsighting at A_1, which is set by measuring a right angle offset to A_1 from A. The right angle at C_1 is measured from A_1 or D_1. No angle is turned at C but the line CD is produced ahead by sighting at D, which has been previously set by right angle offset from D_1. The object of all of this procedure is to establish more precisely the direction ahead by long backsights (A_1B_1, A_1C_1, and CD) rather than to depend on short sights (BB_1, B_1C_1, and C_1C). The shorter sights lessen the precision of the direction so established. A_1 is set as far as possible from B_1, and similarly D_1 is set as far as possible from C_1.

Random Line. Here A and C are established on the ground but C cannot be seen from A. The straight line AB_1C_1 is run

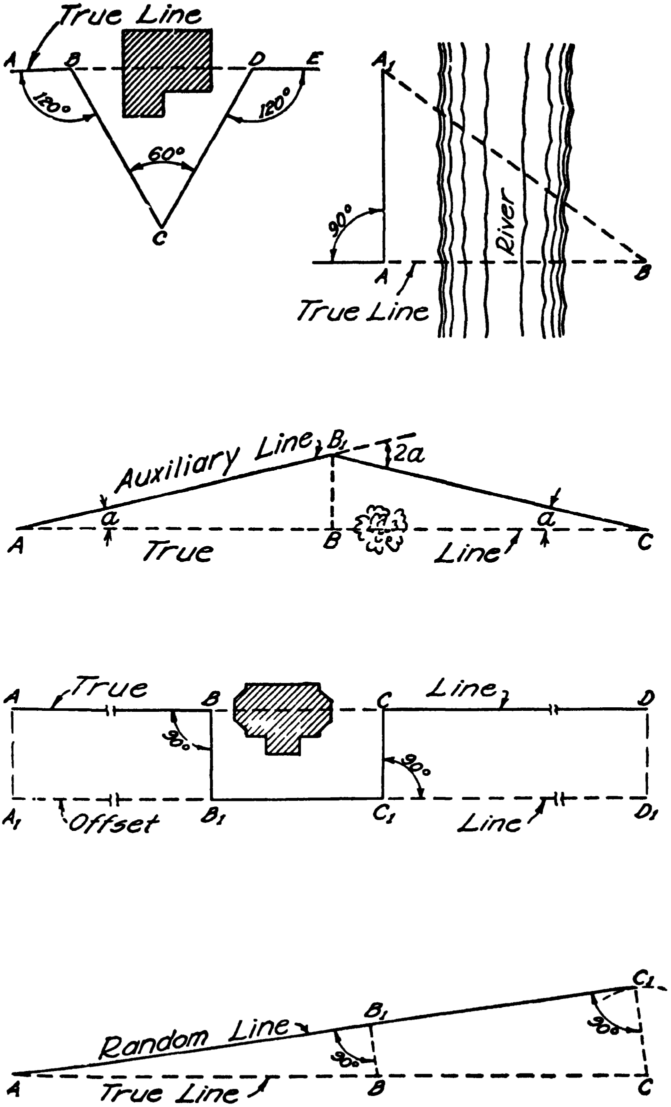


FIG. 104. PASSING OBSTACLES.

in a guessed or random direction towards the assumed position of C. The offset C_1C is measured, after running the random line, and any intermediate point on the random line, for example B_1, may be set over a computed proportional distance B_1B to establish B on the true line. The *swing offset* CC', is measured by swinging a tape from C as center and noting from the transit, say at A, when the shortest offset distance comes tangent to the line of sight. Other methods of measuring the right-angle offsets may be used.

105. Measuring Vertical Angles. Vertical angles are measured in a vertical plane in a manner somewhat similar to horizontal angles except that the transit is not constructed so as to permit repeating the angle, and except that the vertical angle is measured from a horizontal plane in every case.

Since the angle is measured from a horizontal plane, it may be either plus (upward) or minus (downward), and the plus or minus sign must always be recorded in the notes. It is not permissible to omit the plus sign since the person using the notes cannot decide whether the omission was intentional or whether the recorder perhaps neglected to show a minus sign.

Generally the vertical angle is required of a line between two stakes or points over one of which the transit is set up. Here a level rod, stadia rod, or flag pole is held on the stake under the transit and, by leaning the rod outward, a target is set at the height of the horizontal axis. If a target is not available, a piece of paper or card may be attached to the rod with a thumb tack or an elastic band. If the second point is close enough to the transit to be read directly, the target or card may be omitted. Here the transitman may measure the height from the stake under his transit to the horizontal axis with a rod or tape. It is possible to mark 3′, 4′, and 5′ and the tenths of feet downward on one leg of the transit so that the plumb bob may be swung against the leg, and so that the reading on the leg is the height of instrument (H.I.), as shown in Fig. 105. This is a useful device, as the transitman is frequently required to determine the height of instrument without recalling the rodman to the transit.

The rod is then held on the second stake and the center horizontal cross-hair is brought onto the target or otherwise determined H.I. The line of sight of the transit is now parallel to a line connecting the two points and consequently the vertical angle read is equal to the vertical angle from a horizontal plane through the first point, to the second point, as shown in Fig. 105a.

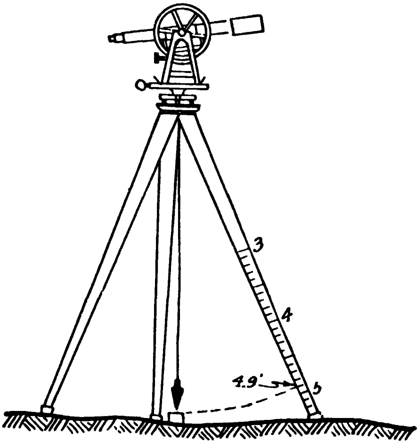


FIG. 105. METHOD OF MEASURING THE H.I.

Of course, not infrequently, the vertical angle to the point itself may be measured.

There are three methods of measuring vertical angles, all appropriate under varying circumstances and requirements:

1. Closely approximate vertical angles are obtained by sighting the center horizontal cross-hair at the distant point (more often at the H.I. on the rod above it) and reading the vertical circle, after it is certain that the plate bubble is centered. The plate bubble is rigidly attached to the frame holding the vernier and they move together. That plate bubble which is parallel to the telescope, therefore, determines the horizontal plane from which the vertical angle is measured. Unless this bubble is in the center, in adjustment, and sensitive, the measured vertical angle will reflect these usually small errors. The adjustment referred to is that mentioned on page 111, which places

the vernier reading at 0° 00′ when the line of sight is horizontal and the plate bubble is in the center.

On some transits, and on most modern plane tables, an extra bubble of about the same sensitiveness as the telescope bubble is firmly attached to the vernier. This bubble and the vernier may be moved together with reference to the plates and standards by means of a slow-motion screw. The vernier is adjusted to read 0° 00′ when the vernier bubble is centered and the line of sight is horizontal. In reading vertical angles, the transit is sighted at the distant point and the vernier bubble is brought to the center with the slow motion screw. It will be noted that the plate and telescope bubbles need not be exactly centered, sensitive, or in adjustment where a properly adjusted vernier bubble is used.

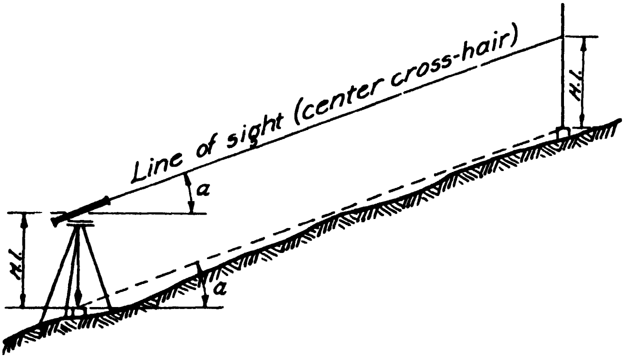


Fig. 105a.

2. When no attached vernier bubble is available, an equally precise method is to level the plates with the plate bubbles, point the telescope to the distant point and read the vertical angle, then raise or lower the telescope until the attached telescope bubble is centered, and again read the vernier. It is possible that an *index correction* of a few minutes, plus or minus, will be read instead of 0° 00′, and this correction must be added to, or subtracted from, the observed vertical angle. If the index correction is due to a misplaced vernier, it will be a constant amount for all vertical angles. If the correction is due to incorrect adjustment of plate bubbles, it will vary as

the transit is rotated about the vertical axis. It is essential that the line of sight be horizontal when the telescope bubble is centered, as described in the two-peg test on page 139. It will be noted that lack of sensitivity or centering of the plate bubble does not enter into this measurement of vertical angle.

3. The surest and most precise method is to level* the transit with the telescope or plate bubbles and to read the vertical circle with the center horizontal cross-hair sighted at the distant point. Then, without releveling, plunge the transit vertically and turn it through 180° horizontally, and again read the vertical circle with the transit sighted at the distant point. The mean of the two readings, one taken with the telescope in its normal position and one with the telescope inverted, is the true value of the vertical angle. This process eliminates all errors of adjustment and minimizes possibility of mistakes since the reading is taken twice. It is suggested that the reader, after studying the adjustments of the transit on pages 135 to 142, follow through the line of reasoning which leads to the conclusion that all errors of adjustment are thus eliminated.

It should be noted in measuring vertical angles that the graduated circle moves as the telescope is raised or lowered, and the vernier remains stationary. This contrasts with the measurement of horizontal angles, where the vernier moves with the telescope as it is rotated horizontally.

106. Checking Angles by Magnetic Needle. In the various forms of field notes shown in Figs. 96a, 97a, 99, 201a, and 204a, it will be observed that provision is made for reading and recording the needle reading. The deflection and direct angle notes contain a column for calculated bearings which are computed to the nearest minute from some line of known bearing, by means of the successive transit angles. Should these calculated bearings not agree, at least roughly, with the needle readings, the discrepancy must be rectified before leaving the set-up.

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* More definitely, this means to make the vertical axis of the transit vertical by one of the following two processes.

1 Leveling the transit with the plate bubbles, reversing 180° in azimuth and bringing the plate bubbles half way back to center with the footscrews.

2. Leveling the transit approximately with the plate bubbles; then centering the telescope bubble, reversing the transit 180° in azimuth, and bringing the telescope bubble half way back to center with the footscrews. The procedure is repeated over the other set of footscrews. This is the more precise method, since the telescope bubble is more sensitive than the plate bubbles.

It will be noted that the bubbles need not be in adjustment. If out of adjustment, they will not be in the center when the vertical axis is vertical; rather they will remain exactly the same distance off center throughout a complete revolution in azimuth of the horizontal plates.

Azimuth notes may or may not contain a column for needle readings, but the needle is occasionally read and compared with the observed azimuth. This computation may be made mentally; for example, an azimuth of 207° 18′ is a bearing of N 27° 18′ E.

Some engineers read the needle at every set-up and some only occasionally, depending on the skill of the transitman, on the time available, and on the difficulty involved in going back to read an angle which may have proved to be wrong.

107. Establishing Points in a Vertical Plane. When the telescope is rotated vertically (about the horizontal axis), the transit will generate a vertical plane only if it is in adjustment and if the plate bubble parallel to the horizontal axis is sufficiently sensitive. The telescope bubble may be used for added precision.

To eliminate all errors of adjustment, level the transit (following one of the procedures of the footnote on page 123) and sight at a point A. Raise or lower the telescope and set point B. Reverse the transit in azimuth and altitude and again sight A. Raise or lower the telescope and set point B' opposite B. A point half way between B and B' is in the same vertical plane with A, regardless of adjustments.

For important vertical plane work, a precise *striding bubble*, which clamps on to the horizontal axis may be obtained.

108. Intersecting Lines. Given two lines on the ground, AB and CD in Fig. 108; required their intersection G.

1. With the transit at C, prolong DC and set stakes E and F (with nails or tacks) a few feet on either side of BA produced (BA being produced by the man setting the stakes who sights along AB by eye).

2. Stretch a string over tacks or nails which have been set not quite flush in the tops of stakes E and F.

3. Set the transit over A and prolong BA to an intersection

ith the string, establishing a tacked stake under the string and on line with the transit, at *G*.

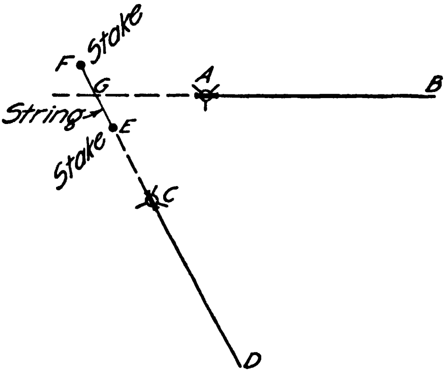

FIG. 108. INTERSECTING LINES.

109. Locating Buildings and Improvements from a Transit Line. The general idea is to locate the building or improvement definitely, preferably with at least one check measurement more than is actually needed to plot it on a map, and to

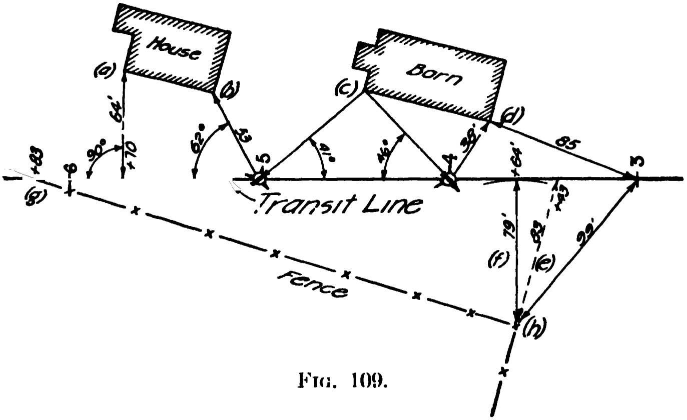

FIG. 109.

do this with minimum work. In taking the field measurement, onstantly keep in mind the way in which these measurements ill be used in plotting the data. One or more of the following easurements will be used, according to circumstances, to cate any required point (Fig. 109):

(a) By station and right angle offset (rectangular coordinates).

(b) By station, angle, and distance (polar coordinates).

(c) By angles from two known points (usually stations).

(d) By distances from two known points (usually stations).

(e) By a *range tie* (sighting along the object from the transit line, which also gives direction) and a distance from a known point.

(f) By a *swing offset* (determining the shortest distance from a point to the transit line) and a distance from a known point.

(g) By intersection with the transit line.

(h) By combinations of the above.

As previously stated, at least one check tie or check dimension should be taken on each object. Such checks are omitted from Fig. 109 so that the various methods may be shown more clearly.

110. Angular and Distance Errors. Surveys involve both angular and distance measurements. It is desirable that some idea of the relative size of the involved errors be kept in mind, since it would obviously not pay to spend considerable time and money in obtaining a much greater degree of precision in angles than in distances, and vice versa.

After a transit is set up, it will usually give the value of an angle to the nearest minute with little or no added time or expense. One minute of angle will move a point some 3400 feet away one foot to the right or left. The taping should therefore have a precision of about one foot in 3400 feet or 1/3400 in order to be of the same order of precision as angles measured to the nearest minute. Since transit angles are usually measured somewhat more precisely than to the nearest minute, due to doubling the angle, compensating errors, and so on, the most common idea as to precision of taping in transit-tape traverses is about one foot in 5000 feet or 1/5000.

A similar line of reasoning, although with different degrees of precision, may be applied to traverses run by transit needle and stadia, or by other methods. Little time or money should be expended in measuring angles more precisely than distances are measured, or vice versa.

PROBLEMS

1. (a) A transit at point A is sighted toward a point B, which is 30 feet away. If either the transit or the sighted point is 0.05 foot out of line, what is the angular error in the direction of the sighted line? (b) The same as (a) except 3000 feet and 0.05 foot? (c) The same as (a) except 40 feet and 0.10 foot? (d) The same as (a) except 1 mile and 1 foot? (e) The same as (a) except 550 feet and 0.75 foot. *Ans.* (a) 5′ 44″

2. It is important to check many engineering and surveying computations roughly and mentally. Do this for Problem 1, using the radian conception, but no tables.

3. (a) In precisely laying out a building 500 feet square on the ground, a base of 500 feet is measured along one side and a perpendicular is erected at one end of the base with an ordinary engineers' transit reading to the nearest minute. On measuring this angle by repetition, we find that its value is 89° 59′ 40″. How much should the end of the line be offset in order to give an angle of exactly 90°? (b) The same as (a) except 90° 00′ 15″? (c) The same as (a) except 89° 59′ 30″? (d) The same as (a) except 90° 00′ 40″? *Ans.* (a) 0.048 foot

4. (a) In order that angular precision may be consistent with taping precision, how closely must angles be measured to correspond with taping having precision of the order of 1/10,000? (b) 1/2000? (c) 1/5000? (d) 1/20,000? *Ans.* (a) 20″

5. (a) Twenty spaces on a transit vernier are equal to 19 spaces on the main circle, the smallest space on the main circle being 20 minutes. What is the least count of the vernier? (b) The same as (a) except 30 spaces equal to 29 with a minimum main circle space of 15 minutes? (c) The same as (a) except 15 spaces equal to 14 spaces, with a main circle minimum space of 5 minutes? (d) The same as (a) except 60 spaces equal to 59 spaces, with a minimum main circle space of 10 minutes?

Ans. (a) One minute

6. (a) A transit is sighted at a point 1800 feet away and turned through an angle of 6° to another point 1800 feet away. What is the arc distance between the two distant points? Use no tables. (b) The same as (a) except 2700 feet and 8°. (c) The same as (a) except 5729.6 feet and 1°. (d) The same as (a) except 5280 feet and 1°. (e) The same as (a) except 1500 feet and 4° 15′. *Ans.* (a) 188.5 feet

7. (a) In Fig. 104, $C_1C = 121.7$ feet, $AC_1 = 2817.4$ feet, and $AB_1 = 1314.6$ feet. In order to set the random stakes over onto the true line, compute AC, AB, B_1B, and the angle at A. (b) The same as (a) except $C_1C = 58.4$ feet, $AC_1 = 3782.3$ feet, and $AB_1 = 1618.8$ feet. (c) The same as (a) except $C_1C = 86.2$ feet, $AC_1 = 849.7$ feet, and $AB_1 = 94.9$ feet. (d) The same as (a) except $C_1C = 8.4$ feet, $AC_1 = 1413.5$ feet, and $AB_1 = 1249.4$ feet. *Ans.* (a) 2820.0 feet, 1315.8 feet, 56.8 feet, 2° 28′

8. In order to form the habit of making rough mental checks of results, compute the approximate values called for in Problem 7 without using tables.

9. (a) Fill in the blanks in the following table. Answers are shown in italics

LINE	AZIMUTH (0 IS S)	DEFLECTION ANGLE	DIRECT ANGLE	BEARING
AB	271 13'	*55 49' L*	*124 11'*	*S 88 47' E*
BC	238 46'	*32 27' L*	*147 33'*	*N 58 46' E*
CD	181 23	etc	etc	etc.
DE	85 31'	Measured Right	Measured clock-	
EF	23 57'	or Left (R or	wise from the	
FA	327 02'	L.)	backsight	

Checks

(b) The same as (a) except use the following azimuths beginning with *AB* and ending with *FA* 213 18', 329 12', 211 28', 317 51', 7 19', and 176 33' (c) The same as (a) except 58 17', 93 28', 315 42', 304 59', 211 36', and 165 17' (d) The same as (a) except 37 58', 199 16', 58 32', 317 51', 295 36', and 108 43'

10. (a) Compute the true value of the angle *ABC* in the following field notes where the angles have been doubled (telescope normal on the first reading and inverted on the second reading) The old transit has eccentric centers, and/or verniers not 180 apart Answers are shown in italics.

Transit at C B

OBJECT	TELESCOPE	NO. OF MEASUREMENTS	VERNIERS A	VERNIERS B	VERNIERS Mean	ANGLE
On A	N	0	0 00'	180 02'	*0 01'*	
On C	N ↷	1	62 10'	242 13'		
On A	*I*					
On C	*I* ↷	2	124 21'	304 23'	*124 22'*	*62 10' 30"*

(b) Same as (a) but using following notes:

(c) Same as (a) but using following notes:

(d) Same as (a) but using following notes:

(b)		(c)		(d)	
0 00'	179 57'	0 00'	180 04'	0 00'	179 58'
48 41'	228 37'	211 31'	31 34'	182 12'	2 11'
97 20'	277 18'	63 03'	243 08'	4 25'	184 25

CHAPTER VII

ADJUSTMENT AND CARE OF INSTRUMENTS

111. Selection and Purchase of Instruments. Instruments should, of course, be selected and purchased by experienced engineers or surveyors, if possible. Their advice should be sought by the novice in much the same way that expert opinion and experience is looked to in selecting any mechanical equipment. Information regarding specifications and prices is available in catalogs and price lists. Used instruments are always available and when selected by experienced men, are usually satisfactory. It is safest, particularly for the inexperienced, to purchase from well-established firms. Instruments purchased from old and reliable makers are almost always dependable.

The *precision of the bubbles* of the instrument should be noted. Most uses of instruments, except the ordinary measurement of horizontal angles, require good bubbles.

The *ease* with which the *graduations* on the circles and on the verniers *are read* is important.

Illumination refers to how well the image appears to be lighted.

Definition has to do with sharpness of detail.

The *magnifying power* of the telescope is the ratio of the size of any object as viewed through the telescope, to the size as viewed by the naked eye. Instruments for leveling or stadia work should have high magnifying power.

The *field of view* is the space which can be seen through the telescope. It is measured as the angle between the outer limits of the field of view, the vertex of the angle being at the telescope. A wider field of view in the usual surveying instrument necessitates lower magnifying power, and vice versa. In mining or underground work, a low magnifying power is sufficient as the sights are short, and instruments of this sort may have a wide field of view.

The type of instrument best suited to the work to be done is best selected upon advice of engineers or surveyors experienced in that particular work.

Many types and grades of instruments are available from many manufacturers. That one should be chosen which is best adapted to the work in hand. The great majority of transits, levels, and compasses are not made for specialized types of surveys, but are designed to give good all-around service.

The various parts of an instrument have usually been designed by the manufacturer so as to give a consistent and uniform degree of precision throughout the instrument. For example, the sensitiveness of the bubble tube and the magnifying power of the telescope on a level should be so designed that a barely perceptible movement of the cross-hair on the level rod, when the rod is held at the usual maximum length of sight, will be accompanied by a barely perceptible movement of the bubble.

112. Care and Use of Instruments. Surveying instruments, particularly the transit and level, are delicate and precise and consequently must be handled carefully. In a building or in timber, the instrument should be carried in the arms, not over the shoulder, to avoid striking it against some object. The instrument should be removed from the tripod and carried in the lap or in its box when transported in an automobile. The screws should be lightly clamped when carrying it so that if it does strike some obstacle, it will be free to turn. If unclamped, the instrument is awkward to carry, since it will be continually swinging to and fro.

In the field, the tripod legs should be spread well apart and sunk securely in the ground. This will minimize danger of damage from overturning and make the instrument more stable, particularly in a wind. The instrument should not be left unguarded where it may be damaged by traffic or by curious livestock.

It is well to carry a light waterproofed silk bag into the field when rain or dust threatens. The eyepiece slide may be closed and the dust cap placed over the objective, but the bag forms an added or better protection in the field. Should the lenses become wet they should not be wiped but must be allowed to dry. Neither should dust be removed from the lens except by a light dusting with a camel's-hair brush or a soft silk rag, preferably the former. Any rubbing will dull the lens. It is best not to unscrew the objective lens, since this necessitates testing and adjusting the line of sight.

The graduations on the horizontal and vertical circles and verniers must not be rubbed with a rag, as this will dull the graduations. Light dusting with a soft brush across the graduations is sometimes helpful. A light rubbing with a soft oiled rag is occasionally useful. Should the glass plate covering the graduations become fogged with dampness, it is sometimes possible to dry it by holding a match or flash light over the glass. If the eyepiece or objective lens becomes damp or fogged, there is no quick process of drying except to place it in warm dry air.

All of the movable operating screws on the level and transit or other surveying instruments are turned with the thumb and forefinger and it is easily possible to turn them so tightly as to injure the threads. They should be turned only so tightly as to hold the instrument from slipping, and this is about as tightly as can be accomplished with the forefinger only.

If the screws or centers do not turn easily, it is necessary to take the instrument apart, clean the parts, lubricate them with a small drop of watch oil or other fine-grade oil, and to wipe off the surplus oil. A special lubricant for very cold weather may be obtained from instrument supply houses. If this does not remedy the difficulty, it is probable that the parts have been strained or scratched by grit, and the instrument must be repaired by the manufacturer or by a competent repairman.

Minor repairs, particularly tape repairs, should be made in the field. A few small tools and supplies are necessary. Even crude repairs will often suffice until a new instrument can be obtained. In shipping by freight or express, the box must be filled with paper or cloth so that nothing can roll around in the box. Small articles should never be packed in the box with the instrument. The instrument box must be enclosed in a larger wooden or corrugated cardboard box and packed with excelsior, hay, or other padding.

113. General Importance of Adjustments. The adjustments of the instruments are of the greatest importance in the study and practice of engineering surveying. It is essential that an instrumentman be capable of testing and adjusting his instrument, since it is difficult or impossible to make most engineering surveys with a poorly adjusted instrument, and no one but the instrumentman himself is ordinarily available to make the tests and adjustments. Most modern instruments are quite stable and do not need adjustment frequently, but it is impossible to predict when such a necessity will arise.

The only safe rule is *to keep the instruments in good adjustment and then to use them as though they were out of adjustment*, that is, to adopt an instrumental procedure which will cause any errors of adjustment to balance or eliminate themselves. Such procedures are suggested later in this chapter after each description of adjustments and also in that chapter which is concerned with the use of the instrument in question. In order so to use an instrument as to eliminate such errors, it is essential to understand the principles of adjustments.

The fundamental principles underlying the adjustment of surveying instruments are the same as those upon which the adjustment of many scientific instruments of precision are based. A mastery of those principles covered in this chapter will give an idea of the construction and the adjustments of precise scientific instruments in general.

Most collegiate, civil service, or other surveying and engineering examinations include questions regarding adjustments, both because of their importance and because such questions bring out the capacity of the candidate to reason clearly and to express somewhat involved processes in good English. Very generally the man without technical training or study is weak in this matter, although he may be proficient in field work of a routine or rule-of-thumb nature.

114. Field and Shop Adjustments. The instruments are designed to permit most of the necessary adjustments to be made either in the field or in the factory. The shop adjustment, made on the bench in the instrument shop, is relatively inexpensive and entirely satisfactory except that since no shop is usually available nearby, considerable delay results due to packing and shipping the instrument. Shop adjustment is usually made in connection with the manufacture or repair of instruments only; all other adjustment is left to the instrumentman in the field.

Should an instrument be dropped or otherwise injured, or if it is considerably worn, it may be difficult or impossible to make sufficiently precise surveys with it because the improper relation between parts cannot be adjusted in the field, but must be remedied by the repairman in the shop. An example of this occurs when the centers or axes about which the instrument revolves bind or become loose or untrue. It is then necessary for a skilled mechanician to turn down the centers on a lathe, or to equip the instrument with new centers; and it is, of course, necessary to return the instrument to the maker or to a well-equipped instrument shop. This work should never be entrusted to an ordinary mechanic. Such defects are seldom found in the better grade instruments when they are new.

115. Focusing the Eyepiece of Any Transit or Level. Due to individual differences in eyesight, it is essential that the eyepiece of the instrument be properly focused to the eye of the person using the instrument. Once the eyepiece is focused, no further attention is needed until another person wishes to use the instrument or until the focus of the observer's eye changes. To secure a good focus, proceed as follows. Point the instrument towards the sky, towards a white wall, or towards a white card or notebook page held a few feet in front of the instrument. Move the eyepiece inward or outward until the cross-hairs appear sharp and black. On some instruments this is accomplished by turning a knurled ring near the eyepiece, in most instruments by turning the eyepiece itself which moves in and out on a spiraled thread, and in a few old instruments by an eyepiece focusing screw similar to that which focuses the *objective* lens at the front end of the telescope.

This results in an approximate focus of the eyepiece which should be further refined to eliminate *parallax* (the apparent movement of the object sighted at when the eye is moved up and down or sidewise) as follows. By means of the focusing screw near the front end of the telescope, focus the objective lens upon a sharply defined object, such as a level rod, located some 300 feet from the instrument. Run the objective lens (at the front end of the telescope) slowly back and forth until there is no apparent movement of the object (no parallax) as the eye is moved up and down, or sidewise. The objective image and the cross-hairs are now in exactly the same plane, and the objective lens is properly focused. Should the object appear sharp, the eyepiece is also properly focused. If not, refocus the eyepiece slightly until the object appears sharp and the cross-hairs will now also appear black. No parallax will now exist

116. Understanding of Adjustments. Frequently the practical surveyor, and unfortunately occasionally the technically trained and educated engineer, make the adjustments in a rule-of-thumb manner without knowing what they are doing or why they are doing it. Such procedure is objectionable. The engineer should visualize each step in the adjustments much as he visualizes a problem in descriptive geometry. To aid in this visualization, each adjustment hereafter will be analyzed and treated in four parts, as follows:

The statement of the adjustment. In the particular adjustment under consideration what relation is necessary between what lines or planes of the instrument?

(a) How is this relation tested and detected? Two pencils manipulated in the hands will frequently aid in understanding these relations better than will an attempt to draw or study a figure.

(b) What screws must be turned, or other adjustment made on the instrument, in order to bring about the required relations?

(c) How may the unadjusted instrument be used so as to eliminate errors of adjustment?

An instrument may appear out of adjustment simply because some part is loose — the wing nuts on the tripod may be loose, the tripod may not be firmly set up, the object glass may be partly unscrewed, an adjusting screw may be only partly tightened, or level bubbles or cross-hairs may occasionally become loosened. Watch for such defects both before and after adjusting the instrument.

Placing the hands on the tripod or moving the feet while sighting may cause the line of sight to move. It is well to try this out so that the necessity for avoiding these pressures may be thoroughly realized. Avoid unequal heating of one end of the bubble tube by breathing or placing the hand on it, since this may cause the bubble to move considerably.

An adjusting pin of proper diameter must be provided to fit the capstan nuts of the instrument under adjustment.

117. Field Transit Adjustments. 1. *To make the plate bubbles* (the tangent to the bubble tubes at the center of their graduations) *perpendicular to the vertical axis* (or spindle), so that the vertical axis will be vertical when the bubbles are centered, and so that the bubbles will stay centered as the instrument is turned horizontally about the vertical axis.

(a) Set up the transit and bring the plate bubbles to their centers, each bubble being parallel to a set of footscrews. Reverse the transit approximately 180° in azimuth (horizontally). If either bubble moves from the center, it is out of adjustment.

(b) Raise or lower one end of the bubble tube by turning the capstan headed screw with the adjusting pin until the bubble is brought *half way* back to the center. The bubble axis is now perpendicular to the vertical axis, although it is not centered. Adjust the other tube in the same manner.

Bring both bubbles back to the center with the footscrews and note if they remain in the center during a complete horizontal revolution of the transit.

(c) The plate bubbles should be kept in adjustment, since this is easily accomplished. If out of adjustment, or if one or both bubbles are broken, the vertical axis may be made vertical by using either plate bubble, or the telescope bubble, first parallel to one set of footscrews and then parallel to the other set of footscrews. In this procedure the bubble is centered, the transit is reversed approximately 180° in azimuth (horizontally), and the bubble is brought half way back to the center with the footscrews. This is repeated over the other set of footscrews. The vertical axis is then vertical and the bubble should remain in the same position (*not centered*) as the instrument is turned through a full 360° in azimuth

To secure maximum precision in verticality of the axis (seldom needed, however), the sensitive telescope bubble should be used In this case, after the bubble has been brought half way back to center with the footscrews, it may be fully centered with the vertical circle tangent screw. The bubble is also centered with the other set of footscrews, after which the vertical axis will be quite precisely vertical, and the telescope bubble should remain centered during a 360° rotation in azimuth.

2. *To make the vertical cross-hair perpendicular to the horizontal axis* of the transit, so that it will be vertical when the instrument is leveled and used.

(a) Sight the upper or lower portion of the vertical cross-hair on a sharply defined point, such as the corner of a roof. Move the telescope upward and downward (rotating it on its horizontal axis) and note whether the point remains on the cross hair from top to bottom of the motion. If so, the adjustment is correct. If not, proceed as follows.

(b) Slightly loosen all capstan screws which hold the *cross-hair ring* (or *reticule*) and rotate the reticule by lightly tapping one of the capstan screws until the vertical cross-hair remains on the point as the telescope is moved up and down. Tighten screws and check.

(c) If these parts are out of adjustment, satisfactory results may be obtained by taking readings at the intersection of the cross-hairs.

3. *To make the line of sight* perpendicular to the horizontal axis;* so that the line of sight will generate a plane when revolved about the horizontal axis.

(a) Set up over *A* (Fig. 117) and center the plate bubbles. Sight on a well defined point *B* not less than 250 feet away. *Transit the telescope* (turn it upside down) and set point *C*. Reverse the telescope approximately 180° horizontally and again sight on *B*. Transit the telescope and set point *D* alongside of *C*. If *C* and *D* coincide, the adjustment is correct; if not, proceed as follows.

(b) Set or mark a point *E* one-quarter of the way from *D* towards *C*. By turning the horizontal capstan screws (slightly loosening the vertical capstan screws if necessary) shift the vertical cross-hair until it falls on *E*, and tighten the capstan screws. Repeat the test until *C* and *D* coincide. Check the verticality of the cross-hair as provided in 2, since it may have shifted.

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* The line of sight of a surveying telescope is a straight line passing through the intersection of the cross-hairs and through the optical center of the objective lens, as in Fig 62.

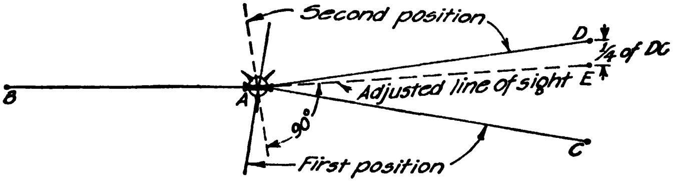

FIG. 117.

Points *A*, *B*, *C*, *D*, and *E* should lie in approximately the same horizontal plane, or they should lie on some plane containing the horizontal axis. It is desirable to have the points *B* and *C* as far as possible from *A*.

(c) Even though the transit is out of adjustment, correct results may be had by setting a point midway between *C* and *D*, as described in § 103, page 116.

4. *To make the horizontal axis perpendicular to the vertical axis*, so that the line of sight will generate a vertical plane as the telescope is revolved in altitude (about the horizontal axis).

(a) Set up and level the transit with the plate bubbles, giving particular attention to the bubble at right angles to the telescope. The plate bubble at right angles to the telescope must remain in the center during both sightings, and it must be in adjustment, in which case the footscrews are not to be touched during the test. For a more precise adjustment, use the telescope bubble to make the vertical axis vertical. If the vertical axis is not vertical, then the transverse bubble must be centered with the footscrews between reversals. Sight at a high point *A*. Depress the telescope and set a point *B*. Reverse the telescope in azimuth and altitude and again sight on the high point *A*. Again depress the telescope and set a point *C* alongside of point *B*. Set a point *D*, by measurement, half way between *B* and *C*. It will be vertically under *A*. If *B* and *C* coincide, the adjustment is correct. If not, proceed as follows.

(b) Sight on *D* and raise the telescope to a point opposite *A*. Raise or lower the end of the horizontal axis by means of the capstan screw at one end of the axis until the vertical cross-hair strikes *A*. Check and repeat until the transit will revolve vertically with *A* and *D* appearing on the vertical cross-hair.

(c) Even though the instrument is out of adjustment, correct results may be had by using the point *D*, provided the vertical axis is vertical.

5 *To make the optical center of the objective move (when focusing) in a straight line which passes through the center of the cross-hairs.*

(a) With the transit clamped in azimuth and in elevation, set points on the line of sight about 10 feet, 300 feet, and 1000 feet from the transit. Also read a level rod on the 10-foot and 300-foot points. Reverse in azimuth and in altitude, sight on the 300-foot point, and clamp in both azimuth and in elevation. If the 10-foot and the 1000-foot points are on the vertical cross-hair, and if with the horizontal cross-hair set on the previous 300-foot rod reading, the same reading occurs on the rod 10 feet away as was previously observed there, no adjustment is necessary. If not, proceed as follows.

(b) Correct part of the error in the vertical cross-hair by the side screws holding the objective slide ring in place, and repeat until no error remains. In most transits, such adjustment is not possible, and the instrument must be sent to a competent instrument repairman.

Correct the error in the horizontal cross-hair by a process similar to the foregoing, or by raising or lowering the horizontal cross-hair by means of the upper and lower cross-hair screws. The shift must be made by successive approximation.

(c) When these parts are out of adjustment, correct results may be obtained by using points midway between the first and second sightings, as mentioned above.

6. *The eyepiece may be centered* on the cross-hairs by turning the screws that hold the eyepiece ring until the cross-hairs appear in the center of the field. An exact centering is not required If no such screws are provided, this adjustment can be made only in the shop.

7. *To make the telescope bubble* (a line parallel to the tangent at the bubble tube at the center of its graduations) *parallel to the line of sight,* so that the line of sight will be horizontal when the telescope bubble is centered.

(a) Perform the *two-peg test.* Set two pegs or stakes between 200 and 300 feet apart. Set the transit midway between them.* Take a rod reading on each peg with the telescope bubble centered at each reading (Fig. 117a). The difference in the two rod readings (3.94) gives the *true* difference in elevation between the pegs, regardless of whether or not the instrument is in adjustment.

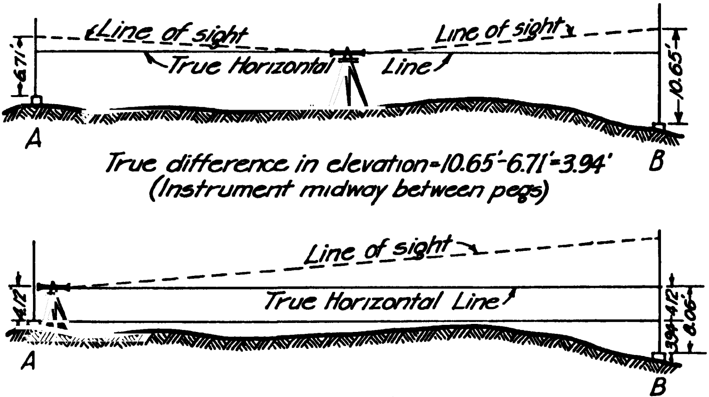

FIG. 117a. TWO-PEG TEST.

Now set the instrument at *A*, preferably the high peg, and read the rod (which is held against, or within half an inch of the eyepiece) by noting the rod reading opposite the center of the eyepiece hole. Some prefer to read backward through the telescope. This reading, plus or minus the *true* difference in elevation between the pegs as previously determined, gives the *true* reading on *B* (in this figure, 8.06 feet) which makes the line of sight horizontal. With the horizontal cross-hair on this *true* reading on the rod at *B*, the instrument is in adjustment if the telescope bubble is in the center. If not, proceed as follows.

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* If the instrument is thought to be in good adjustment and only needs testing, then the midpoint may be estimated by eye. If it is thought to be in fair adjustment, pacing is sufficient. If in very poor adjustment, the midpoint should be set by rough taping.

(b) Raise or lower one end of the telescope bubble tube with the adjusting pin, by turning the capstan nut, until the telescope bubble is centered. Finally check the *true* reading at *B*.

(c) If the transit is used when it is out of adjustment, correct differences in elevation are obtained by setting it midway between observed points.

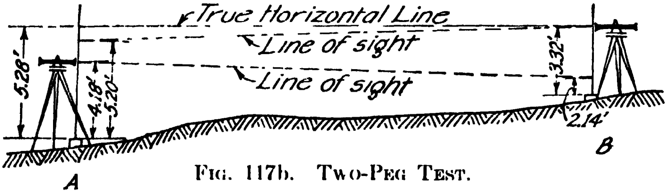

FIG. 117b. TWO-PEG TEST.

Some engineers prefer another *two-peg test* of Fig 117b, as follows.

(a) With the transit at one peg and the bubble centered, take readings on both pegs. Subtract one reading from the other to secure the apparent difference in elevation between the pegs.

Move the instrument to the other peg and repeat.

The mean of the two apparent differences is the *true* difference in elevation between the pegs. The last near reading, plus or minus (depending on which peg is the higher) the *true* difference in elevation as previously computed, gives the *true* reading on the distant rod required to make the line of sight horizontal. For example:

With instrument at *A* rod reads	4.18 on *A*	
" " " " " "	2.14 " *B*	
Apparent difference in elevation	2.04	
With instrument at *B* rod reads	5.20 on *A*	
" " " " " "	3.32 " *B*	
Apparent difference in elevation	1.88	
True difference in elevation	$= \frac{2.04 + 1.88}{2}$	= 1.96
Last reading on near rod		3.32
Reading on *A* (with instrument at *B*) which makes the line of sight truly horizontal		5.28

CAUTION. In this second method, the two pegs must be selected so as to have a considerable difference in elevation. If the *difference in elevation* is less than the *error of adjustment*, then the foregoing explanation and example will not give correct results. In such a case (with a small difference in elevation between the pegs), only a very careful study of a figure and a very careful computation, with careful attention to algebraic signs, will give correct results.

With the horizontal cross-hair on this *true* reading on the distant rod, the instrument is in adjustment if the telescope bubble is in the center.

(b) If not, proceed as in (b), p. 140.

(c) If the transit is used when it is out of adjustment, correct differences in elevation are obtained if it is set midway between points observed.

The first two-peg test (with the transit or level midway between pegs) is simpler to conceive in theory. Frequently only one extra set-up is needed since in most leveling operations the instrument is set up midway between two points, and the test may be performed in connection with such operations. It is best used when the instrument is thought to be in fair adjustment so that the midpoint may be estimated by eye or by pacing.

The second two-peg test (with the instrument first at one peg and then at the other) may require two extra set-ups beyond ordinary leveling operations. The theory is difficult to understand, especially as mentioned in the foregoing CAUTION of this section. It is best used when the instrument is badly out of adjustment, in order to avoid measuring the midpoint distances; or for very precise work.

8. *To make the vertical circle vernier read zero when the line of sight is horizontal, when the telescope bubble is centered, and when the plate bubble parallel to the telescope is centered*, so that correct vertical angles may be read directly.

(a) Level the transit, paying particular attention to the bubble parallel to the telescope. Center the telescope bubble carefully. It must be in adjustment. If the vernier of the circle reads zero the instrument is in adjustment. If not, proceed as follows.

(b) Slightly loosen the capstan screws holding the vernier and shift the vernier slightly by tapping lightly with a pencil until the zeros coincide. It should also be noted whether or not the 30 on the vernier corresponds with 29 divisions on the main vertical circle, at both ends. Thin paper inserted between the vernier and the main scale gives the proper space without binding. Tighten screws and check.

(c) When the transit is used when it is out of adjustment, correct vertical angles are obtained as described on pages 122 and 123.

9. *Needle adjustment.* See the adjustments of the compass (page 115).

118. Shop Transit Adjustments. The foregoing adjustments are the ones usually made in the field. They may also be made quickly and cheaply in well-equipped surveying-instrument repair shops.

Another set of tests and adjustments, rarely necessary, are usually made in the shop. The necessity for such adjustments usually arises after the instrument has been damaged, as by a fall, or after it is worn. In either of these cases, the instrument should be gone over in the shop. A complete description of these tests and adjustments may be found in some texts and in some manufacturers' manuals but here are only mentioned as:

1. *Centering of the graduated circle around the vertical axis,* so as to eliminate eccentricity. If uncorrected eccentricity exists, the mean of both verniers will give correct results.

2. *Setting the verniers exactly 180° apart.* If a vernier is out of place, correct results may be obtained by always reading the same vernier, or by taking the mean of both verniers.

3. *Centering the eyepiece* as described in § 117, test 6.

4. *Centering the objective slide,* as mentioned on page 138.

119. Field Adjustments of the Dumpy Level. 1. *To make the bubble tube* (the tangent to the bubble tube at the center of its graduations) *perpendicular to the vertical axis.*

(a) Proceed as in transit adjustment 1 of page 135, but make the tests with the telescope bubble alternately over both pairs of footscrews.

(b and c) Proceed as in transit adjustment 1 of page 135.

2. *To make the horizontal cross-hair truly horizontal* (perpendicular to the vertical axis).

(a) Level up, sight on a point, and rotate the telescope horizontally about the vertical axis. The horizontal cross-hair should follow the point, from end to end. If not, proceed as follows.

(b) Loosen the cross-hair reticule screws and rotate the reticule slightly by tapping with a pencil, until it conforms to (a).

(c) If the level is used when out of adjustment, correct results may be had by using the intersection of the cross-hairs when sighting.

3. Use the *two-peg test* described under transit adjustment 7 on page 140, *except* that instead of adjusting the telescope bubble, that bubble is centered with the footscrews and the horizontal cross-hair is raised or lowered by means of the cross-hair reticule screws, until the correct reading is attained on the distant rod.

4. The eyepiece may be centered as described under transit adjustment 6 on page 138.

5. The objective slide must be adjusted in the shop.

120. Field Adjustments of the Wye (*Y*) Level. 1. The same as dumpy level adjustment 2, except that the cross-hairs of the *Y* level are not disturbed; instead, a stop-screw on one of the *Y*s is adjusted.

2. *To make the line of sight coincide with the axis of the Ys,* sometimes called the axis of the collars or rings.

(a) Loosen the *Y* clips and sight on a fixed point, such as the center of a tack, several hundred feet away. Rotate the telescope in the wyes about its axis, through 180° (upside down), and note if the intersection of the cross-hairs remains on the point. If not, proceed as follows.

(b) Correct half of the error by the cross-hair reticule adjusting screws.

(c) If the level is used when it is out of adjustment, correct differences in elevation may be obtained if the level is set midway between rod positions.

3. *To bring the tangent to the bubble tube into the same plane with the line of sight.*

(a) Center the bubble and rotate the telescope back and forth a few degrees in the wyes (around the axis of the wyes as an axis). The bubble should stay centered. If not, proceed as follows.

(b) Adjust by means of the lateral bubble adjusting screws until the bubble remains centered during such rotation.

(c) The same as in 2 (c), preceding.

4. *To make the bubble tube* (the tangent to the bubble tube at the center of its graduations) *parallel to the bottom element of the wyes.*

(a) Center the bubble over a pair of footscrews, *carefully* reverse the telescope end for end in the wyes, and note whether the bubble returns to the center of the tube. If not, proceed as follows.

(b) Correct half of the error by means of the vertical adjusting nuts at the end of the bubble tube.

(c) The same as in 2 (c), preceding.

At this point it is well to note that adjustments 1, 2, 3, and 4 of the *Y* level center the line of sight in the axis of the wyes, bring the bubble tube parallel to the bottom element of the wyes, and bring the bubble tube into the same plane with the axis of the wyes. If the axis and the bottom element of the wyes are parallel (if the wyes and collars are true and unworn), then the line of sight is parallel to the bubble tube and in the same plane with it, and the *Y* level conforms to the major adjustments therefor. If the wyes and collars are not true, or are worn, then this *entire indirect method of adjustment fails and it is necessary to adjust the wye level according to the program of adjustments described for the dumpy level.* It is always *safest* to complete the *Y* level adjustments with a two-peg test, unless it is known that the wyes are in good condition.

5. *To bring the bubble tube and the line of sight perpendicular to the vertical axis of the level,* so that the bubble remains centered as the telescope is rotated horizontally around the vertical axis. This adjustment is a convenience, not a necessity.

(a) Level up over a diagonal pair of footscrews. Revolve the telescope 180° horizontally about the vertical axis and note if the bubble remains centered. If not, proceed as follows.

(b) Bring the bubble half way back by means of the *Y* nuts at the lower ends of the *Y*s). Then fully center the bubble using both pairs of footscrews, and note if it remains centered during a complete horizontal revolution of the telescope.

(c) If the instrument is out of adjustment, correct results may be obtained by keeping the bubble centered with the footscrews while sighting on the rod.

6. The eyepiece may be centered as described under transit adjustment 6, or by centering it so that it remains steadily upon an observed object while the *Y* clips are loosened and the telescope is rotated 360° around the line of sight.

The objective slide may be adjusted, especially for the horizontal cross-hair, as in transit adjustment 5 (page 138).

121. Field Adjustments of the Compass. 1. The same as adjustment 1 of the transit.

2. *To straighten the needle.*

(a) Steady the south end of the needle with a stick or wire and read the north end. Touching the needle with the hands weakens the magnetism. Reverse the needle 180° until the north end is now at the original position of the south end, and read the south end, which is now nearly in the position of the first reading on the north end of the needle. If the two readings agree, the needle is straight. If not, proceed as follows.

(b) Remove the needle from the pivot and straighten an amount equal to one-half of the difference in readings.

(c) If the compass is slightly out of adjustment, approximately correct bearings may be read by using one end of the needle only

3. *To center the pivot point.*

(a) If the difference in readings of the two ends of the needle is constant for different positions of the needle around the circle, the pivot is centered. If not, proceed as follows.

(b) Find the position in which the difference in readings of the two ends is a maximum. Remove the needle and bend the pivot with a small pair of pliers at right angles to the needle direction for a distance of half of the difference, and continue this process, alternately testing and bending until there is no difference between end readings of the needle at various positions (various positions because the pivot may not have been bent at right angles) around the circle.

(c) It is not possible to secure correct readings if the pivot is out of center.

4. *To correct sluggishness of the needle.*

(a) If the needle moves into position too slowly and settles too quickly, either it needs remagnetizing, or the pivot point needs sharpening.

(b) It may be remagnetized by drawing a bar magnet several times from the center of the needle towards its end, then continuing this motion several inches beyond the end of the needle. The magnet must be lifted a foot or so above the needle when it is being returned to the center of the needle for another stroke. Use that end of the bar magnet on the end of the needle which it attracts. Reverse the bar magnet and magnetize the opposite end of the needle.

A blunt pivot point may be sharpened with a fine-grained oil stone.

(c) No procedure will give good results if the needle is sluggish.

122. Field Adjustment of the Hand Level and of the Clinometer. *To make the line of sight horizontal when the bubble is centered.*

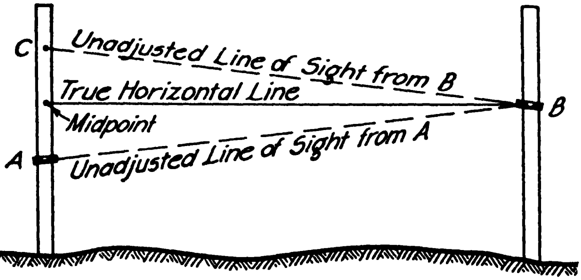
C
Unadjusted Line of Sight from B
True Horizontal Line
B
Midpoint
Unadjusted Line of Sight from A
A

FIG 122. ADJUSTING THE HAND LEVEL OR THE CLINOMETER.

(a) With the hand level or clinometer held against a post or tree at A, as in Fig. 122, center the bubble and mark a point B on another post or tree about 100 feet away. Move the instrument to B and set C above or below A. When the clinometer is being tested, the vertical circle must read 0°. If A and C are the same elevation, the instrument is in adjustment. If not, proceed as follows.

(b) Mark a mid-point between *A* and *C*. With the cross-hair on the mid-point, usually the bubble is adjusted to the center.

(c) If the instrument is used when it is out of adjustment, correct differences in elevation are obtained if it is midway between rod positions.

123. General Procedure. Adjustment by a skilled technician in the shop may be advisable rather than field adjustments by one who is inexperienced. The adjustments all require careful manipulation.

Making some adjustments may throw other relationships out of adjustment; consequently certain tests must be repeated. When the instrument is badly out of adjustment, it is best to make these related adjustments (as for example those in which the cross-hairs are moved) approximately at first and then finally to perfect them. In general, it is desirable to make the adjustments in the order given above.

PROBLEMS

True-False Questions

1. At the second set-up of a two-peg test of the dumpy level, the bubble line and the line of sight are made parallel by raising or lowering one end of the bubble tube.

2. In adjusting the bubble attached to the transit telescope, the bubble line and the line of sight are made parallel by moving the cross hairs.

3. Errors due to curvature and refraction are ordinarily negligible in engineering surveys.

4. On a level or transit, the shorter eyepiece shows the image right side up.

5. On the horizontal circle of the transit, all errors of eccentricity are eliminated by taking the mean of the two vernier readings.

6. In measuring a vertical angle with a transit, all errors of adjustment are eliminated by pointing the telescope, reading the vertical angle, lowering the telescope until the telescope bubble is centered, and reading and applying the index correction as shown on the vernier.

7. All errors of adjustment are eliminated by placing the level midway between the points whose difference in elevation is being determined.

8. The plate bubbles of a transit which is out of adjustment are centered, the instrument is reversed 180° on its vertical axis, and the bubbles

are brought half way back to the center by raising or lowering one end of the bubble tube. The vertical axis is now vertical.

9. The telescope bubble should be used to secure maximum precision in making the vertical axis vertical.

10. The line of sight of a transit is not perpendicular to the horizontal axis of a transit, due to lack of adjustment. In rotating the telescope about the horizontal axis, the line of sight generates an inclined plane.

11. In 10 above, a straight line is produced by *double sighting*. The true error of adjustment is one-half of the distance between the two points set ahead.

12. A transit is in adjustment except that the horizontal axis is not horizontal when the plate bubbles are centered. When the telescope is revolved about the horizontal axis, the line of sight generates a cone.

13. When dropping a point vertically by double reversing (in both azimuth and altitude), the transit should be releveled between reversals, unless you are sure the vertical axis is vertical. All other adjustments are correct.

14. It is desirable to keep the instrument in good adjustment and then to use it as though it were out of adjustment.

15. In adjusting the *Y* level, the cross-hairs must be brought into the axis of the *Y* rings before the bubble is adjusted by raising or lowering its end.

16. In the indirect method of adjusting the *Y* level, one cannot be sure that the instrument is correctly adjusted except by applying the two-peg test.

Other Adjustment Questions

17. Is there any single test which may be applied to the wye level, the dumpy level, the transit, the hand level, and the clinometer, which guarantees satisfactory performance in ordinary leveling operations?

18. Define the "line of sight" of a telescope of a surveying instrument.

19. (a) Which of the several transit adjustments (give numbers) as given in this chapter may be changed in order of performance, without affecting the validity of the adjustment? (b) The wye-level adjustments? (c) The dumpy-level adjustments? (d) The compass adjustments?

20. In transit adjustment number 4, after point *C* has been set alongside of point *B*, why is it not possible to complete the adjustment by tilting the horizontal axis until the vertical cross-hair is on *D*?

21. (a) What adjustments (by number) are substantially the same for the transit and for the dumpy level? (b) For the transit and wye level? (c) For the transit and compass?

22. In a two-peg test, with the level midway between pegs, the rod on peg *A* reads 3.93 feet, and on peg *B* 5.81 feet. When the instrument is moved to *A*, it is found to be 4.21 feet above peg *A*, and the rod held on peg *B* reads 6.17 feet. What is the true difference in elevation? Is the line of sight inclined upward or downward? With the instrument remaining at *A*, what must be the rod reading on peg *B*, in order that the line of sight be horizontal?

3. In a two-peg test with the level at *B*, the instrument is found to be feet above *B*, and the rod on *A* reads 6.21 feet. With the instrument at *A*, the rod held on *A* reads 3.59 feet, and when held on *B* 3.22 feet. Answer the same questions as in 22.

24. This is the same problem as 23 except that the final reading at *B* is 3.72 feet. Answer the same questions as in 22. See CAUTION on page 144.

25. In performing adjustment number 4 of the wye level, the 20-second (see page 144) bubble moves 3 divisions towards the objective end of the telescope when the telescope is reversed, end for end, in the *Y*s. Immediately before the test was made, uphill levels were run between B.M.$_1$ and B.M.$_2$, in which work the total length of the backsights exceeded the total length of the foresights by 600 feet. What error, between B.Ms., was caused by this lack of adjustment? Is the elevation determined for B.M.$_2$ too high or too low?

CHAPTER VIII

OBSERVATIONS FOR MERIDIAN

124. The Scope of This Chapter. Astronomical observations are occasionally needed in field survey work. A complete theoretical explanation of this subject, involving astronomy and spherical trigonometry, would require an entire volume. Fortunately, however, the following relatively limited treatment, coupled with a little practice, will enable the engineer to do all that he will ordinarily be called upon to accomplish. For the rarer solutions, seldom if ever encountered by most engineers, the reader is referred to Chapter XXI, Engineering Astronomy, and similar treatments. The method of one of the following sections will usually suffice to determine a meridian within a minute or two of its true direction. Greater precision is seldom required.

125. Determining a Meridian by Observation on Polaris at Elongation. The star Polaris is also known as the Pole Star and as Alpha Ursa Minor. The meridian determination here described, using this star, is simpler, more precise, and less subject to mistakes in observation and in computation, than is any other astronomic method of establishing a meridian. The star Polaris can be used only in the northern hemisphere.

The daily rotation of the earth on its axis causes every star to appear to rotate in a circle about the earth's axis. The axis prolonged passes near Polaris, and consequently this star appears to rotate counter-clockwise in a small circle about the axis, completing the rotation in about 23 hours and 56 minutes. The angular radius of rotation, called the *polar distance*, is a little over one degree, as measured from the center of the earth. When Polaris appears exactly above the *Pole* (or earth's axis prolonged) it is said to be at *Upper Culmination;* when below the Pole, *Lower Culmination;* at the extreme eastern position of its apparent movement, *Eastern Elongation;* and at the extreme western position, *Western Elongation.*

126. Computations for the Observation at Elongation. A lescope pointing at Polaris when it is at either culmination will be in a true meridian. When this star is observed at elongation (as here described and used because the star at elongation does not appear to move in horizontal angle for over half an hour), it will be necessary to turn the telescope through a horizontal angle of something over a degree in order to bring it into a true meridian. This angle may be found (for any given date) in an *Ephemeris,** or it may be computed from the formula

(126) sine of the bearing of Polaris

$$= \frac{\text{sine of the polar distance of Polaris}}{\text{cosine of the latitude of the place of observation}}.$$

The *polar distance*† of Polaris for the date of observation is given in an Ephemeris. Sometimes only the *declination*† of Polaris is given, in which case the polar distance may be secured by subtracting the declination from 90°. The latitude (correct within two or three minutes) may be taken from a good map.

The Ephemeris gives the civil date and time (ordinary watch time at Greenwich, England, where the longitude is 0°) for the elongation of Polaris, if observed along the Greenwich meridian. This tabular entry must be corrected as explained in the Ephemeris, and as shown in Fig. 127a, in order to secure the Standard (ordinary watch) time of elongation at the point of actual observation. It will suffice if this computation is within ten or fifteen minutes of the true value. That elongation (eastern or western) is chosen which comes after sunset and before sunrise.‡

127. Field Work for a Polaris Observation at Elongation. About half an hour before the computed time of elongation, set up the transit over a stake and carefully make the vertical axi vertical, as described on page 123. It is not sufficient simply to center the plate bubbles, unless they are sensitive and in good adjustment. Focus the telescope on some bright star. Focusing on Polaris is facilitated if the objective slide position is marked, when the telescope is focused on a star.

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* An *Ephemeris,* containing astronomic data for the current year, may be secured for 5 cents from the General Land Office of the United States at Washington, D. C. Most manufacturers of, and dealers in, surveying instruments will furnish an Ephemeris free, or for a few cents.

† The declination of any star, or of the sun, is the angle (at the center of the arth) from the plane of the equator, to that body. The angle from that body o the Pole (the north end of the earth's axis, prolonged) is called the polar istance. Declination plus polar distance = 90 degrees.

‡ The observation may be taken just after sunrise or just before sunset with high-powered telescope on which the ordinary sunshade has been replaced by black paper tube some six inches long, held in place with an elastic band aylight observation is more difficult since it is not easy properly to focus the lescope so as to locate and see the star.

Clamp the horizontal plates on 0°, or on some other definite reading, with the transit pointing at a reference stake. Preferably the stake should be far enough from the transit so that it will not be necessary to change the objective from its focus as previously made on a star. It is a little difficult to focu properly on a star; consequently this focus should not be changed, once it has been accomplished. It will be necessary to reflect light down the telescope, through the objective lens, in order to illuminate the cross-hairs, and this is accomplished with a mining lamp or a pocket flash light. This reflection of light is facilitated by replacing the sunshade with a 3-inch or 4-inch white paper shade which is held in place with a rubber band. A light is also needed behind the reference stake.

The position of Polaris in the sky may be determined by using the sketch of Fig. 127. If the latitude of the place of observation be set off on the vertical circle of the transit, Polaris (when at elongation) will be found near the horizontal cross-hair. The transit telescope may be pointed with the compass needle (allowing for the declination of the needle and for the small angle of formula (126)) so that Polaris may be found near the vertical cross-hair.

With the vertical cross-hair on Polaris, its movement is followed with the upper motion tangent screw until it reaches elongation (where it will not move horizontally, sufficiently for detection with an ordinary transit, for fifteen minutes before and fifteen minutes after true elongation). Read the horizontal angle on the plates, using both verniers.

Reverse the telescope in azimuth and in altitude (turn it around horizontally and upside down) and again sight on Polaris and read the horizontal angle on the plates. The transit should *not* be releveled between pointings. After a few minutes, Polaris will be observed to move away from the cross-hair. Leaving the telescope in this reversed position, check the original

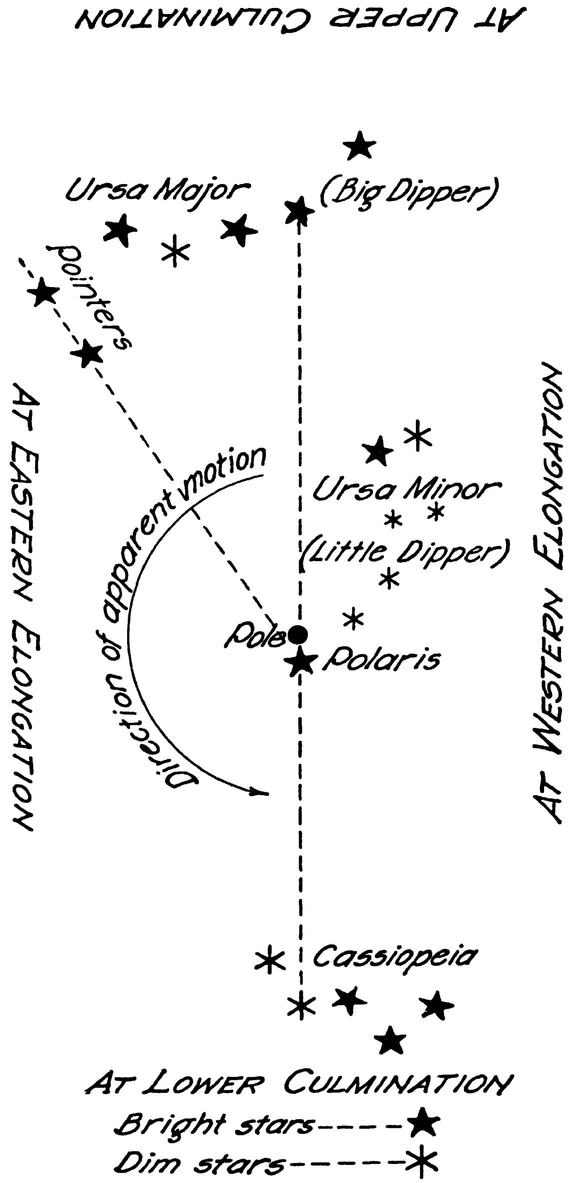
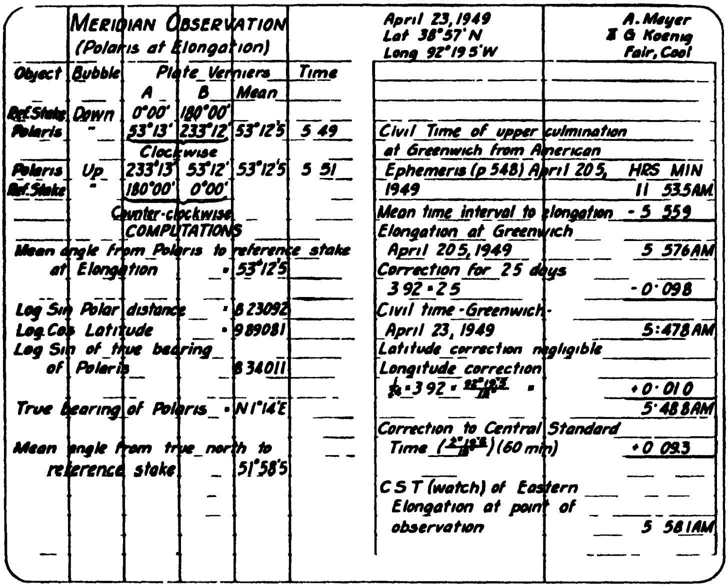


FIG. 127. LOCATING POLARIS

reading of the horizontal plates with the transit pointing at the reference stake. The mean of the two horizontal angles between the reference stake and Polaris, corrected by the angle of formula (126), will give the angle from the reference stake to true north. An example of the computations and field notes is given in Fig. 127a.



MERIDIAN OBSERVATION
(Polaris at Elongation)

Object	Bubble	Plate Verniers A	B	Mean	Time
Ref. Stake	Down	0°00′	180°00′		
Polaris	"	53°13′	233°12′	53°12′.5	5 49
		Clockwise			
Polaris	Up	233°13′	53°12′	53°12′.5	5 51
Ref. Stake	"	180°00′	0°00′		
		Counter-clockwise			

COMPUTATIONS

Mean angle from Polaris to reference stake at Elongation = 53°12′.5

Log Sin Polar distance = 8 23092
Log Cos Latitude = 9 89081
Log Sin of true bearing of Polaris 8 34011

True bearing of Polaris = N 1°14′ E

Mean angle from true north to reference stake 51°58′.5

April 23, 1949
Lat 38°57′ N
Long 92°19.5′ W

A. Meyer
I G Koenig
Fair, Cool

Civil Time of upper culmination at Greenwich from American Ephemeris (p 548) April 20.5, 1949 — HRS MIN 11 53.5 AM
Mean time interval to elongation − 5 55.9
Elongation at Greenwich April 20.5, 1949 — 5 57.6 AM
Correction for 2.5 days 3.92 • 2.5 — −0·09.8
Civil time - Greenwich - April 23, 1949 — 5:47.8 AM
Latitude correction negligible
Longitude correction $\frac{1}{24}$ • 3.92 • $\frac{92°19.5}{15}$ = — +0·01.0
5·48.8 AM
Correction to Central Standard Time $\left(\frac{2°19.5}{15}\right)$(60 min) — +0 09.3
C S T (watch) of Eastern Elongation at point of observation — 5 58.1 AM

FIG. 127a. POLARIS OBSERVATION.

The foregoing procedure constitutes a *set*. This will suffice for the experienced transitman, but the inexperienced man should take at least one additional set as a check.

Instead of using a reference stake as just described, some engineers prefer to lower the telescope and set a stake vertically under Polaris (when at elongation). The transit is then reversed in altitude and azimuth and a second point set vertically under the star. Next day the midpoint between them is established and the angle of formula (126) turned off in the proper direction to give the true meridian. In this alternate procedure, it is not essential that the vertical axis of the transit be truly vertical, provided the plate bubble at right angles to the telescope is centered before observation and again centered after the transit is reversed in azimuth and altitude, just prior to the second observation. A striding bubble placed on top of the horizontal axis will give still better results

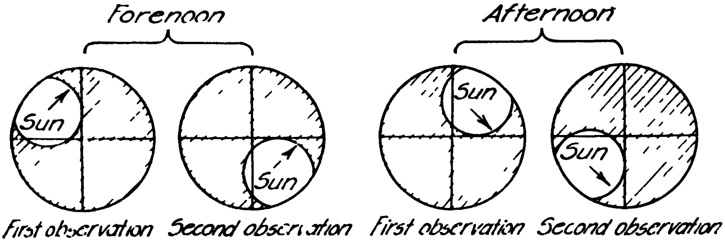
128. Determining a Meridian by Direct Observation on the Sun. This observation should not be taken within two hours of noon, since in that period the horizontal angle of the sun is changing rapidly and the elevation of the sun is changing slowly. Neither should the observation be taken early in the morning or late in the evening (usually expressed as when the vertical angle to the sun is less than 15 degrees), since at smaller vertical angles there is an excessive and variable refraction correction. The following procedure is that for the Northern Hemisphere. It must be slightly modified in the Southern Hemisphere.

Courtesy W. and L. E. Gurley, American Museum of Natural History, and Asia Magazine

FIG. 128. A SOLAR OBSERVATION IN THE GOBI DESERT.

129. Field Work for a Direct Solar Observation. Level the ransit carefully over a stake. If the plate bubbles are out of adjustment, reverse the transit 180° in azimuth and bring the ubbles half way back to the center with the footscrews. This will make the vertical axis vertical, which is the condition needed for this observation; and the bubbles will remain out of center by the same amount during a complete horizontal revolution of the transit. Some engineers prefer to make the vertical axis vertical with the telescope bubble (see page 123), and this is good practice, especially if the plate bubbles are not precise. It is *essential* that the vertical axis be vertical.

The telescope is sighted at a reference stake or point, with the plates clamped at 0°, or at some other definite angle. The upper motion is loosened and the transit pointed at the sun by drawing the eyepiece out to its full length and holding a white card a few inches from the eyepiece, so that the images of the sun and of the cross-hairs appear clearly on the card. The objective is used in focusing. Nothing can be seen by sighting through the telescope at the sun and it may injure the eyes.



Position of sun and cross-hairs just before tangency

FIG. 129. SUN'S DISK WHEN TRANSIT HAS AN INVERTING EYEPIECE.

Only the sun's disk, and that portion of the cross-hairs on the disk, can be seen, the shaded portions of Fig. 129 not being visible. Also, since the sun is changing in both vertical and horizontal angle, difficulty is experienced in securing a precise pointing on the sun. For these reasons, the procedure of Fig. 129 is used. For example, in the first forenoon observation, the sun is kept tangent to the vertical cross-hair by following its azimuth movement with the upper motion tangent, or slow motion, screw. The center horizontal cross-hair is set so as to cut a small segment from the bottom of the sun's disk, and is not again moved. When the sun has risen to tangency on the horizontal cross-hair, all motion of the telescope is stopped and e readings are taken. A similar procedure is followed in the cond forenoon observation except that the rising sun is kept tangent to the horizontal cross-hair and the transit is clamped in azimuth so as to cut off a small segment of the sun with the vertical cross-hair.

On transits of low magnifying power (with a wide field of view) the full circle of the sun's disk will show on the card, but with higher-powered telescopes, frequently only a portion of the disk is visible, as indicated in Fig. 129. Great care must be exercised to avoid mistaking one of the stadia hairs for the center horizontal cross-hair. The stadia cross-hairs are purposely omitted from Fig. 129.

With the image attaining tangency as just explained and as shown in Fig. 129 for the *first observation*, the standard (ordinary watch) time is recorded and readings of the horizontal and vertical circles are taken and recorded. Both of the horizontal verniers are used. The telescope is then reversed in azimuth and altitude, if there is a full vertical circle on the transit, and the same readings are repeated with the sun attaining tangency as explained for the *second observation* of Fig. 129. This constitutes a *set* from which mean values of time and of angles are used in subsequent computations, providing that not more than 15 minutes has elapsed between the two pointings at the sun. The transit is not releveled during a set. With the transit remaining in this reversed position, a second sight is taken on the reference point, at which time the plates should read exactly 180° different from the original reading on the reference point.

A second set should be taken by an experienced transitman; and a third, and even a fourth set, by a less experienced man. *Each set must be computed separately.*

Fewer sets will suffice if each set consists of two readings (of vertical angle, horizontal angle, and time) taken for the *first observation* position; then two readings, with the transit reversed, for the *second observation* position; and the mean of the four readings used in computing the set.

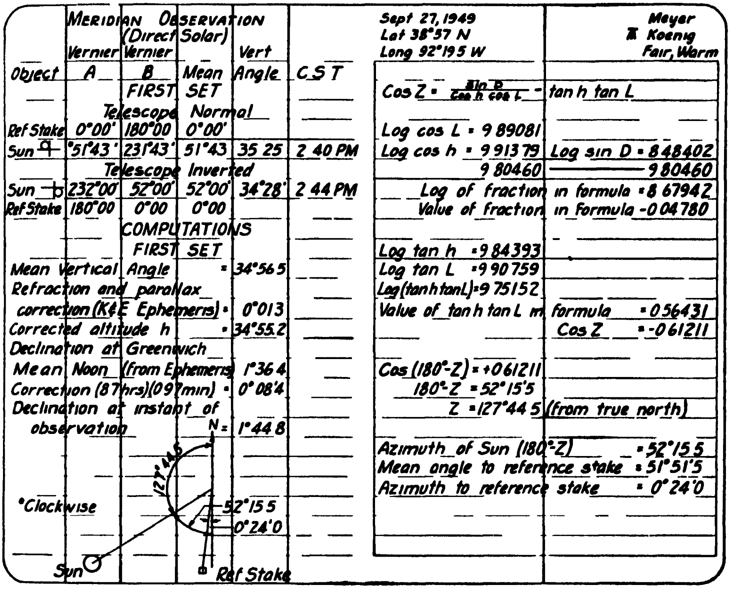
130. Other Field Methods of Direct Solar Observation. While the foregoing field procedure is recommended, some engineers prefer not to use a card. In this case a special attachment of colored glass is fitted over the eyepiece, and the telescope is sighted through, and pointed at the sun in the usual manner of using the telescope. The sun is placed first in one quadrant, and then in the diagonally opposite quadrant, as explained in the preceding section. The attachment is seldom available, and it is difficult properly to smoke a piece of glass which could be used as a substitute.

When using this colored glass attachment for the eyepiece, a few engineers bisect the sun with both the horizontal and vertical cross-hairs. This requires following the sun's movement with both cross-hairs. If the work is skillfully done, and if several bisections are made in each set, the mean values may be closely correct. The practice is not recommended, especially for inexperienced men.

131. Computations for a Direct Solar Observation. This computation involves the solution of the equation

$$(131) \qquad \cos Z = \frac{\sin D}{\cos h \cos L} - \tan h \tan L.$$

The declination D of the sun is taken from the Ephemeris and is corrected so as to give the declination at the moment of observation. It is *not corrected for refraction* as may sometimes be inferred incorrectly from the Ephemeris. The Ephemeris gives the declination for either Greenwich mean *noon* or for 0^h Greenwich *Civil Time*. These are the two instants of time, noon and midnight, when the mean sun is on the Greenwich meridian. Greenwich *apparent* noon differs from Greenwich *watch, or standard, or local mean noon*, by the *equation of time*, the value of which rarely exceeds 15 minutes, and which may be neglected in a direct solar computation. Hence, for the purposes of this computation, Greenwich apparent noon and Greenwich watch, or standard, or local mean noon may be considered to be the same thing. If the Ephemeris gives the sun's declination at Greenwich standard midnight (0^h), consideration of apparent time or of the equation of time is also unnecessary. In Fig. 131, the average watch (standard) time of observation, 2:42 P.M., is 8 hours and 42 minutes, or 8.7 hours after Greenwich standard noon. The correction to the declination for this 8.7 hours which has elapsed since Greenwich noon, is applied as shown in Fig. 131 or as explained in the Ephemeris. The declination is plus if the sun is north of, or above, the equator, as during the summer months. It is minus if the sun is south of, or below, the equator, as during the winter months.



	MERIDIAN OBSERVATION (Direct Solar)				
	Vernier	Vernier		Vert	
Object	A	B	Mean	Angle	C S T
		FIRST SET			
		Telescope Normal			
Ref Stake	0°00'	180°00	0°00'		
Sun ♀	*51°43'	231°43'	51°43	35 25	2 40 PM
		Telescope Inverted			
Sun ⊸	232°00'	52°00'	52°00'	34°28'	2 44 PM
Ref Stake	180°00	0°00	0°00		

COMPUTATIONS
FIRST SET
Mean Vertical Angle = 34°56 5
Refraction and parallax correction (K&E Ephemeris) = 0°01 3
Corrected altitude h = 34°55 2
Declination at Greenwich Mean Noon (from Ephemeris) 1°36 4
Correction (8 7 hrs)(0 97 min) = 0° 08'4
Declination at instant of observation = 1°44 8

*Clockwise

Sept 27, 1949
Lat 38°57 N
Long 92°19 5 W

Meyer
& Koenig
Fair, Warm

$\text{Cos}\,Z = \frac{\sin D}{\cos h \cos L} - \tan h \tan L$

Log cos L = 9 89081
Log cos h = 9 91379 Log sin D = 8 48402
9 80460 ———— 9 80460
Log of fraction in formula = 8 67942
Value of fraction in formula -0 04780

Log tan h = 9 84393
Log tan L = 9 90759
Log(tan h tan L) = 9 75152
Value of tan h tan L in formula = 0 56431
Cos Z = -0 61211

Cos (180°-Z) = +0 61211
180°-Z = 52° 15'5
Z = 127°44 5 (from true north)

Azimuth of Sun (180°-Z) = 52°15 5
Mean angle to reference stake = 51°51'5
Azimuth to reference stake = 0° 24'0

FIG. 131 DIRECT SOLAR OBSERVATION.

The mean for the set of the observed vertical angles to the sun, diminished by a correction from the Ephemeris for parallax and refraction, gives the value h to be used in the formula. It is always plus.

The angle L is the latitude of the place of observation, correct within a minute, and always plus in the Northern Hemisphere. It may be taken from a reliable map, or obtained otherwise. Astronomic methods for determining the latitude are given in many books on surveying, although it is rarely necessary to make such an observation.

The angle Z is the horizontal angle, east or west, from true north to the sun. The value of cos Z is negative, as in Fig. 131, here it is $-$ 0.61211. Since $-\cos Z = +\cos(180° - Z)$, the angle whose cosine is $+ 0.61211$ is looked up in trigonometric tables. Then $180° - Z = 52° 15'.5$, or $Z = 127° 44'.5$. Of course Z must be so considered as to show the sun east of the meridian in the morning, or west of the meridian in the afternoon.

The value of Z as computed from formula (131) is combined with the mean reading of the transit plates (for the set), to secure the bearing or azimuth of the line from the transit to the reference point. An example of field notes and computations is given in Fig. 131.

Five-place logarithms are convenient and sufficiently precise for solving formula (131).

PROBLEMS

1. (a) Compute from the Ephemeris the declination of the sun for your location, for the current year, at 4:00 P.M. (Standard time), August 3. (b) For 10:00 A.M. December 6. (c) For 2:30 P.M. March 21. (d) For 9:30 A.M. September 23.

2. (a) Compute from formula (126) the bearing of Polaris at elongation for your locality and for the current year, for that date on which the bearing is a maximum. Check by the Ephemeris table. (b) For the date on which the bearing is a minimum. (c) For the current date. (d) For March 7.

3. (a) For your locality and for the current year, compute from the Ephemeris the watch time for a night elongation of Polaris on January 16. Is Polaris east or west of the meridian? (b) June 21 (c) October 16. (d) December 4.

4. (a) From the following notes, compute the watch time of a night elongation of Polaris for the current date, and for your locality. Is that elongation east or west of the meridian? Compute the angle of formula (126) and check by the Ephemeris table. Compute the azimuth from the transit to the reference point. (b) For February 22, of the current year. (c) For July 7, of the current year. (d) For November 29, of the current year.

Observation	Bubble	Plate Verniers			Remarks
		A	B	Mean	
1	Down	0° 00′	180° 00′		On reference stake
	"	18° 49′	198° 50′		Clockwise, to Polaris at elongation
2	Up	198° 51′	18° 50′		On Polaris at elongation
	"	180° 01′	0° 00′		On reference stake

5. (a) For the current date and your locality, compute the azimuth of a line from the transit to the reference point, for the following set of direct solar observations. While this is not a bona fide set of data for various localities and times, yet the computations may be carried through satisfactorily. (b) For Latitude 47° 25′ north, Longitude 122° 18′ west. (c) For Latitude 29° 11′ north, Longitude 82° 07′ west (d) For Latitude 27° 49 north, Longitude 97° 52′ west.

OBJECT	PLATE VERNIERS			VERTICAL ANGLE	WATCH (STANDARD) TIME
	A	*B*	Mean		
	Telescope	*Normal*			
Ref. stake	0° 00′	180° 00′			
Sun $^{\circ}\vert-$	23° 18′ (clockwise)	203° 18′		34° 03′	9 : 16
	Telescope	*Inverted*			
Sun $\vert_{\circ}$	203° 25′	23° 25′		35° 01′	9 : 19
Ref. stake	180° 01′	0° 01′			

NOTE. Of course, the best problems are the actual determination of a meridian (which may be checked) in the field. Both the solar and Polaris methods should be used. The Polaris method is particularly valuable, since it gives practice in using an instrument in the dark.

CHAPTER IX

STADIA SURVEYING

132. The Stadia Theory. The upper and lower of the three horizontal hairs seen in a transit telescope are the stadia hairs In Fig. 132, in the similar triangles FAB, FA_1B_1 and Fab, $ab = i =$ actual distance between stadia hairs. AB and A_1B_1 are rod intervals when the rod is y or y_1 distance from F.

$$\frac{f}{i} = \frac{y_1}{R_1} = \frac{y}{R}$$

$$y = \frac{f}{i}R \text{ and } y_1 = \frac{f}{i}R_1$$

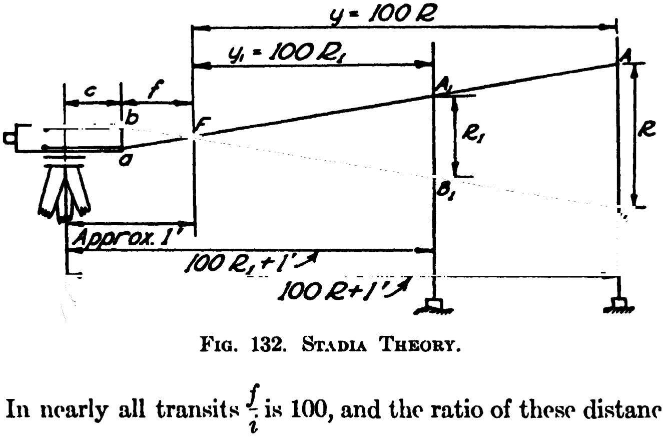


Fig. 132. Stadia Theory.

In nearly all transits $\frac{f}{i}$ is 100, and the ratio of these distances is 100 to 1. The respective horizontal distances or altitudes of the triangles for each position of the rod will be 100 times the rod interval as read between the stadia hairs at that point. This is shown on Fig. 132.

For all practical purposes, the distance $f + c$ measured from F backward to the center of the transit may be considered 1 foot. It is more precisely given on the transit box by the instrument maker; or it may be obtained by measuring two distances on the transit and adding them together, namely, from the objective lens to the cross-hairs (f) and from the objective lens to the center of the transit (c). The point F and the stadia hairs are each located at, or close to, a focus of the objective lens. In the modern internal focusing telescopes, this distance $f + c$ is practically zero.

In general, then, we may say that if the telescope is horizontal the horizontal distance from the rod to the center of the transit is

$$H = 100R + 1.^* \tag{132}$$

Stadia hairs are also furnished and used in plane table alidades, as described in § 217. They are frequently provided on the more precise levels.

133. Inclined Sight Formulas.

Usually the line of sight cannot be kept horizontal, but the telescope is inclined at some angle with the horizontal, α. The points E and O are at equal heights above the respective stakes. The rodman holds the rod vertically and it is therefore necessary to modify the preceding formula. In Fig. 133, the interval actually read on the stadia rod, AB or R, multiplied by $\cos \alpha$, gives CD (angles C and D being assumed 90°), or

$$CD = R \cos \alpha.$$

The slant distance from the instrument to the rod is S and

$$S = EO = 100CD + 1^* = 100R \cos \alpha + 1.^*$$

In right triangles OEG and $O'E'G'$, $S = S'$, $H = H'$ and $V = V'$. These sides with angle α give the following trigonometric relations:

$$H' = H = EO \cos \alpha,$$

$$H = S \cos \alpha = 100R \cos^2 \alpha + \cos \alpha,^* \tag{133}$$

$$V' = V = EO \sin \alpha$$

$$V = S \sin \alpha = 100R \cos \alpha \sin \alpha + \sin \alpha^*$$

$$V = 100R(\tfrac{1}{2} \sin 2\alpha) + \sin \alpha^* \tag{133a}$$

In Fig. 133 $E'O'$ is parallel to EO and the computed value of V is equal to the difference in elevation.

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* This term is omitted from the formula when internal focusing telescopes are used.

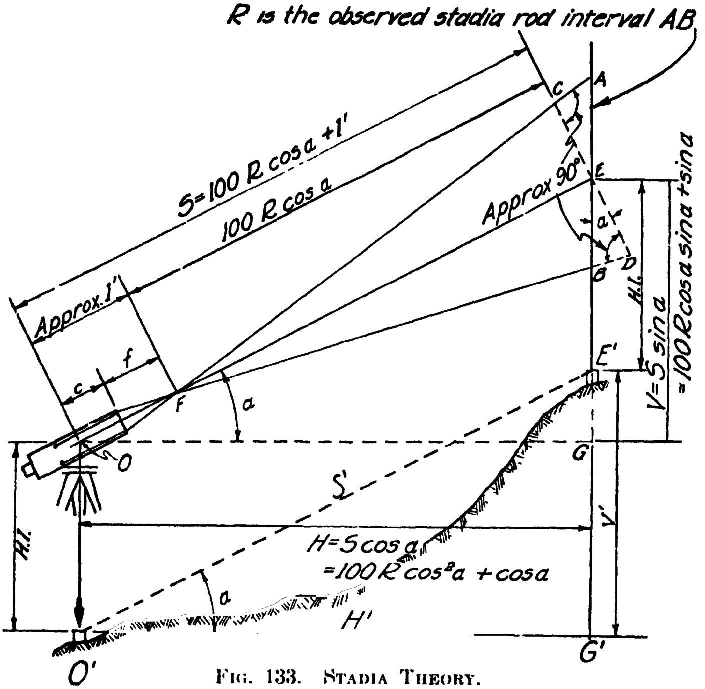


FIG. 133. STADIA THEORY.

134. Reduction of Field Notes. Horizontal distances H and differences in elevation V may be computed theoretically by substitution in formulas (133) and (133a), but this would be too slow and tedious for the many reductions necessary in practice. Stadia tables (Table V) and diagrams and slide rules will increase the speed, and will simplify these reductions.

Table V contains stadia coefficients of horizontal distance $(100)(\cos^2 \alpha)$ and difference in elevation $(100)(\frac{1}{2} \sin 2\alpha)$. The following procedure is helpful and the use of this table gives all needed precision:

(a) Add 0.01 foot to the observed rod interval R. Omit this 0.01 foot addition when internal focusing telescopes are used.

(b) Under the proper vertical angle in Table V, obtain the stadia coefficients.

(c) Multiply these coefficients by (a) to obtain the horizontal distance and the difference in elevation.

As an example illustrating the use of this table, assume the vertical angle to be 5° 32′ and the observed rod intercept to be 7.23 feet. The stadia coefficients for this angle are horizontal distance 99.07 and difference in elevation 9.60. The horizontal distance is $7.24 \times 99.07 = 717$ feet, and the difference in elevation is $7.24 \times 9.60 = 69.5$ feet.

Stadia diagrams showing differences in elevation and horizontal distances are a more rapid means for reducing stadia notes, but they are less precise. They may be obtained from supply houses, or they may be plotted to suit the particular conditions of use.

The most expedient, and sufficiently accurate, method for reducing stadia notes is by using the stadia slide rule.

135. Approximate Formulas. For most purposes approximate horizontal distances and differences in elevation are sufficient. Closely approximate results, correct to within two feet horizontally and 0.1 foot vertically, may be obtained without tables, diagrams, slide rules, or other aids, as suggested below. The orders of limitations for use of the formulas to obtain the foregoing degree of precision are:

α less than 2° and R less than 15 feet;
α less than 3° and R less than 8 feet;
α less than 4° and R less than 4 feet;
α less than 5° and R less than 3 feet.

Under such limitations and with R in feet and α in degrees, the horizontal distance in feet is

$$H = 100R, \qquad \text{approximately,} \tag{135}$$

and the difference in elevation in feet is

$$\begin{aligned} V &= 1\tfrac{3}{4}R\alpha, && \text{approximately,} \\ V &= 2R\alpha - \tfrac{1}{4}R\alpha, && \text{approximately.} \end{aligned} \tag{135a}$$

Similar formulas have been in use for many years,* with a rod graduated in yards and α in minutes; the difference in elevation in feet, for α less than 5°, is then approximately

$$V = \frac{7}{8}\left[\frac{(\text{Dist. in yds.})(\alpha \text{ in min.})}{1000}\right]. \tag{135b}$$

---

* Courtesy of Professor W. S. Williams, University of Missouri, and the Mississippi River Commission.

For distances up to 2000 feet and angles under 15°, the horizontal correction (to be subtracted from $100R + 1$), within one foot of its true value, is given, in feet and degrees, by the formula

(135c) Horizontal Correction $= 0.03R\alpha^2$, approximately.

The ordinary slide rule is convenient for solving formulas (135a), (135b), and (135c). The computation may sometimes be made mentally by using R to the nearest foot and α to the nearest degree, one larger and one smaller than the exact values.

Formulas (135a) and (135b) may be used for longer distances and larger vertical angles, provided the horizontal distance, rather than the distance read on the rod, is used in the formulas.

Many engineers do not consider the $(f + c)$ constant, of about 1 foot (almost zero for internal focusing instruments), since it is relatively small and since it tends to balance the horizontal correction.

136. Beaman Stadia Arc. An occasional transit or plane table alidade is equipped with a Beaman (or other) stadia arc, which is designed to minimize the computations of H and V. They are pictured and described in instrument makers' catalogs. It is doubtful if they are better than the ordinary vertical circle which is used with the methods of computation previously described.

137. Stadia Multiplying Constant. The spacing of the cross-hairs determines this constant. As previously mentioned, on most transits, levels, or plane table alidades they are spaced to give a multiplying constant of 100.0. It is difficult to place the cross-hairs precisely on the reticule or metal ring which holds them. It is possible that the first figure following the decimal point in 100.00 is not exact and it is probable that the second figure following the decimal point is not exact. For example, the stadia constant may be 99.8, or 100.04, or some such figure. If the cross-hairs have been replaced by other than a skilled workman, the error may be considerably larger.

The multiplying constant is best determined by carefully taping a level or nearly level line and marking points exactly 100, 200, 300, 500, 700, and 900 feet from the principal focus in front of the objective. The rod interval is then read on these successive points (preferably by setting targets on the rod at least three times at each distance) and the multiplying constant is then obtained by dividing *H* by *R*. The average value for the several distances is used. It is somewhat less satisfactory to use irregular distances in determining the constant.

The difference of the multiplying constant, say 99.3, from 100.0 is applied as a correction of 0.7 foot per 100 feet of observed distance, rather than used as a multiplier.

For the better grades of stadia work, the multiplying constant should be determined more precisely with targets. One target is fixed and the average of several settings of the second target, read to two or three decimal places, is obtained for each distance. Since the cross-hairs have an appreciable width on long shots, it is sometimes advisable to read the top or bottom of the hair, using the same edge on both upper and lower hairs.

In the foregoing determination of the stadia multiplying factor, it may rarely be desirable also to determine the upper and lower intervals separately, so that when necessary they may be used separately.

Refraction of the lower line of sight, when near the surface of the earth or of water, does not introduce appreciable error for ordinary stadia work but may do so for long sights or for precise measurements. Under such requirements, the multiplying constant should be determined with the same conditions of refraction that will occur during measurement; for example, at about the same time of day and with the lower line of sight at about tne same distance above the ground.

138. Field Procedure. In most stadia work, several *shots* or sets of readings are taken on different points from one setup of the transit. The usual field procedure is as follows:

1. After the transit is set up, the stadia rod is held alongside it and the vertical distance from the stake under the transit to the center of the horizontal axis is measured, or this distance may be measured with a regular tape or with a 5-foot tape, or by marking one leg of the tripod as described on page 121 (Fig. 105). Sometimes, although not necessarily, this reading is recorded in the notes as the H.I., for example H.I. = 4.9 feet.

2. The rodman holds the rod on some required point and the transitman sets the middle cross-hair approximately on the H.I. above mentioned. He then shifts the telescope through a small vertical angle until one stadia hair is on a full foot mark and reads and records the stadia interval between this full foot mark and the other cross-hair. It will be noted that the inclination of the telescope is not exactly the true vertical angle, but the effect of this error on the observed rod interval is negligible.

If, for example, the stadia interval was 8.91 feet, it may be recorded as 8.91, 8.92, 891, or 892. Perhaps 891 is most used. Each transitman has his preference, but the notes *must* state whether the extra 0.01 foot or the extra one foot is included. The extra 0.01 foot or the extra one foot is added to care for the constant $f + c$. It is not applied for internal focusing telescopes.

The half stadia intervals between the center hair and the upper and lower hairs may be read separately, by shifting the telescope vertically, if the sight is so long, say over 1300 feet, that the full stadia interval does not fall on the rod; or if part of the sight is obstructed. Should test show the half intervals to be exactly equal, either may be read and doubled.

3. The transitman then sets the middle cross-hair exactly on the H.I. and turns the plates slightly in azimuth, if necessary to bring the vertical cross-hair exactly on the rod. The rodman is signaled to proceed to the next point and while he is walking to it, the instrumentman reads and records the vertical angle and azimuth. The azimuth may be read approximately by sighting at the center of the stadia rod and estimating the azimuth to the nearest 5 minutes without reading the vernier. For more precise azimuth, the rod should be turned edgewise and carefully centered over the tack, or one prearranged edge of the rod may be held over the tack.

The precise extra vernier bubble mentioned on page 122 is a decided aid in precisely determining the vertical angle. In much stadia work, however, this added precision is unnecessary.

The horizontal distance, H, and the vertical distance, V, are computed from these field data, as an office or field computation, and the azimuth of the point is read on the transit plates.

These give the polar coordinates by which the point may be plotted on a map, and also the elevation of the point. The foregoing procedure determines azimuth, distance, and elevation. If only part of this information is required, as for example only *H* and not *V*, the procedure is correspondingly simplified.

This procedure is repeated on each of the several points regarding which data are desired.

139. Vertical Angles to Any Point on the Rod. It is occasionally necessary to sight the center cross-hair at some point on the rod other than the H.I. previously mentioned, for example, when the H.I. is obscured by brush. The difference in elevation as usually computed (44′) must then be increased or decreased by the difference between the rod reading on the center horizontal cross-hair and the H.I. It is convenient to sight at some exact number of feet above or below the H.I. This process is illustrated by Fig. 139.

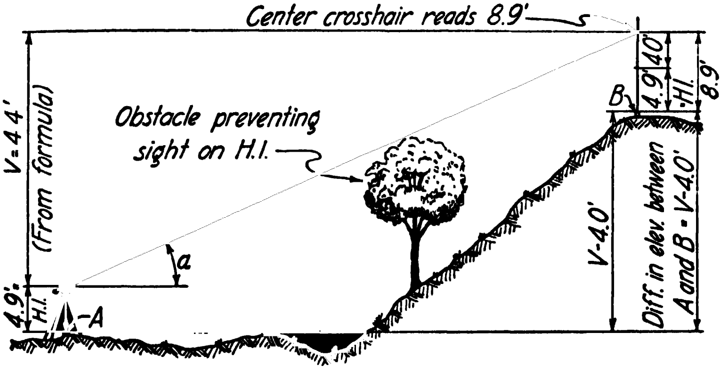
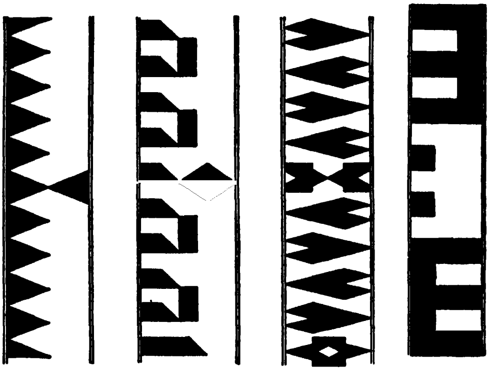

Center crosshair reads 8.9'
Obstacle preventing sight on H.I.
V=4.4'
(From formula)
a
4.9'
H.I.
A
B
4.9
4.0'
H.I.
8.9'
V-4.0'
Diff. in elev. between A and B = V-4.0'

FIG. 139.

140. Stadia Rods. The ordinary level rod with graduations to hundredths of a foot will serve for stadia work where the maximum length of sight is 300 or 400 feet. Such short shots occasionally suffice, particularly for side distances on line surveys (Fig. 201a). For the longer sights of the usual stadia work it is necessary to use a coarsely graduated rod on which no attempt is made to show the separate hundredths of a foot.

It will be noted on Fig. 140 that most of these major graduations are only to tenths of a foot, yard, or meter.

Many instrumentmen prefer that the rods be graduated to decimals of a yard or meter, rather than of a foot, since such graduations are about three times the size of the decimals of a foot, and are therefore easier to read.



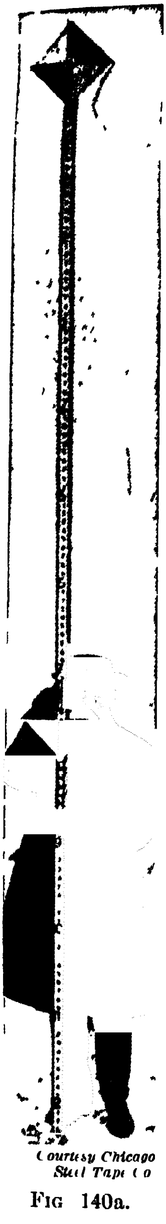
Courtesy of Keuffel and Esser Co

FIG. 140. STADIA GRADUATIONS.

Where the rods show only tenths of a foot, yard, or meter, the graduation is coarse and is used primarily for long shots. It will be noted that most such rods provide sloping lines upon which hundredths may be easily estimated, and the rod consequently may be used for either short or long shots.

Figure 140 shows the more common graduations. Other types are available through instrument houses and a great variety have been made by individual engineers. The rods are usually made of a single stick 12 to 15 feet long, but are occasionally found with slip or hinge joints. Stadia rod ribbons are to be had from instrument supply houses.

At one time some engineers graduated the stadia rod to correspond exactly with the interval between cross-hairs on a particular instrument. Hence the graduations were not exactly one foot, yard, or meter apart. This practice has been abandoned, because such a rod could not be used with other instruments, or for leveling, and considerable confusion frequently resulted.



Courtesy Chicago Steel Tape Co

FIG 140a.

Where targets are used, the stadia rod is sometimes graduated from the top downward. A fixed target is clamped at zero at the top of the rod and a movable target is set and read by the rodman, lower down on the rod, as in Fig. 140a.

141. Advantages of Stadia. In the past, the stadia method has been largely used in mapping, as described in §§ 216 to 217. It is increasingly used for mapping, traversing, hydrographic surveys, and other not-too-precise determinations of horizontal and vertical distances.

Stadia measurement offers a more rapid and a somewhat less precise method than does taping. In rough country where the taping must be up and down hill, while the stadia readings may be taken across ravines and gullies, the precision of stadia compares favorably with that of taping. This is particularly true in traversing where the errors of the stadia-rod intervals tend to compensate. Careful stadia measurement across a deep depression such as a canyon is nearly as precise as the simpler triangulation described on page 118. Taped or triangulated measurements may frequently be checked to advantage by stadia.

Stadia elevations are quickly, but only approximately, obtained, as in Fig. 133. If the elevations are measured while the stadia and azimuth are being used for distance and direction, only a neglıgible cost is incurred for securing elevation data.

The stadia method is less satisfactory in timber or underbrush since extensive clearing is required. It is particularly difficult, almost impossible, to clear a line of sight through the tops of trees where the line of sight is a considerable distance above the ground. Stadia may here be used by picking natural openings in the foliage and sighting through these openings. Occasionally it is possible to move the instrument rather than the rod; for example, the rod may be left standing in a clearing in the valley while the transit is moved along the adjacent ridges and stadia readings and needle bearings are taken on the fixed rod.

Much of the mapping detail of plane-table surveys is obtained by the stadia method.

142. Common Mistakes. 1. Reading the interval from the center hair to one of the outer hairs, instead of between the two outer hairs. This is to be avoided by special care in observing, by estimating the distance by eye, or by reading a half-interval as a check.

2. Mistaking a fence wire or a telephone wire for a stadia hair.

3. Mistaking or misrecording a plus for a minus vertical angle, or vice versa.

4. Miscounting the yards, feet, or meters in the rod intercept. This is to be avoided as in 1.

143. Precision of Stadia Distances. For any one stadia shot by ordinary methods, precision is limited by the smallest graduation which may be read or estimated on the rod. At from 200 to 400 feet, it is possible, without targets, to read the rod interval on the rod to hundredths of a foot, corresponding to one foot in distance; this gives a precision of 1/200 to 1/400. At 600 to 1200 feet, it is possible to read the nearest hundredth of a yard, or of a meter, which is about 0.03 foot or 3 feet of distance; this gives approximately the same precision as previously stated. This, then, is the precision to be expected in the usual single stadia measurement.

With favorable visibility and using targets, perhaps taking the mean of several target readings, we can raise this precision to the order of 1/1000. Of course the multiplying constant must be carefully and precisely determined.

A higher degree of precision is obtained with small vertical angles than with large vertical angles.

Since the precision is largely dependent on ability to read the precise rod interval, greater precision will result from taking the mean of several readings, since they will tend to compensate according to the theory of least squares. If, for example, a rod interval is read nine times, then the probable error of the mean will be approximately $1/\sqrt{9}$ or 1/3 of the probable error of a single measurement. Such repetition will of course also tend to prevent mistakes, readings that are evidently erroneous being disregarded. Any error due to an erroneous multiplying constant will of course be present in its full amount in the final result.

In a stadia traverse where nine successive distances, each different, are measured, there will remain in the total distance an approximate probable error (due to lack of precision in reading the rod interval) of $\sqrt{9}$, or three, times the probable error of a single measurement. On the contrary, the error due to an erroneous multiplying constant will occur with the same algebraic sign in each measurement and will therefore accumulate, in the total, to 9 times the error of a single measurement.

It is apparent from the preceding discussion that correct determination of the multiplying constant is essential to prevent a cumulative error, whereas the errors of reading the rod interval tend to balance or compensate.

It is of course essential that the rod be held truly vertical. The error due to an inclined rod may be plus or minus and will, to some extent, accumulate. It increases with the vertical angle of the transit, hence *the rodman must exercise unusual care in plumbing the rod on large vertical angle sights*, using a rod level, or a plumb bob, if necessary.

The telescope for stadia measurements should be somewhat better than for most other purposes. A magnifying power of from 20 to 30 diameters is needed; and a sharp, clear image of the rod is essential. The vernier bubble mentioned on page 121 facilitates the work.

144. Precision in Stadia Elevations. The precision in stadia elevations is controlled by the following elements.

1. The various factors affecting horizontal distance as treated throughout this chapter; any error in distance gives rise to a corresponding error in elevation.

2. The errors in vertical angles; these generally tend (for

traverses) to compensate. Properly reading vertical angles forward and backward on traverses eliminates all errors of adjustments, curvature of the earth, and constant refraction.

The ordinary transit reads to the nearest minute on the vertical circle vernier but this circle is not large or well protected and it is probable that the usual error in measuring a vertical angle on an old transit is of the order of 1′ to 2′. At 1000 feet this will produce an error in elevation of 0.3 to 0.6 foot. Errors in large vertical angles cause greater errors than in small ones.

3. Errors due to curvature and refraction, see page 87.

In stadia traversing without large vertical angles, where forward and backward readings are taken on each line, it is possible with ordinary care to carry forward elevations with an error on the order of 1/2 foot times the square root of the distance in miles.

PROBLEMS

1. (a) Compute the difference in elevation and the horizontal distance by formulas (133) and (133a). Use a multiplying constant of 100.0, and $f + c$ as 1 foot. Answers are shown in italics.

Point	Rod Interval	Vertical Angle	Difference in Elevation	Horizontal Distance
1	3.72	1° 09′	*7.5*	*372.8*
2	8.47	6° 12′	*91.0*	*838.1*
3	12.91	20° 18′	*420.4*	*1136.6*

(b) For 4.13 and 1° 21′; 9.47 and 8° 14′; and 12.05 and 24° 17′.
(c) For 3.17 and 0° 39′; 8.63 and 7° 47′; and 11.86 and 28° 42′.
(d) For 2.89 and 0° 53′; 7.54 and 7° 01′; and 12.06 and 26° 13′.

2. The same as 1 but use Table V. *Ans.:* 1. 7.5, 372.8
2. 91.1, 838.1
3. 420.4, 1136.4

3. The same as 1 but use stadia diagrams.

4. The same as 1 but use formulas (135) and (135a), and note errors.
Ans.: 1. 7.5, 372
2. 91.9, 847
3. 458.6, 1291

5. The same as 1 but use stadia slide rule.

6. The same as 1 but use formula (135c) for horizontal distance only.
Ans.: 1. 372.9
2. 837.2
3. 1131.4

CHAPTER X

TOPOGRAPHY

145. Definition. Topography may be broadly defined as the shape or configuration of the earth's surface, together with the changes that have been made by man. In engineering projects extending over a considerable area or for a considerable distance, it is necessary for the engineer to fit his proposed construction onto the topography. To accomplish this, it is usually necessary for him to prepare and work with maps showing the topographical features.

146. Hachures. A generation ago it was not uncommon to see topography represented on the map by hachures, which were short lines of varying widths drawn in the direction of greatest slope. On level stretches of territory the hachures were omitted. By their shading effect, they convey to the layman some relative idea of the topography but they do not in any sense give actual elevations. Hachures were used on German military maps at the beginning of the World War but were shortly replaced by more definite contour maps described in the next section.

147. Contours. A *contour* is an imaginary line on the ground, every point of which has the same elevation. The shore line of a lake or of a body of still water is a contour. If the water is raised or lowered, the new position of the shore line will be another contour.

When *contour lines* are drawn on maps, it is apparent that wherever a pencil is touched upon a contour line, the elevation of the point may be read from the map. In such a use of maps the ground is assumed to slope uniformly between adjacent contours and it is consequently possible to estimate closely, by proportion, the elevation of any point on the map.

In examining contours on the ground, or in the construction and use of maps, the rivers, streams, and drainage lines, together

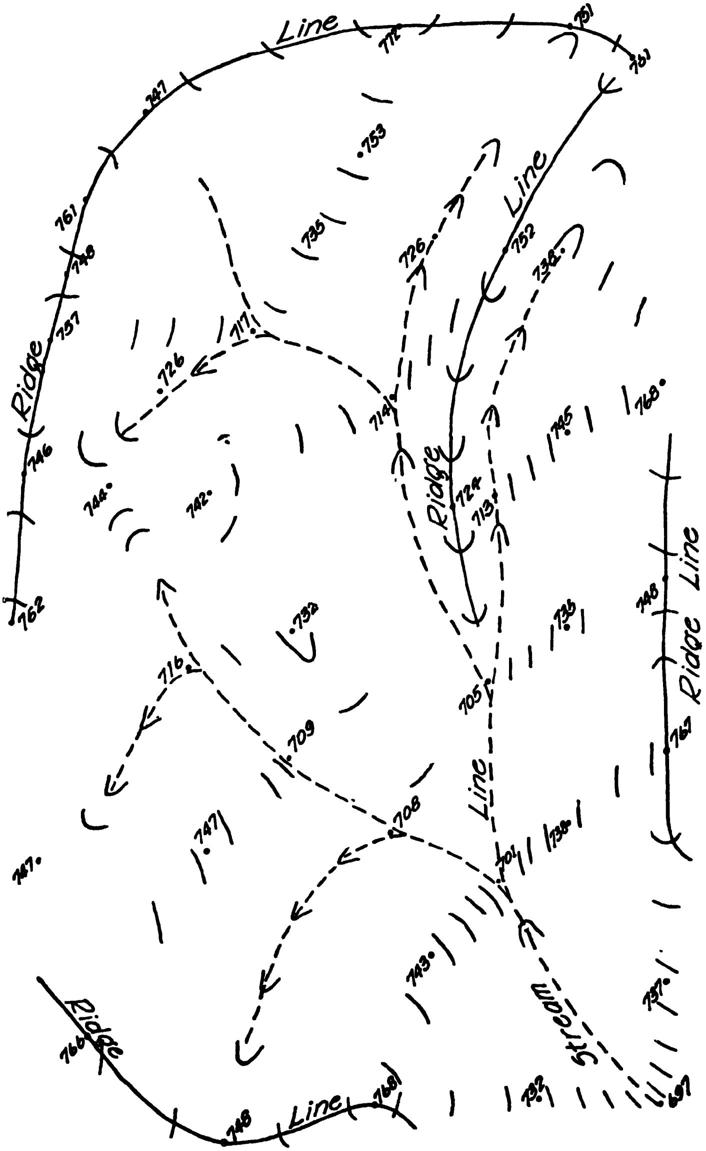


FIG. 147. FIRST STEP IN MAPPING CONTOURS

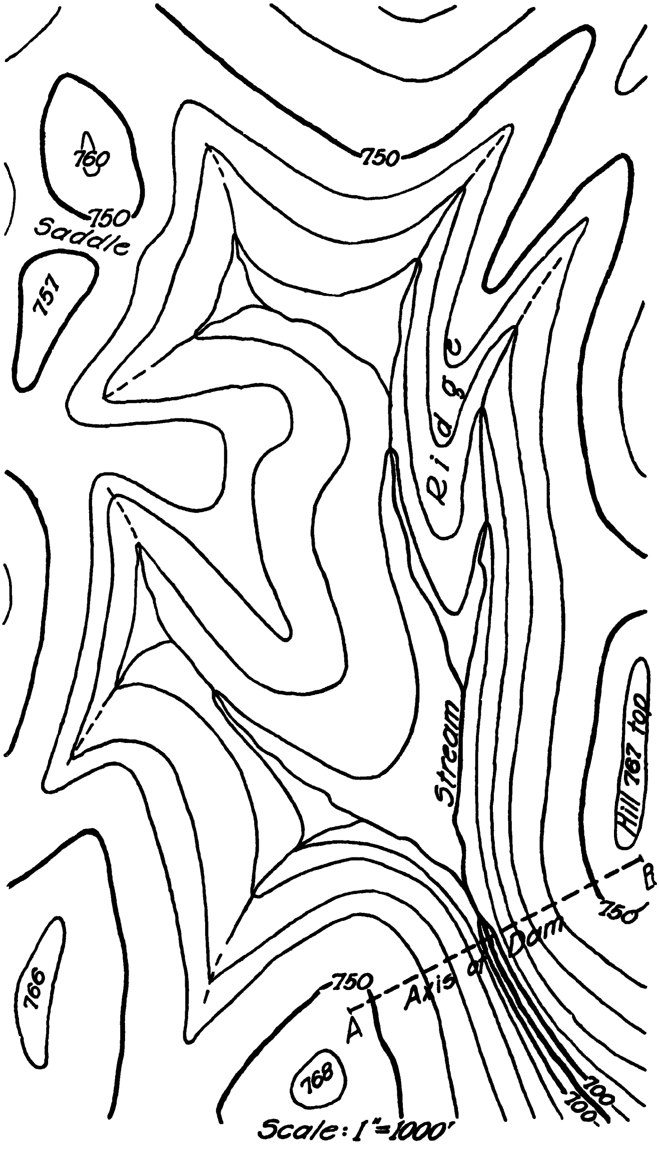


FIG. 147a. CONTOURS.

with the summits and ridges, are the controlling features. Once these are fixed in mind and on the map and their elevations are known, it becomes a comparatively simple and routine matter to sketch the contours on the map with any required degree of precision. The number of additional points of definite location and elevation which may be required depends on the precision necessary on the map. These steps are shown in Figs. 147 and 147a.

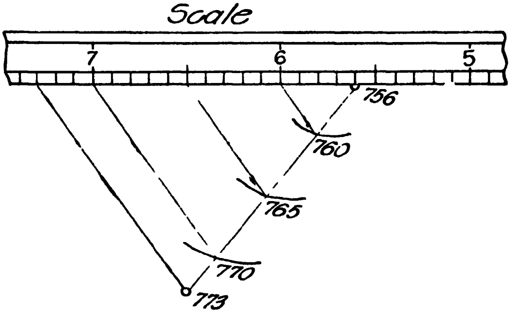


Fig. 147b.

Contours are usually interpolated by eye between known elevations on the map. Where greater precision is necessary, any convenient decimal (Engineers') scale may be used, as shown in Fig. 147b.

Contours are close together on steep slopes and widely spaced on gentle slopes, equally spaced and parallel on a plane, at right angles to valley and ridge lines, perpendicular to the line of steepest slope, superimposed on one another only on some vertical surface, and only rarely cross one another, as for example under an overhanging cliff.

Contours always close, either within or beyond the limits of the map. Closed contour lines on a map represent a summit, unless they are shown with short lines drawn inside of, and perpendicular to, them. In that case, they represent a depression. The vertical distances between contours is called the *contour interval.* Contour lines are best drawn with a *contour pen.* Each fifth line is usually made somewhat heavier than the others.

Some persons seem naturally to sense, or to *draw in,* contours and general topography better than others, although sufficient practice and effort will enable any engineer to do satisfactory work. The field and office work involved in the preparation of topographic contour maps is treated in Chapters XIII and XIV.

148. Use of Topographic Maps. Such maps are essential to the modern engineer, geologist, and military commander. The engineer formerly located his new project — a railroad, canal, highway, pipe line, reservoir, industrial plant, or whatever it might be — directly on the ground without the aid of topographic maps. This type of location was more of an art than a science. The locating engineer of that day was necessarily a man of natural topographic aptitude and extensive experience. The project was surveyed on the ground, and if it did not then seem to be satisfactory, it was relocated in whole or in part; this system constituted a cut and try method on the ground. The engineer was usually hurried by the owners and it was quite possible to overlook a better location or to accept a less satisfactory one, rather than to incur the considerable delay and expense necessary to revise the surveys on the ground. Such inadequate surveys and projects seemed reasonably satisfactory for many years, and their defects only became really apparent to the owners as competitors adopted the better locations which had been overlooked by the earlier engineers.

The modern practice is to look over all possible locations roughly in a reconnaissance, and then to make surveys for preparing a topographic map of all of the promising territory. After this, the design of the location is made in the office on the topographic map, where one man can try out several alternative plans on the map until he finds the best one.

This map or paper location offers several advantages over the older ground location. One man in the office can compare alternative locations more cheaply and much more rapidly than can be done by large survey parties in the field. Less topographic aptitude, and more of the modern technical attitude of mind, is required.

The mind may be concentrated better on the essential engineering and economic features of the project in the office than in the field, where the detail and bustle of the survey organization distract thought from the more important matters

Finally, modern industry is becoming more and more concentrated into large corporations or units, with their more highly skilled officers and specialists in the general offices. While the maps and projection of designs are frequently made in the field offices, nevertheless, the maps may be easily carried or mailed into the general office and the entire matter quickly threshed out with all of the interested general officers. Changes may then be decided upon and made from the topographic map without the almost prohibitive loss of time that is involved in having these officers travel hundreds or thousands of miles to the project, and then to wait for alternate surveys to be made on the ground.

149. Maps Available. Mapping is expensive, and the engineer first avails himself of those maps that are already made. The Federal Government, through the United States Geological Survey, has mapped about half of the United States on scales of 1 to 62,500 or 1 to 125,000. This is approximately 1″ = 1 mile or 1″ = 2 miles. More recently, larger scales have been used. The contour interval varies from 10′ in flat country to 100′ in mountainous country. These surveys are gradually being extended. Figure 57a shows that part of the United States covered up to 1948.

As explained in Chapter XII, the Federal Government has sectionalized most of the United States except the original thirteen colonies, Kentucky, Tennessee, and Texas, and land still belonging to the government, such as forest and Indian reservations. The sectionalization covers that portion of the United States shown in Fig 179. Township plats, similar to Fig. 181a may be secured for all of this territory on the scale of 1 inch = 1/2 mile. While no contours are shown, the streams, drainage lines, and occasional notes of hills will give some idea of topography.

The Post Office Department has combined these township plats into larger sheets and has somewhat improved them, particularly as to roads, so that the larger sheets may be used in connection with their rural mail delivery service.

Various other branches of Federal and State governments have prepared less extensive maps for special purposes, frequently on larger scales. Drainage and irrigation districts as well as other minor governmental units have mapped large isolated territories. Maps of airways, railways, highways, transmission lines, and so on, are occasionally obtainable. Good maps are usually available covering cities, towns, and thickly-settled communities.

A *Digest of Geodetic Publications for the State of*———— gives triangulation, leveling, and magnetic compass information of the United States Coast and Geodetic Survey, by states.

The organizations of the Federal Government engaged in mapping and surveying are:

Army Map Service, War Department;
U. S. Coast and Geodetic Survey, Department of Commerce;
U. S. Geological Survey, Department of the Interior;
General Land Office, Department of the Interior;
Division of Topography, Post Office Department;
Bureau of Soils, Department of Agriculture;
Bureau of Reclamation, Department of the Interior;
Bureau of Public Roads, Department of Agriculture;
Office of Indian Affairs, Department of the Interior;
Mississippi River Commission, War Department;
International (U. S.-Canada) Boundary Comm., Department of State;
Forest Service, Department of Agriculture;
U. S. Hydrographic Office, Navy Department;
Military Intelligence Department, General Staff, War Department;
Federal Power Commission;
Air Service, War Department;
Bureau of Aeronautics, Navy Department;
U. S. Lake Survey, War Department;
Aeronautics Branch, Department of Commerce;
Division of Maps, Library of Congress.

Engineers working in a particular territory are usually familiar with the available maps. County, city, or state engineers, as well as the Bureau of Mapping* in Washington, D. C., can furnish maps or mapping advice. Aerial surveys, made under the Agricultural Adjustment Administration, provide aerial photographic maps of most of the settled portions of the country.

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* Office of Map Information, U. S. Geological Survey, Department of Interior, Washington 25, D. C.

The Surveying and Mapping Division of the American Society of Civil Engineers publishes manuals of specifications and suggestions for mapping and surveying.

An effort should always be made to get the best available maps before any extensive mapping or engineering project is undertaken. The maps already made are usually on so small a scale, or otherwise so unsuitable, as to serve only in the reconnaissance stage of the work for rough estimates of cost and feasibility. It is usually necessary to make surveys and larger scale maps for the detail planning.

150. Specific Information from Contour Maps. The profile of Fig. 150 was made from the contour map of Fig. 147a along the line *AB*. After the line is drawn along the map and the stations scaled onto the line, the elevations of each station and of any necessary intermediate elevations may be read by interpolation between contours.

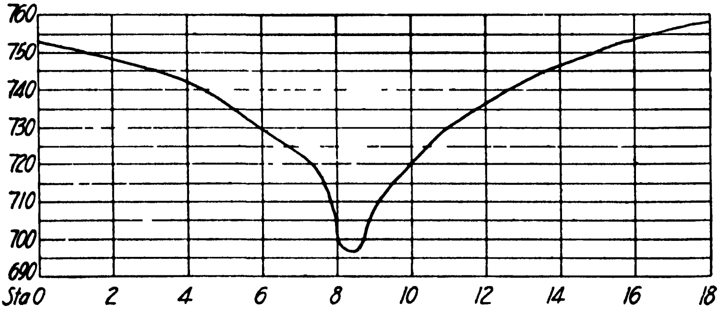


Fig. 150. Profile along Axis of Dam.

Some engineers prefer to scale the station and plus at which each contour crosses the line, and then to plot each contour; the plotted points varying in elevation from one another by the contour interval. This latter method is somewhat more precise than the former, although the maps seldom justify such great precision.

It is evident that map profiles may be made much more rapidly than field profiles, and that they will serve equally well for many purposes, even though they are not quite so precise.

If the project under consideration involves an area rather than a line survey, for example, a large building, industrial plant, or stadium, the plan of the structure may be sketched tentatively on the map and the necessary elevation data may be obtained by interpolation between contours. Estimates of quantities of excavation and embankment may be made as outlined for borrow pits in § 209.

In fact, since the topographic map is a small scale representation of the ground, all of the required topographic information which could be obtained by a survey on the ground, may be read and scaled approximately from the map. For example, in projects involving the storage of water (Fig. 147a), information regarding the proposed dam, the quantity of water stored, and the drainage area may be taken from the map

While the precision of contour maps varies, in general it may be assumed that if the pencil is placed at any point on a topographic map, the elevation of the ground will correspond to the elevation interpolated from the contours, to within less than 1/2 of the contour interval.

PROBLEMS

1. Sketch in contours on Fig. 147.

2. (a) Referring to the map of Fig. 147a: *AB* is the center line of an earth dam to impound water, top width 20 feet; earth slope on water side 3 to 1, and on dry side 2 to 1. A concrete core wall 2 feet thick extends from 15 feet below ground to the top of dam.

Compute and tabulate the quantities of earthwork and of concrete for a dam with a top elevation of 735 feet. Compute and tabulate the number of acre-feet of water stored, with the water 5 feet below the top of the dam.

With earthwork at 30 cents per cubic yard and concrete at $20 per cubic yard, what is the cost per acre-foot of water stored?

(b) The same as (a) except with top of dam at 740 feet.

(c) The same as (a) except with top of dam at 745 feet.

(d) The same as (a) except with top of dam at 750 feet.

Answers to 2(a): 31,487 cu. yds. earth, $9446;
1564 cu. yds. concrete, $31,280;
1835 acre-feet;
$22.20 per acre-foot

CHAPTER XI

TRIANGULATION

151. Definition. *Triangulation* may be defined as that branch of surveying which utilizes triangles and which involves a minimum of distance measurement and a maximum of angular measurement in the determination of distances and directions over the earth's surface.

152. Minor Triangulation. Minor triangulation is sometimes necessary in engineering surveys where a distance across a wide body of water, a deep canyon, or a ravine must be determined. Here direct measurement of distances by taping is impracticable. Stadia measurement may be used, but this is frequently not sufficiently precise and is awkward for distances of over a half mile. Some such triangles as are shown in Figs. 152 and 104, with a base line measured in a convenient place, will permit the calculation of the inaccessible distance. This type of triangulation is sometimes necessary in surveying a transit line around an obstacle, as described on page 118.

In Fig. 152, and on page 130 of the Appendix, if only the horizontal projections of AC and/or BC are required, the base line AB is carefully measured and the horizontal angles at A and B are measured with a transit and repeated. Considerable precision is sometimes necessary. Then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

A convenient tabulation of logarithms for the solution of a triangle by the sine law follows. The angles A, B, and C, and the side c are known. The sides a and b are required.

(1) $\log c$ (1899.20) = 3.2785707
(2) $\operatorname{colog} \sin C$ (57° 58′ 21″) = 0.0717099
(3) $\log \sin A$ (83° 40′ 09″) = 9.9973435
(4) $\log \sin B$ (38° 21′ 30″) = 9.7927962

$$(1) + (2) + (3) = \log\left[\frac{c \sin A}{\sin C}\right] = \log a = 3.3476241$$

$$(1) + (2) + (4) = \log\left[\frac{c \sin B}{\sin C}\right] = \log b = 3.1430768$$

$$a = 2226.51$$
$$b = 1390.20$$

If the differences in elevation from C to A, and from C to B, are required, then the vertical angles α and β are measured;

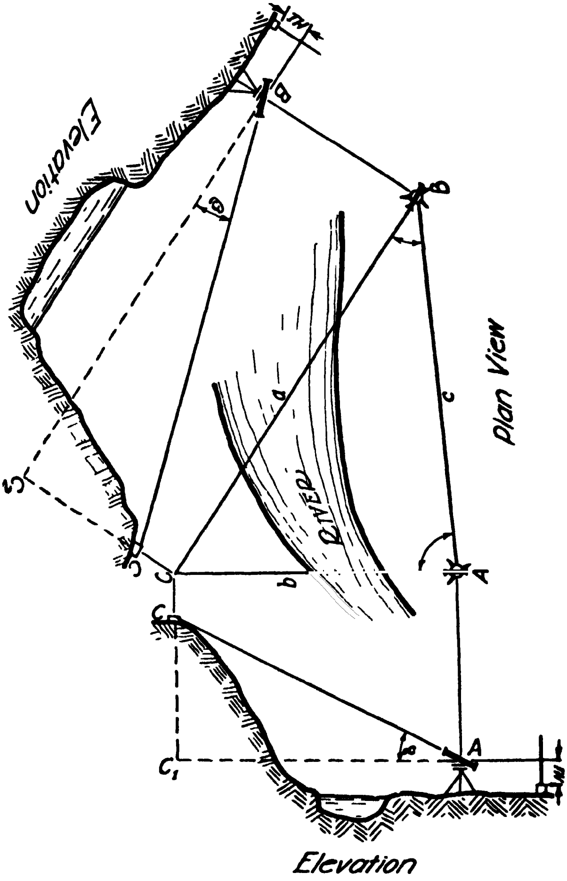


FIG. 152.

and from the above computed horizontal projections *b* and (AC_1 and BC_2) the vertical distances CC_1 and CC_2 are computed, using the tangents of the respective vertical angles.

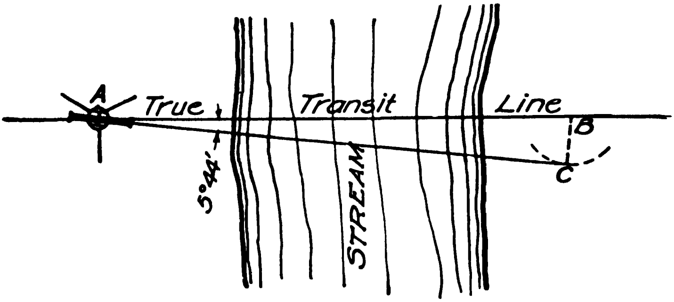


Fig. 152a.

An approximate and rapid triangulation along a line survey is shown in Fig. 152a. The method depends on the fact that the sine of 5° 44′ is 0.09990 or very nearly 1/10. Under some circumstances, it is better to use an angle of 11° 32′ whose sine is 0.19994 or very nearly 1/5. The line *AB* is approximately 10, or 5, times the taped *swing offset BC* which is the shortest swung measurement to the transit line of sight *AC*. Lack of precision in this method is largely due to the poorly shaped triangle.

153. Triangulation Systems. Several systems or nets are shown in Fig. 153. The system (a) is used for long narrow triangulation of relatively poor precision, such as along a narrow body of water or valley; (b) gives better precision and covers a wider area; while (c) with overlapping triangles, or quadrilaterals, is generally used for long and fairly narrow surveys of a high degree of precision.

Usually at least two base lines are taped; these are located near the ends of the system. The lengths of sides of the various triangles are then computed and checked on some common side, preferably midway between the bases.

When the taped bases are shorter than the sides of the main triangulation system they must be *expanded* onto a main side by a series of triangles called a *base net*, as shown in Fig. 158.

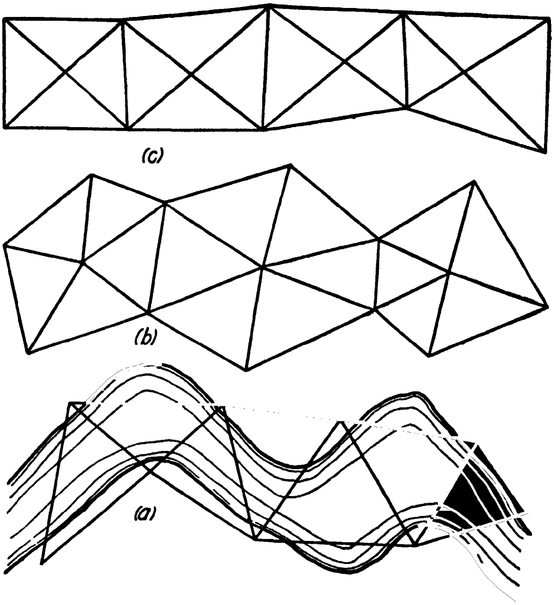


FIG. 153. TRIANGULATION SYSTEMS.

154. Shape of Triangles. An approximately equilateral triangle is ideal for triangulation. Angles of less than 30° or more than 120° opposite to known or required sides will tend to decrease the precision of the system, and should be avoided. The triangles are computed from the sine law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

where a, b, and c are the sides of the triangle and A, B, and C are the angles opposite to the respective sides.

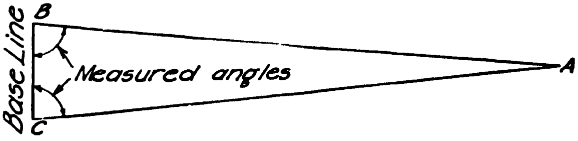


FIG. 154.

In Fig. 154 it is apparent that in drawing the figure and in aling the angles and distances, the intersection established at *A* is not precise because the angle *A* is too acute. The lengths *AB* and *AC* cannot be determined accurately by either scaling or computation.

It is possible here to generalize somewhat the foregoing statements. In the consideration of geometric and trigonometric problems of all natures it is easier to grasp the involved interrelations by drawing the figures to scale, or at least roughly to scale, or, in simple cases, by considering the figures mentally Any relations which develop in drawing the figure will exist similarly in the computation. For example, if enough data are given to draw the figure, then enough are given to compute the figure; or, as another example, if the figure cannot be drawn with precision, then it cannot be computed with precision.

155. Computations. In triangulation computations it is customary first to test the measured angles to see if they satisfy geometric and trigonometric requirements. The observed angles, which, of course, do not meet these conditions exactly, are adjusted:

(a) Approximately for ordinary engineering usage or for simple triangulation of low precision.

(b) By the theory of least squares for more elaborate and precise triangulation.

These adjustments are described in books on higher surveying or geodetic surveying. The computations are then carried forward by the sine law mentioned in the preceding section to any required degree of precision. For precise surveys over very large areas, account must be taken of the spherical excess over 180° of the sum of the angles of a triangle (1 second for a triangle having 76 square miles of area), but such conditions are seldom encountered in engineering surveys.

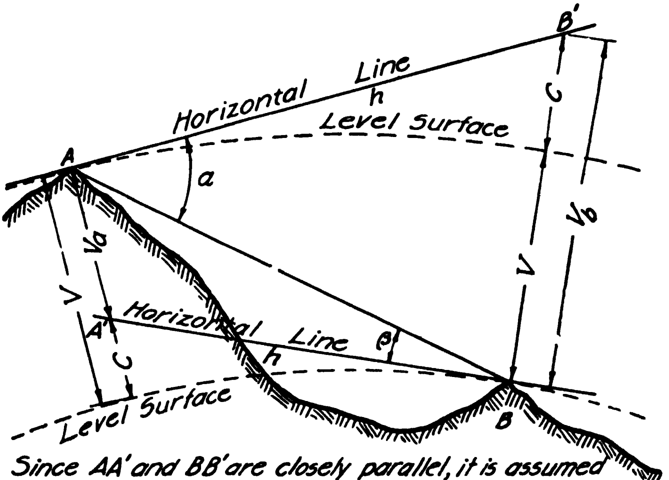
156. Measurement of Base Lines and Angles. This type of work is best left to experienced men. Base lines are measured as mentioned in Chapter II, to any required degree of precision. Angles are repeated four to twelve times with transits or *repeating theodolites* whose verniers may read to within from 1 minute to 10 seconds. It is customary to repeat the angle about six times with the telescope normal and then to use six repetitions in measuring the explement (360° – the angle) of the angle with the telescope upside down, as mentioned on page 106, and as shown in Fig 94 Frequently several such sets are measured Such precise triangulation is described in works on higher surveying

Courtesy of A Lietz Co

FIG. 156 TRIANGULATION

Much precise angle measurement is made with a *direction instrument* which measures angles to perhaps one or two seconds by means of a micrometer screw instead of a vernier Such instruments are not arranged so that the angles may be repeated as with a transit They are seldom used except in Federal Government work

157. Elevations by Triangulation. Figure 157 shows a method of obtaining elevations in connection with horizontal triangulation In any triangulation work elevations may be computed if the vertical angles are measured in addition to the horizontal angles. The elevations so computed are limited in accuracy by the precision of the vertical angles. For example, an error of 0° 01′ in the vertical angle would make an error of about 1½ feet in the elevation of a point one mile distant. Correction must be made for curvature and refraction as mentioned on page 87, for single observations.



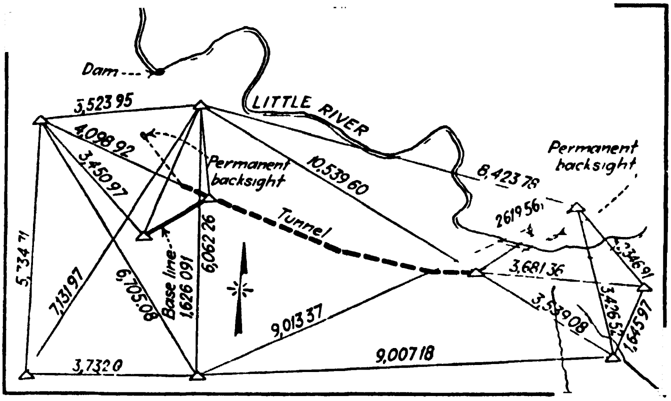
Since AA' and BB' are closely parallel, it is assumed that AB'B and AA'B are 90°, and that AB' = BA' = h.

$$\begin{aligned} \therefore\ V &= h\tan\alpha - C \qquad \text{where } C = \text{correction for curvature and refraction.}\\ \text{and}\ V &= h\tan\beta + C\\ 2V &= h(\tan\alpha + \tan\beta)\\ \text{or}\ V &= \frac{h(\tan\alpha + \tan\beta)}{2} \end{aligned}$$

Fig. 157.

Errors of curvature, of adjustment, and of refraction are eliminated if the mean tangent of α and β, measured simultaneously at both ends of the line is used. If the double measurements are not made simultaneously, then errors due to change in refraction remain. If the observations are taken when the refraction is nearly constant, as on cloudy days, or at night or from 2 to 4 P.M. on sunny days, it is unnecessary that they be made simultaneously.

158. Construction Triangulation. Triangulation is used in the surveying and construction of tunnels, bridge piers, subways, and the like, where distances with a precision on the order of



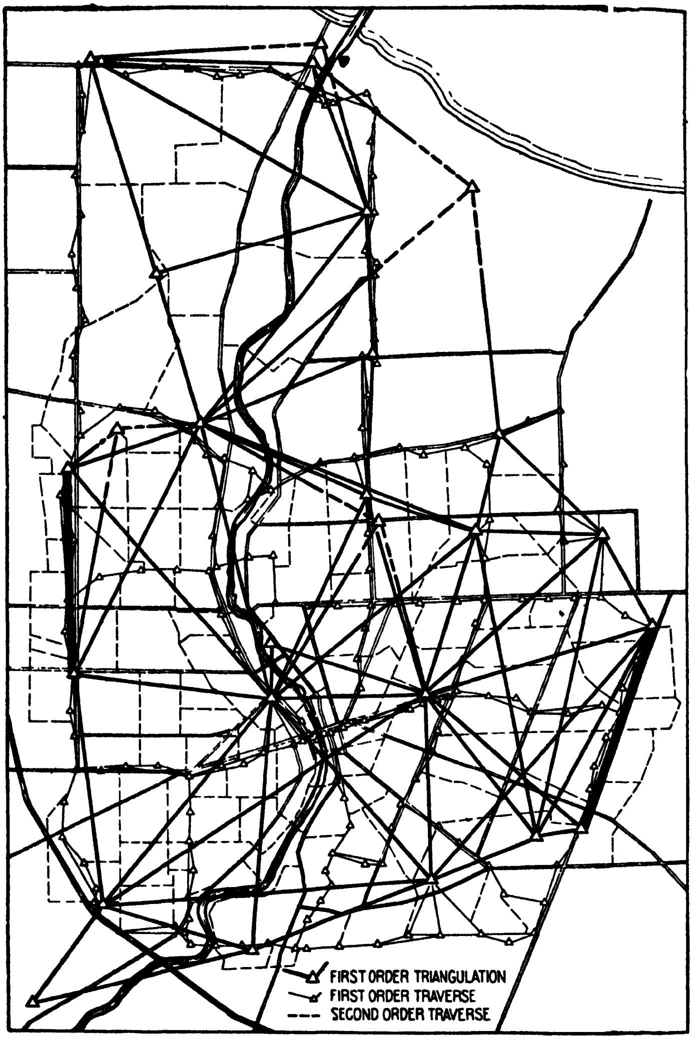
From Engineering News-Record, Courtesy of McGraw-Hill Co

FIG. 158 EXTENSIVE TRIANGULATION SYSTEM CHECKED TRAVERSE LINE ESTABLISHING CONTROL OF PRESSURE TUNNEL.

1 25,000 are required and direct measurement is awkward or impossible. Figure 158 is an example of such a triangulation system.

159. City Triangulation. Because of the high degree of precision of surveys required for expensive city property, and for the further reason that traversing in cities is somewhat awkward on account of traffic, and that high buildings or structures are available for triangulation stations, triangulation is frequently used as the basis, or skeleton, of city surveys. A recent example is that of Fig. 159.

160. Hydrographic Surveys. Simple triangulation is generally nployed in sounding wide bodies of water, such as rivers, kes, and along the sea coast. The man taking the soundings usually in a boat. The position of the boat at the moment sounding is frequently obtained by two transits on shore at opposite ends of a base line as in Fig. 160, or by the use of extant in the boat. Many modifications of this method are d.



Courtesy of U. S. Coast and Geodetic Survey

FIG. 159. SCHEME OF FIRST AND SECOND ORDER CONTROL FOR A CITY SURVEY.

Triangulation frequently offers the most rapid and precise method of conducting shore surveys along streams, lakes, or inner passages along the sea coast. If such surveys are made as traverses (Fig. 153), they are difficult and expensive since they must follow the winding shore line, frequently over rough or timbered country. As may be inferred from Fig. 153, a triangulation party with a motor boat will accomplish work rapidly.

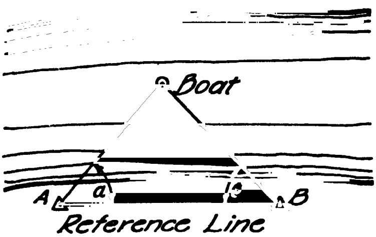


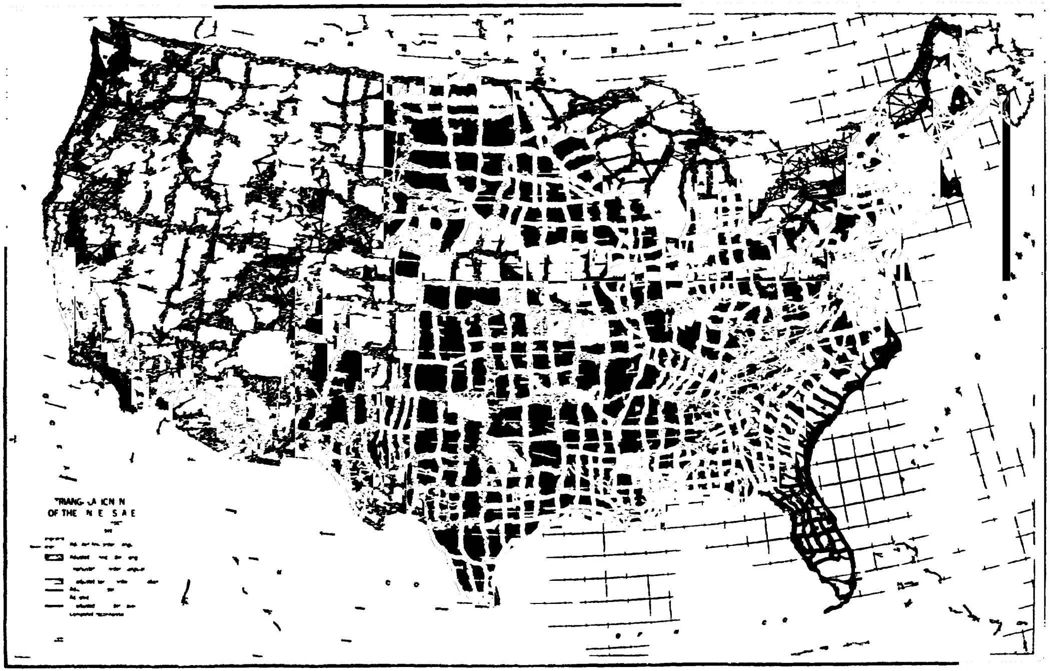
FIG. 160.

161. **Precise Triangulation.** The most precise triangulation is done by the United States Coast and Geodetic Survey in extending the first order triangulation net of Fig. 161 across the continent. The highest order of precision is obtained. Base lines are measured with errors on the order of one in a million or better, and angles are measured to a fraction of a second.

The computed sides of the triangles, many miles in length, are required to check with a precision on the order of 1/25,000. Allowance is made for the curvature of the earth. Such work is described in the *Manual of First Order Triangulation* published by the United States Coast and Geodetic Survey and in books on geodetic surveying. This is a specialized type of survey work and is only accomplished efficiently by men with highly specialized experience.

162. **Azimuth Mark Set for Use in Local Land Surveys.** Establishment of an azimuth mark at a distance of 300 to 400 yards from a triangulation station has been the practice of engineers of the U. S. Coast and Geodetic Survey for several years past, according to the June 1933 Bulletin of the Association of Field Engineers, U. S. Coast and Geodetic Survey. These marks are so set that they can be seen from the ground at the station, in order to enable local engineers and surveyors to start their traverses from the triangulation stations.

163. **Conclusions.** Triangulation offers the cheapest and most precise method of controlling extensive surveys, particu-



Courtesy of the United States Coast and Geodetic Survey

Fig. 161

larly in rough country, but it is seldom used for small areas and for ordinary degrees of precision, except where traversing is unusually difficult. It is best done by men who have had such experience, especially where precise results are needed.

The following extracts from *Topographic Instruction of the United States Geological Survey*, Bulletin 788–B, 1926, are applicable to the usual engineering triangulation, in which errors are of the order of a few seconds in angle and less than 1/5000 in distance.

"Stations should be selected and signals built before any observing is done, and to this end the triangulator and his assistant should make a reconnaissance over the area to be controlled. This reconnaissance should be thorough enough to disclose every practical scheme of triangulation and the angles at each point should be measured with a pocket compass and platted with the protractor so that the size and proportions of the figures may be ascertained. All preparatory work, such as setting tablets and posts, erecting signals and scaffolds, clearing lines of sight, and obtaining consent of owner, if on private land, should be completed during this reconnaissance, so that the final observing may be performed with economy and dispatch. The reconnaissance affords the triangulator opportunity to acquaint himself with the territory. . . . The most desirable groups of triangles consist of either quadrilaterals with both diagonals sighted or central-point figures with four to seven sides. The triangles composing these figures should be well proportioned, angles measuring not less than 30° nor more than 120° each. The scheme should not be allowed to dwindle down to single, unsupported triangles, and especial care should be taken to connect the work done with other work by means of well-proportioned triangles. Overlapping figures r an excess of observed lines beyond those necessary to insure double determination of each length are undesirable, although diagonal through some figure may occasionally be valuable a check. As a rule additional lines of this kind only compli- te the main scheme without materially adding strength, and merous observations made for them are discarded by the mputers as superfluous. Judgment is to be used in this mat- , however, for in many regions the atmospheric conditions are exceedingly uncertain, and the observer cannot always count on being able to see in both directions over every line that may be essential to the main scheme. In such regions it is well to err on the safe side and obtain too many data rather than too few. . . . Two or more permanent reference marks should be established about each station mark."

CHAPTER XII

LAND SURVEYS

GENERAL

164. Introduction. The surveying of land boundaries antedates recorded history. It is doubtless truly stated that practical geometry and surveying were largely developed in the distant past to redetermine the position of land boundaries which were obliterated annually by the overflow from the Nile River in Egypt. For thousands of years it has been necessary continually to re-establish old boundaries and to establish new ones.

Land surveys are the stock in trade of the surveyor, involving as they do the determination of lengths, directions, and areas. The engineer sometimes makes such surveys, especially in his younger years; but his interest is more frequently in those broader professional phases mentioned in the following paragraph.

Most business men and executives have occasion, from time to time, to conduct transactions involving real estate and the surveys and descriptions connected therewith. The writing of descriptions of land boundaries and the computation of areas, such as occur in deeds, mortgages, right-of-way documents, and the like, is properly the function of the engineer or surveyor, especially if complications are involved. Difficulties frequently arise when others attempt this function. The modern corporation has many dealings in land, and questions regarding boundaries are referred to the engineering department. The better type of real-estate transaction is based in part on a survey on the ground, both to show the new owner or mortgagee what the actual boundaries and area are, and to assure him that he has not inadvertently acquired an old boundary dispute or a law suit. Many engineering constructions and improvements must be built with reference to property lines which must be definitely located on the ground. The layman has only a hazy and imperfect idea regarding these matters.

165. Records. Courts have ruled that any deed, mortgage or agreement as to property lines, or any document affecting land titles is not legally effective as against another previously filed and conflicting document; in other words, that any such document should be promptly filed with the proper official This may be the county registrar or recorder of deeds or other official, depending on the statute covering the matter in the particular state. This official copies the document into his records and it is there available for public inspection. Because of the large number of documents thus chronologically recorded, it is necessary to maintain an index. To use such an index, one must know the approximate date of recording, the nature of the document, and the names of the parties to and/or from whom the document was made. These names may be the grantor or grantee in cases of a deed, the mortgagor or mortgagee in the case of a mortgage, or the owner in certain special cases. When the engineer or surveyor looks up these records, it is usually with the purpose of securing from them the exact legal description of the property. However, he frequently secures this information from some document in the hands of the owner or from the professional searcher of records who is described in the following section.

In addition to the preceding records, the Office of Records will contain most of the frequently needed maps which have been filed to show the *subdivision,* or dividing up, of property into small acreage tracts or city lots, as in Fig. 226. Frequently, also, it may contain field notes and records of subsequent surveys useful in locating missing corners.

166. Record Searchers. The indexes kept in the city or county offices bearing on land titles are not arranged conveniently for the layman to secure a summary of all documents filed regarding a particular piece of property. Private title-searching companies, or *abstract companies,* are better prepared for such work and will furnish, for a fee, a complete series of sheets of paper upon each of which is abstracted, or briefed, the essential features of one of the filed documents regarding the property in question. These are commonly known as *abstracts of title* but, unfortunately, the layman cannot understand them and it is necessary to submit the abstract to an attorney versed in land transfers, who will render an opinion regarding the validity of the title to the property.

The functions of the abstractor and attorney have in recent years been combined and an insurance policy added, so that the modern title insurance company, or title guarantee company, performs all of these functions. Most real estate transactions are now handled for the principals by such companies. Their policies insure the title of the property, but exclude certain features; for example, they assume no responsibilities regarding the location of the property lines as determined by surveys. The best modern practice is for both parties to a transaction to place all papers and monies in the hands of a reliable company of this nature, in *escrow* — a sort of temporary trusteeship. The title company thereupon, for a fee, investigates all questions regarding title, regarding the survey and determination of boundaries, and so on. If everything can be made agreeable to the terms of the escrow, the title company then files the documents and pays the money, thus completing the transaction, without risk of misunderstanding or fraud.

167. Courts. Many disputes regarding land boundaries may best be settled by agreement or arbitration between affected parties and such an agreement ***should be filed*** with the recording office. This failing, recourse may be had to proceedings in the proper court. Here the engineer or surveyor enters as an expert witness only. He has no power to decide the matter, even though he may hold official position as a county surveyor or city engineer.

Special Land Courts, patterned after the Australian Torrens Act, are in operation in California, Massachusetts, Minnesota, Ohio, Hawaii, and the Philippines. Some twelve other states have enacted similar statutes which are not in effect as yet, however. Claimants refer questions regarding land surveys and tles to these courts. A moderate fee is charged. Detailed rveys conforming to definite specifications must be submitted the court, which is empowered to render a decision. These e seldom reversed by a higher court.

168. General Considerations Governing Land Surveys. *Most modern surveys are resurveys* It is held by the courts that the function of the surveyor is to reestablish the corners of the property in their *original location on the ground* rather than according to exact and accurate surveys and descriptions of property. In other words, it is *not* the function of the surveyor to change the position of corners by correcting the original surveys, but only to locate the corners in the position which they originally occupied on the ground.

One exception may be noted. In most states exclusive, actual, continuous, visible, hostile, and notorious possession of land, or the position of a land boundary, which has existed for a statutory period, normally 20 years (in some states 10 years), gives title to the land. Missouri and Maine give the minimum weight to this legal principle of *adverse possession.* A somewhat similar doctrine of *acquiescence* is in effect in some states, notably in Iowa.

Where *color of title* exists, that is, "the semblance or appearance of legal or equitable title which is in fact no title"; then *adverse possession* ripens into title in one-half of the time normally required, in some states. It is not possible to take over a public road by adverse possession except that a few states permit this after a period of double the normal statutory requirement.

Where adverse possession controls a corner, no time is therefore to be spent in relocating the original corner. Even where the period of possession is less than the statutory one, it is frequently undesirable to attempt to disturb long-established property lines which are generally accepted, as evidenced, for example, by roads, fences, buildings, and the like.

If the original survey was made with the compass, using magnetic north, then the relocating survey must allow for the change in declination of the needle since that time. Should the old-fashioned surveyors' or engineers' chain have been used, the distances are apt to be recorded as too short due to the lengthened chain which resulted from wear of the chain links. Most of the older surveys are quite inaccurate, or at best are not precise.

169. Marking Corners. Property corners are usually defined on the ground by stakes (sometimes with charcoal or other permanent material deposited at their bottoms), mounds of earth, notched stones, iron pipes, fence corners, bolts leaded into bed rock, mounds of rock, blocks of concrete, and so on.

It was a common practice to establish a *witness corner* in a given direction (usually along the line of the survey) and at a given distance from a property corner that was subject to loss or disturbance. The more common witness corners were blazed trees or corners similar to the true property corners. The distances and bearings to these witness corners are recorded, as in the field notes on pages 216 and 217.

In modern surveys, it is the practice to *tie in* the corners to nearby objects which can be readily distinguished. Such ties consist of bearings or angles and distances, as in Fig. 203.

It is not uncommon to find the survey line between corners blazed on trees on or near the line of the survey. A blaze may be distinguished for many years, even after the bark has grown back over it.

170. Difficulties of Land Resurveys. To the usual difficulties of surveying are added the following, largely occasioned in attempting to relocate old corners:

1. Poor original surveys.
2. Unknown bearings due to lack of known declination of the needle during the original survey.
3. Missing or inadequate descriptions.
4. Errors in transcribing data from surveys and in copying and recording.
5. Lost or obliterated corners, and/or reference points.
6. Lack of funds to do satisfactory work.
7. Conflicting court decisions, or lack of decision, regarding the exact point at issue.
8. Enmity between adjoining property owners.
9. Lack of detailed records of surveys of abutting or nearby property and of pertinent court decisions in the vicinity.
10. The surveyor's inadequacy as an expert witness in court.
11. The objection on the part of the owner to pay an adequate fee.

It is thus evident that land resurveys are something of a specialty which will best be left to those qualified for such work in the particular locality. Frequently this may be accomplished by the corporation engineers' making the basic surveys with their own organization (usually in connection with the surveys of their engineering project) but calling upon the local surveyor, as, for example, the county surveyor or private surveying company in a city, to identify corners and to advise upon the restoration of those which have been lost or obliterated The local surveyor usually has considerable standing in the community and its local courts, and the best available information. For all these reasons, the engineer without local experience should avoid all but the simplest land surveys. The engineer or surveyor is legally liable for damage due to incorrect work in which he has failed to exercise that degree of prudence, judgment, and skill which may be reasonably expected of a member of his profession.

In all cases the engineer or surveyor must realize that his primary function is to aid in determining the boundary to the satisfaction of the interested parties. Every effort at conciliation and arbitration should be exhausted before a client is permitted to go to court. The surveyor must consider himself simply as an adviser, a technician, or an expert witness; not as having authority to settle a dispute. An excellent discussion of *The Surveyor and His Legal Equipment,* by Prof. A. H. Holt, is given in the Transactions of the American Society of Civil Engineers, Vol. 99, 1934, page 1155.

171. Weight Accorded Various Factors in Surveys. A land description is, of course, to be interpreted as forming a closed survey. Natural and ascertained objects on the ground are normally given the greatest weight, then artificial monuments then a statement of adjoining property (which in a sense is an independent designation of the boundary). As between distance and bearing of lines, that should be given the greater weight which was probably the more accurately measured. Area is given least weight in determining land boundaries unless, as is sometimes the case, area is the essence of the deed. Errors in bearings or distance are usually proportioned over the lines and distances intervening between known monuments. It must be remembered that the original location of the corners on the ground is desired.

172. Procedure in Resurveys. The proper descriptions, maps, and field notes must be secured; then the existing or easily identified corners found on the ground, using local witnesses. Only after a thorough search, including digging for corners not evident on the surface, and all possible testimony from local people, should the attempt be made to reestablish the corner by surveys. Following the surveys, a new search for the corner is sometimes successful since the probable location is then more definitely established.

Should it be necessary to reestablish a corner by surveys, this is usually accomplished by *proportionate* measurement and other procedures described later in this chapter.

Every effort should be made to secure an agreement between interested property owners without recourse to the courts, and friendly relations should be fostered. However, in case of legal action the surveyor, because of his knowledge and experience, is quite often called upon to testify as an expert witness.

173. Subdivisions. The subdivision of property into city lots is described in § 226. Somewhat larger acreage tracts are similarly surveyed.

174. Metes and Bounds Surveys. Most of the land surveys in the original thirteen colonies, Kentucky, and Tennessee, and occasional surveys elsewhere, are executed and described as a closed traverse of successive bearings and distances known as *metes and bounds.* They are rarely encountered in or near cities, since such property is divided on the plats described in the previous section. Neither are they often encountered where the United States Government has sectionalized the country as shown on Fig. 179 (page 211); although an exception is to be noted in that metes and bounds descriptions are occasionally found on sectionalized land where Spain, France, England, or our own government granted land before the sectionalizing was done; also in mineral claims, and the like.

It is legal and permissible, although frequently not advisable, convenient, or desirable, to describe any tract of land by metes and bounds. A thorough and complete metes and bounds description follows, although many such surveys are less adequately described. This description is from Figs. 253 and

253a, and conforms to recommendations of the Am. Soc. C. E. in their June 1941 Proceedings. "Beginning at a concrete monument located 1253.3 feet south and 49.6 feet west from the cross on the stone at the intersection of the center lines of Maryland and Rollins Avenues in the City of . . ., County of . . ., State of . . .; thence

North 81° 44′ west along the southerly line of Kentucky Avenue for a distance of 939.0 feet to an iron pin on the easterly line of Providence Road;

South 10° 04′ west along the easterly line of said Road for a distance of 833.7 feet to an iron pin;

South 63° 18′ east along the northerly iron fence of the Stadium for a distance of 182.7 feet to a cross cut on a stone;

South 77° 42′ east continuing along said iron fence for a distance of 310.9 feet to an iron post;

North 67° 47′ east continuing along said iron fence for a distance of 638.2 feet to the southwest corner of a 15-inch square concrete fence-corner post;

North 1° 40′ east along a woven wire fence and the property now, or formerly, belonging to Charles Dearing for a distance of 593.1 feet to the point of beginning;

all bearings being referred to the true meridian; the tract containing 18.59 acres, more or less; and being shown on the plat drawn by John Doe, Civil Engineer, dated . . ., which is attached hereto and made a part hereof (or state place of filing if not attached to the document)."

175. Law of Land Surveying. In this brief treatment of the legal aspects of land surveying and boundaries, it must be emphasized that the intention is only to indicate the extent and complexity of the subject, conflicting court decisions, and the like. Perhaps the best published references are *Surveying and Boundaries*, by Frank Emerson Clark, Bobbs-Merrill Company, 1939; *Boundaries and Adjacent Properties*, by R. H. Skelton, Bobbs-Merrill Company; and *Public Land Surveys*, by L. O. Stewart, Collegiate Press, Ames, Iowa, 1935. Much other information is available in scattered sources, especially in periodicals, and this may best be located through the Engineering Index or the Industrial Arts Index. The literature is voluminous, especially if court decisions are included. That section of the *Manual of Instructions for the Survey of the Public Lands of the United States, 1947,* issued by the Bureau of Land Management of the Department of the Interior, which refers to lost and obliterated corners, should be consulted. The surveyor constantly encounters perplexing problems which must be solved, at least tentatively, in the field with no reference volumes or legal advice available. He must bear in mind that the courts will decide boundaries and surveying questions in accord with *legal precedent,* only a part of which is directly concerned with surveying.

Where interested parties have fairly and knowingly, without fraud or mistake, agreed upon boundaries and have occupied their respective lands under this agreement, the courts will approve the same. Such agreements should be drawn and recorded under adequate legal direction. One court ruling states: "This court has frequently passed on questions of boundaries, and, as has been frequently cited, 'these settlements of boundaries are common, beneficial, approved, and encouraged by the courts, and ought not to be disturbed, though it was afterwards shown that they had been erroneously settled. Convenience, policy, necessity, justice, all unite in favor of such a settlement.' While a large majority of cases passed upon in this State present questions of acquiescence, partition, compromise, and arbitration, still agreements of a recent date, where there is doubt, and whether the parties are right or wrong in their belief that the line established and agreed upon as the boundary of their land was precisely as it should be, have been encouraged, favored, and upheld in a number of cases."

However, if such agreement is not possible, then the best legal and surveying counsel is necessary in attempting to secure a court decision.

As an example of the varying decisions rendered by courts in the various states, the reader is referred to page 220, where the general law regulating boundaries adjoining navigable bodies of water and other important rivers and lakes is stated in an Iowa decision. In contrast to the Iowa decision, some jurisdictions hold that the riparian (land bordering on water) proprietor owns the bed and the right to the water resting thereon to the center line or the "thread of the stream" of both navigable and non-navigable streams, subject, of course, to the rights of the public to navigate and use said streams. Non-navigable lakes are governed by a similar rule in those states. In other cases, as in Missouri, the court has held that a riparian proprietor owns to low water mark on navigable rivers. A great many other variants could be cited.

As another instance, a large number of varying decisions have been rendered regarding *avulsion* (the apparent loss of land because of the sudden and perceptible change of the course of a river, as when a cutoff develops suddenly), *revulsion* (a reversal of this process), and *accretion* (where land is built up slowly due to deposits).

The engineer or surveyor is bound to exercise that degree of care which a skilled civil engineer of ordinary prudence would have exercised under similar circumstances, and the engineer is liable for damages under these circumstances, whether he be a private engineer, a city engineer, or a county surveyor.

The law pertaining to libelous things written, or scandalous remarks made, of a surveyor in his professional capacity, is practically the same as in other professions. If libelous writing or scandalous statements tend to injure him in his profession, an action for damages would lie. The recent trend of court decisions is toward the admissibility of expert testimony from a surveyor or engineer regarding almost all phases of questions regarding boundaries. However, he should consult an attorney as to whether certain types of expert testimony will be received by the courts from him, since in the past courts have refused to admit some types of such testimony.

ORIGINAL FEDERAL SURVEYS

176. United States Public Land Surveys. All of the quotations and figures which follow in this chapter are extracted from the *Manual of Instructions for the Survey of the Public Lands of the United States, 1939*, issued by the General Land Office of the Department of the Interior, or from a revision of the same manual published in 1947 by the Bureau of Land Management. The reader is referred to the Manual for a more complete treatment of the details involved. *Only the usual present procedure is here described,* many older practices and special conditions being described, or referred to, in the Manual or in special instructions. *The engineer must familiarize himself with the instructions in effect when the original survey which he is retracing was made, since slight changes have been made from time to time.*[*]

It is only rarely that the engineer will be connected with the original surveys of the public lands, but a knowledge of the methods employed in such surveys is necessary in order that he may relocate missing corners and properly subdivide the property into smaller units.

The compass, which was formerly used, has been replaced by the transit. All distances in the early United States Public Land Surveys were measured and recorded in 66-foot Gunter's chains, this chain containing 100 links each 0.66 of a foot long. While the units of measurement remain the same, the awkward and inaccurate chain has been replaced by the modern steel tape from 2 to 8 chains long, with link subdivision.

177. Lands So Surveyed. Practically all of the United States is divided by the Federal Government into *sections* approximately one mile square except the original thirteen colonies, Texas, Kentucky, Tennessee, and those lands yet belonging to the United States Government such as forest reserves, Indian reservations, and so on. Texas has sectionalized its own lands on a similar but not identical basis. The territory covered by United States Public Land Surveys is shown on Fig. 179.[†]

"The term public lands has been applied broadly to this area insofar as the lands have been subject to administration, survey, and transfer of title under the public-land laws of the United States, wherein the jurisdiction is now vested in the Director for the Bureau of Land Management, under the direction of the Secretary of the Interior.

"Beginning with the territory northwest of the Ohio River, the public land surveys have been extended, or are now being continued within the 29 states and the Territory of Alaska that has constituted the public domain."

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[*] As an example, in Missouri the Guide Meridians and Standard Parallels are usually 30 (instead of 24) miles apart. Also, double sets of corners were sometimes established on township and range lines similar to the standard and closing corners established under present instructions on Standard Parallels.

[†] For the source of the following quotations, see § 176.

The 29 states referred to in this quotation, the custodians of records, and the periods of maximum activity are as follows:

Alabama, Secretary of State, Montgomery, 1802–1850
Arizona, Public Survey Office, Phoenix, 1867–
Arkansas, Commissioner of State Lands, Little Rock, 1817 1849
California, Regional Survey Office, San Francisco, 1851–
Colorado, Public Survey Office, Denver, 1861–
Florida, Commissioner of Agriculture, Tallahasse, 1822–1880
Idaho, Public Survey Office, Boise, 1867
Illinois, Auditor of State, Springfield, 1813–1849
Indiana, Auditor of State, Indianapolis, 1807–1849
Iowa, Secretary of State, Des Moines, 1836–1858
Kansas, Auditor of State and Register of State Lands, Topeka, 1854–1877
Louisiana, Register of State Lands, Baton Rouge, 1806–1880
Michigan, Department of Conservation, Lansing, 1803–1860
Minnesota, Secretary of State, St. Paul, 1848–1900
Mississippi, Commissioner of State Lands, Jackson, 1802 1860
Missouri, Secretary of State, Jefferson City, 1810–1850
Montana, Public Survev Office, Helena, 1867–
Nebraska, Commissioner of Public Lands and Buildings, Lincoln, 1854–1895
Nevada, Public Survey Office, Reno, 1860–
New Mexico, Public Survey Office, Santa Fe, 1854–
North Dakota, State Engineer, Bismarck, 1861–1910
Ohio, Auditor of State, Columbus, 1785–1843
Oklahoma, Bureau of Land Management, Washington, D. C., 1871–1900
Oregon, Regional Land Office, Portland, 1850–
South Dakota, Commissioner of School and Public Lands, Pierre, 1861–1900
Utah, Regional Survey Office, Salt Lake City, 1855–
Washington, Public Survey Office, Olympia, 1854–
Wisconsin, Commissioners of Public Lands, Madison, 1833–1865
Wyoming, Public Survey Office, Cheyenne, 1869–
Alaska, Public Survey Office, Juneau, 1899–

178. Historical and Legal.* "The survey of the public lands has been inseparably associated with the many questions of title relating to the lands which were turned over to the Federal Government by the Colonial States, also in a similar relation to those larger areas that were acquired later by purchase from, or treaty with the native Indians, or with the foreign powers that had previously exercised the sovereignty. Under the laws of the United States the navigable waters have always been and shall forever remain common highways. This includes all tidewater streams, and other important permanent bodies of water whose natural and normal condition at the date of the admission into the Union was such as to classify the same as navigable water."

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* For the source of these quotations, see § 176.

The rectangular system of survey of the public lands was inaugurated by a committee appointed by the Continental Congress. Various acts from May 7, 1784, provided, with occasional modification in detail, that the public domain be subdivided as described in this chapter. The Congressional Act of March 4, 1909, provides "Whoever shall willfully destroy, deface, change or remove to another place any section corner, quarter-section corner, or meander post, on any Government line of survey, or shall willfully cut down any witness tree or any tree blazed to mark the line of a Government survey, or shall willfully deface, change or remove any monument or bench mark of any Government survey, shall be fined not more than $250, or imprisoned not more than six months, or both."

The various laws passed may be summarized in the following general rules:

"1. That the boundaries and subdivisions of the public lands as surveyed under approved instructions by the duly appointed engineers, the physical evidence of which survey consists of monuments established upon the ground, and the record evidence of which consists of field notes and plats duly approved by the authorities constituted by law, are unchangeable after the passing of the title by the United States.

"2. That the physical evidence of the original township, section, quarter-section, and other monuments must stand as the true corners of the subdivisions which they are intended to represent, and will be given controlling preference over the recorded directions and lengths of lines.

"3. The quarter-quarter-section corners not established in the process of the original survey shall be placed on the line connecting the section and quarter-section corners, and midway between them, except the last one-half mile of the section lines closing on the north and west boundaries of the township, or on other lines between fractional or irregular sections.

"4. That the center lines of a regular section are to be straight, running from the quarter-section corner on one boundary of the section to the corresponding corner of the opposite section line.

"5. That in a fractional section where no opposite corresponding quarter-section corner has been or can be established, the center line of such section must be run from the proper quarter-section corner as nearly in a cardinal direction to the meander line, reservation, or other boundary of such fractional section, as due parallelism with section lines will permit.

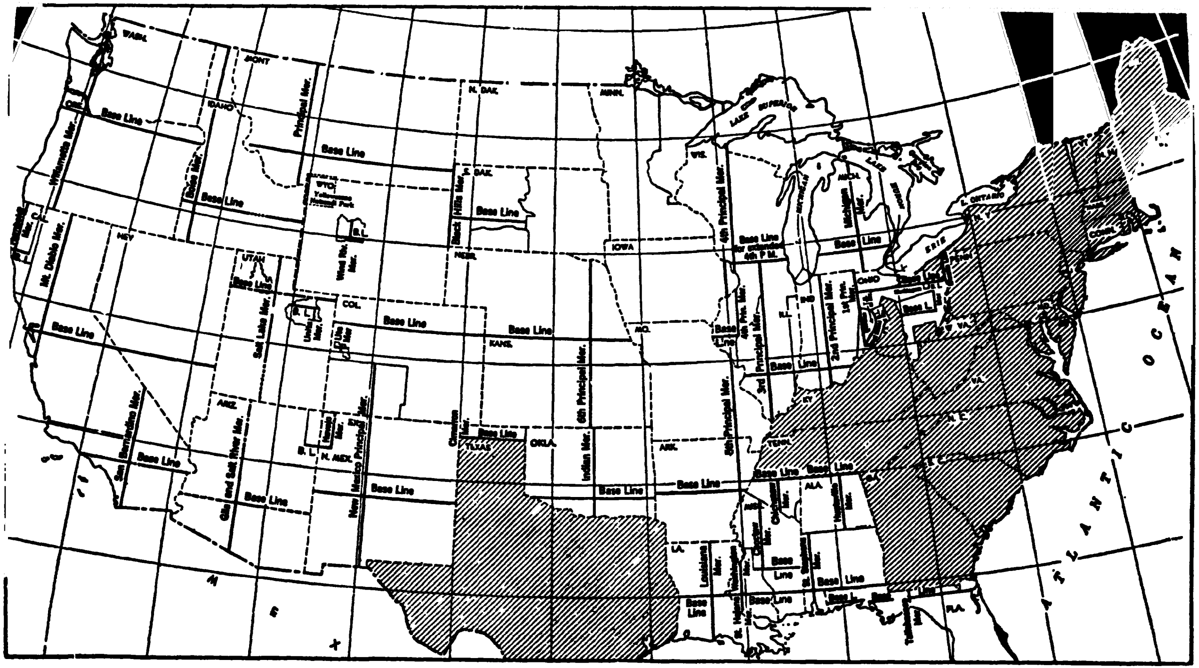
"6. That lost or obliterated corners of the approved surveys must be restored to their original locations whenever it is possible to do so. Actions or decisions by surveyors, Federal, State, or local, which may involve the possibility of changes in the established boundaries or patented lands, are subject to review by the State courts upon suit advancing that issue."

179. Initial Points, Meridians, Base Lines, Guide Meridians, and Standard Parallels. The public land surveys have been extended as the country became settled. An *Initial Point* was first chosen in a location convenient to control the lands which were to be passed from the Government into private hands. Those Initial Points already chosen and in use are shown on Fig. 179. From each Initial Point a *Principal Meridian* was run due north and/or south and a *Base Line* was run due east and or west.

According to the *Manual of Instructions*, the *guide meridians* and *standard parallels* are now run in a similar manner at intervals of 24 miles as shown on Fig. 179a. However, *guide meridians* are often found to be 30 or more miles apart. In rare instances additional *auxiliary standard parallels* and *auxiliary guide meridians* are established.

All of the foregoing meridians are run truly north and south with corners set 40 chains or 1/2 mile apart; except that where Guide Meridians intersect previously established Standard or Auxiliary Standard Parallels, the most northerly 1/2 mile will contain all of the accumulated discrepancy in the 24 miles (or more) of meridional chaining and consequently will not be exactly 1/2 mile.

Since the meridional boundaries of the 24-mile tracts (or larger tracts) converge as they go northward towards the north pole where they meet in a point, offsets occur on the standard parallels. Each guide meridian is run north to an intersection



Courtesy of Am. Soc C E., Civil Engineering, page 1384, Dec. 1931

FIG. 179. AREAS NOT COVERED BY FEDERAL LAND SURVEYS ARE CROSS-HATCHED.

with the standard parallel at which points *closing corners* are set on the ground. Then a new guide meridian is started from the 24-mile point previously set by east and west chainage along the standard parallel. This is shown in Fig. 179a.

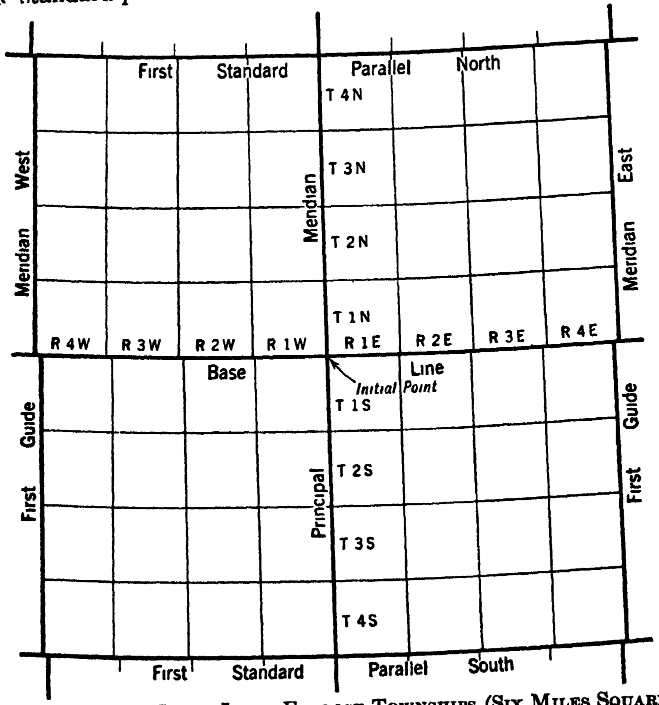


FIG. 179a. THE LIGHT LINES ENCLOSE TOWNSHIPS (SIX MILES SQUARE).

The base lines and standard parallels are run due east and/or west as parallels of latitude to establish the true curve of the parallel of latitude on the ground. These curved lines are everywhere at right angles to a true meridian of that point, but since the meridians are not parallel, the lines must evidently be curved, as shown in the figures.

As each of the base lines and standard parallels is run east and/or west, corners are set 40 chains (1/2 mile) apart and these are the true corners, or *standard corners*, of the surveys to the north, while the closing corners previously mentioned control the surveys to the south. Two sets of corners, usually several hundred feet apart, will therefore be found on base lines and standard parallels, the *standard corners* governing surveys to the north and the *closing corners* governing surveys to the south.

180. Township Exteriors.* In the foregoing description of the 24-mile (usually larger) tracts, the lines were surveyed as true meridians and as true parallels of latitude, that is, they were fitted to the spherical shape of the earth. In dividing these larger tracts into *townships* which are approximately six miles square, the north and south lines, called *range lines*, are run as meridians but those east and west, called *township lines*, are run as straight lines and not as curved parallels of latitude. Township boundaries are shown in Fig. 179a.

"Whenever practicable the township exteriors will be surveyed successively through a quadrangle in ranges of townships, beginning with the townships on the south. The meridional boundaries of the townships will have precedence in the order of survey and will be run from south to north on true meridians; quarter-section and section corners will be established alternately at intervals of 40 chains, and meander corners at the intersection of the line with all meanderable bodies of water; a temporary township corner will be set at a distance of 480 chains, pending a determination of the controlling factor upon which its final position will be governed, whereupon the temporary point will be replaced by a permanent corner in proper latitudinal position. The latitudinal township boundary will be run first as a random line, setting temporary corners, on a cardinal course, from the old towards the new meridional boundary, and corrected back on a true line if ideal conditions are found to obtain. Where both meridional boundaries are new lines or where both have been previously established, the random latitudinal boundary will be run from east to west. In either case, if defective conditions are not encountered, the random line will be corrected back on a true line, upon which will be established regular quarter-section and section corners at intervals of 40 chains, alternately, counting from the east, and meander corners at the intersection of the true line with all meanderable bodies of water. The bearing of the true line will be calculated on the basis of the falling of the random, and the fractional measurement will be placed in the west 1/2 mile. A meridional township exterior will be terminated at the point of its intersection with a standard parallel, placing the excess or deficiency in measurement in the northernmost half mile."

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* For the source of these quotations, see § 176.

181. Subdivision of Township into Sections. These surveys are made in the order shown on Fig. 181, the section lines on the interior of the township being run as nearly as may be parallel to the south and east boundaries thereof. Line 1 is parallel to the east line of section 36 and determines its northwest corner as well as the west 1/4 corner of section 36. Line 2 is run as a random east and corrected so as to hit the previously established northeast corner of section 36. A 1/4 corner is set midway on this corrected north boundary, usually by offset from the mid-point on the random line.

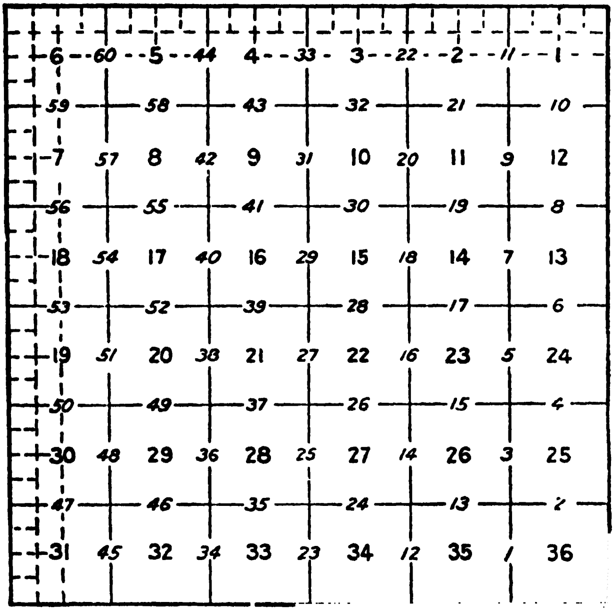


FIG 181. A TOWNSHIP.

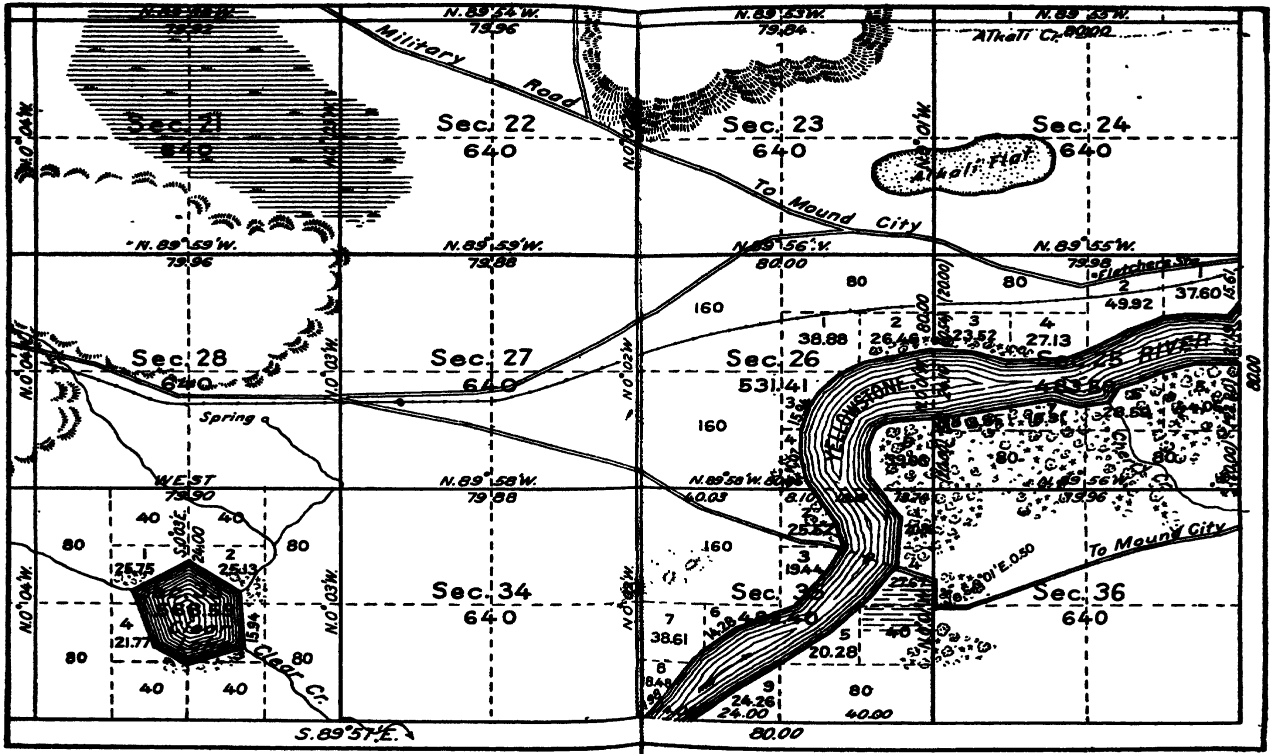


FIG. 181a. PART OF A TOWNSHIP PLAT.

Distance and bearings correspond to theoretic values except where otherwise shown.

The survey is continued north and east in a similar manner until line 11 is run as a random line parallel to the east line of section 1, and corrected to hit the previously established northwest corner of section 1. When line 11 reaches a standard parallel, it is not so corrected, but the 1/4 corner is set at the full half mile on line 11 and all error of chaining is thrown into the last approximate half mile on the north.

This procedure is continued northward and eastward from the south township boundary in the order shown by the numbers on the lines. For the west tier of sections random lines are run west and corrected to the previously set section corners on the west boundary of the township, throwing all errors of chaining into the west approximately 1/2 mile.

All of the interior meridional section lines of the township are run approximately parallel to the east boundary of the township and are therefore not true meridians. They are not always exactly parallel; if, for example, the length of line 2 does not correspond with that of the south line of section 36, then line 3 is inclined slightly to the east or west as may be required to make line 4 correspond in length with the south line of section 36.

It is attempted to make the east and west interior section lines of the township parallel to the south boundary of the township, but since this is accomplished by chaining along the meridional lines, it is not very well accomplished.

It should be noted that in subdividing it, the township is treated as though it were laid out on a plane and not as a part of a spherical surface.

All sections, *except those on the north and west boundaries of the township and except those other sections that contain lots because of bodies of water or other irregularities*, contain approximately 640 acres and are approximately 1 mile square. They may be divided into 1/4 sections of 160 acres; 1/4 1/4 sections of 40 acres; and so on.

The method of numbering the sections in the township shown by Fig. 181 should be memorized.

Township plats, similar to Fig. 181a, are available for all lands which have been sectionalized by the Government, at a nominal cost. Such plats are essential in retracing the original surveys, and are useful for much engineering investigation.

The detailed field notes are also available but are somewhat expensive and not so generally used. The following notes cover a mile of the survey of Fig. 181a.

Chains.	I commence the subdivisional survey at the cor. of secs. 1, 2, 35, and 36, on the S. bdy. of the Tp., which is a sandstone, 8x6x5 ins. above ground, firmly set, marked and witnessed as described in the official record.
	N. 0° 01′ W., bet. secs. 35 and 36.
	Over level bottom land.
20.00	Enter scattering ash and cottonwood.
29.30	SE. cor. of field; leave scattering timber.
31.50	A settler's cabin bears West, 6.00 chs. dist.
39.50	Set an iron post, 3 ft. long, 1 in. diam., 27 ins. in the ground, for witness ¼ sec. cor., with brass cap mkd. W C ¼ S 35 | S 36 1925 dig pits, 18x18x12 ins., N. and S. of post, 3 ft. dist.
	Enter an ungraded road, bears N. along section line, and E. to Mound City.
40.00	True point for ¼ sec. cor. falls in road.
	Deposit a sandstone, 14x8x5 ins., mkd. X, 24 ins. in the ground.
50.50	NE. cor. of field.
51.50	Road to Bozeman bears N. 70° W.
57.50	Enter heavy ash and cottonwood, and dense undergrowth, bears N. 54° E. and S. 54° W.
72.00	Leave undergrowth.
80.00	Set an iron post, 3 ft. long, 2 ins. diam., 27 ins. in the ground, for cor. of secs. 25, 26, 35, and 36, with brass cap mkd. T 15 N R 20 E S 26 | S 25 S 35 | S 36 1925 from which

A green ash, 13 ins. diam., bears N. 22° E., 26 lks. dist., mkd. T 15 N R 20 E S 25 B T.
A green ash, 23 ins. diam., bears S. 71¼° E., 37 lks. dist., mkd. T 15 N R 20 E S 36 B T.
A green ash, 17 ins. diam., bears S. 64° W., 41 lks. dist., mkd. T 15 N R 20 E S 35 B T.
A cottonwood, 13 ins. diam., bears N. 21¼° W., 36 lks. dist., mkd. T 15 N R 20 E S 26 B T.

Land, level bottom; northern 20 chs. subject to overflow.
Soil, alluvial, silt, and loam; 1st rate.
Timber, green ash and cottonwood; undergrowth, willow.

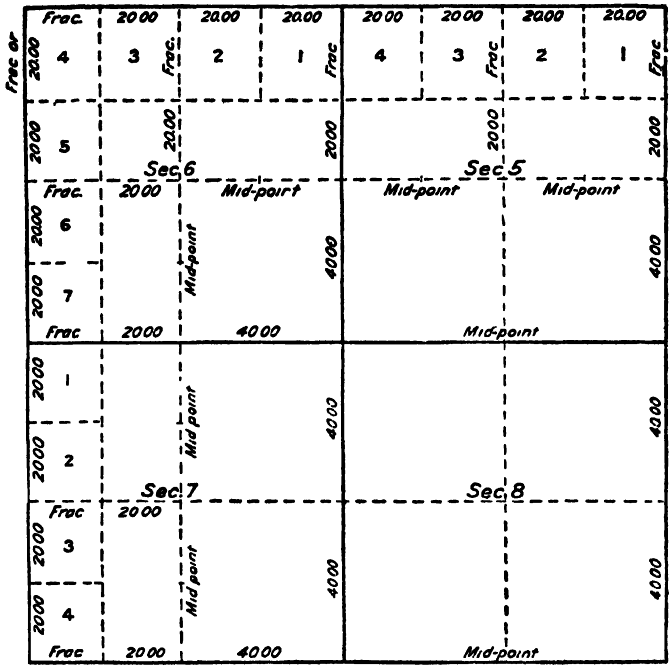
Field notes, township plats and surveys are always more irregular and inaccurate than in the preceding ideal examples.

182. Irregular Boundaries. At certain points in the public domain, some of the old controlling boundaries from which additional surveys were to be made, have been found erroneous beyond permissible limits. In other instances, natural obstacles have interfered. In these cases *auxiliary* or *correction* lines have been run to facilitate the further subdivision and are so reported in the field notes and so designated on the maps.

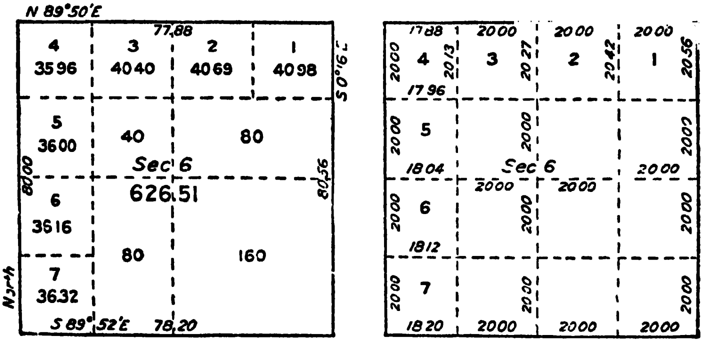
183. Order of Surveys. The various surveys of a single type, as, for example, township exteriors, are not usually all run at one time but several types may be in progress simultaneously from the same camp.

184. Subdivision of Sections. In general, the United States public land surveys set only the section, quarter-section, and meander corners on the exterior lines of the sections and no corners are set inside of the section. However, the township plats of the survey (Fig. 181a) do show each section divided into quarter-sections, these interior section lines being drawn, but not surveyed, by the Government. Along the north and west boundaries of the township and at other locations where irregularities occur, as, for example, bodies of water, the plats show *lots* which are introduced to show that irregular tracts, not full 1/4 sections, exist at these points. The lot lines are not surveyed by the Government. Figure 184 shows the subdivision of a section.

It is thus evident that any survey inside of the sections, either to locate the subdivisions shown on the plats or to



Showing normal subdivision of sections.



Showing areas. Showing calculated distances.

FIG. 184 SUBDIVISION OF A SECTION

further subdivide the platted areas, must be made by competent *private* surveyors or engineers. The law governing such subdivision surveys is well established, to the end that property may be simply, definitely, and uniformly described as illustrated by the following examples:

1. The north-east quarter (NE 1/4) of section thirteen (13), Township six (6) north, Range three (3) west, of the Fifth (5) Principal Meridian, County of . . ., State of . . ., containing one hundred and sixty (160) acres, more or less, according to the United States Survey.

2. The south half (S 1/2) of the south-west quarter (SW 1/4) of the north-east quarter (NE 1/4) of section thirteen (13) . . . and so on as above.

3. Lots one and two (1 and 2) of section thirteen (13) . . . and so on as above.

In fractional sections, that is, where some of the quarter-section corners have not been set on the ground or map because of bodies of water or other obstacles or reasons, subdivisional lines are controlled precisely as though the section lines had been completed regularly. This is accomplished by running lines in one of the two following ways.

1. In a mean direction between two adjacent lines, or occasionally parallel to a single boundary of the section, as conditions may require.

2. By *proportionate measurement*, which is a "measurement having the same ratio to that recorded in the original field notes as the length of the line by re-measurement bears to its length as given in the record." This process is also necessary in locating lot boundaries of the sections along the north and west boundaries of the township, and in *meander corners*, mentioned later.

"By way of recapitulation . . . thus will be produced in the field the figure represented upon the plat, every part of the former in true proportion to the latter, where the elements of solute distance and area have given way to corresponding oportional units as defined by fixed monuments established the original survey."*

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* For the source of these quotations, see § 176.

185. Meandering.* "All navigable bodies of water and other important rivers and lakes (as hereinafter described) are to be segregated from the public lands at mean high-water elevation. The traverse of the margin of a permanent body of water is termed a *meander line.*

"The running of meander lines has always been authorized in the survey of the public lands fronting large streams and bodies of water, but the mere fact that an irregular or sinuous line must be run, as in case of a reservation boundary, does not entitle it to be called a meander line except where it closely follows the bank of a stream or lake. The legal riparian rights connected with meander lines do not apply in case of other irregular lines, as the latter are strict boundaries.

"Low-water mark is the point to which a river or other body of water recedes, under ordinary conditions, at its lowest stage. High-water mark is the line which the water impresses on the soil by covering it for sufficient periods to deprive it of vegetation. *Raide v. Dollar,* 203 P. 469 (1921). The shore is the space between the margin of the water at its lowest stage and the banks at high-water mark. *Alabama v. Georgia,* 64 U.S. 505 (1859).

"Numerous decisions in the United States Supreme Court and many of the State courts assert the principle that meander lines are not boundaries defining the area of ownership of tracts adjacent to waters. The general rule is that meander lines are not run as boundaries, but to define the sinuosities of the banks of the stream or other body of water, and as a means of ascertaining the quantity of land embraced in the survey, the stream, or other body of water, and not the meander line as actually run on the ground being the boundary. When by action of water the bed of the body of water changes, high-water mark changes and ownership of adjoining land progresses with it. *Lane v. United States,* 274 Fed. 290 (C.C.A., 1921); *Harper v. Holston,* 205. P. 1062 (Wash. 1922).

"Meander lines will not be established at the segregation line between upland and swamp or overflowed land, but at the ordinary high-water mark of the actual margin of the river or lake on which such swamp or overflowed lands border. . . .

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* For the source of these quotations, see § 176.

"At every point where either standard, township, or section lines intersect the bank of a navigable stream, or any meanderable body of water, corner monuments at such intersections will be established at the time of running these lines. Such monuments are called *meander corners.* In the survey of lands bordering on tide waters, meander corners may be temporarily set at the intersection of the surveyed lines with the margin of mean high tide, but no monument should be placed in a position exposed to the beating of the waves and the action of ice in severe weather. In all such cases, a witness corner on the line surveyed, at a secure point near the true point for the meander corner, will be established. The crossing distance between meander corners on the same line will be ascertained by triangulation or direct measurement, and the full particulars will be given in the field notes. . . .

"*Meander lines* are run in surveying fractional portions of the public lands bordering on navigable rivers, not as boundaries of the tract, but for the purpose of defining the sinuosities of the banks of the stream, and as the means of ascertaining the quantity of land in the fraction subject to sale, which is to be paid for by the purchaser. . . .

"The field notes of meanders will show the corners from which the meanders commenced and upon which they closed, and will exhibit the meanders of each fractional section separately. . . . Navigable rivers and bayous, as well as all rivers not embraced in the class denominated *navigable,* the right angle width of which is three chains and upward, will be meandered on both banks. . . . Rivers are not classed as navigable will not be meandered above the point where the average right angle width is less than 3 chains, except when duly authorized.

"Shallow streams and intermittent streams without well-defined channel or banks will not be meandered, even when more than 3 chains wide. Tide-water streams will be meandered at ordinary mean high tide as far as navigable even when less than 3 chains wide.

"The meanders of all lakes of the area of 25 acres and upwards will be commenced at a meander corner and continued, as above directed, for navigable streams."

186. Permissible Limits of Errors and Closure in United States Public Land Surveys.* "Under the general subjects of 'township exteriors' and 'subdivision of townships' certain definite limits were prescribed beyond which previously established surveys are classed as 'defective' or in the case of new surveys corrective steps are required. Such limits constitute the standard of accuracy of the United States rectangular surveys, and, for convenience, have been referred to as the 'rectangular limit,' 'limit for the control of new surveys,' 'limit relating to defective exteriors and section lines,' 'limits for subdivision,' etc., each expression having been formed to suit the descriptive exigency of the text. A more general requirement known as the 'limit of closure' will be applied as a test of the accuracy of the alignment and measurement of all classes of lines embraced in any closed figure incident to the public-land surveys, and corrective steps will be required wherever this test discloses an error beyond the allowable limit.

"The 'error of closure' of a survey may be defined, in general terms, as the ratio of the length of the line representing the equivalent of the errors in latitude and departure (as found by a table of latitudes and departures) to the length of the perimeter of the figure constituting the survey; but, with due regard for the controlling coordinate governing lines of a rectangular survey, pronounced accuracy in latitude will not be permitted to offset gross exterior in departure, or vice versa, and, in order to be consistent with the fundamental theory, a double test must be applied in place of the one expressed in general terms. The 'limit of closure' fixed for the United States rectangular surveys may be expressed by the fraction 1/452 provided that the limit of closure in neither latitude nor departure exceeds 1/640, and where a survey qualifies under the latter limit the former is bound to be satisfied; thus an accumulative error of 12½ links per mile of perimeter, in either latitude or departure, will not be exceeded in an acceptable survey. The limit of closure as thus expressed may be applied to various specific conditions as heretofore stated.

"The latitudes and departures of a normal section shall each close within 50 links; of a normal range or tier of sections, within 75 links; and of a normal township, within 300 links. The boundaries of each fractional section including irregular claim lines or meanders, or the meanders of an island or lake in the interior of a section, should close within a limit to be determined by the fraction 1/640 when the error in either latitude or departure is considered separately; the same rule will be applied to all broken or irregular boundaries.

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* For the source of these quotations, see § 176.

"Engineers are required to compute all doubtful closings while in the field in the immediate vicinity of a particular line, or series of lines, in question, and to accomplish all necessary corrective work before concluding a survey.

"Further attention is required on the subject of the accuracy of the current surveys when considered apart from the basic limit of 21′ departure from the cardinal, or 50 links per mile allowable tolerance in measurement, when these are applied as a test of acceptable rectangularity, especially in initiating or closing new surveys from, or upon, the approved older surveys that may lack modern requirements as to accuracy. . . .

"This is a test that bears directly on the improved technique which is now practiced in the making of public land surveys. The latter has its own limit in the region under survey."

The public land surveys have been made with increasing precision as time has passed, as land has become more valuable, and as better technique has been developed. It will frequently be found that the older surveys greatly exceed the limits of error specified above, while the few made recently will usually fall well within such limits.

187. Marking Corners and Lines and Noting Resources.* "The marking of a survey upon the ground in such a manner as to fix forever the position of the legal lines in relation to the earth's surface is the final step in the field work, and is accomplished in three ways . . . (a) The regular corners of the public-land surveys are marked by fixed monuments of specified character . . .; (b) the relation of the officially surveyed lines to natural topographic features is recorded in much detail . . .; and (c) the locus of the legal lines, wherever living timber is encountered, is plainly marked upon the forest trees, which is accomplished by the process of *blazing* and by *hack* marks. . . . Lines are also to be marked by cutting away enough of the undergrowth to facilitate correct sighting of instruments. . . . The undergrowth will be especially well cut along all lines within distances of 5 chains of corner monuments and within 2 chains of arteries of travel."

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* For the source of these quotations, see § 176.

The following technical and topographic features are to be carefully observed and recorded in the field during the progress of public land surveys:

"1. The precise course and length of every line run, noting all necessary offsets therefrom, with the reason for making them, and method employed.

"2. The kind and diameter of all bearing trees, with the course and distance of the same from their respective corners, and the markings; all bearing objects and marks thereon, if any; and the precise relative position of witness corners to the true corners.

"3. The kind of material of which corners are constructed, their dimensions and markings, depth set in the ground, and their accessories.

"4. Trees on line: The name, diameter, and distance on line to all trees which it intersects, and their markings.

"5. Intersections by line of land objects. The distance at which the line intersects the boundary lines of every reservation, townsite, or private claim, noting the exact bearing line of such boundary lines, and the precise distance to the nearest boundary corner; the center line of every railroad, canal, ditch, electric transmission line, or other right-of-way across public lands, . . .

"6. Intersections by line of water objects. All unmeandered rivers, creeks and smaller water-courses which the line crosses; . . . All intermittent water-courses, such as ravines, gulches, arroyos, draws, dry-drains, etc.

"7. The land's surface; whether level, rolling, broken, hilly or mountainous.

"8. The soil; whether rocky, stony, gravelly, sandy, loam, clay, etc.

"9. Timber; the several kinds of timber and undergrowth, in the order in which they predominate.

"10. Bottom lands to be described as upland or swamp and overflowed, as contradistinguished under the law, noting the extent and approximate position of the latter, and depth of overflow at seasonal periods. . . .

"11. Springs of water, whether fresh, saline, or mineral, with the course of the stream flowing therefrom. . . .

"12. Lakes and ponds, describing their banks, tributaries, and outlet, and whether the water is pure or stagnant, deep or shallow.

"13. Improvements; towns and villages; post offices; Indian occupancy; houses or cabins, fields, or other improvements; mineral claims; mill-sites; . . .

"14. Coal banks or beds, all ore bodies, with particular description of the same as to quality and extent; all mining surface improvements and underground workings; and salt licks. . . .

"15. Roads and trails, with their directions, whence and whither.

"16. Rapids, cataracts, cascades, or falls of water, in their approximate position and estimated height of their fall in feet.

"17. Stone quarries and ledges of rocks, with the kind of stone they afford.

"18. Natural curiosities, petrifactions, fossils, organic remains, etc.; . . .

"19. *The magnetic declination.* — To be included in the transcribed field notes. . . . The value at the township or section corner at the southeasterly point of the survey, corrected to the mean magnetic declination, will be shown on the plot.

"20. *General Description.* — The above information will be summarized by townships in a general description which will be made the concluding part of the field notes of every survey. The general description will be made to embrace many more comprehensive details in regard to the characteristics of the region than it is feasible to cover as an intimate part of the technical record of the survey, as follows: —

"*Land.* — A general outline of the drainage and topographical features of the township and approximate range of elevation above sea level.

"*Soil.* — The prevailing and characteristic soil types. . . .

"*Timber.* — The predominant forest species, age, size, condition, etc.

"*Evidence of Mineral.* — All known bodies of mineral, and lands whose formation suggests mineral-bearing characteristics, . . .

"*Watering Places.* — The areas embracing all streams, springs, or water holes as may be of special value as public watering places, . . .

"*Settlement.* — The extent of the settlement at the time of the survey.

"*Industry.* — The industrial possibilities of the township, especially as to the adaptability of the region to agricultural pursuits, stock raising, lumbering, mining, or other profitable enterprise.

"*Special.* — All exceptional steps in the technical process of the survey, and other special matters required in paragraphs Nos. 1 to 19, inclusive, of the above summary, not otherwise suitably recorded will be reported in the general description.

"In addition to the field notes the engineers are required to prepare, as the work progresses, an outline diagram showing the course and length of all established lines with connections, and a topographical sketch embracing all features usually shown upon the completed official township plat. These maps will be made to scale, drawn in pencil only, if desired, and will be kept up with the progress of the field work. The interiors of the sections will be fully completed; the topographical features will be sketched with care while in the view of the engineer, and the position within the section of the various details which are to be shown on the completed plat will be located with an accuracy commensurate with their relative importance. The design of the specimen township plat will be followed closely in the preparation of the outline diagram and topographical sketch plat, except that it will generally be desirable to employ a separate sheet for each of the two purposes. These maps will then form the basis of the official plat, the ultimate purpose of which is a true and complete graphic representation of the public lands surveyed."

It will be noted that much information useful in engineering promotion and development is available from these records.

"The courts attach major importance to evidence relating to the original position of the monument, such evidence being given far greater weight than the record relating to bearings and lengths of lines. It is the purpose that the monuments shall serve every necessary requirement for the identification of the survey. The legal importance, as thus briefly outlined, makes it mandatory upon the chief of the field party to exercise constant diligence in the workmanlike construction of lasting monuments. . . .

"If the engineer is called upon to alter the condition of a previously established monument, the utmost regard should be shown for the original location. . . . A complete record will be kept of the description of the old monument as identified, and all alterations and additions thereto. . . .

"The description of the monument will embrace (*a*) the significance of its position; (*b*) length and diameter of an iron post; or the kind and length, width, and breadth of a stone; or the species and breast height diameter of a tree; (*c*) depth set in the ground, with mention of additional support, if any; (*d*) the markings upon the monument; and (*e*) the nature of the accessories including character, size, position, and markings.

"The Bureau of Land Management has adopted a standard iron post for monumenting the public-land surveys, which will be generally used unless exceptional circumstances warrant the use of other material."

188. Irregularities in Original Federal Surveys.*

IRREGULARITIES IN SURVEYS

Due to less precise instruments and methods, and to cheap work, many of the early Federal land surveys contain errors of appreciable magnitude. A contract system for making Federal land surveys was in effect from their beginning until 1910. This system was thoroughly unsatisfactory, since the contracts were occasionally let to incompetent and dishonest men. The worst surveys were made between 1875 and 1890, when scandalous frauds were perpetrated by the contractors. Due to increasing rigorous inspection, the surveys improved from 1890 to 1910. Since 1910 they have been performed in excellent manner by Civil Service employees under the General Land Office.

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* See the references on page 201.

Despite the occasional fraud in the early surveys, investigators have found that most of those in the eastern states were well done, in view of the conditions existing at the time, and gross fraud, including fictitious notes with little field work, was found in perhaps less than one per cent of the surveys.

IRREGULARITIES DUE TO CHANGING INSTRUCTIONS

In order to show the maximum variation from present practice, the following instructions are quoted regarding surveys in California, where it is provided that "whenever, in the judgment of the Secretary of the Interior, a departure from the rectangular mode of surveying and dividing the public lands in California would promote the public interest, he may direct such changes to be made in the mode of surveying and designating such lands as he deems proper with reference to the existence of mountains, mineral deposits, and the advantages derived from timber and water privileges. . . ."

As another example, prior to 1836 section lines were required to meet the section corners on the township and range lines of the south and east sides of a township, but not on the north and west sides thereof. This rule produced what are known as double corners on all boundaries of townships. The practice was followed in some surveys as late as 1854, or perhaps later. Some instances exist where triple corners were established. Under present instructions, double corners are found as a general rule only on base and correction lines.

Between 1835 and 1851, and at earlier and some later dates, section lines were run due north, rather than parallel to the east boundary of the township as at present.

The foregoing examples simply illustrate the many unusual instructions which have been issued from time to time. In order that the surveyor may intelligently retrace the original surveys, it is apparent that he must familiarize himself with the instructions in effect at the time of the original surveys which relate to the lines which he is now retracing.

189. Convergency of Meridians.* "The linear amount of the convergency of two meridians is a function of their distance apart, of the length of the meridian between two reference parallels, of the latitude, and of the spheroidal form of the earth's surface.

"In table 11, Standard Field tables,† are tabulated the linear amounts of the convergency of meridians, 6 miles long and 6 miles apart, for each degree of latitude from 25° to 70° N., together with the angle of convergency of the same meridians. These amounts of linear convergency are at once the proper corrections to apply to the north boundary of a regular township in the computation of the closing error around a township, or other computation by which a theoretical length of a north or south boundary of a township is compared with the length of the opposite boundary; the tabulated linear amounts of convergency are equal to double the amounts of the offsets from a tangent to the parallel at 6 miles for the same latitudes. Simple interpolation may be made for any intermediate latitude, and the amount of the convergency for a fractional township or other figure may be taken in proportion to the tabulated convergency as the fractionl area is to 36 square miles.

"The tabulated angle of convergency represents at once the deviation in azimuth of the tangent from the parallel at 6 miles; and 1/6, 1/3, 1/2, 2/3, and 5/6 of the tabulated angles of convergency represent at once the amounts of the correction in the bearing of meridional section lines to compensate for convergency within a township.

"In the same table are given the differences of longitude for 6 miles in both angular and time measure, also the differences of latitude, for 1 or 6 miles, in angular measure, in the various tabulated latitudes."

190. Special Surveys and Instructions.* "The detailed specifications for every survey assignment and the basic information necessary for accomplishing the field work are set out in a written statement designated: 'Special Instructions.' The special instructions are an important unit of the record relating to the survey and are prepared by the officer in administrative charge of the work. Emphasis will be given to any provision in the Manual which may be more or less unusual in application, but the purpose is more especially to outline the extent of the work intended and the method and order of procedure. The special instructions will ordinarily be written in the third person, and, coupled with the Manual, will contain the necessary specifications and information for executing the survey."

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* For the source of these quotations, see § 176.

† Table VI in this text.

Instructions are available in the Manual relating to the duties of United States mineral surveyors and to the field and office procedure to be observed in the execution of mineral surveys and the filing of returns.

ORIGINAL STATE SURVEYS

191. State Systems of Plane Coordinates. Led by New Jersey, many states are adopting rectangular coordinate control systems of a precision of the order of 1/10,000, upon which monuments are established throughout the state. Through their use, difficulties of property surveys will be largely eliminated. These systems are being developed and extended so rapidly that descriptions of them must be sought in current technical publications. The multiple benefits accruing therefrom make their early adoption desirable.

PRIVATE RESURVEYS

192. Identification of Existent Corners.* "In the extension of the rectangular surveys it devolves upon the cadastral engineer of the Bureau of Land Management to identify the initial lines of his group and to replace all lost corners thereof. On the other hand in the subdivision of sections and in the location of property lines generally, it falls to the county or other local surveyor to identify the official corners, and where a required corner is missing, the local surveyor will be called upon to recover the point. Thus it will be seen that local surveyors as well as cadastral engineers of the Bureau of Land Management are constantly called upon to search for existing evidence of original monuments, and in this work the surveyors will be guided by the same general methods. Should the search for a monument result in failure, the appropriate restorative surveying process to be observed by either surveyor will be based upon the same rules as hereinafter outlined."

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* For the source of these quotations, see § 176.

193. Restoration of Lost Corners.* *"A lost corner is a point of a survey whose position cannot be determined, beyond reasonable doubt, either from traces of the original marks or from acceptable evidence or testimony that bears upon the original position, and whose location can be restored only by reference to one or more interdependent corners.*

"An obliterated corner is one at whose point there are no remaining traces of the monument, or its accessories, but whose location has been perpetuated, or the point for which may be recovered beyond reasonable doubt, by the acts and testimony of the interested landowners, competent surveyors, or other qualified local authorities, or witnesses, or by some acceptable record evidence.

"The engineer is not prepared to consider the restoration of a lost corner until he has exhausted every other means of identifying its original position, and at this stage of his work he should have determined upon an approximate position of the original monument based upon his findings resulting from retracements leading from known corners to the lost corner, from one, two, three, or four directions in accordance with the plan of the original survey. The principle of proportionate measurement, which most nearly harmonizes surveying practice with the legal and equitable considerations involved in controversies concerning lost land boundaries, enters into the problem at this stage, and this plan of relocating a lost corner will always be employed unless outweighed to the contrary by physical evidence of the original survey. In cases where the relocated corner cannot be made to harmonize with all the calls of the original field notes, due to unexplained discrepancy which is made apparent by the retracement, the engineer is required to determine which calls will be given major control, and those which must be subordinated.

"The preliminary retracements furnish the only possible means of arriving at the discrepancies of the courses and distances of the original survey as compared with those derived in the process of re-running the lines, and the whole problem of proportionate measurements is one involving the adjustment of said discrepancies. The restoration of the lost corners cannot proceed until the retracement of the original survey has been completed. The retracement will be based upon the courses and distances returned in the field notes of the original survey, or the equivalent by calculation, initiated and closed upon known original corners. Temporary stakes for future use in the relocation of all lost corners may be set when making the retracements.

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* See pamphlet issued by U. S. Gen. Land Office, 1939.

"*A proportionate measurement is one that gives concordant relation between all parts of the line, i.e., the new values given to the several parts as determined by the remeasurement shall bear the same relation to the record lengths as the new measurement of the whole line bears to that record.*

"*The term 'single proportionate measurement' is applied to a new measurement made on a line to determine one or more positions on that line.*

"*The term 'double proportionate measurement' is applied to a new measurement made between four known corners, two each on intersecting meridional and latitudinal lines, for the purpose of relating the intersection to both.*

"It will almost invariably happen that discrepancies will be developed between the new measurements and the original measurements recorded in the field notes. When these differences occur the engineer will generally be required to adopt a proportionate measurement based upon a process conforming to the method followed in the original survey.

"The principle of the precedence of one line over another of less original importance is recognized, relative to single or double proportionate measurement, in order to harmonize the restorative process with the method followed in the original survey, thus limiting the control.

"*Standard parallels will be given precedence over other township exteriors, and ordinarily the latter will be given precedence over subdivisional lines; section corners will be relocated before the position of lost quarter-section corners can be determined.*"

A valuable paper, *Recovering and Identifying Original Government Section Corners*, by J. S. Dodds will be found on pages 1383–1387, Civil Engineering, Dec., 1931.

*Federal Resurveys.** "Certain important considerations are involved in the execution of Government resurveys of an entirely different character from those relating strictly to the making of original surveys. . . . There is a twofold object of a resurvey: First, the adequate protection of existing rights acquired under the original survey in the matter of their location on the earth's surface, and second, the proper marking of the boundaries of the remaining public lands.

"As already noted, . . . the Congress has authorized under certain conditions, the re-marking of the public-land surveys. The acts relating to resurveys contemplate a restoration of the monuments in those townships, (*a*) where because of obliteration the boundaries can be identified only through extensive retracements in units of whole townships or more, by surveyors experienced in this specialized work, and (*b*) where field investigation has shown that the conditions on the ground are in such disagreement with the representations of the official field notes and plat that the subdivisional lines cannot be identified with certainty as to location, in consequence of which that plat must necessarily be annulled as the basis for the disposal of the remaining public land. . . .

"The authority for engaging in a general resurvey where public lands are involved can issue only through the Director, Bureau of Land Management. Other Federal agencies, vested with the administration of the lands, may initiate the action by showing the justification, addressing the Director through the usual official channels. The settlers and owners of the land may take the preliminary steps through a report of the facts submitted to the regional administrator. . . .

"A State court of competent jurisdiction (in cases involving property boundaries) may make application to the Bureau of Land Management to cause a general resurvey of a township.

"The Bureau of Land Management has exclusive jurisdiction over all matters pertaining to surveys and resurveys affecting the public lands. As between private owners of lands the title to which has passed out of the United States, final determination in the matter of fixing the position of disputed land boundaries rests with the local court of competent jurisdiction.”

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* For the source of these quotations, see § 176.

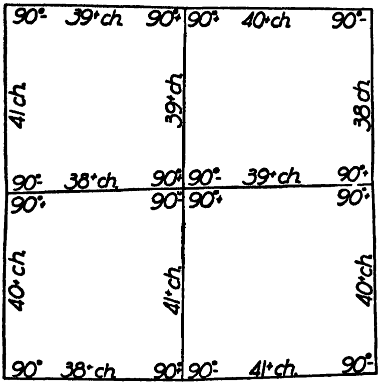


FIG. 194. EIGHT-SIDED SECTION.

194. Eight-Sided Section. Due to errors in the original federal surveys, many sections encountered by the resurveyor show small errors in angles at the quarter-section corners, and these quarter-corners are not always equidistant from the adjacent section corners, as shown in Fig. 194. It is therefore important generally to consider the section actually found on the ground as a figure of more than four sides.

PROBLEMS

1. (a) Draw a sketch or sketches showing the south-east quarter (SE 1/4) of the north-east quarter (NE 1/4) of the south-west quarter (SW 1/4) of section twenty-two (22), Township four (4) south, Range two (2) west of the Initial Point of the Third (3) Principal Meridian. What are the dimensions of this tract, in feet and in chains? What is its acreage?

Ans. (a) 660 ft. × 660 ft.
10 ch. × 10 ch.
10 Acres

(b) The same for the south-west quarter (SW 1/4) of the north-west quarter (NW 1/4) of the south-east quarter (SE 1/4) of section thirteen (13), Township three (3) north, Range four (4) east of the Initial Point of the Fourth (4) Principal Meridian.

(c) The same for the north one-half (N 1/2) of the south-west quarter (SW 1/4) of the north-east quarter (NE 1/4) of section thirty-two (32), Township five (5) south, Range three (3) east of the Initial Point of the Boise Meridian.

2. Write a metes and bounds description for a deed from an assigned map or from an assigned set of field notes.

3. With all Government section corners and quarter corners in place on the ground, how would you proceed to survey the property of question 1?

4. (a) How would you relocate the missing north quarter corner of section 6?

(b) The west quarter corner? (c) How locate the center of the section?

5. In question 1, assuming that the north-west corner of the section and the north quarter corner of the section are missing, how would you relocate them?

CHAPTER XIII

ROUTE SURVEYS

195. Route Surveys. These surveys are made along a line or along a narrow strip of territory such as would be necessary for railways, highways, canals, levees, pipe lines, transmission lines, flumes, and similar engineering construction. A survey around the boundary of a large piece of property may be considered as a line survey, and so conducted, provided it gives little or no information about the country inside of the boundary. The term *route surveys* is therefore contrasted with *area surveys*, which cover and give information about a broad area of territory.

196. The Reconnaissance. A *reconnaissance* is a rapid and rough survey or examination of the territory through which it is proposed to run one or more line surveys. Since there are usually several possible routes, the reconnaissance is actually an examination of the entire area through which the line surveys may pass, and it partakes of the nature of a rough examination of an area.

The first step in such work is to secure the best available maps of the territory under consideration, such as the United States Geological Survey topographic maps, the rural route maps of the Post Office Department, the township maps of the original United States land surveys or sectionalizing surveys, aerial maps, and such other maps as may be available in the offices of the city, county, or other engineer, or corporation operating in that vicinity (page 180). Few of these maps are precise or give sufficiently definite information regarding elevation, though such information is sometimes included. Inference as to relative elevation may be suggested by the streams, which are always shown.

With these maps in hand, together with an aneroid barometer (described on page 93) to secure approximate elevations, the territory may be gone over on foot or horseback, by automobile or airplane, and several of the more promising routes may be selected.

The reconnaissance generally includes a business survey of the territory with a view to determining whether the project will be profitable, which of the several possible routes will be most suitable from the viewpoint of development of market, and so on.

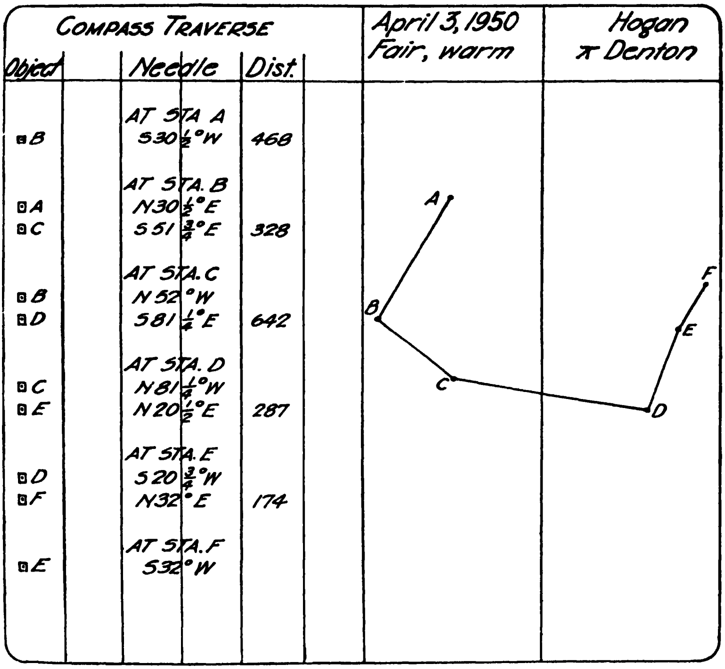
The field notes for a reconnaissance are usually recorded on the maps and in narrative form in a notebook. It is seldom possible to tabulate them as is done with most engineering surveys, except that aneroid barometer readings must be tabulated in the usual manner.

Certain points along the proposed routes are usually fixed, such as mountain passes, bridges, towns, and the like. The reconnaissance begins at one of these fixed points and proceeds in the general direction of the adjacent one. It is advantageous to go over the project first in one direction and then in the other direction, both to secure a different viewpoint and as a check. The entire area should be examined rather than special routes only.

The objective of the reconnaissance is realized when one or more of the several routes have been selected for further detailed study, and all others have been eliminated from further consideration. Rough estimates of first cost, operating cost, and income of the project under consideration are frequently required. The reconnaissance must be conducted by the most mature and experienced engineer available, one who has commercial and operating judgment, as well as engineering judgment. His work is seldom thoroughly checked and should he recommend against the best route, it is difficult or impossible to rectify the mistake, since the subsequent detailed surveys will not discover a major route which was overlooked or decided against in the reconnaissance, and since, also, the project will be too far advanced to change. The project will then be perpetually burdened with a poor location. This is a most serious handicap, since it hampers the project and invites competitors to develop a better location. Such an outcome frequently results in a loss of part or all of the investment.

197. Traversing. *Traversing* (which is extensively used in route surveys) is the process of obtaining a connected set of angles, or directions, and of distances along a line survey, sufficient to plot the survey on a map.

198. Compass Traversing. For approximate surveys, one of the several types of compass is used, the distances being obtained by some of the less precise methods mentioned in Chapter II. To insure against local attraction and other compass errors (page 56), it is necessary to set up the compass at every change in direction, or angle point, and to take both forward and reverse bearings at each set-up, as in Fig. 198.



COMPASS TRAVERSE

Object		Needle	Dist.	
		AT STA A		
□B		S30½°W	468	
		AT STA. B		
□A		N30½°E		
□C		S51¾°E	328	
		AT STA. C		
□B		N52°W		
□D		S81¼°E	642	
		AT STA. D		
□C		N81¼°W		
□E		N20½°E	287	
		AT STA. E		
□D		S20¾°W		
□F		N32°E	174	
		AT STA. F		
□E		S32°W		

April 3, 1950
Fair, warm

Hogan
⊼ Denton

FIG. 198.

A more rapid but less certain method is to occupy each alternate point of the traverse with the compass and to read the forward bearing of one line and the reverse bearing of the other, or backsight, line. It will be noted that this method gives no check on the bearings, either for mistakes in reading or for local attraction.

199. Transit Traversing. Either of the preceding methods of traversing may be accomplished by using the compass of the transit only, without measuring transit plate angles.

However, in nearly all transit traversing, the angles at each angle point are read on the horizontal plates of the transit to a precision of about one minute.

In the better engineering traversing, it is customary to determine the true bearing or azimuth of the first and last line of the traverse either by astronomic observations or by connecting with lines of known direction. In long traverses, it is desirable to secure additional directional checks not more than 10 miles apart, or not more than 50 angle points apart if the traverse has many short courses. From the original bearings or azimuth, the calculated bearings are computed forward in the field notes by using the measured transit plate angles, and checked against the known, or determined, bearings as occasion permits. Allowance must be made for the convergence of meridians as explained on page 229.

Distances are obtained with good or excellent precision with the steel tape; or for somewhat less precision, by stadia. Figures 96a, 97a, and 99, on pages 109 to 112, give examples of the notes for such surveys.

Taping is carried in units of 100 feet, or stations; thus, station 16 + $75^{\underline{2}}$ means 1675.2 feet from the beginning of the survey.

The computation and checking of traverses is covered in Chapter XVII.

200. Aerial Surveys. While it is recognized that there are limits of precision in measurements taken from aerial photographs, experience within the last decade has indicated that frequently they may be used to advantage in connection with route surveys. Aerial photographic surveying is generally let by contract to firms specializing in this work; if the pictures meet contract specifications. the errors are generally negligible, since the accuracy, as compared to precise ground measurements, is usually within one per cent. In some cases, results from aerial surveys check more closely with the actual ground than do those from ordinary ground surveys. A limitation of aerial photography is the inability accurately to define elevations or relief, and to indicate features hidden under natural cover. Offsetting these disadvantages, there is a wealth of detail, absence of personal errors, and uniformity of accuracy in all parts of the map in aerial pictures far beyond that given by a map made by ground methods. With intelligent use, aerial surveys provide a valuable adjunct to the older methods of route surveying. They are particularly advantageous in covering forbidden property and inaccessible country which is either rough or covered with water.

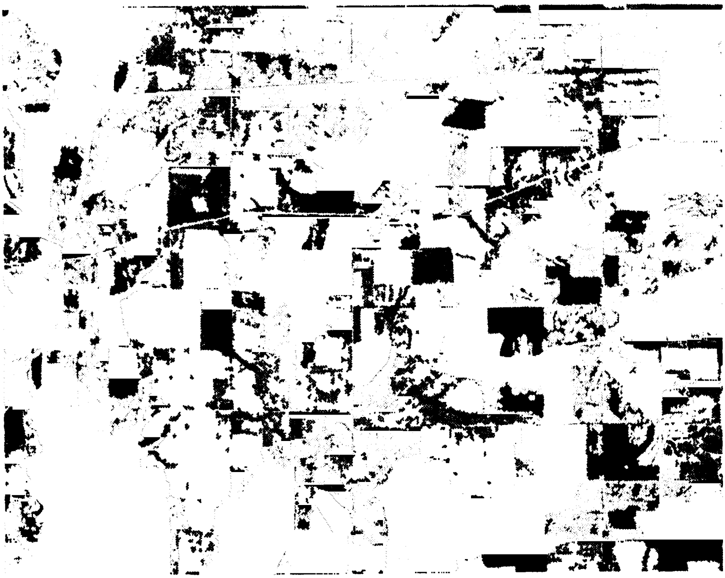


FIG. 200. AERIAL MOSAIC.

Perhaps as much as one-half, or even more, of the country has been photographed from the air in recent years, largely in connection with governmental projects, and many of these photographs are available.

Aerial photography has been used successfully in the reconnaissance and/or preliminary surveys for electrical transmission lines, for highways, for pipe lines, and for river and harbor improvements. The usefulness of the aerial survey begins with the inception of the work and continues throughout the construction period. Map locations may be made directly on aerial mosaics or aerial maps;* right of way is more easily obtained with little or no use of ground surveys and with a minimum of negotiations with land owners; minute studies may be made of drainage, structures, and other improvements; stream flow and drainage areas may be calculated without the use of field operations; and throughout the progress of the work, the aerial survey may furnish a basis for, and a constant check on, the more exact ground surveys. In many instances, the time saved and the aid furnished in securing rights of way are alone a sufficient justification for aerial surveys. The cost is not out of line with that of ground surveys. Where little information is available from previous surveys or maps, it is usually economical to make an aerial survey. See also § 221

201. The Preliminary Survey. Following the reconnaissance, a more careful instrumental survey or surveys of the selected route or routes is made with a view to comparing their advantages more precisely. The preliminary surveys are made by one of the processes of traversing previously mentioned. Generally the traverse line is run only approximately in the position of the final location which the construction will occupy; rather it is so run as to serve as a base line from which a topographic map of a narrow strip of territory may be prepared. This map is wide enough to cover all possible positions of the final location; in some places perhaps it covers a width of only 100 feet, in rare instances perhaps a mile.

In the older types of preliminaries, still used on many long surveys, the transit traversing party is followed by a level party, which in turn is followed by a topographic party. The organization and duties of these parties may be inferred from the notes of Figs. 79, 79a, 97a, and 201, all of which refer to the same railroad preliminary. Tapes 300′ or 500′ long will nearly double the speed of such preliminaries if they can be used instead of the usual 100′ tape. In Fig. 201, the elevation data were taken with an engineers' level or with a hand level, and the slope angles were taken with a clinometer. However, the clinometer alone might have been used, since, with its vernier clamped at zero, the clinometer becomes a hand level. Three

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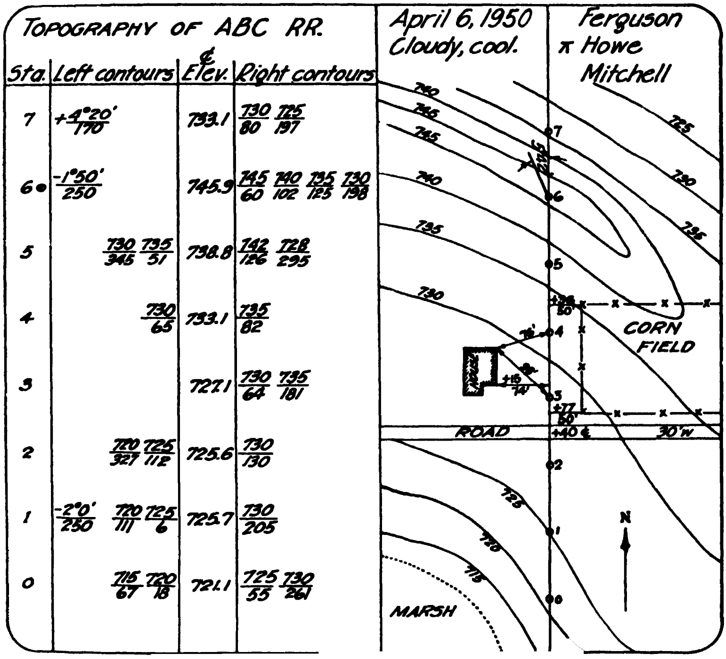
* In aerial photography, a mosaic is a composite picture made up of reduced aerial photographs or of "contact prints." The aerial map is usually traced from the mosaic, omitting some of the details. Approximate contours as obtained by means of the aerial survey, or more precise contours from ground surveys, may be added to the map.



TOPOGRAPHY OF ABC RR.

Sta.	Left contours	℄ Elev.	Right contours
7	$+\frac{4^\circ 20'}{170}$	733.1	$\frac{730}{80}$ $\frac{725}{197}$
6●	$\frac{-1^\circ 50'}{250}$	745.9	$\frac{745}{60}$ $\frac{740}{102}$ $\frac{735}{125}$ $\frac{730}{198}$
5	$\frac{730}{345}$ $\frac{735}{51}$	738.8	$\frac{742}{126}$ $\frac{728}{295}$
4	$\frac{730}{65}$	733.1	$\frac{735}{82}$
3		727.1	$\frac{730}{64}$ $\frac{735}{181}$
2	$\frac{720}{327}$ $\frac{725}{112}$	725.6	$\frac{730}{130}$
1	$\frac{-2^\circ 0'}{250}$ $\frac{720}{111}$ $\frac{725}{6}$	725.7	$\frac{730}{205}$
0	$\frac{715}{67}$ $\frac{720}{18}$	721.1	$\frac{725}{55}$ $\frac{730}{261}$

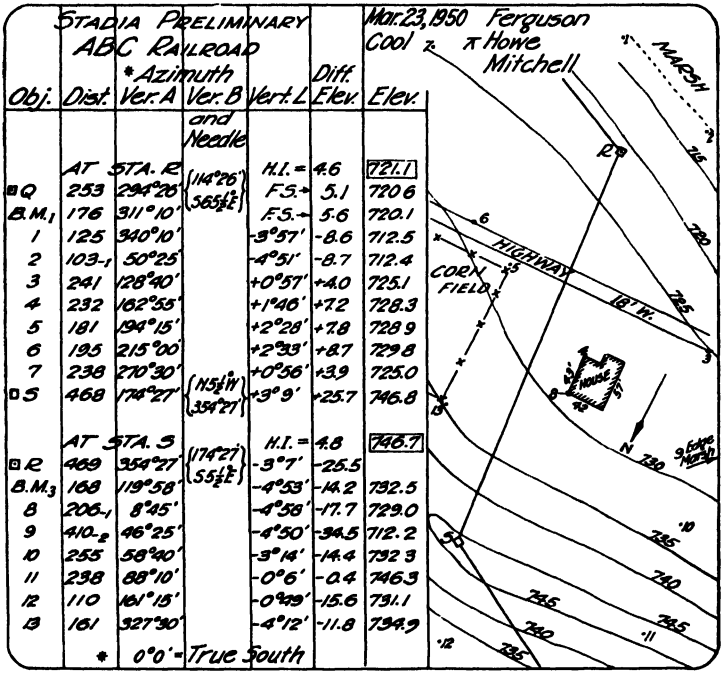
April 6, 1950
Cloudy, cool.

Ferguson
π Howe
Mitchell

FIG. 201.

methods of locating contours are shown on the figure: (1) by locating the contour interval points (even 5-foot points); (2) by locating breaks in the slope (as to the right of station 5); and (3) by slope angle taken from the last measured distance. While these cross lines are usually 100 feet apart, they may be spaced closer in broken country, or they may be spaced perhaps 300 feet apart in smooth country. If the wide spacing is permissible, then the 300-foot or the 500-foot tape may be used to advantage in the transit traverse.

The more modern *stadia preliminary survey* gathers alignment, elevation, and topographic data at the same time and thus avoids the necessity of going over the ground successively with several parties. Such a survey is rapid and cheap and requires better personnel in the party, especially transitman and notekeeper. It is not quite so precise as the older type of preliminary and it is impracticable in timbered country.



STADIA PRELIMINARY
ABC RAILROAD

Mar. 23, 1950 Ferguson
Cool ⊼ Howe
Mitchell

Obj.	Dist.	*Azimuth Ver. A	Ver. B and Needle	Vert. ∠	Diff. Elev.	Elev.
	AT STA. R		(114°26' S65½°E)	H.I. =	4.6	721.1
□ Q	253	294°26'		F.S.→	5.1	720.6
B.M.$_1$	176	311°10'		F.S.→	5.6	720.1
1	125	340°10'		-3°57'	-8.6	712.5
2	103-1	50°25'		-4°51'	-8.7	712.4
3	241	128°40'		+0°57'	+4.0	725.1
4	232	162°55'		+1°46'	+7.2	728.3
5	181	194°15'		+2°28'	+7.8	728.9
6	195	215°00'		+2°33'	+8.7	729.8
7	238	270°30'		+0°56'	+3.9	725.0
□ S	468	174°27'	(N5½°W 354°27')	+3°9'	+25.7	746.8
	AT STA. S		(174°27' S5½°E)	H.I. =	4.8	746.7
⊡ R	469	354°27'		-3°7'	-25.5	
B.M.$_3$	168	119°58'		-4°53'	-14.2	732.5
8	206-1	8°45'		-4°58'	-17.7	729.0
9	410-2	46°25'		-4°50'	-34.5	712.2
10	255	58°40'		-3°14'	-14.4	732.3
11	238	88°10'		-0°6'	-0.4	746.3
12	110	161°15'		-0°49'	-15.6	731.1
13	161	327°30'		-4°12'	-11.8	734.9
	*	0°0' = True South				

FIG. 201a.

Figure 201a gives a set of stadia preliminary notes covering the same territory as was covered by the older type preliminary of the field notes of Figs. 79, 79a, 97a, and 201. In Fig. 201a, the distances are entered in the notes as 100 times the observed rod interval. While some of the distances so recorded (on flat angles and/or short sights) should theoretically have 1 foot (the $f + c$ constant) added in order to secure the horizontal distance, this minor correction is frequently omitted. On points R_2 and S_8, the horizontal correction of 2 feet (from Table V) is diminished by 1 foot ($f + c$) and entered in the distance column as a minus 1 subscript, which means that the true corrected horizontal distances are 102 feet and 205 feet, respectively. Similarly, on point S_9, the tabular horizontal correction of 3 feet is entered in the distance column as minus 2, and the true corrected horizontal distance becomes 408 feet.

Verniers *A* and *B* and the needle are read only on transit stations. Side shot azimuths are estimated by the zero index on one vernier only to the nearest 5 minutes, without troubling to note the coincidence of vernier lines. The basic elevation of each transit station, to which the differences in elevation are applied, is enclosed in a square. The elevation of transit point *S* is obtained by using a mean of the differences in elevation along *RS* and *SR*.

The F.S. readings on *Q* and on B.M.$_1$ are level readings taken with the bubble centered.

The object of any of these preliminary surveys is to get data for the *preliminary map*, which is usually drawn to a scale of one inch equals one hundred, two hundred, or four hundred feet. The contour interval is commonly 1 foot, 5 feet, or 10 feet. The larger scale maps are prepared for those types of construction which involve considerable grading or other detail. Perhaps a 400-foot scale and a 5-foot contour interval have the greatest usage.

On this map, an example of which is shown in Fig. 201b, the *final location* (defined in the following section) is projected in the office by a trial and error method; that is, what appears to be a good location is laid out with a thread on the map, a profile is made from the map along this line, and a grade line for the construction laid out with a thread on the profile. Needles are stuck at the intersections of the tangents on the map and at the grade-line intersections on the profile. The original map location is then shifted where necessary to secure an improved location; the profile is modified accordingly; the grade line is adjusted on the profile; and this process of revision on paper is continued until the location seems satisfactory, when both are penciled.

Frequently an additional location, or several locations which differ so far from the first as not to be simply a modification of it, must be worked up on map and profile. First costs, operating costs, and other features of desirability are computed, and the most favorable of the several locations is then selected. Even after this, it is always necessary to make minor alterations when the line is staked in the field, and frequently major changes are required. Since such changes are possible up to and even during construction, it is apparent that a well-constructed location will usually be obtained.

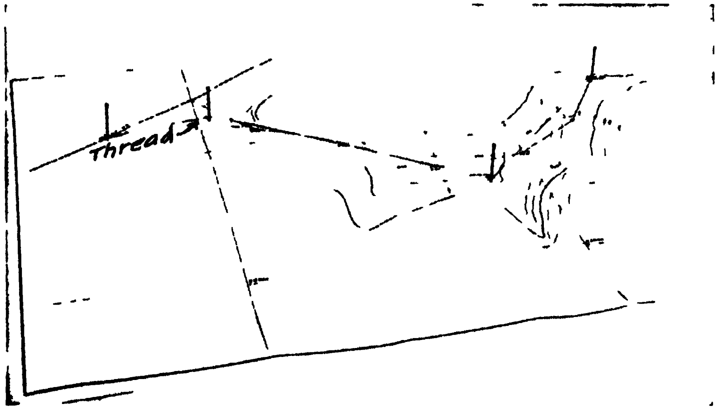


Fig. 201b. Preliminary Map, with Thread Location.

Where the location is a simple one — for example in transmission lines — it is not necessary to take topography, the preliminary survey alone, together with some little information regarding roads, improvements, and property lines being sufficient for the preparation of the map. Contours or topography will not be necessary, except perhaps at a few points. In the more simple surveys of this nature it is sometimes possible to run the preliminary line in the exact final location, at least part of the way, and thus to eliminate the necessity of another survey line for the final location mentioned below.

Opinions differ as to whether preliminary lines should be run as precisely as is the final location. A precise preliminary offers some advantage in surveying and checking the final location. If any portion of the preliminary is to be used as the final location, then it should, of course, be run precisely. On the other hand, a less precise preliminary is faster and less expensive to run and is therefore better where several preliminaries are needed. The desirability of running preliminaries over *all* routes which may offer an advantageous location must not be overlooked or omitted because of the slight delay or expense involved. Such omission may cause embarrassment in much the same way as similar omission in the reconnaissance; or they may cause financial failure of the project.

The preliminary map (Fig. 201b) is drawn in accordance with the suggestions of Chapter XVI.

202. The Final Location. This is the exact location of the center line (sometimes a line parallel to the center line) of the engineering construction which is to be undertaken. It is usually run with a precision of from 1/2000 to 1/5000 and consists of a traverse with fairly permanent stakes set 100 feet apart, and with hubs at all transit and angle points. On curves, the stakes are rarely set closer than 100 feet. On transmission-line location, the interval between stakes may be 300 feet or even 500 feet where long tapes are used, and it is frequently not essential that they be set on exact 100-foot stations.

On such work as highways, railways, canals, and pipe lines, where curves are required, these are now laid out for the first time on the ground, although they have previously been drawn on the map in the office.

In the older and less desirable practice, the location was sometimes made in the field immediately following the reconnaissance, and without a preliminary. This is not recommended except in very simple locations. Frequently highway location that follows existing roads may be made without a preliminary.

The final location on the ground is usually made from the previously mentioned *paper location,* minor changes being made as the staking proceeds.

A careful profile is run over the line with an engineers' level nd, after the quantities and detailed plans are worked up in he office, the project is in condition to justify securing bids om contractors, or starting construction.

203. Construction Stakes. At this point, the larger party that s been engaged in survey work is usually split up into smaller parties, each of which will set the construction stakes and supervise construction on a *residency* or *project* several miles long.

The first duty of the resident or project engineer and his small party is to find the final location stakes, check them more or less thoroughly depending upon their probable reliability, and to reset missing stakes from the field notes and maps. It is particularly desirable to check elevations, since trouble from this source is not infrequent and cannot be detected by casual inspection.

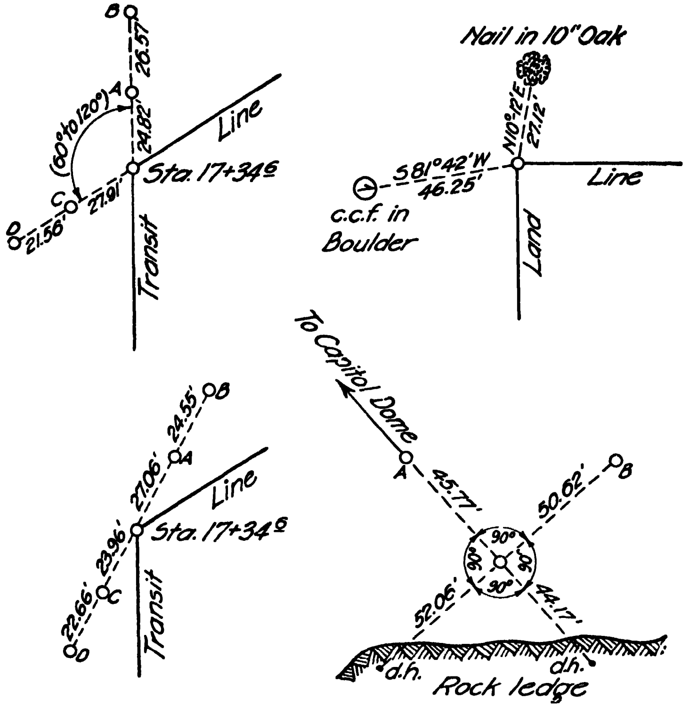


FIG. 203. REFERENCING POINTS.

He should then set additional stakes or reference points at the intersections of all straight lines or tangents, as indicated in Fig. 203. These reference points will be used to re-stake the final location when the location stakes are disturbed or obliterated during construction.

The next step is to set construction stakes to give detailed information to the contractor or the construction force. If grading is involved, cross-section stakes will be set as described on pages 251 to 257. Stakes for structures such as bridge abutments, culverts, towers for transmission lines, and so on, will be set as described for structures in §229. In general, it is necessary to set all the stakes which are difficult or impossible for the foreman or construction superintendent to set. In addition, it is desirable to set those stakes which may be set by the foreman or superintendent, but which can be more cheaply set by the engineer corps. Should the work be handled by contract, it is occasionally agreed therein that the contractor shall pay the cost, or furnish the men, material and transportation for setting the construction stakes. In nearly all contracts, it is provided that the contractor shall pay for re-setting stakes which his forces have carelessly disturbed.

In addition to setting stakes, the *resident engineer* or *project engineer* also inspects and passes on the construction work.

204. Horizontal Curves. For a more complete treatment of this subject the reader is referred to books* covering curves and earthwork in detail. The basis of computations necessary for staking a rather sharp simple circular curve is given in Fig. 204. In the United States, the sharpness of a curve is designated by the *degree of curve*, rather than by the radius, as in many other countries. The degree of curve is defined as the angle at the center of the curve, subtended by the two radii from the ends of

1. A 100-foot chord (railway definition and usage) or
2. A 100-foot arc (highway definition and usage).

There is no uniformity of practice in regard to which of the two definitions shall be used in other types of engineering work; adoption of the 100-foot arc or the 100-foot chord idea apparently being dependent upon the past experience of the engineer or engineers in charge. The chord definition is used by most railways and by many highway departments. It is perhaps to be preferred. The two definitions give somewhat different radii, tangents, externals and lengths of curve for the same degree of curve, as shown in Table C, where for the same angle between the tangents and the same degree of curve, the various necessary values for a curve have been computed for the two definitions.

---

* See, e g., Harry Rubey, *Route Surveys*, Macmillan, 1938.

TABLE C

P.C. = 181 + 13.7 $D = 16° 42'$ $I = 97° 13'$

	100-Foot Chord (Railroad) Definition	100-Foot Arc (Highway) Definition
Radius	344.3	343.1
Tangent	390.6	389.3
External	176.4	175.8
Length of curve	582.1 (around chords)	582.1 (around arc)
P.T.	186 + 95.8	186 + 95.8
First chord	86.3	86.1
Last chord	95.8	95.5
Chord corresponding to 100-foot arc		99.6
First deflection angle	7° 12′	7° 12′
Deflection angle for final chord	8° 00′	8° 00′

Tables are provided in railroad and highway handbooks which shorten the computations of Fig. 204 and Table C. All of the formulas of Fig. 204 apply to both definitions of degree of curve except that the degree of curve and radius for the highway definition are found from the formulas

(204) $$D = \frac{5729.6}{R}$$

or

(204a) $$R = \frac{5729.6}{D}.$$

These formulas are exact for the highway definition of degree of curve and approximate for the railroad definition of degree of curve.

The usual practice in engineering line surveys is to lay out the straight lines or tangents on the ground first, then to establish their intersection (called the P.I. or point of intersection), to choose a radius or degree of curve which best fits the requirements of the construction and of the topography, and finally to make the computations and notes of Figs. 204 and 204a. The positions of the P.C. and P.T. (point of curve and point of tangent, also sometimes designated as the T.C. and C.T. for tangent-curve and curve-tangent) are obtained on the ground

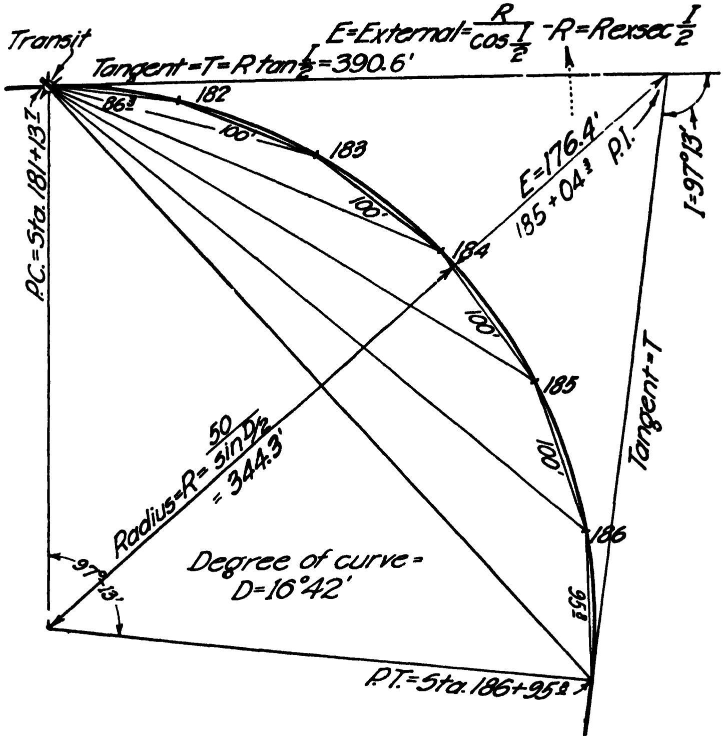
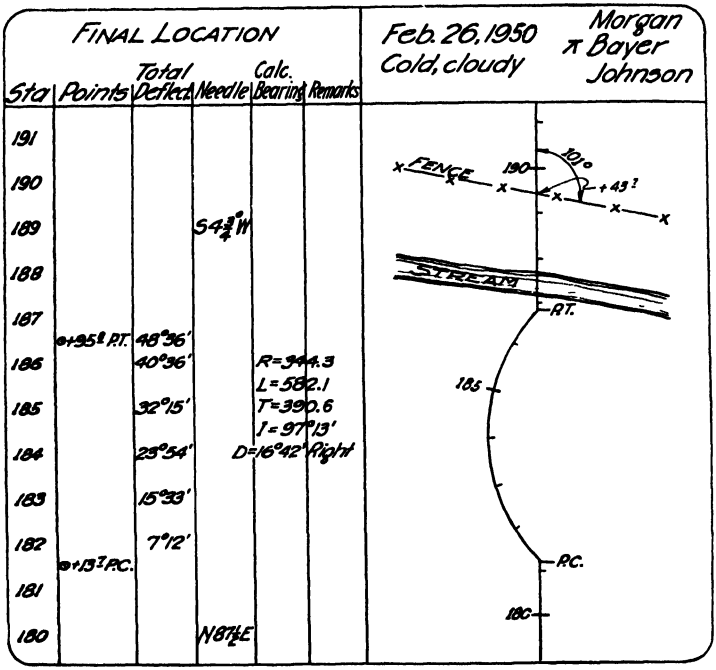


FIG. 204. HORIZONTAL CIRCULAR CURVE.

by measuring T (the tangent distance) from the P.I. in both directions. The transit is set up at either the P.C. or P.T., or ter at any intermediate point; and the plates are set on angles cording to this "rule of thumb." *Whenever the transit is ghted on a point the plates must read the total deflection angle nd opposite that point in the field notes.* Successive full stations 00 feet apart (rarely 50 feet or even less) are established by rning off the aforesaid angles and measuring the chord distances (100-foot chords if the railway definition is used, or chords sufficiently less than 100 feet to give a 100-foot arc if the highway definition is used). At each end of the curve, shorter chords of the required computed length are used so that the full



FINAL LOCATION

Feb. 26, 1950
Cold, cloudy

Morgan
π Bayer
Johnson

Sta	Points	Total Deflect.	Needle	Calc. Bearing	Remarks
191					
190					
189			S 54¾° W		
188					
187					
	⊙+95^{1} P.T.	48°36'			
186		40°36'		R=344.3	
				L=582.1	
185		32°15'		T=390.6	
				I=97°13'	
184		23°54'		D=16°42'	Right
183		15°33'			
182		7°12'			
	⊙+13^{7} P.C.				
181					
180			N 87½ E		

FENCE
190
101°
+43?
STREAM
P.T.
185
P.C.
180

Fig. 204a. Curve Notes.

100-foot stations used on the tangents may be carried around the curves. Field notes are given in Fig. 204a, corresponding to the calculations of Fig. 204 and the angular calculations below.

In using the highway, or arc, definition of degree of curve, some engineers prefer to use 50-foot chords in staking curves sharper than about 7 degrees; 25-foot chords for curves sharper than about 14 degrees; and so on; thus making their taping tend to follow the arc of the curve on the ground.

The deflection angles from the tangent are calculated as follows:

First station to be set 182 + 00.0
P.C. 181 + 13.7
First chord (or arc) length 86.3 feet.

Length of curve $= \left(\frac{I}{D}\right)(100) = \left(\frac{97°\ 13'}{16°\ 42'}\right)(100) = 582.1$ feet.

P.C. = 181 + 13.7 181 + 13.7
P.T. = 186 + 95.8
which makes the last chord (or arc) = 95.8 feet.

With the transit at the P.C., sighted along the tangent, turn off a first deflection angle to line in station 182 of

$$\left(\frac{86.3}{100}\right)\left(\frac{16°\ 42'}{2}\right) = \qquad 7°\ 12'$$

To line in station 183 add $\frac{D}{2}$ 8° 21'
and use 15° 33'
To line in station 184 add $\frac{D}{2}$ 8° 21'
and use 23° 54'
To line in station 185 add $\frac{D}{2}$ 8° 21'
and use 32° 15'
To line in station 186 add $\frac{D}{2}$ 8° 21'
and use 40° 36'
To line in the P.T. = 186 + 95.8
use $\frac{I}{2}$ 48° 36'

Leaving an angle corresponding to
the last chord (or arc) of 8° 00'
As a check on all deflection angles

$$\left(\frac{95.8}{100}\right)\left(\frac{16°\ 42'}{2}\right) = \qquad 8°\ 00'.$$

When it is unnecessary to lay out the curves precisely, or hen a good foreman is in charge of construction and the rves are short, it is sometimes sufficient to establish the P.C., he P.T., and the center of the arc, by measuring along the tangents and along E (the external). In such cases it is not necessary to move the transit from the P.I.

Curves of radii less, and occasionally more, than 100 feet may be laid out by establishing the center of the curve and swinging the arc with a tape or wire. However, even with short radii this method is frequently impracticable because of the difference in elevation of the various points, because of obstructions, or because the center is inaccessible.

205. Vertical Parabolic Curves. At points where the grade changes (point V of Fig. 205), it is necessary for most constructions to use a *vertical curve.* It is convenient, although not necessary, to arrange that V shall fall at a full station and that the vertical curve shall extend an equal number of full stations in both directions from V. The curve used is a parabola whose equation is of the form $y = 4kx^2$, where k is a constant. The skewed axes are either AVC and a vertical through A, or BV and a vertical through B. Vertical offsets may be computed from either AVC or BV. Vertical distances are y coordinates and distances along AVC from A or along BV from B are x coordinates.

As is seen in Fig. 205, $VE = 1/2CB$ (from similar triangles). From the equation of the parabola, it follows that $VD = (1/2)^2(CB) = 1/4CB$ and consequently that $VD = 1/2VE$. It also follows from the equation of the parabola that

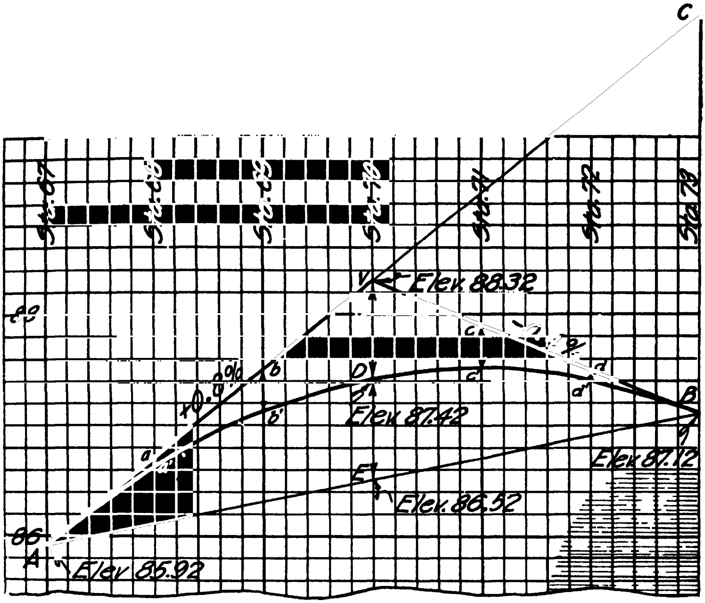
$$aa' = dd' = (\tfrac{1}{3})^2VD = \tfrac{1}{9}VD$$

and that

$$bb' = cc' = (\tfrac{2}{3})^2VD = \tfrac{4}{9}VD,$$

since the vertical offsets from either tangent to the parabola vary as the square of the distances along the tangents AVC or BV, beginning at A or B, respectively. A numerical example is shown in Fig. 205. Other methods may be used to secure the same results. The slide rule is helpful.

In rare instances a *horizontal parabola* may be needed, for example, to secure a pleasing appearance in landscaping. The same general method of laying out the parabola may be used by considering Fig. 205 as lying in a horizontal plane.



A = 85.92 88.32 = V
B = 87.12 86.52 = E
2 | 173.04 2 | 174.84
E = 86.52 87.42 = D 88.32 = V
86.52 87.42 = D
Check VD = 0.90 0.90

Also

$$VD = \frac{\text{Stations} \times \text{Diff of Gradiants}}{8}$$

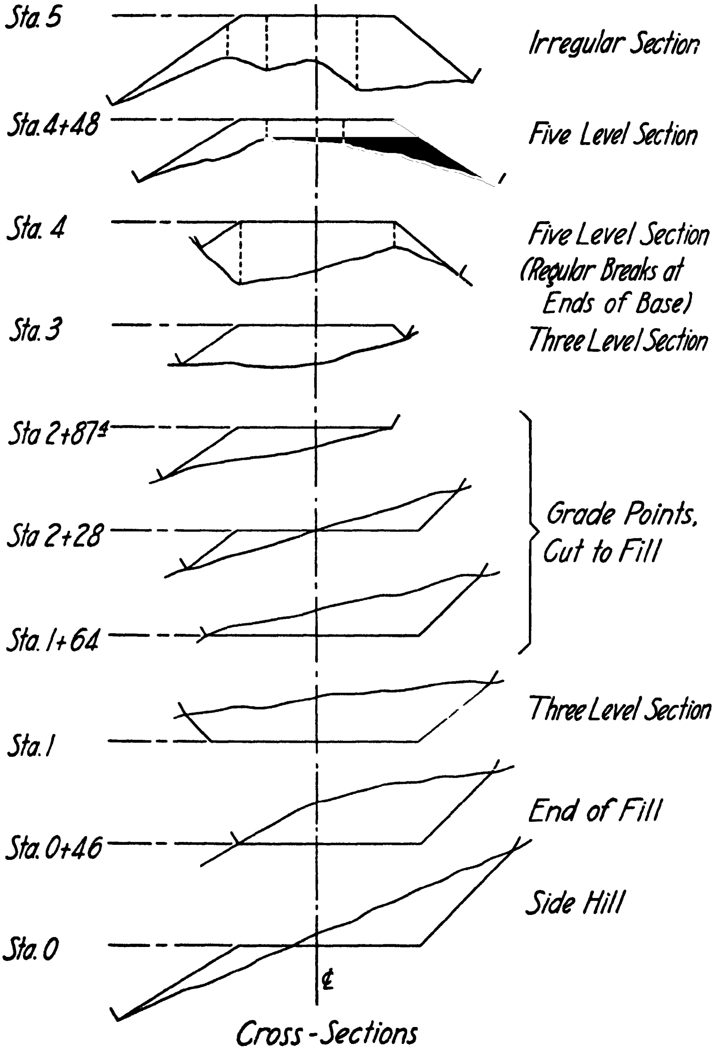
$$= \frac{6 \times 1.2}{8} = 0.90$$

Sta.	Tangent elevations along AV & VB	Vertical offsets from AV & VB	Elevations on vertical curve
67	85.92	0.00	85.92
68	86.72	$\frac{1}{9}(0.90) = 0.10$	86.62
69	87.52	$\frac{4}{9}(0.90) = 0.40$	87.12
70	88.32	0.90	87.42
71	87.92	$\frac{4}{9}(0.90) = 0.40$	87.52
72	87.52	$\frac{1}{9}(0.90) = 0.10$	87.42
73	87.12	0.00	87.12

Fig. 205. Vertical Curve.

206. Grading. Approximate estimates of the quantities of excavation and embankment may be made from center heights of the profile (Fig. 207a). Such estimates are based on the assumption that the ground is level, or nearly level, transversely to the center line. For more precise computation of earthwork quantities, and to stake out the work for construction, procedure is described in §§ 207, 208, and 209.

207. Slope Stakes. Figures 207 and 207a show typical earthwork cross-sections and the corresponding profile. Field notes are shown in Fig. 207b. Slope stakes are set where the side



Cross-Sections

FIG. 207.

lope of the construction intersects the natural ground line. If the transverse slope of the ground is not uniform between

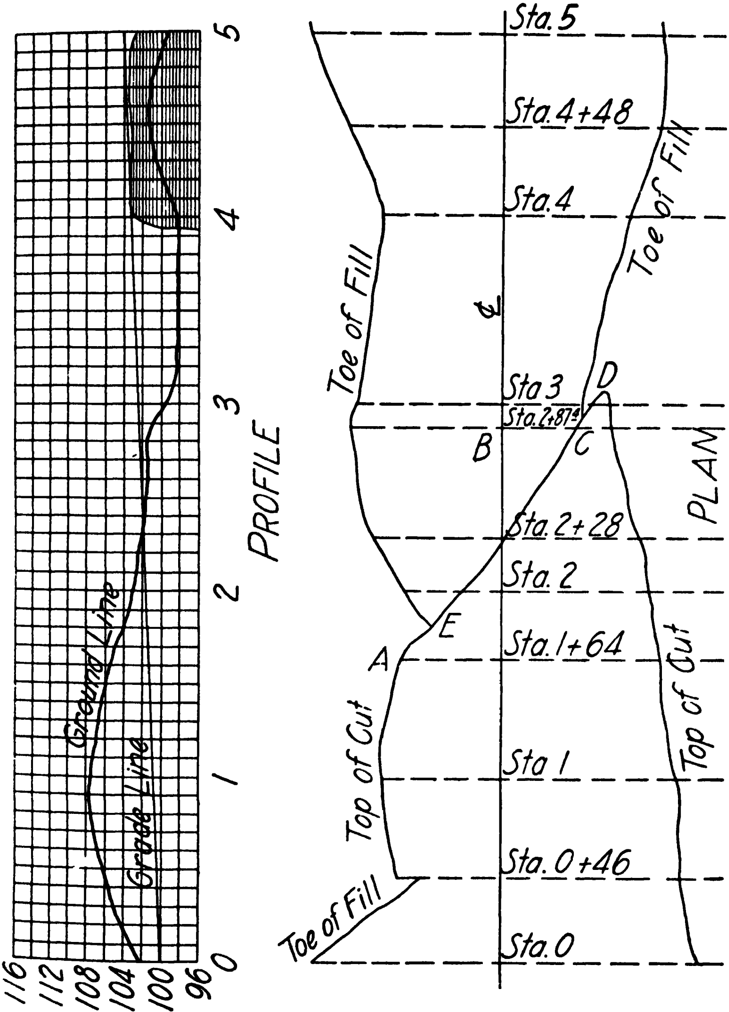
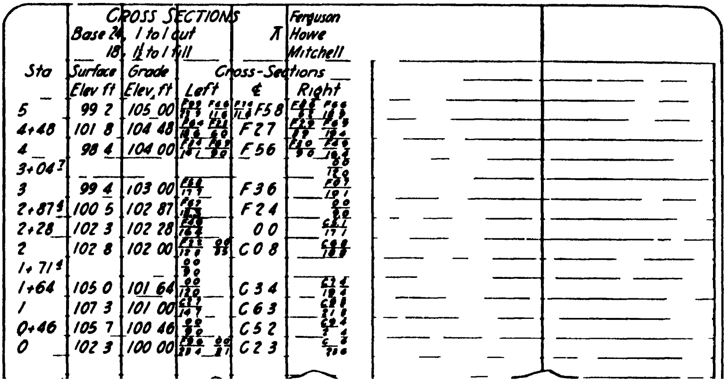


FIG. 207a.

he center stake and the slope stakes, additional elevations are ken. The elevation at the center stake is always taken. ross-sections of this nature are taken every 100 feet along the line survey and at such intermediate (or plus) points as may be necessitated by non-uniform slope of the ground along the line.

Slope stakes are usually set with a 50-foot metallic tape and an engineers' level. Occasionally a hand level or a long straight edge with a bubble, called a cross-section rod, replaces the engineers' level. The slope stakes are placed by a trial and error method. A rodman, levelman, and occasionally a notekeeper constitute the cross-section party. The zero end of the tape is held on the center stake by the notekeeper; or it is attached in some way to the stake. The levelman reads the rod as held on the center stake and computes the cut or fill, which is then marked on the stake. From this computed cut or fill it is possible to estimate the cut or fill for the slope stake (Fig. 207c), and from this estimate a corresponding horizontal distance is figured. The rod is then held at the computed distance from the center stake and the exact cut or fill may be computed from the rod reading. Should this exact value correspond with the estimated cut or fill, the slope stake is driven at that point and the cut or fill is marked on it. If the computed value does not correspond within a tenth or two-tenths of the assumed value, then the rod is moved towards or away from the center stake, a new rod reading is taken, and a new cut or fill compared with the actual distance from the center stake. Usually the



CROSS SECTIONS

Base 24, 1 to 1 cut

18, 1½ to 1 fill

Cross-Sections

π Ferguson

Howe

Mitchell

Sta	Surface Elev ft	Grade Elev, ft	Left	℄	Right
5	99 2	105 00	[illegible]	F5 8	[illegible]
4+48	101 8	104 48	[illegible]	F2 7	[illegible]
4	98 4	104 00	[illegible]	F5 6	[illegible]
3+04 7					[illegible]
3	99 4	103 00	[illegible]	F3 6	[illegible]
2+87 5	100 5	102 87	[illegible]	F2 4	[illegible]
2+28	102 3	102 28	[illegible]	0 0	[illegible]
2	102 8	102 00	[illegible]	C0 8	[illegible]
1+71 5			[illegible]		
1+64	105 0	101 64	[illegible]	C3 4	[illegible]
1	107 3	101 00	[illegible]	C6 3	[illegible]
0+46	105 7	100 46	[illegible]	C5 2	[illegible]
0	102 3	100 00	[illegible]	C2 3	[illegible]

FIG. 207b. CROSS-SECTION NOTES.

second or third trial will locate the slope stake with sufficient precision. All cuts or fills are figured from the horizontal plane of the base of the roadbed and the distances from the center stake are computed as shown in Fig. 207c. The result of this field procedure is entered in the notes of Fig. 207b. Where the ground does not slope uniformly from the center stake to the slope stake, it is necessary to take an extra reading as indicated on stations 0, 2, 4, 4 + 48, and 5 of Fig. 207b.

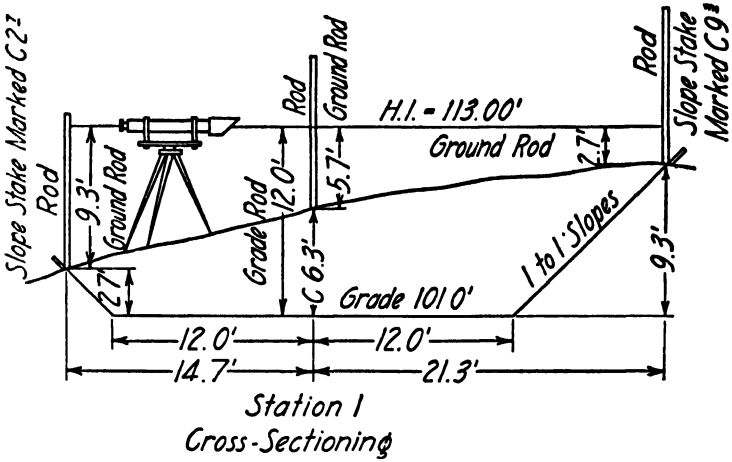


Fig. 207c.

The height of instrument for cross-sections is carried forward by differential leveling, the notes for which are kept on a separate page. The *grade rod* is the difference between the height of instrument and the *grade elevation.* The *ground rod* is the actual rod reading. Grade rod minus ground rod equals *t* or *fill.*

Another less accurate method of setting slope stakes is to un profiles at right angles to the center line and plot these on oss-section paper. Scaled distances from the center line to he slope stake points may then be determined by plotting the ut or fill at the stations on the cross-section paper. Cut and ill templates formed to scale may be used to advantage.

208. Computation of Grading. The reader is referred to special books on earthwork* for a complete treatment of this subject. Only the simpler and more fundamental facts will be discussed here. The following formula is used to obtain the area of each cross-section and from these areas, the volume between adjacent cross-sections. The ground is assumed to slope uniformly from one cross-section to the next.

A closely approximate volume is obtained by averaging the areas at the two ends of a section (usually 100 feet apart) and multiplying this average by the length between the planes of the two end areas. This is expressed by the formula

$$V = \frac{L}{27}\left(\frac{A_1 + A_2}{2}\right), \quad \text{approximately,} \tag{208}$$

where V is the volume in cubic yards, and A_1 and A_2 are the areas in square feet of two adjacent cross-sections which are a distance L feet apart. One cubic yard equals 27 cubic feet.

A mathematically exact *prismoidal formula* is

$$V = \frac{L}{27}\left(\frac{A_1 + 4A_m + A_2}{6}\right), \tag{208a}$$

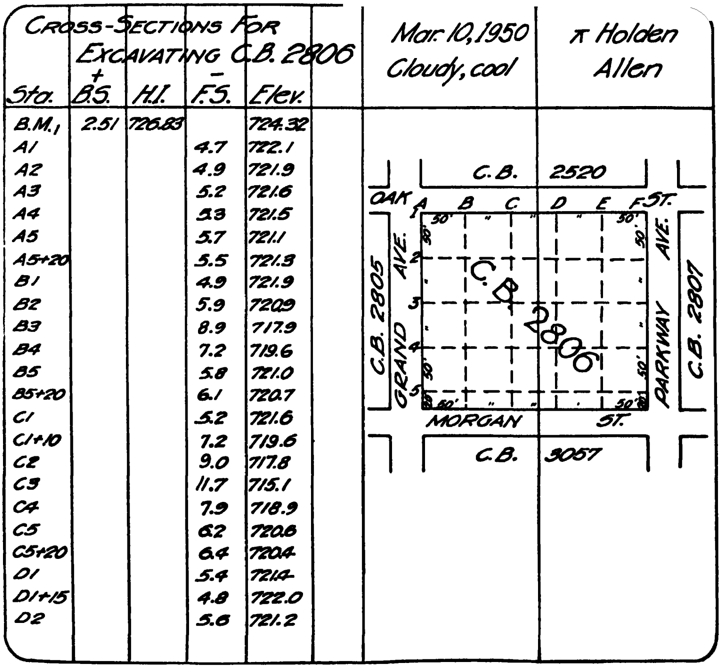
where A_m is the area of the cross-section half way between A_1 and A_2. No measurements of A_m are taken on the ground, but the lineal dimensions of this section are computed as the means of the corresponding dimensions of the end sections. This, of course, assumes that the ground slopes uniformly between A_1 and A_2. The area of A_m is *not* the mean of A_1 and A_2.

The prismoidal formula is of broad application in engineering. It applies to all prismoids, the prismoid being defined as any solid having for its two ends parallel plane figures of the same number of sides, all of the sides of the solid being plane figures also. The earthwork solids mentioned above are not exactly prismoids since the surface of the earth is more nearly composed of warped surfaces than of planes, yet the prismoidal formula also applies in this case. It may be used properly for exact computation of the frustra of pyramids, wedges, and conic sections of revolution, including the cone, sphere, ellipsoid of revolution, paraboloid of revolution, and the hyperboloid of revolution; hence it is of quite general value in various engineering computations. It is in a form which is much easier to use and to remember than other formulas for volume computations.*

---

* See, e.g., Harry Rubey, *Route Surveys*, Macmillan, 1938.

End areas for use in formulas 208 and 208a are either plotted on 1 10-inch cross-section paper and planimetered, computed geometrically or by coordinates from the notes of Fig. 207b, or taken from earthwork tables or diagrams. Detailed descriptions, tables, and diagrams for these methods are beyond the scope of this text but are treated fully in Rubey's *Route Surveys* and similar volumes.



CROSS-SECTIONS FOR EXCAVATING C.B. 2806

Sta.	+ B.S.	H.I.	− F.S.	Elev.
B.M.₁	2.51	726.83		724.32
A1			4.7	722.1
A2			4.9	721.9
A3			5.2	721.6
A4			5.3	721.5
A5			5.7	721.1
A5+20			5.5	721.3
B1			4.9	721.9
B2			5.9	720.9
B3			8.9	717.9
B4			7.2	719.6
B5			5.8	721.0
B5+20			6.1	720.7
C1			5.2	721.6
C1+10			7.2	719.6
C2			9.0	717.8
C3			11.7	715.1
C4			7.9	718.9
C5			6.2	720.6
C5+20			6.4	720.4
D1			5.4	721.4
D1+15			4.8	722.0
D2			5.6	721.2

Mar. 10, 1950
Cloudy, cool

⊼ Holden
Allen

C.B. 2520
OAK ST.
A
B
C
D
E
F
1
2
3
4
5
50'
"
GRAND AVE.
C.B. 2805
C.B. 2806
PARKWAY AVE.
C.B. 2807
MORGAN ST.
C.B. 3057

FIG. 209.

209. Borrow Pits. Borrow pits adjacent to railways and highways are best staked and computed by extending the cross-sections outward to include the borrow pit.

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* The formula is indeed precisely accurate whenever the area of the cross-section varies as any quadratic function of the distance to that cross-section, measured along a line perpendicular to the cross-sections, from any point in that line. This is true for the solids just mentioned and for many others.

Where, however, the excavated material is to be taken from an area which does not adjoin the railway or highway, or where excavation is being made for a building or industrial plant, the work is best laid out and computed by dividing the area into 50-foot or 100-foot squares as shown in Fig. 209. Each of the rectangles shown on this figure is the top view of a truncated rectangular prism whose volume is the area of the rectangle multiplied by the average (of the four corners) height. The ground surface and the excavated surface need not be planes. Occasionally truncated triangular prisms will be encountered and their volumes are likewise computed. Extra points may be taken within the rectangles. Some short cuts and special cases are treated in Rubey's *Route Surveys* and similar books.

PROBLEMS

1*. (a) Given I = 64° 13′, D = 10°, P.I. = station 98 + 47.0; compute the data necessary to stake this curve in the field, using the railroad (chord) definition of degree of curve. (b) The same with I = 58° 27′, D = 6°, P.I. = station 107 + 92.8. (c) The same with I = 108° 10′, D = 32° 12′, P.I. = station 13 + 84.6.

2*. The same as 1 but using the highway (arc) definition of degree of curve.

3*. (a) Compute the station elevations for a vertical parabolic curve (elevation of V = 748.31 feet) beginning at station 42 on a − 3 per cent grade and ending at station 46 on a − 1 per cent grade. (b) The same beginning at station 67 on a + 2 per cent grade and ending at station 73 on a + 1 per cent grade. (c) The same beginning at station 18 on a + 2.4 per cent grade and ending at station 26 on a − 1.3 per cent grade.

4. (a) A traverse is run in the neighborhood of the 43° parallel of latitude and extends N 45° E for a distance of 14 miles. What correction due to convergency of meridians should be applied to azimuths as carried forward from the beginning to the end of this traverse?

Ans. (a) + 34.3″ per mile of line

(b) The same for the 36° parallel and extending N 60° W for a distance of 20 miles.

(c) The same for the 48° parallel and extending S 45° W for 25 miles.

5. (a) In Fig. 209, excavation is to be made to elevation 710 feet. What is the volume (cubic yards) in the approximate rectangular prism A1, C1, C3, A3? (b) A3, C3, C5, A5? (c) A1, C1, C5, A5? *Ans.* (a) 3751 cu. yds.

6. Compute the earthwork yardage between adjacent stations in Fig. 207b.

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* See pages 247 to 253 for solutions of similar problems.

CHAPTER XIV

AREA SURVEYS *

210. Area Surveys. This chapter will treat of surveys of areas which are broad in relation to their length, as contrasted with route surveys of the preceding chapter, which deal with long, narrow areas.

211. Maps Available. These are mentioned on pages 180 to 182. Frequently no suitable map of the territory under consideration is available and it is consequently necessary to prepare one as described in this chapter. An organization may use its own engineers for this purpose, or it may contract with engineering or mapping companies for this service.

212. Control. Since these surveys extend over considerable area, it is desirable first to extend a skeleton or control survey of distances and angles of the required precision over the territory. This is accomplished by triangulation or traverse or by a combination of them. Elevations are usually determined with the engineers' level, rarely by vertical angles in connection with triangulation. Bench marks are set.

213. Triangulation. This is treated in Chapter XI.

214. Traverse. A traverse, as described in the preceding chapter, may be run in any convenient place which will serve as a basis for filling in the details described later, for example, in city streets, along roads or railroads, in valleys or along ridges, or through cleared ground. Where the country has been divided into sections by the Government, it is frequently desirable to run the traverses along or immediately adjacent to the section lines, both to facilitate location of the section corners and because the section lines are frequently located in the center of roads, or are easily traversed. A control consisting of triangulation and traverse is shown in Fig. 159.

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* See also Chapter XVIII.

215. Filling in Details. Usually following the control (seldom accompanying or preceding it because lack of elevation and orientation data hinders the work) comes the detailed location of streams, drainage lines, ridges, and contours, as well as the various improvements of man, such as buildings, roads, fences or property lines, railroads, canals, transmission lines, pipe lines, and the like. These may be located in several ways, the methods most frequently used being described in the following paragraphs.

216. Transit-Stadia. In this method the transit is set up on a previously established traverse or triangulation point and the instrument is *oriented* from a previously determined azimuth, that is, the telescope is so manipulated that it would point due south (occasionally north) when the plates read zero.

The rodman or rodmen then select points on the ground which will be needed for mapping and the transitman reads the rod interval, the vertical angle, and the azimuth of each of the points. The rodman holds the rod at corners of buildings and along roads, streams, drainage or ridge lines, and so on. In addition, for determining contours, he holds the rod on all high or low points which determine the topography, including all points at which the slope of the ground changes materially. This choice of points requires considerable judgment. Too few or poorly located points will give an unsatisfactory map, while too many points will make the work slow, tedious, and costly.

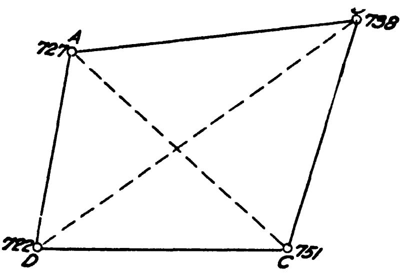

Fig. 216.

In the choice of points the rodman must keep constantly in mind the viewpoint of the draftsman who is to make the map. The contour points should be so selected that the draftsman may assume that the ground slopes uniformly between adjacent points. In Fig. 216 *A*, *B*, *C*, and *D* are points of known elevation which have been plotted on the map. Were this a plane surface, the ground would slope uniformly between any two points, but it is more generally a warped surface or composed of two planes. Correct interpolation may be made along *AB*, *BC*, *CD*, and *DA*, but interpolation along the diagonals *AC* and *BD* is uncertain. Nevertheless, the preceding assumption by the draftsman is the best possible one and the ground points selected must conform to it.

A rarely used and expensive method of contouring is actually to trace out the contour locations on the ground with level and rod and then to locate the successive rod points with a transit or plane table or by azimuth and distance.

The rodman paces or measures with a pocket tape or with his rod, many short distances which will save time for the instrumentman, or which will give more satisfactory results. For example, from any single set-up of the transit, it is impossible to see all four corners of a building, and even if regular transit-stadia data were taken on all four corners from more than one set-up, the dimensions of the building would not be accurate and would not be available except by scaling the plotted map or by prohibitively laborious computation. It is therefore customary to take transit-stadia shots on two or three corners of the building and for the rodman to measure the dimensions of the building. It is best practice to secure at least one surplus stadia shot or one measurement to serve as a check.

The following different methods of filling in detail are used.

FOLLOWING CONTROL SURVEYS

METHOD 1. As in Fig. 201a, the data are tabulated on the left-hand page of the notebook and a sketch made on the right-hand page, corresponding points being numbered for identification. The transit is set up on a point previously established in the control survey and oriented, that is, pointed due south when the plates read zero. Figure 201a gives the notes for such a survey, the letters *Q*, *R*, and *S* indicating occupied stations and the numbered points indicating successive points around *R* and *S* used for locating details. On each point at which the rodman holds his rod, the transitman reads the azimuth (estimated to 5′ without using vernier), vertical angle, and rod interval. The sketch on the right-hand page, which shows all details and occasionally even sketched contours, is made *before* the field data are taken and thus serves to indicate to the notekeeper whether or not the rodman has secured all necessary points. The map is made in the office.

METHOD 2. In this method, the sketch which was made in the notebook in Method 1 is drawn to scale on a sketching board in the field; in fact the map is completely penciled in the field. It has the advantage of allowing the field draftsman to draw the map with the ground before his eye. He can thus prepare an equally good map with fewer points actually located on the ground. For example, it is not necessary to locate every point at which the slope of the ground changes since the draftsman can see this with his eye and consequently needs fewer points. The penciled map is immediately available.

METHOD 3. This method is the same as Method 1 except that *no* sketch is made on the right-hand page, only a worded description or small sketch showing the location of each point. This method is definitely inferior to Method 1 except for very simple surveys and where the notekeeper draws the map in the office very shortly after completing the field work, so that he has the ground and field work in his memory.

CARRYING CONTROL SIMULTANEOUSLY

METHOD 4. This method is similar to Method 1 except that no control is available. Here it is necessary to carry forward azimuth, distance, and elevation from one instrument set-up to the next; as well as to fill in the detail at each set-up. One objection to this method is that no checked location of the points *Q*, *R*, and *S* (Fig. 201a) can be plotted, or their checked elevations known until the entire survey is completed or checked on some final point, for example, on the point of beginning. The map must be made in the office.

METHOD 5. This is similar to Method 2, except that no control is available and consequently the contours are only sketched as to form but not numbered. The continual traverse is carried forward by stadia as the survey progresses and checked for traverse closure and for elevations at the time of final closure. Traverses may be checked by plotting, by latitude and departures, or by coordinates. The map must be prepared in the office.

METHOD 6. This method is similar to Method 5 except that the detail around each point is plotted on a separate polar coordinate sheet in the field. Since checked azimuths, distances, and elevation for control are not available and since this later checking frequently requires adjustment, the map is not completed in the field nor are the contours definitely numbered; only their shape is sketched. The sheets are made to scale and may easily be oriented and placed together in the office and the numbered contours may there be drawn.

In general, it is better to do as much of the mapping as possible in the field since the best of field notes are hard to interpret when the ground is not visible. If the map is largely drawn in the office, it is usually necessary to take the completed map into the field and check it over by comparing it with the ground.

In comparatively level country, it is sometimes possible to avoid the necessity of reading vertical angles by leveling the bubble under the telescope and taking an ordinary level foresight on the rod. Such notes are taken at *Q* and B.M.$_1$ from Station *R* in Fig. 201a.

217. The Plane Table. This instrument (page 266) is used for obtaining detailed topographic data and a completely penciled map with much the same result as in Method 2 or Method 5 above. Little traversing is done with the plane table because of the time required to set it up. Small plane tables, or sketching boards which may be carried on the arm or used on a light tripod, are frequently used for work of low precision; as, for example, on rough or small scale maps and in military mapping.

The plane table is described in books on higher surveying. It is seldom used by private surveying organizations because most of them have neither the plane tables nor good plane-table men to operate them. Good plane-table men are rare, but may be developed from apt men after considerable training.

218. Rectangular Control. The foregoing transit-stadia or plane-table methods are more economical in country with little or no high heavy brush or timber. In wooded or brushy country it is usually better to run a series of parallel traverses, usually one-sixteenth, one-eighth, one-quarter, or one-half mile apart, depending upon the precision required in the contours. If section lines are used for control, then these traverses should divide the sections approximately equally

Levels are run along these traverses with an engineers' level and some side elevations are taken where necessary. If the

Courtesy of W. and L. E. Gurley

FIG. 217. PLANE TABLE

traverse lines are a considerable distance apart and some little detail is required between them, it is usually obtained by pacing or taping at right angles to the traverse, and by using a hand level or clinometer in a manner similar to that used in taking topography along a line survey, as described on page 241.

219. Timber Land Surveys. Surveys of timber land are made in the manner of § 218, modified to give cheaper and less precise results, and are used to plan logging operations. Such surveys are apt to be drawn to a scale of about 1 inch = 1000 feet and to have a contour interval of about 25 feet. The control lines, perhaps one-quarter or one-half mile apart, are run with a compass, and are paced or taped, the elevations being determined with an aneroid barometer. One party does all of this work, and it is accomplished rapidly, although not very precisely. If a tape is used, the custom is to use 300-foot or 500-foot tapes, sometimes with extra length on the end to allow for slope, as described on page 22.

220. Precise Topography on Small Areas. This type of large scale map is used by the landscape architect or engineer as a basis for both design and construction. A scale of 1 inch = 40 feet with contour intervals of from ½ foot to 2 feet is frequently adopted because it permits accurate delineation of details on the map, and the use of either engineers' or architects' scales.

Such surveys are made by establishing rectangular control systems as indicated in Fig. 209, and by determining the ground elevations at the stakes with an engineers' level. The sides of the rectangles vary from 50 feet to 100 feet or more. Much of the labor and time of stake driving may be saved by using a larger interval between adjacent stakes and by reading the ground elevations at the centers of the rectangles and at the midpoints of the sides. It is also necessary to determine additional elevations at other points of topographic importance. Details such as trees, roads, and fences are located with reference to these stakes.

In addition to their uses for design purposes, such surveys and maps may be used for yardage estimates, earthwork volumes being computed as on page 258.

221. Photographic and Aerial Mapping.* Photographs taken with special cameras, either from the ground or from an airplane, form a basis from which maps satisfactory for many purposes may be prepared. It is a highly specialized type of work requiring special training and special equipment, and cannot ordinarily be undertaken economically or effectively by the usual engineer without such training. Many private mapping companies will take contracts for work of this nature.

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* See also § 200.

Photographic surveys are based on a control of triangulation, traverse, or other surveys, such as the United States sectionalizing surveys, which have been made by ordinary ground methods. With elevations of critical points known from ground control, relative elevations and contours can be determined with considerable accuracy on rectified aerial photographs by means of a stereoscope equipped to measure parallactic displacement.

The United States Geological Survey and the Tennessee Valley Authority are obtaining topographic maps from aerial photographs comparable to those obtained from their ground survey work.

Aerial photography with a minimum of ground control is also being used successfully by the Department of Agriculture to produce maps from which areas of farm lands can be measured fairly accurately with a planimeter.

This method of mapping is progressing very rapidly and doubtless will be used to an increasing extent on large area work of the future.

Considerable published data are available in periodicals. Some of the best recent references are *Bulletins on Aerial Photogrammetry*, Earl Church, Syracuse University, 1931 to date, and *Photogrammetric Engineering*, American Society of Photogrammetry.

222. Scale and Use of Maps. The use of the map should govern the scale to which it is to be drawn, and the scale of the map then determines the type and precision of field surveys necessary. The smallest scale maps in general engineering use are those of the United States Geological Survey, which are drawn to scales of 1/62,500 or 1/125,000, that is, one inch equals approximately one mile or two miles. Recently the Survey is using a larger scale ($\frac{1}{24,000}$). Such maps are valuable for the general plan or reconnaissance of large projects, and for making rough estimates of their feasibility and cost.

For detailed estimates and planning of construction, it is customary to make maps on scales of 1 inch = 50 feet, 1 inch = 100 feet, or 1 inch = 200 feet. Scales of 1 inch = 400 feet and

1 inch = 1000 feet are used for somewhat less detailed planning and for general maps of small projects. Maps of 1 inch = 1000 feet scale are large enough to show property lines, except in cities, but will not adequately portray smaller features, nor will the contours be sufficiently numerous or precise for close estimates. A scale of 1 inch = 40 feet lends itself either to the engineers' or architects' scale.

With reasonable precaution it is possible to scale a map to the nearest 1/50 inch, which would mean one foot on the ground if the map scale were 1 inch = 50 feet, or proportionally for other scales.

The contour interval varies from one-half foot to 100 feet, in the majority of cases being 1 foot, 5 feet, or 10 feet. It is customary to assume, in using a contour map the detailed precision of which is unknown, that if the pencil is placed at any particular point on the map, over 90 per cent of the elevations as interpolated from the map should represent the true elevations of the ground with a maximum error of from one-quarter to one-half of the contour interval. That is, on a 5-foot contour map we should be able to estimate elevations within 1¼ feet to 2½ feet. As a consequence, the contour interval should not be less than the field data warrant. If the contour interval is too small, the contours will have to be drawn so closely together on the map as to run together.

223. Plotting the Data. The control or skeleton survey is first plotted on the sheets (preferably by rectangular coordinates, §§ 240 and 251) and the details are then plotted with straight edge, triangle, scale, and protractor. Since few, if any, dimensions are given on a map, all data must be scaled from it; and it is consequently necessary that maps be more carefully drawn to scale than is the case in machine or architectural drawings, where all dimensions are given in figures. Steel straight edges and triangles should be used instead of the less precise celluloid ones.

While any type of protractor may be used for plotting details, one of the better ones is shown in Fig. 223. It speeds and facilitates the work, where any considerable quantity of work is to be done.

For laying off angles precisely, as is frequently necessary in establishing the control lines on the map, it is customary to use the tangent method of Fig. 35e. A ten-inch base is convenient since it permits the necessary multiplication by shifting the decimal point. Precise maps are usually plotted by the coordinates of §§ 240 and 251.

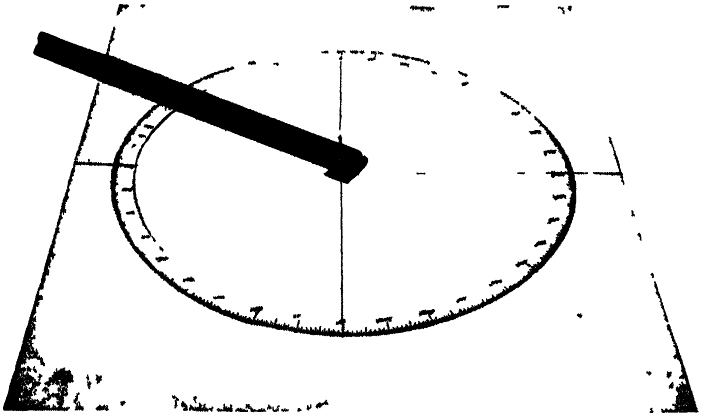


FIG 223 PROTRACTOR AND SCALE FOR PLOTTING DETAILS.

After computation is completed in the transit-stadia notebook, the azimuth and distance of the several shots around the stadia station are plotted and the elevation is lightly penciled adjacent to the point. The plotted point is often used as the decimal point of the elevation. The contours are interpolated as described on page 178. The penciled transit-stadia sheets are sometimes penciled in the field as previously suggested and are always completely penciled if the plane table is used. This, of course, minimizes the office mapping

224. Mapping by Contract. It has become increasingly the custom in recent years to let contracts for mapping to companies which specialize in such work (particularly aerial photography) and thus to secure the services of a trained, organized, and skilled group of technicians. Even engineering companies or governmental units which wish to concentrate their attention on the design and economic features of a project, frequently shift the burden for detailed surveys and maps to a mapping company. Here it is necessary first to agree upon specifications governing the final surveying results needed and also to some extent to specify the order in which the work shall be done so that progressive information may be furnished the designing engineers in order that their work may progress with the surveys. In order to check the accuracy of such contract surveys, profiles are run on the ground and compared with corresponding map profiles. Also, a small portion of the map may be re-surveyed as a check.

PROBLEMS

1. Plat a topographical map from assigned field notes, for example, those of Fig. 201a, or for some more elaborate survey

2. On an assigned area in the field for which controlling elevations are furnished, sketch in 5-foot contours, buildings, roads, improvements, and so on by eye, checking important points or dimensions by pacing. Check the completed sketch by superimposing a true tracing on it.

3. Make a topographical map of an assigned area in the field by one or more of the methods mentioned in this chapter. Sometimes part of the map may be made by methods outlined in the chapter on Route Surveying.

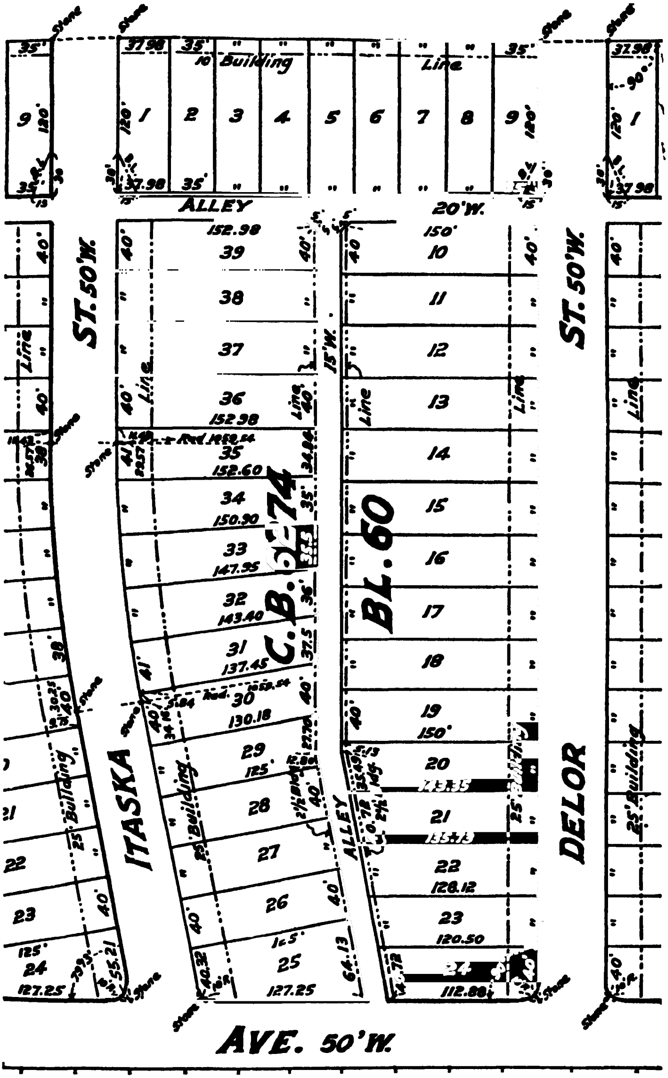
CHAPTER XV

CITY SURVEYS

225. The City Engineer and the Private Engineer. Certain surveys are made by the city engineer or his staff for the purpose of laying out construction improvements to be undertaken by the city, such as streets, sewers, water mains, conduits, and the like. Here the surveyor is not concerned with lot boundaries, since his only purpose is so to locate street and alley lines as to avoid encroaching on private property with his construction. The sidewalks most nearly approach private property and they are often set one foot into the street away from the private property line. The city engineer is not concerned with establishing lot boundaries for private owners, and any lot boundaries so established by him have no greater value or authority than those established by any competent private surveyor. Railway, telephone, gas, power, and other corporations make surveys of this same general nature within a city, usually without special reference to lot boundaries.

In contrast to these public or semi-public surveys are those private surveys which are made to determine private property or lot boundaries. The original subdivision of acreage into city lots is the work of the private engineer or surveyor, as is the future relocation and preservation of these boundaries; all, of course, are subject to appeal to the courts. It is commonly found that private engineering and surveying companies have better and older records and a more competent staff for the location of lot boundaries than are available in the official organization of the city engineer.

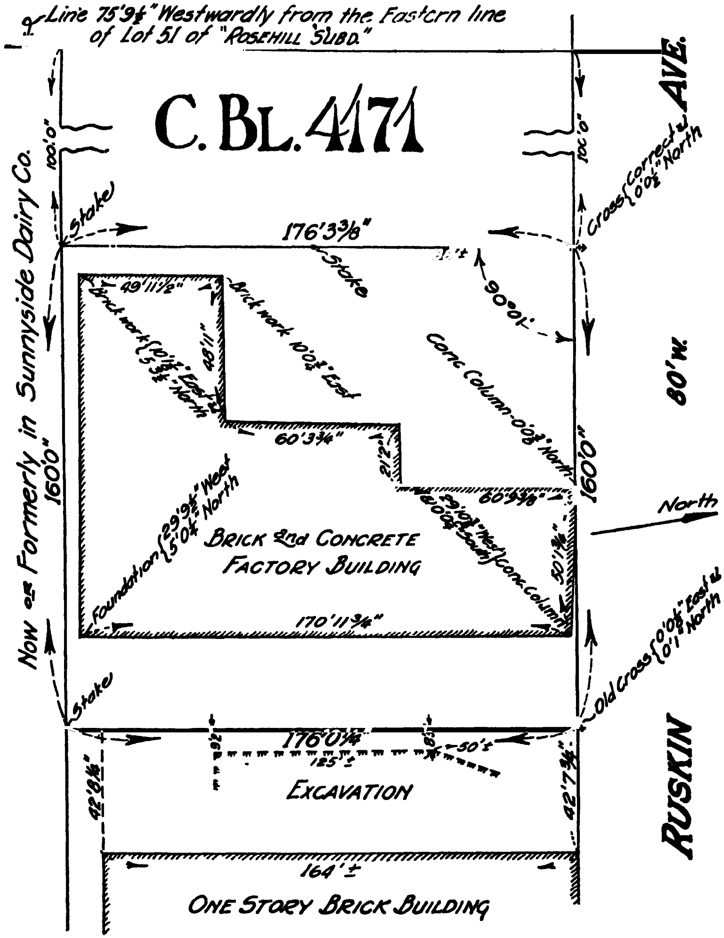
226. Lot Surveys and Subdivision Plats. Subdivision of city acreage into lots is mentioned in Chapter XII. One of the better types of subdivision is shown in Fig. 226. Most of our older cities were either laid out haphazardly or on the rectangular system. In a few cases, as a reaction against this regu-



Courtesy of Pitzman Engineering Company, St Louis

FIG. 226. CITY SUBDIVISION.

larity, subdivisions were made with all streets curved regardless of topography. Somewhere between these two extremes is the ideal city plat, with curved streets where topography permits


Line 75'9¼" Westwardly from the Eastern line of Lot 51 of "ROSEHILL SUBD."
C. BL. 4171
AVE.
100'0"
100'0"
Cross {Corrected 0'0¼" North
Stake
176'3⅜"
Stake
90°01'
49'11½"
Brick work {10'1¼" East 5'3¾" North
48'11"
Brick work 10'0¾" East
Conc Column 0'0¾" North
80'W.
60'3¾"
21'2"
60'9⅜"
160'0"
Now or Formerly in Sunnyside Dairy Co.
160'0"
North
Foundation {29'9¾" West 5'0¾" North
BRICK and CONCRETE FACTORY BUILDING
29'10½" West 0'0¾" South {Conc. Column
50'1⅝"
170'11¾"
Old Cross {0'0¾" East 0'1" North
Stake
176'0¼"
50'±
125'±
EXCAVATION
42'8⅛"
42'7¾"
RUSKIN
164'±
ONE STORY BRICK BUILDING

Courtesy of Pitzman Engineering Company, St Louis

FIG. 226a. CITY LOT SURVEY.

or demands them, diagonal highways for boulevards or arterial streets, and some rectangular blocks to suit those purchasers who prefer them. Detailed information from both the aesthetic viewpoint and the practical viewpoint is found in books on city planning.

The relocation of lot boundaries is made primarily from the filed subdivision plats, of which Fig. 226 is a very good example. These are filed with the city recorder or similar official. Due consideration must be given to court decisions and to long occupancy of property (adverse possession, page 200) in the neighborhood. When the resurvey is made, it is generally found that the measured distances and angles do not agree exactly with those on the filed plat. Errors of distance are usually proportioned over all of the lots lying between existing monuments. Other types of error must be adjusted as occasion demands, and it is here that the law of land surveying must be known and applied. Much of the procedure is similar to that given on pages 230 to 234.

The older lot-survey firms, and occasionally the office of the city engineer, have in their files plats of each city block, upon which are recorded all previously made surveys and court decisions affecting that block. The records of the private firms are not available to the public, and it is apparent that the young or newly-arrived engineer should hesitate to make lot surveys in the older communities. He may well make lot surveys of less expensive property where the original subdivision was accurately made in recent years and where no court decisions or erroneous surveys complicate the matter.

The survey of a city lot is shown in Fig. 226a.

227. Surveys by the City Engineer. The city engineer occasionally makes surveys of almost every conceivable kind, but the majority of his work is in laying out such public improvements as streets, sewers, water mains, parks, and the like.

It is the duty of the city engineer to set and to maintain permanent street monuments to which all subsequent surveys may be tied or connected. In unpaved streets, these monuments are usually set at the intersecting center lines of the streets, and consist of a large stone, a block of concrete, or an iron pipe buried sufficiently below the surface to prevent disturbance from above or upheaval due to freezing. A definite point is, of course, marked on the monument. Where the streets are paved, it is customary to place the point of intersection of the street lines in the bottom of a small cast-iron box set flush with the pavement surface.

In some cities the center lines are offset so as to throw the intersections and survey lines outside of the paved area and in the parking or sidewalk area.

It is customary to place only stakes when the survey is being made, and the permanent monument is placed later by means of strings or offsets from temporary marks or stakes, as shown in Fig. 227. If there is possibility of the monuments being disturbed, they should be carefully referenced to three or more reference points as in Fig. 203. The subdivision plat of Fig. 226 shows the location of such monuments. Their descriptions and rectangular coordinates, and the bearings and lengths of lines connecting them are available to the public in the office of the city engineer.

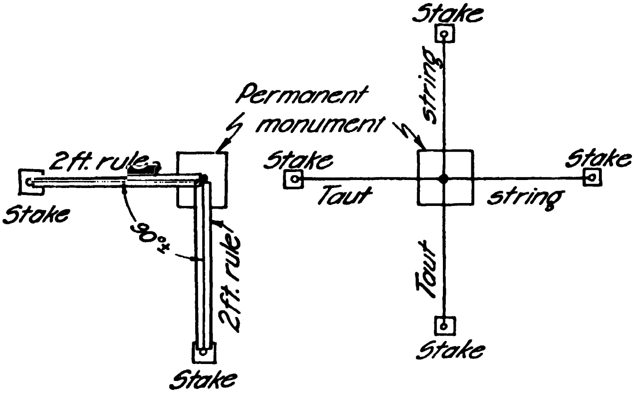


Fig. 227. Replacing Stakes with Monuments.

Bench marks, for elevation, must be accurately established by the city engineer. Only a few set-ups of the level should be necessary to carry elevations from these bench marks to any part of the city. The elevations of the bench marks are available to the public at the office of the city engineer. Bench marks are established on the water table at the top of the foundation of buildings, on solid curbs, and on other improvements that are not apt to settle or to be upheaved by frost. Steps of buildings are not generally useful because they are frequently built on very light foundations.

Triangulation is frequently used as a basis of control for city surveys, as mentioned on page 191.

228. Precision of City Surveys. In small towns or outlying districts where property is not very valuable, a precision of 1/5000 may be sufficient, but most city surveys require a precision of 1/10,000 or better. In valuable business districts, a precision of 1/25,000 may be necessary.

In the less valuable districts, and especially in residence sections where the buildings do not touch the property lines, an error of a few tenths of a foot may not be serious, although an attempt should be made to confine such error to the nearest one-tenth or two-tenths. Such errors, however, would not be tolerated in expensive business property where the lot lines must be located within a few hundredths of a foot.

In surveys of lesser precision, it is acceptable practice to measure angles to the nearest minute and to tape by the better ordinary methods, recording distances to hundredths of feet. The tape is leveled by eye, pulled by estimate, and only roughly corrected for temperature. In the more precise surveys, it is necessary to use standardized tapes read to the nearest hundredth of a foot and corrected for temperature, pull, sag, and variation from the horizontal. Angles are measured by repetition to the nearest one-half, one-quarter, or one-sixth of a minute. It is evident that this type of survey requires skilled personnel.

The foregoing precision refers to lot-line location as usually handled by private engineering or surveying firms. The lesser degree of precision, about 1/5000, is ordinarily sufficient for most of the public improvement surveys made by the city engineer since these improvements do not usually adjoin private property.

229. Staking out Structures. In setting stakes for construction, too many stakes are expensive and apt to be confusing to the construction men, while too few stakes require the foreman to set the missing ones, often not precisely or economically. A happy mean should be chosen between these extremes, preferably in consultation with the construction superintendent.

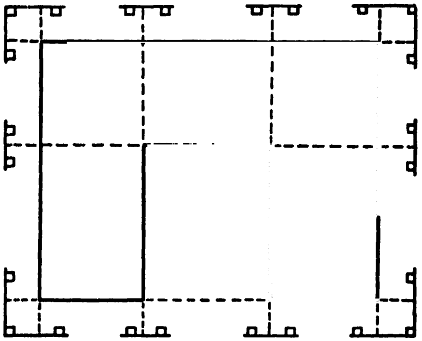


FIG. 229. BATTER BOARDS.

Buildings are staked in accordance with the plans of the architect or engineer or in line with the wishes of the owner after the property lines are established. It is not possible to retain stakes set at the corners of structures since such stakes would be dug out in the excavation for foundations. Batterboards (Fig. 229) consisting of 2″ × 4″ posts driven into the ground across which are spiked horizontal 1″ × 4″ strips, are

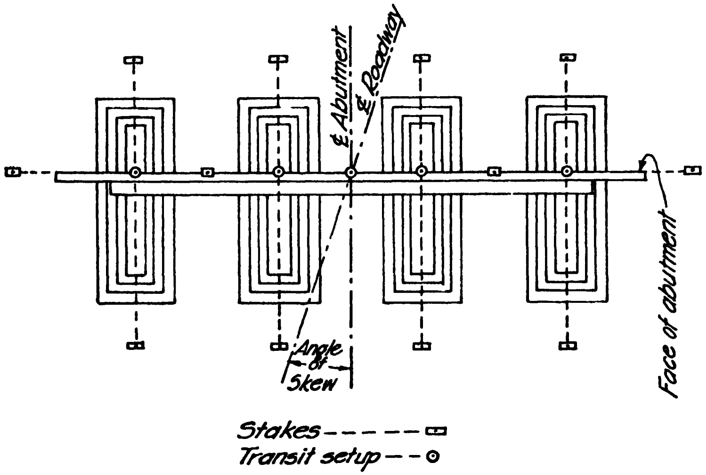


FIG. 229a. STAKING A BRIDGE ABUTMENT.

set a few feet outside of the structure lines. Heavier strips and posts are necessary for important structures. Nails are driven into these horizontal strips on the prolongation of the lines of the building so that a wire or chalk line stretched over the nails will define the line of the structure throughout its length. It is sometimes convenient to set these batterboards at some even number of feet above or below the top of foundation or floor level, and in such case the tightly drawn wire or chalk line will give approximate elevation data. Sag prevents such data from being exact except near the batterboards.

Figure 229a shows stakes necessary for a modern open type of bridge abutment. The stakes must be set out far enough so as to avoid disturbance during rough excavation, pile driving, concreting, and so on. Moreover, they must be set in such positions that the controlling points of the structure may be reproduced on the ground, on the concrete forms, or on the structure at any time during construction, regardless of intervening obstacles, such as falsework or piles of materials.

CHAPTER XVI

OFFICE WORK

230. General Information. The office work and computations in surveying vary in scope and complexity from the simple computations which are made in the field, through the more complex surveying computations and mapping which are best done in the office, and eventually reach those computations and drawings which are a part of the design of the project.

In a survey of considerable extent or complexity, it is desirable that the computations and preparation of maps follow immediately after the field work while the matter is fresh in mind and while the field party is still available to correct errors and omissions. In those rare cases where the office work must be postponed for a considerable time, it is necessary to modify field methods so as to minimize such errors and omissions.

If only a few men are engaged in the survey, the work may be computed and plotted in the evening or on rainy days; or one man may be left occasionally in the office. With several or more field men, it is advisable to leave a field draftsman in the office or in camp. He not only works up the data at the end of each day, but he also furnishes maps and other data to the field parties before they enter the field on the following day, and thus enables them to work more effectively. Since field work is slow and expensive relative to office work, the office man or men are an economy.

231. Errors and Omissions. The notekeeper makes every effort to avoid errors and omissions, but they will creep into any extensive survey. It is the duty of the office man or draftsman to scrutinize the notes immediately and to work them up so that the field party may be advised at the earliest date, preferably on the next morning, what added or corrected data are necessary. The office man, in conference with the chiefs of party and the engineer in charge, frequently outlines the field duties for the following day, laying out the work so that the results from the several parties will be correlated and will progress effectively.

232. Organization of the Office. While the office man may receive more or less salary than the transitmen, he should rank with the transitmen. One office man will usually be required for each one to three parties, and where several parties are working from different quarters, each party may have its own draftsman with it.

So far we have spoken of the field draftsman directly attached to the field parties. In the headquarters of the project, the office force is composed of draftsmen, computers, designers, a timekeeper, and a blueprinter. The size of the force will depend on the nature of the survey and on the amount of design work that is handled in connection with the computations and mapping.

233. Office Records. It is customary to provide computation books in which the figures may be kept permanently. The most popular form uses an 8½-inch × 11-inch yellow sheet with light ¼-inch cross-section ruling on the sheet. While these are available in loose leaf form, the bound books are frequently used. In both field books and in office computation books the loose leaves are apt to become lost or misplaced and are more easily, and improperly, altered.

It is necessary that some systematic arrangement be provided for filing field notebooks, computation books, maps, and other office records. Each field book is invariably indexed in the front of the book (Fig. 39). The books are numbered in some way so as to designate, at least roughly, the nature of the notes. For example, transit books may bear the letter *T* and level books the letter *L*, or the transit books may be numbered from 1 to 1000 while the level books are of a series 1000 to 2000, and so on. Where a considerable number of books accumulate, it is desirable to use a cross-indexed set of cards similar to those of a library. The cards may be cross-indexed by subject, geographic location, owner, or other division. These divisions may be further subdivided; for example, under subject we might have reconnaissance, preliminary, location, construction, maintenance, and property surveys.

Computation books, maps, and plans should be similarly filed and indexed. Drawings are most conveniently filed flat in drawers or hung vertically on racks, although some engineers prefer to roll them, especially if the number is small. Computations, maps, and plans usually carry a file number on their face. In some instances the file number may be the same one that is used for correspondence on that subject.

It pays to store the more valuable field notebooks, computation books, maps, and plans in fire-proof vaults, since the records have cost many thousands of dollars and, in addition, their loss may cause delays and disputes involving considerable expense. Since steel filing cabinets and office equipment are now as cheap as wood, they may be provided to reduce fire hazard. Entries or alterations in field notes which are made in the office should be in red ink.

234. Tables and Diagrams. For offices extensively engaged in surveying, it is usually found desirable to prepare tables and diagrams for facilitating both field and office work. Some of these may be secured from publishers and dealers, such as traverse tables, stadia tables and diagrams, earthwork tables and diagrams, slide rules for stadia, and so on. On a project of considerable size, it is customary to prepare many other such aids to fit the special conditions encountered on that project.

235. Reproduction of Maps and Plans. Drawings on thick opaque paper can be reproduced only by tracing or by photography. Copies can, of course, be made to any desired scale by photography.

Drawings made on thin paper or on tracing cloth may be reproduced to very closely the same scale (not precisely the same because of shrinkage or expansion of the tracing and of the print) through a process of blueprinting. According to the kind of paper used, prints may show:

1. A white line on a blue background (the ordinary blueprint),
2. A white line on a black background,
3. A black line on a white background,
4. A reddish line on a white background,

5. A transparent white line on a brown background (a Van Dyke), from which in turn prints may be taken,
6. A black line on transparent tracing cloth (not paper) which itself may be used as a tracing,
7. Other unusual combinations.

If a drawing is reduced or enlarged by photographing it, the resulting copy is called a photostat. When a drawing is made with the intention of reducing or enlarging it by photography, the thickness of the lines and the size and weight of the lettering should be made so as to show properly on the photographed copy. Imperfections in the original are, of course, exaggerated by enlargement and minimized by reduction. Certain colors will not show on the usual photostat unless a color filter is used.

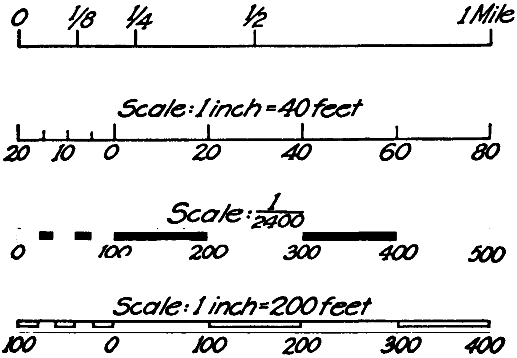


FIG. 235. GRAPHICAL MAP SCALES.

Photography is the basis of planographing, the most modern method of printing engineering drawings and reports.

By these various processes, almost any type of reproduction of drawings in any quantity can be secured from commercial blue-printing companies on short notice. These processes have developed rapidly in recent years.

Since any of the reproduction processes may somewhat alter the scale of the drawing and since there are few dimensions given on maps and most of the information must be scaled therefrom, it is desirable that maps have a graphical scale (Fig. 235) shown on them. This scale will, of course, be proportionally reproduced on the reproduced map. Coordinate lines on a map are the most precise way of caring for this distortion of scale in the reproduction process.

236. Suggestions to Computers.* "Do not crowd your work, paper is comparatively cheap.

"Do your work in a systematic manner. If it permits tabular arrangement, always use the forms approved by other computers unless you can convince them that yours are better The Survey has printed forms for many purposes; these should be used whenever possible, for by their use the work is made more mechanical, and the more mechanically the work is done the less chance there is for error.

"A computer who is inexperienced or out of practice should check his work in every way possible. He should check logarithms either of numbers or of circular functions by using first a tabular value for a quantity less than the given one and then a greater tabular value, so that the differences in one case may be added and in the other subtracted. This operation may be reversed when the logarithm is given and numbers or angles are required.

"Many errors are made by taking out the first three figures of a logarithm from the wrong line of a seven-place table where a dash over the fourth figure indicates that the first three should come from a lower line.

"As the algebraic signs of cosines and sines are so frequently required, the rules governing them should be firmly fixed in the mind; as an aid to this remember the general rule that distances measured upward or to the right on the conventional plat of the quadrants of the circle are considered positive, others negative. The wrong use of signs is a very common source of error.

"Each step in a long computation, if it is not at once automatically checked, should be checked by repeating the computation.

"Check the copying of angles, distances, etc. taken from adjusted results for use in new computations; also check figures carried from page to page.

"Gross errors are sometimes made by using the sine when a cosine is required, or by writing a product in the wrong column, as east for west, in transit-traverse computations.

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* From Bulletin 788, 1926, United States Geological Survey.

"Placing the decimal point in the wrong place is a common mistake. This may in many cases be corrected by a mere inspection of the quantity to see whether it appears of proper value.

"When a large closure error is found in a transit-traverse line look first for a compensating error of 1° or 10° in the azimuths or angles.

"Good judgment should be exercised in the degree of accuracy sought for a given result. For the preliminary computation of geographic positions, for example, six-place logarithms will suffice; these can be taken from a seven-place table with only a rough interpolation. A four-place logarithm can often be used to advantage. The accuracy of the results obtained should equal the requirements; more than this involves a waste of time.

"When computers are duplicating work and a difference is found, each should recompute the result before correcting either as errors have frequently been made by changing the correct figures.

"When two persons are comparing a copy with the original, if the reader occasionally calls out a wrong figure or word intentionally and notes whether the error is caught up, it tends to keep the listener more intent on the work."

237. Office Equipment. A first-class mapping and drafting office will contain the following equipment, either belonging to the office or to the draftsmen there employed. Temporary offices will, of course, not contain all of these items.

1. The usual set of drafting instruments, including a swivel or contour pen, a bow-pen for making small circles with a whirling motion (sometimes called a rivet pen or a piledriver), and a railroad pen for drawing double lines or borders.
2. Architects' and engineers' scales, the latter used for most mapping except in landscaping and in building-site maps.
3. T-squares and triangles, including at least one steel traight edge and one steel triangle.
4. Various devices for assisting in lettering, such as rubber r metal stamps for titles; Payzant, Wrico, or Leroy lettering ns; and so on.

5. Irregular curves, a flexible curve or spline, a set of highway or railway curves, and a beam compass for drawing curves of long radii or for use as long span dividers.

6. A polar planimeter for measuring areas with a precision somewhat better than 1 per cent, and a rolling planimeter for considerably better precision than this. The size and shape of the figure and the skill of the operator also affect precision.

7. Slide rules of the ordinary type and of the stadia reduction type.

8. Computing machines for addition, subtraction, multiplication, and division, of the type of the Monroe or Marchant machines.

9. One set of seven-place logarithms and trigonometric functions, and several sets of five- or six-place tables.

10. A survey traverse table for computing latitudes and departures. The larger and better tables, such as Gurden, are easier and faster to use, while the smaller tables, such as Louis and Gaunt, require more time but are cheaper and easier to carry about.

11. Special instruments, such as a drafting machine, a blueprinting machine, a pantograph, large proportional dividers, and so on, that are somewhat rarely used but occasionally useful.

12. Equipment for reducing maps photographically.

238. Map Papers. The map may be drawn on buff detail paper, on heavy white high-grade paper, on linen tracing cloth, or on tracing paper.

Buff detail paper is most used. It is not expensive, lasts well, and will stand considerable abuse. Occasionally this map is inked and used permanently, but more frequently it is penciled or partially inked and then traced on linen. It is much easier to trace provided the buff detail sheet has been inked, at least in part. White paper is more expensive, more durable, makes a better looking map, and is generally used where the map is to be inked on the paper and used permanently or for exhibition purposes. Various weights of paper are used. Any of these papers, as well as blueprint paper, is stronger if mounted on cloth.

Oiled transparent paper or pencil tracing cloth is increasingly used. The drawing may be only penciled, it may be inked, or the less important lines may be left in pencil and the important ones inked. They are not as durable as tracing cloth or heavy detail paper, but their transparency permits use in preparing blueprints. Sometimes this eliminates the process of tracing on linen, although, if permanent records are needed, it is perhaps best to trace the map from the tracing paper onto tracing linen One advantage of making the original map on pencil tracing paper or cloth is that it hastens completion of the map. Also, it permits the preparation of blueprints as the map progresses and such partially complete maps are of great assistance to the field parties, to the designing engineers, and to the officials, who are thus enabled to plan and design before the map is finally completed. Such use of the partially complete map frequently alters the nature of the yet uncompleted field work. The disadvantage of oiled tracing paper lies in its inability to stand hard usage. The plain tissue tracing paper used by architects is not used in mapping since it is even less able to stand hard usage. Improved tracing papers are recently available.

Linen tracing cloth makes the best blueprints and is generally used to trace, with ink, maps which have been penciled. In rare instances, the map may be drawn with soft pencil on the dull side of the linen, but such lines are easily blurred and do not blueprint well. Tracing linen is more durable, is now available waterproofed, and permits erasures and alterations better than any paper.

A 4H, or harder, pencil is used in mapping so that the lines will be sharp and permanent. Mapping must be done more accurately than ordinary engineering drawing.

239. Placing the Drawing on the Paper. Profiles are drawn from left to right, with the title at the left end so that it will be seen when the unrolling is started.

Maps are drawn with their *north to the top* unless their shape demands that they be otherwise turned in order better to fit the paper. Lettering reads from the bottom and right, as in most engineering drawings.

Some general idea of the dimensions of the map and its extent in all directions should be visualized and estimated so that all work may lie within the limits set for the map. It is better not to draw the final border lines until the map has taken form on the paper and its limits and the position of the title are established. If it is traced later, the border lines may then be shifted, but it is preferable to have them properly placed on the pencil drawing.

240. Coordinate Axes. If the coordinates have been figured for the survey, as mentioned in § 247, they should be used in plotting all coordinated points. For precise mapping of any considerable extent, the coordinates must be computed, if only for plotting purposes. The coordinate lines are carefully constructed with steel straightedge and triangles, or by geometrically constructing a right angle with a beam compass. The coordinate squares, usually 10 inches by 10 inches, are checked by measuring all sides and diagonals, since the precision of the map will depend upon their accuracy. Wooden or celluloid T-squares, straightedges, or triangles should not be used. If no other equipment is available, then the wooden instruments must be carefully reversed as in Fig. 240, to eliminate the warp which is generally present.

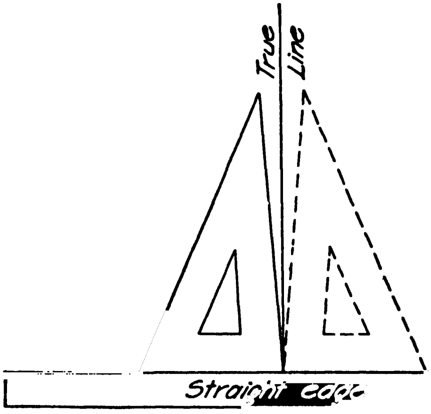

FIG. 240.

Coordinates are plotted as in Fig. 240a. In all plotting or scaling by coordinates, it is desirable to place the 0 of the scale on one coordinate line and the 10 (or other definite division) of the scale on the adjacent coordinate line. The amount of shrinkage or expansion of the paper is thus noted and an allowance is made for it in plotting or scaling.

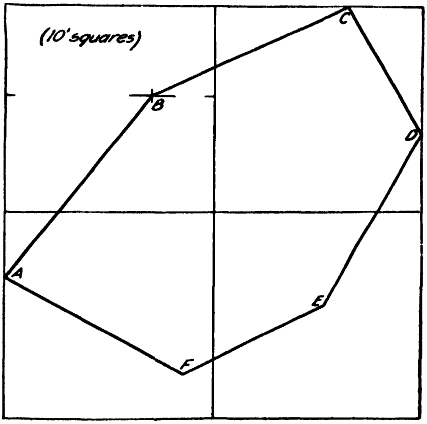

FIG. 240a. PLOTTING BY COORDINATES.

In the usual method of plotting at *B*, the short horizontal marks on the adjacent vertical coordinate lines are measured with the 0 of the scale on the lower adjacent horizontal coordinate line, and the longer horizontal line at *B* is drawn between them. The short vertical intersecting line at *B* (which accurately locates the point) is located with the 0 and the 10 (or other definite division) of the scale on the adjacent vertical coordinate lines.

The draftsman must compare the scaled distance between successive plotted points with the surveyed length, thus securing an excellent check.

241. Other Plotting Methods. If coordinates are not available, the angles and distances are laid off by one of the following ess satisfactory methods.

1. *By Tangents.* With a 10-inch base and the natural tangent f the required angle, the tangent is multiplied by ten by shifting the decimal point, and the right angle offset from the base

so calculated is laid off, thus permitting the angle to be plotted precisely, as in Fig. 35c.

2. *The Chord Method.* The chord method is of about the same precision but a bit more awkward, and is less frequently used It is shown in Fig. 35d.

3. *By Protractor.* By some one of the various protractors. This is the easiest, quickest, and least precise method.

4. *By Radians,* as in Fig. 35e. The results may be made satisfactory in precision for ordinary work and no trigonometric tables are needed.

242. Scales. The scale and contour interval should be chosen as suggested on page 268, and the purpose of the map and the precision of the survey work should be borne in mind.

243. Finishing the Map. Even though the map is later to be traced on linen, it is desirable that it be completely and carefully penciled, both to make the map available for a permanent record and to facilitate tracing. Since it is frequently difficult to trace pencil lines, the detail drawings, or at least their principal lines, are often inked.

The finished map should include a title giving the name and the location of the area mapped, a statement of scale, the date, and the names, preferably the signatures, of persons responsible for the work. There also should be a legend explaining the symbols used, and the orientation should be shown by a simple north point.

CHAPTER XVII

COMPUTATION OF TRAVERSES, LATITUDES AND DEPARTURES, COORDINATES AND AREAS

244. Checking Angles of a Traverse. It is customary for the transitman to check his angular work and to correct any appreciable errors in it before leaving the field, or at least before turning over his notes to the office force. The angles may be checked in several ways.

1. The sum of the interior angles of a polygon is equal to the number of angles minus two, multiplied by 180°. The derivation of this simple and useful geometric relation (Fig. 244) is easier to remember than is the rule or theorem. It will be noted that, if the polygon be divided into triangles, there are two less triangles than there are polygon sides. Each triangle comprises 180° and the sum of the angles of all the triangles is equal to the sum of the interior angles of the polygon.

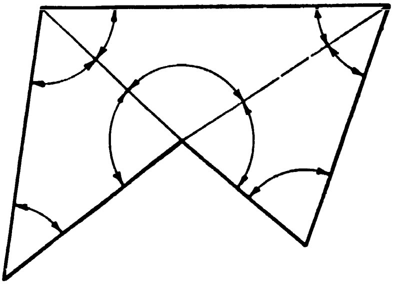

FIG. 244.

2. In a closed deflection angle traverse which does not cross itself, the algebraic sum of the angles is 360°.

3. Where azimuths are used, they are checked on the plates whenever the transit is set up on a line of known azimuth. In a closed traverse, this check occurs at the final set-up, as explained on page 112.

4. When a traverse does not close, it is the practice to determine the bearing or azimuth of the first line from some known direction, or by solar or polaris observation, and then to carry forward the azimuth on the plates or to compute the bearings of subsequent lines from the measured angles, as shown in

Figs. 96a and 97a. At the end of the traverse the bearing or azimuth of the last line is again determined from a known bearing or azimuth, or from astronomic observation, and this value should agree with the value which has been carried forward. These checks are made every few miles. Allowance must be made for convergence of the meridians (Table VI) which amounts to 1 minute per mile of east and west direction in latitude 49°, and also varies with the latitude.

Angular transit errors are usually small, amounting to only a few minutes. Small angular errors of a minute or two may be added to or subtracted from one or two of the angles which are apt to have been erroneously measured. For example, the angles whose sides are short are usually not precisely measured because the slight displacement of a point at either end of a short line changes an angle more than at the end of a long line. Frequently the total angular error may be distributed equally among several angles or among all of them.

Any large error in angle must be corrected by remeasurement before the party leaves the field. Most large errors or mistakes are apparent while the transit is yet at a point, provided the calculated bearings or the plate azimuths (Figs. 96a, 97a, and 99) are compared with the needle readings.

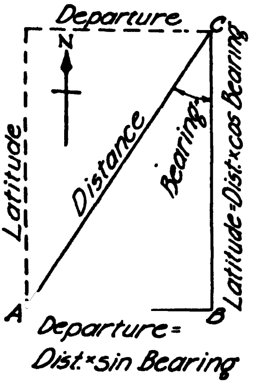

FIG. 245.

The angles are *usually either remeasured or adjusted to close* before the work of calculating latitudes and departures is begun.

245. Latitudes and Departures. Figure 245 shows the relation between a line and its latitude and departure. The *latitude* (length × cosine of bearing) of a line is its north and south component. The *departure* (length × sine of bearing) is the east and west component. It will be noted that the latitude and departure are obtained by the above multiplication, regardless of the quadrant in which the bearing occurs.

The sine or cosine of the azimuth may be used instead of the sine or cosine of the bearing, but it is seldom used since trigonometric tables arranged to read from 0° to 360° are rare.

In any closed traverse, as in Fig. 245a, it is apparent that the sum of the latitude of lines bearing in a northerly direction must be equal to the sum of the latitudes of lines bearing in a southerly direction, since the survey goes as far north as it goes south. Similarly the sum of the west departures must be equal to the sum of the east departures.

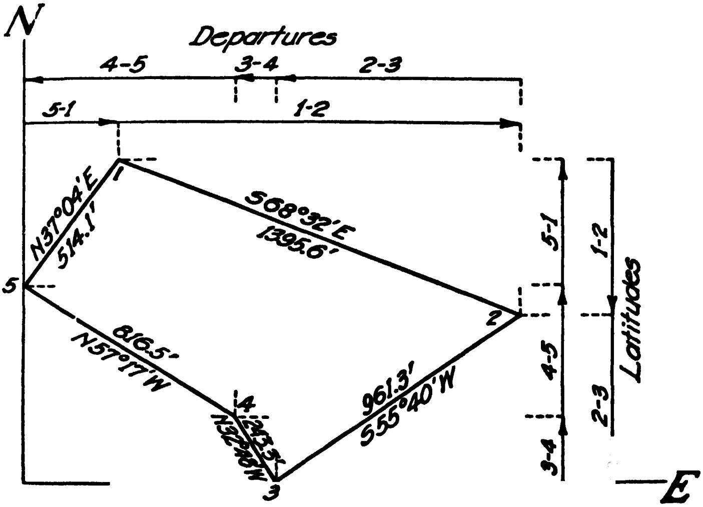

FIG. 245a. A CLOSED TRAVERSE.

If, then, the north latitudes be computed and added, their sum must equal that of the south latitudes unless some error has occurred in the field work or in the computation. The same is true for east departures and west departures. Actually some error always exists in the field work and not infrequently in the computation. Sometimes this error is large enough to require correction in the field; at other times it may be adjusted in the office, as described in the following section.

The computed distance between the first and final points of a closed traverse, which would be zero if the field and office work were perfect, is called the *linear error of closure* and is shown on Fig. 245b. It is approximately determined by plotting the traverse to scale, or more exactly by roughly computing the hypothenuse of a right angle triangle whose sides are the error in latitudes and the error in departures, respectively. The error of closure is expressed as a fraction; for example, if the hypothenuse is 5.0 feet and the traverse is 3930.8 feet long, the error of closure is expressed as 1/786.

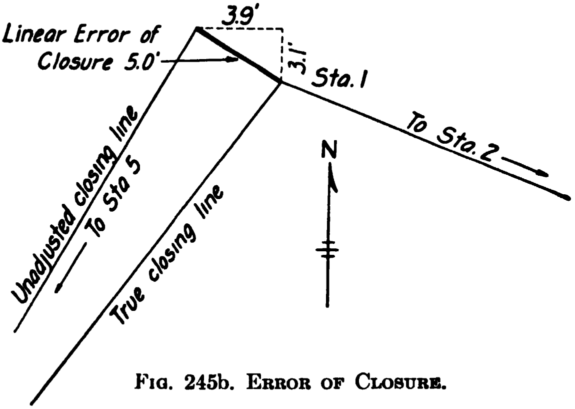

FIG. 245b. ERROR OF CLOSURE.

The computation for Figs. 245a and 245b follows.

LINE	LENGTH IN FEET	BEARING	LATITUDES		DEPARTURES	
			North	South	East	West
1 2	1395.6	S 68° 32′ E		510.7	1298.8	
2 3	961.3	S 55° 40′ W		542.2		793.8
3 4	243.3	N 32° 48′ W	204.5			131.8
4 5	816.5	N 57° 17′ W	441.3			687.0
5 1	514.1	N 37° 04′ E	410.2		309.9	
Sum	3930.8		1056.0	1052.9	1608.7	1612.6

Error in latitude is 3.1 feet. Error in departure is 3.9 feet.
Linear error of closure is $\sqrt{(3.1)^2 + (3.9)^2} = 5.0$ feet.
This error of closure is expressed also as $5.0/3930.8 = 1/786$.

The following arrangement of logarithmic computation is best. For each adjusted bearing, take the logarithmic sine and logarithmic cosine simultaneously from a single line of Table III (Logarithms of Trigonometric Functions).

The Gurden Traverse Tables, the Louis and Gaunt Traverse Tables, and others are somewhat better than ordinary tables of logarithms for the computation of latitudes and departures. If

LINE	1-2	2 3	3-4	4-5	5-1
Latitude	510.7	542.2	204.5	441.3	410.2
Log. lat.	2.70819	2.73414	2.31071	2.64474	2.61302
Log. cos.	9.56343	9.75128	9.92457	9.73278	9.90197
Log. dist.	3.14476	2.98286	2.38614	2.91196	2.71105
Log. sin.	9.96878	9.91686	9.73377	9.92498	9.78013
Log. dep.	3.11354	2.89972	2.11991	2.83694	2.49118
Departure	1298.8	793.8	131.8	687.0	309.9

calculating machines such as the Marchant, Monroe, and others are available, they provide the easiest method of computing latitudes and departures.

246. Adjustment of Traverses. A single large mistake, as for example 100 feet, may frequently be located on the line where it has occurred by inspection of the amounts of errors in the latitudes and departures. For example, if only a single large mistake in taping is involved, then the error in departure divided by the error in latitude will give the tangent of the bearing of the line upon which the mistake occurred. After the mistake has been so located it is usually necessary to check the line in the field.

If the error of closure, as shown by the failure of the algebraic sums of the latitudes and departures to total zero, is within the limits of precision set for the survey — in other words, if only small errors and no large mistakes exist — then it is customary to adjust these small errors (called *balancing* the survey or traverse) by slightly changing the latitudes and departures so that their algebraic sums will exactly equal zero. The small angular errors are adjusted to give a perfect angular closure before the latitudes and departures are computed.

It may be that the length of a particular side or sides is suspected of containing most of the error because of difficulty encountered in field measurement. In this case it would be justifiable to change the latitude, the departure, or both for this side or sides with a view to making the algebraic sums of the latitudes and departures zero.

Since taped distances are usually recorded slightly longer than their true length, as described on page 29, some engineers prefer to leave the columns of latitudes and departures which total the lesser amounts, without correction, and to apply all of the correction to the columns of latitudes and departures which total the greater amounts. It may rarely occur that the recorded lengths are thought to be too short, in which case the latitudes and departures whose sums are too large are left uncorrected, and those which total less than a closing value are increased by the full amount of the error.

Such balancing as suggested above is made on a *field-conditions* basis, throwing the error where judgment dictates. Frequently, however, the computer has no idea where the error should be placed and in many cases it is logical to assume that the distance errors are greater than the angular errors, particularly where the angles are measured with a transit and checked, as has been suggested above. The customary procedure under such an assumption, known as the *transit rule*, is to proportion the total error in latitude over all of the latitudes in accordance with the lengths of the respective latitudes, and to make a similar adjustment in the case of the departures, as shown in the following tabulation.

If the error is due as much to angular errors as to distance errors, as might be the case in a compass traverse, it is somewhat better to make the corrections in proportion to the lengths of the sides by the *compass rule*, as in the following tabulation.

Since the assumptions for either the transit rule or for the compass rule are neither exact nor, in many cases, even approximately correct, it is sufficient to make these proportional corrections roughly *by inspection*.

A comparison of adjustments by these rules follows.

LINE	LATITUDE ADJUSTMENT BY		DEPARTURE ADJUSTMENT BY	
	Compass Rule	Transit Rule	Compass Rule	Transit Rule
1-2	+1.1	+0.7	+1.4	+1.6
2-3	+0.8	+0.8	−1.0	−0.9
3-4	−0.2	−0.3	−0.2	−0.2
4-5	−0.6	−0.7	−0.8	−0.8
5-1	−0.4	−0.6	+0.5	+0.4
Total adjustments	3.1	3.1	3.9	3.9

It is interesting to note that if a line runs north and south, it will have no departure adjustment, and if it runs east and west, it will have no latitude adjustment, provided the transit rule is used.

When the foregoing transit rule adjustments are applied to the tabulation of page 294, the adjusted latitudes and departures are as follows

LINE	ADJUSTED (TRANSIT RULE)			
	Latitudes		Departures	
	North	South	East	West
1-2		511.4	1300.4	
2-3		543.0		792.9
3-4	204.2			131.6
4-5	440.6			686.2
5-1	409.6		310.3	
	1054.4	1054.4	1610.7	1610.7

More complicated and theoretically more nearly correct methods of making these corrections for transit surveys have been derived from the theory of least squares, but they are seldom used

It will be noted that the adjustment of the latitudes and departures will slightly alter the length and bearing of the original sides of the traverse This alteration is sometimes computed and made, as for example in a metes and bounds survey of property. Where the latitudes and departures are computed only to determine the closure of the survey, to find the area, or to compute coordinates of survey points, it is not necessary to make this somewhat tedious computation.

It must be borne in mind that only small errors, within the range of precision specified for the survey, are adjusted in the ways indicated above; frequently some rough combination of them is used rather than an exact mathematical application of any one method. This adjustment precedes the computation of areas or coordinates. Larger errors or mistakes require a resurvey in the field.

It must also be realized that the error of closure shows only the accidental errors and the mistakes. If cumulative errors exist, such as result from the tape's being too long, then the error of closure as computed may be little or nothing, but each line will be recorded as less than its true length. In other words, the computed traverse has a true *shape* but is smaller than it should be.

247. Coordinates. The use of coordinates in surveying computations has been widely adopted during the last thirty years and is now standard practice for much survey work. It was first generally used in mining surveys, perhaps the most complicated and difficult class of engineering surveys.

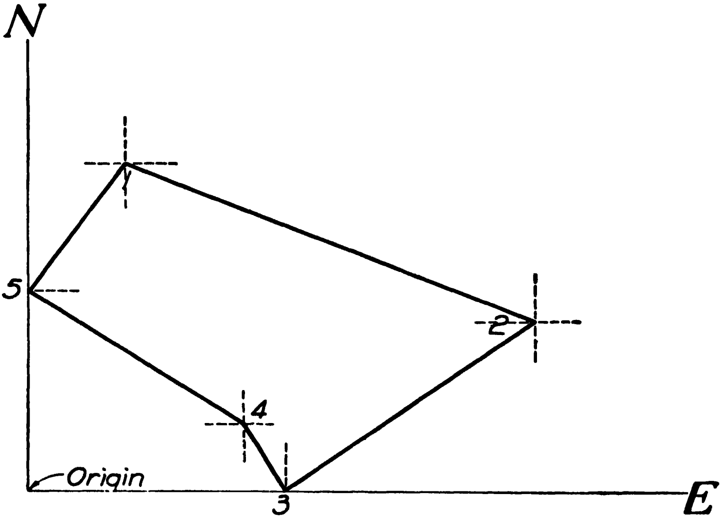

Fig. 247. Coordinates.

The origin of coordinates is generally selected so as to throw the entire survey into one quadrant, usually the northeast quadrant, and thus to avoid negative coordinates. Occasionally the origin is a definite monument on the ground, but more frequently it is only an imaginary point, as in Fig. 247.

In Fig. 247 the coordinate of point 5 is assumed as E 0.0, in which case the axis passes north and south through 5. Similarly, the coordinate of point 3 is assumed as N 0.0 and the axis passes east and west through that point. All coordinates are positive, or +. Or, these coordinates might have been assumed as N 1000.0 and E 1000.0, in which case both axes would be 1000 feet removed and the entire figure would again have positive or + coordinates.

After the latitudes and departures are computed and adjusted (pages 291 to 298), the coordinates may be computed by addition and subtraction. In a closed traverse, the computation of coordinates is checked as shown.

Line	Adjusted (Transit Rule)				Coordinates		
	Latitudes		Departures				
	North	South	East	West	North	East	Point
					1054.4	310.3	1
1 2		511.4	1300.4				
					543.0	1610.7	2
2-3		543.0		792.9			
					0.0	817.8	3
3 4	204.2			131.6			
					204.2	686.2	4
4 5	440.6			686.2			
					644.8	0.0	5
5 1	409.6		310.3				
	1054.4	1054.4	1610.7	1610.7			

East	
Pt.	Coordinates
5 =	0.0
	+ 310.3
1 =	310.3
	+ 1300.4
2 =	1610.7
	− 792.9
3 =	817.8
	− 131.6
4 =	686.2
	− 686.2
5 =	0.0

check

North	
Pt.	Coordinates
3 =	0.0
	+ 204.2
4 =	204.2
	+ 440.6
5 =	644.8
	+ 409.6
1 =	1054.4
	− 511.4
2 =	543.0
	− 543.0
3 =	0.0

check

Coordinates are used in plotting (page 288), in calculating areas, and in many special computations where they require the applications of trigonometry, analytic geometry, and descriptive geometry. Their increasing use in recent years will doubtless continue.

248. Area by Coordinates. With known coordinates numbered consecutively around Fig. 248 the area of the figure is

$$(248)\quad (\text{trapezoid } y_1, 1, 2, y_2) + (\text{trapezoid } y_2, 2, 3, 0) - (\text{trapezoid } 0, 3, 4, y_4) - (\text{triangle } y_4, 4, 5) - (\text{triangle } 5, 1, y_1)$$

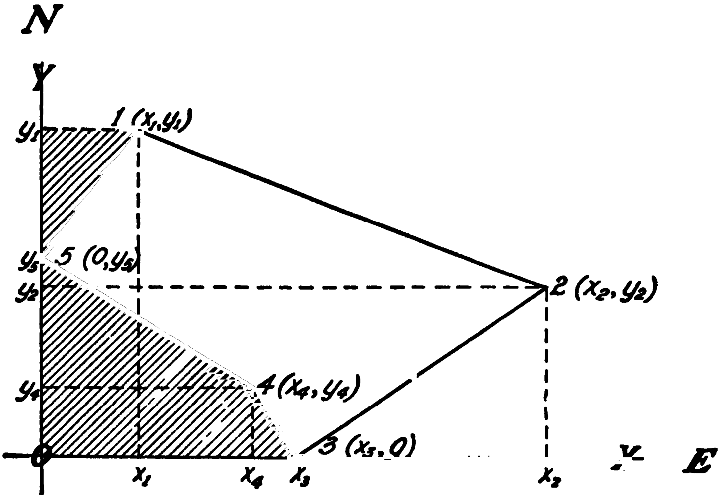

FIG. 248. AREA BY COORDINATES.

This may be reduced to the conventional determinate form as derived in analytic geometry

$$(248a)\quad \text{Area} = 1/2\left(\frac{x_1}{y_1} \times \frac{x_2}{y_2} \times \frac{x_3}{y_3} \times \frac{x_4}{y_4} \times \frac{x_5}{y_5} \times \frac{x_1}{y_1}\right)$$

Here the coordinates connected by solid lines are multiplied together; those connected by dashed lines are multiplied together; the solid line products are added together and the dashed line products are added together. Then one-half of the difference of these sums represents the area.

A second, and perhaps preferable method, derived from formula 248a, is

$$(248b)\quad \text{Area} = 1/2[y_1(x_2 - x_5) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_5 - x_3) + y_5(x_1 - x_4)]$$

which may be stated as the rule: *Multiply each y by the difference between the following and preceding x values (being careful* *to use the correct algebraic sign for this difference), and take half of the algebraic sum of the products.* Applying this rule to the tabulation of § 247, we obtain the area as follows.

POINT	NORTH y	EAST x	DIFFERENCE IN ADJACENT x VALUES	DOUBLE AREAS	
				+	−
1	1,054.4	310.3	+1,610.7	1,698,322	
2	543.0	1,610.7	+ 507.5	275,572	
3	0.0	817.8	− 924.5		0
4	204.2	686.2	− 817.8		166,995
5	644.8	0.0	− 375.9		242,380
			Sum = 0.0	1,973,894	409,375

$\frac{782{,}259}{43{,}560} = 17.96$ acres

1,973,894
− 409,375
2)1,564,519
782,259 square feet

249. Area by Double Meridian Distances. Referring to Fig. 249, we pass a meridian through the most westerly point of the figure. The distance from the meridian to the center of each side is the *Meridian Distance* (M.D.) of that side. For convenience in computation, double the M.D. or the D.M.D. (***Double Meridian Distance***) is used. D.M.Ds. are computed and checked as follows, the adjusted departures of Fig. 249a

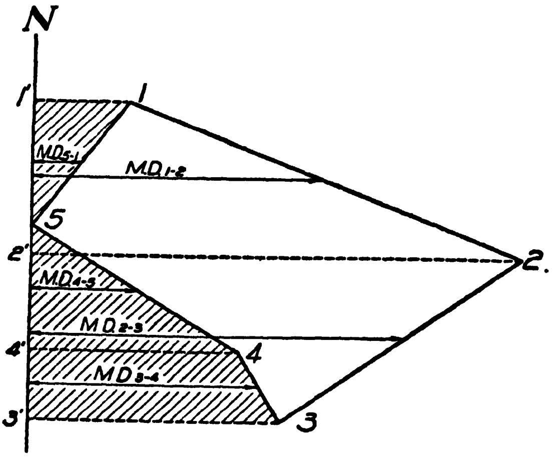

FIG. 249. MERIDIAN DISTANCES.

being used. It is to be noted that this method of computing D.M.Ds. gives a complete and inclusive check by starting on the meridian and returning to it.

In Figs. 249 and 249a, 5 is the most westerly point and:

1. The D.M.D. of the first line, 5–1, is the departure (310.3) of that line.

2. The D.M.D. of any other line is equal to the D.M.D. of the preceding line plus the departure of the preceding line plus the departure of the line itself.

Examples: For line 1-2: 310.3 + 310.3 + 1300.4 = 1921.0
For line 2–3: 1921.0 + 1300.4 − 792.9 = 2428.5

3. The D.M.D. of the last line is equal to the departure of that line, but of opposite sign.

Example: For line 4–5: 1504.0 − 131.6 − 686.2 = 686.2

As is seen in Fig. 249, the double net area 1, 2, 3, 4, 5 is composed of the double trapezoidal area 1′, 1, 2, 2′ plus the double trapezoidal area 2′, 2, 3, 3′ minus the shaded double trapezoidal area 4′, 4, 3, 3′ minus the shaded double triangular areas 5, 4, 4′ and 1′, 1, 5. Each of the double trapezoidal or triangular areas is the product of the latitude of the line multiplied by its D.M.D.

The numerical example in Fig. 249a shows the computation of the area of Figs. 245a or 249. D.M.Ds. are computed as shown above. It is not necessary to identify the triangles and trapezoids of Fig. 249 if care is taken to enter the products of the south latitudes and accompanying D.M.Ds. in the South Double Area columns, and similarly to enter the products of the north latitudes and the accompanying D.M.Ds. in the North Double Area columns. The true area is half of the difference between the North Double Area sum and the South Double Area sum.

250. Comparison of Methods of Computing Areas. The D.M.D. method of computing areas is best, provided the traverse can be run along the property lines.

In the great majority of cases, and especially if it has been necessary to compute the coordinates for some purpose other than area, it is usually simpler to compute the area by coordinates rather than by D.M.Ds.

While the D.M.D. procedure has been the traditional one

Line	Length feet	Bearing	Unadjusted				Adjusted (Transit Rule)				DMDs	Double-Areas	
			Latitudes		Departures		Latitudes		Departures				
			N	S	E	W	N	S	E	W		North	South
5-1	514 1	N37°04'E	4102		3099		4096		3103		310 3	127,099	
1-2	1395 6	S 68°32'E		510 7	1298 8			511 4	1300 4		1921 0		982,399
2-3	961 3	S 55°40'W		542 2		793 8		543 0		792 9	2428 5		1,318,676
3-4	243 3	N 32°48'W	204 5			131 8	204 2			131 6	1504 0	307,117	
4-5	816 5	N57°17'W	441 3			687 0	440 6			686 2	686 2	302,340	
Sum	3930 8		1056 0	1052 9	1608 7	1612 6	1054 4	1054 4	1610 7	1610 7		736,556	2,301,075

$$\begin{array}{r} 736{,}556 \\ 2\,\underline{\big|1{,}564{,}519} \\ 782{,}259 \text{ sq ft} \end{array}$$

or

$$\frac{782{,}259}{43{,}560} = 17\,96 \text{ acres}$$

Error in latitude = 3.1'

Error in departure = 3.9'

$$\text{Error of closure} = \frac{\sqrt{(3\,1)^2 + (3\,9)^2}}{3930\,8} = \frac{5\,0'}{3930\,8} = \frac{1}{786}$$

Fig 249a Area by D M Ds

for computing areas, it is used decreasingly at present. Except for the first use mentioned above, it should be largely abandoned in favor of the more generally useful coordinate method.

251. Checking and Plotting by Coordinates.

Checking. Cutoff lines and/or tie lines permit checking of the work by using closed traverses. These cutoff lines are usually run at any convenient point during the original survey, or occasionally they may be run after errors have developed. They facilitate the location and correction of errors and reduce the work accordingly. Figure 251 shows how such cutoff lines or tie lines may be used in checking line or closed traverses.

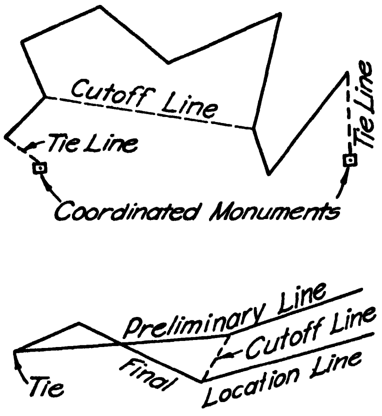

FIG. 251. CHECKING TRAVERSES.

Plotting. In plotting survey points, whether from traverse or triangulation, coordinates give the most precise results.

A short traverse or triangulation system may be plotted by laying off the angles and distances in much the same way as the measurements are taken in the field. In longer traverses the objections to this method are that errors are carried through subsequent plotting and may tend to accumulate. As a further objection, the points are less accurately plotted.

Ten-inch squares are usually laid out on the paper (Fig. 240a). The 12″ engineers' scale will then straddle two lines, and the amount of shrinkage or expansion of the drawing will be noted. Scaled distances may thus be corrected and new points plotted in true proportionate position.

A map a yard square on detail paper will shrink or expand as much as 1/2 inch under extremes of humidity and temperature. If traced on cloth and blue-printed under conditions tending to accumulate error, the final print may be nearly 1 inch larger or smaller, usually smaller, than is indicated by the scale. Such a change is immediately apparent and may be allowed for by proportion when a scale is laid across the 10-inch coordinate lines.

As explained on page 269, it is necessary that maps be more accurately prepared and scaled than is the case with most engineering drawings.

252. Missing Measurements. Where only two parts of a closed traverse are missing, either distance or bearing of a line being considered as a part, the missing parts may be computed. In this case there is no inclusive check on the field and office work. If a check is desired, there can be not more than one missing part of the closed traverse and it is preferable that no parts be missing. Several cases follow.

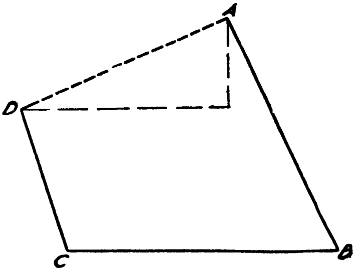

Fig. 252.

(1) *One side (DA) missing, both in bearing and in distance* (Fig. 252). The latitude and the departure of *DA* may be computed by addition and subtraction, since it is known that the summation of all latitudes (for any closed traverse) equals zero, and that the summation of departures also equals zero. In the right triangle of the figure, the bearing and length of *DA* may be computed from the latitude and departure of this line, by using logarithms and by checking the length by both divisions.

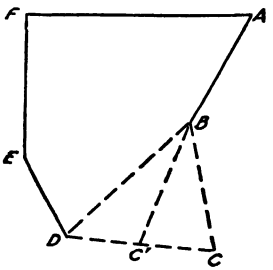

Fig. 252a.

$$\text{Bearing of } DA = \tan^{-1}\frac{\text{departure}}{\text{latitude}}$$

$$\text{Length of } DA = \frac{\text{departure}}{\text{sin bearing}}$$

$$= \frac{\text{latitude}}{\text{cos bearing}}.$$

(2) *In the adjacent sides BC and CD of* Fig. 252a *there may be missing one bearing and one length, two bearings, or two lengths.* A closing line *BD* between termini of completely known sides

may be computed as in (1). There are then sufficient data known to solve the triangle *BCD*. In some cases two solutions (*BC* and *BC′*) are possible. In that case the general shape of the figure must be known.

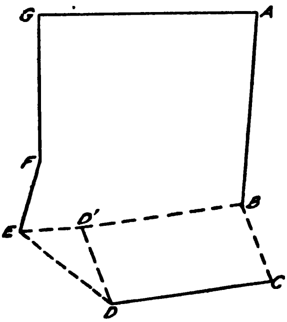

Fig. 252b.

(3) *In the non-adjacent sides BC and DE of Fig. 252b, there may be missing one bearing and one length, two bearings, or two lengths.* A continuous traverse of fully known lines is obtained by shifting *CD* to *BD′* (parallel and equal in length) and computing the closing line *D′E* as in (1). *BC* is shifted to *DD′* (parallel and equal in length). Sufficient data are then known to solve the triangle *DED′* by trigonometric relationships. Here also two solutions are possible and the general shape of the figure must be known in order to choose the proper one.

(4) *Other cases.* These are solved by similar methods, or more laboriously by simultaneous equations of the forms

$$AB \cos \text{bearing} + BC \cos \text{bearing} \ldots + YA \cos \text{bearing} = 0,$$
$$AB \sin \text{bearing} + BC \sin \text{bearing} \ldots + YA \sin \text{bearing} = 0.$$

253. Off-Line Traverses. It is seldom that a survey follows the exact property lines because these lines are usually occupied by buildings, fences, or hedges which necessitate making the field traverse adjacent to, rather than along, the property lines. In such cases it is necessary to measure the angle and distance of *side shots* from the traverse point to the property corner, as shown in Fig. 253. Figure 253a shows the complete computation of the lengths and bearings of the property lines in such a case. The survey and computation are much simplified if part or all of the survey can be made along the property lines.

254. Coordinates Available. Most thorough surveys are coordinated and consequently there is available a mass of coordinate data from the United States Coast and Geodetic Survey, the United States Geological Survey, state, city, and county engineers, and from some private corporations such as

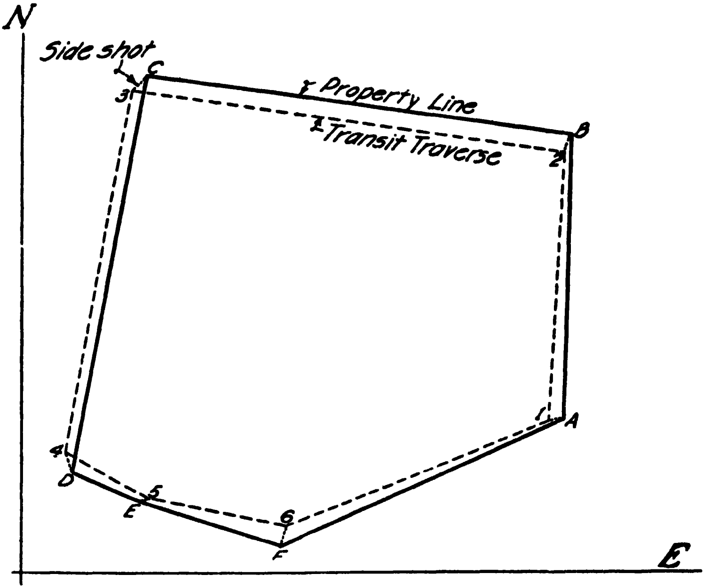

FIG. 253. TRAVERSE ADJACENT TO PROPERTY LINES.

the railways. It is desirable to connect the coordinates of any important surveys with those already available, preferably the North American Datum, information regarding which may be secured from the two Surveys just mentioned, or as is further explained on page 181.

PROBLEMS

1. (a) Fill out this table and obtain the area in acres. The values are in feet. The answer to 1 (a) is in italics.

Course	Latitude	Departure	D.M.D.	Double Areas	
				N. Products	S. Products
AB	N 300	E 1200	*1200*	*360,000*	
BC	S 200	E 400	*2800*		*560,000*
CD	S 200	W 200	*3000*		*600,000*
DA	*N 100*	*W 1400*	*1400*	*140,000*	

$\frac{330{,}000}{43{,}560} = 7.58$ *Acres*

500,000 *1,160,000*

500,000

2)660,000

330,000

sq. ft.

TRANSIT TRAVERSE

Course	Bearing	Dist.	Latitudes N+	Latitudes S-	Departures E+	Departures W-	Balanced Coördinates Lat.	Balanced Coördinates Dep.	Balanced Coördinates X(E)	Balanced Coördinates Y(N)	Pts.
1—2	N2°36'E	584.2	583.6		26.5		+583.7	+ 26.5	1151.2	327.6	1
2—3	N81°39'W	945.3	137.3			935.3	+137.4	-935.5	1177.7	911.3	2
3—4	S10°14'W	800.4		787.7		142.2	-787.5	-142.2	242.2	1048.7	3
4—5	S60°40'E	207.5		101.7	180.9		-101.7	+180.8	100.0	261.2	4
5—6	S78°40'E	302.8		59.5	296.9		- 59.5	+296.8	280.8	159.5	5
6—1	N68°22'E	617.2	227.5		573.7		+227.6	+573.6	577.6	100.0	6
		3457.4	948.4	948.9	1078.0	1077.5	0.0	0.0			

SIDE SHOTS

Course	Bearing	Dist.	Latitudes N+	Latitudes S-	Departures E+	Departures W-
1—A	N72°16'E	15.4	4.7		14.7	
2—B	N21°07'E	14.9	13.9		5.4	
3—C	N45°34'E	16.4	11.5		11.7	
4—D	S20°33'E	23.4		21.9	8.2	
5—E	S76°05'W	9.7		2.3		9.4
6—F	S15°30'W	9.4		9.1		2.5

Error of closure (linear)

$$= \sqrt{0.5^2 + 0.5^2} = 0.7 \text{ feet}$$

Error of closure

$$= \frac{0.7}{3457.4} = \frac{1}{4939}$$

PROPERTY LINES

Course	Bearing	Dist.	Latitudes N+	Latitudes S-	Departures E+	Departures W-	Coördinates X(E)	Coördinates Y(N)	Pts.
A—B	N1°40'E	593.1	592.9		17.2		1165.9	332.3	A
B—C	N81°44'W	939.0	135.0			929.2	1183.1	925.2	B
C—D	S10°04'W	833.7		820.9		145.7	253.9	1060.2	C
D—E	S63°18'E	182.7		82.1	163.2		108.2	239.3	D
E—F	S77°42'E	310.9		66.3	303.7		271.4	157.2	E
F—A	N67°47'E	638.2	241.4		590.8		575.1	90.9	F
			969.3	969.3	1074.9	1074.9			

FIG. 253a. OFF-LINE TRAVERSE COMPUTATION.

(b) The same except AB, N 200, E 300; BC, S 300, E 100; CD, S 200, W 300; DA, N?, W?.

(c) The same except AB, N 700, W 100; BC, N 200, E 800; CD, S 900, E 300; DA, N?, W?.

2. Pass axes through the south and west points of the traverse and compute the area of the figure in Problem 1, using coordinates and the tabulation of page 301.

Point	North y	East x	Difference in Adjacent x Values	Area	
				+	−
A	*100*	*0*	*− 200*		*20,000*
B	*400*	*1200*	*+1600*	*640,000*	
C	*200*	*1600*	*+ 200*	*40,000*	
D	*0*	*1400*	*−1600*	*0*	
			Sum = 0	*680,000*	*20,000*

$$\frac{330{,}000}{43{,}560} = 7.58 \text{ Acres}$$

680,000
− 20,000
2)660,000
330,000 square feet

3. (a) Two mine shafts at A and D are connected by a surface traverse: AB is S 12° 14′ E for a distance of 1291.3 feet, BC N 83° 12′ E, 1317.4 feet, CD S 89° 41′ E, 1819.8 feet. What are the bearing and length of a tunnel to connect these two shafts? *Ans.* S 71° 50′ E. 3579.9 feet

(b) The same except AB, S 13° 43′ E, 721.5 feet; BC, N 74° 18′ E, 2432.6 feet; CD, N 32° 19′ E, 417.6 feet.

(c) The same except AB, N 18° 14′ W, 121.7 feet; BC, S 81° 19′ W, 2046.4 feet; CD, S 39° 17′ W, 427.6 feet.

4. (a) In the following closed deflection angle traverse, calculate the bearings and compute the latitudes and departures. What is the linear error of closure in feet; also, what is the error when expressed as a vulgar fraction? The bearing of AB is S 10° 32′ E.

Line	Distance Feet	Bearings	Deflection Angle	Point
AB	227.2	S 10° 32′ E	0° 43′ R	A
BC	226.6		95° 28′ L	B
CD	483.0		1° 27′ L	C
DE	1013.0		127° 47′ L	D
EA	573.3		136° 05′ L	E

(b) Balance the latitudes and departures in (a) by the compass rule.

(c) The same as (b) but use the transit rule.

(d) Compute the area from (c) by the D.M.D. method.

(e) Calculate the coordinates from (c) and compute the area by coordinates.

(f) How can the transitman check the angles in a closed deflection angle traverse before leaving the field? Give all of the methods that you can think of.

5. (a) Given the following closed traverse, compute the length and bearing of the closing side.

(b) Compute the area by D.M.Ds.

(c) Compute the area by coordinates.

Line	Bearing	Distance Feet
AB	N 1° 10′ W	751.0
BC	N 63° 04′ E	392.0
CD	S 10° 50′ E	561.3
DA	?	?

6. (a) Compute the missing measurements in the following traverse.

Line	Bearing	Distance Feet
AB	S 14° 28′ E	296.8
BC	S 87° 51′ E	?
CD	N 20° 05′ W	472.5
DA	S 77° 15′ W	?

(b) Compute the area by D.M.Ds.

(c) Compute the area by coordinates.

7. (a) Compute the missing measurements in the following traverse.

(b) Compute the area by D.M.Ds.

(c) Compute the area by coordinates.

Line	Bearing	Distance Feet
AB	N 50° 32′ W	748.9
BC	S 36° 10′ W	?
CD	?	750.0
DA	N 41° 00′ E	624.7

8. Can a check be made where one measurement of angle or distance is missing? If so, how?

9. A survey traverse along the surface and underground is recorded in the following notes and shown on Fig. 254.

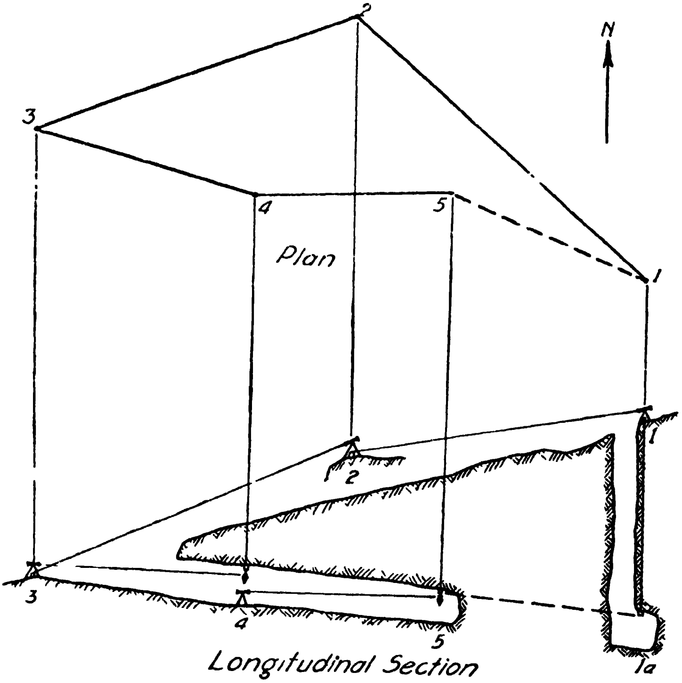

FIG. 254.

POINT		HOR ANGLE		DISTANCE		VERT ANGLE	H PT SIGHTED	H.I.	DESCRIPTION
Occ	Sighted	Vern A	Vern B	On Slope	Hor				
1	Due south	0° 00′	179° 58′					+5.21	E. edge of shaft, on surface.
	2	135° 23′	315° 24′	215.53		− 9° 45′	0.0		On surface.
2	1	0° 00′	179° 59′					+5 03	
	3	118° 03′	298° 06′	201 92		−23° 32′	0.0		On surface.
3	2	0° 00′	179° 58′					+5.12	
	4	36° 58′	216° 58′	128 27		− 2° 18′	+4 17		In roof.
4	3	0° 00′	179° 59′					−8.73	
	5	162° 15′	342° 16′	108 64		0° 00′	+2.84		In roof.

Find the bearing and the horizontal length of a line connecting points 5 nd 1a (1a being directly under 1) so that the heading at 5 may be connected with the shaft at 1a. Having given the elevation of point 1 as 200.00 feet and the elevation of point 1a as 81.43 feet, find the difference in elevation of the points 5 and 1a. Note in the foregoing tabulation that all horizontal angles are turned clockwise from the backsight, that eccentricity exists in the transit, and that the horizontal distances must be computed from the slope distances by using the trigonometric functions of the vertical angles. The H.I. is plus if the transit is above the stake. The H. Pt. sighted is plus when the stake is above the end of the line of sight, as at points 4 and 5. H. Pt. signifies the height between the point sighted at, and the survey monument.

CHAPTER XVIII

ORGANIZATION OF SURVEYS

255. Planning the Survey. Surveys are a comparatively inexpensive part of the cost of engineering projects — on the order of from 1 per cent to 3 per cent of the construction cost. Mistakes in the surveys or inefficient surveys may cost many times this amount, in some cases sufficient to throw the company into a receivership. It therefore does not pay to slight the surveys at the expense of increased construction cost or financial hazard. Speed in surveys is frequently more essential than low cost, since completion of the project is, of course, held up pending surveys, and large overhead expense accrues. In general, carefully planned speed tends to give low costs, but hasty, ill-planned surveys are usually expensive.

Neither speed nor low cost should be sought at the expense of the necessary quality and precision of the survey. If we assume that these necessary qualities are attained, low cost, and especially speed, will tend to establish a good reputation for the engineer; in fact this has been the basis of many successful careers. They are to be obtained by:

(1) Establishing the character of information desired and the limits of required precision;

(2) Organizing to meet these requirements with a minimum of time and cost. Under modern conditions, it is not advisable to overwork the men for long hours. Careful preplanning of the survey methods is the better way.

The more survey parties employed, the greater the speed but usually with higher cost, since each party must be broken in along the lines necessary for that particular survey. Frequently this increased cost is justified by the loss which the corporation would incur through delay. On the other hand, it is somewhat difficult to correlate and check the work of a large number of hastily organized parties. Serious difficulties and errors have occurred in the past from this source.

Perhaps the greatest waste in time and expense required in making surveys results from lack of careful and competent determination of requirements and methods *before* the survey starts. It is not uncommon that surveys are made over and over again with increasing precision and adaptability to requirements, but with corresponding loss of time and money. Frequently time and money are wasted in securing much more precise results than is necessary.

Another source of waste is to allow the ordinary surveyor to attempt a more complicated or precise survey than those for which he is qualified, such, for example, as triangulation and plane table surveys. Competent men who specialize in such work may often be secured from the Federal Government. Aerial surveys are another such specialty, but it is customary to contract for such work with qualified private companies.

256. Simple Surveys. No particular organization is here necessary since ordinarily only one or two parties are employed and they are easily managed. Frequently only one trained engineer or surveyor is used and he is assisted by college students, active laborers, or young men of no special training. The engineer making the survey frequently makes his own maps and computations from it. It is usually possible to estimate the time required. This time multiplied by the daily, weekly, or monthly expense of the party and of the office and overhead expense, gives the total cost.

257. Extensive Surveys. It is difficult or impossible to estimate exactly what a survey will cost or exactly what organization and method will work best unless conditions are identical with some previous experience. It is frequently necessary to consider or actually to try out through the earlier stages of the survey, perhaps two of several organizations and methods, and then to pick the better one. The best thought must be given to this matter both before the survey is started and during its early progress. With these qualifying remarks, an attempt will be made to point out some general principles and facts regarding costs and organization.

The first steps in a survey are to learn definitely its purpose, how quickly it must or should be completed, what maximum errors may be permitted, and what precision is desired. The engineer in charge of the surveys should be experienced in, and thoroughly familiar with, the technical, economic, administrative, and commercial requirements so that his work may conform to them. Books and periodicals giving these viewpoints for specific kinds of engineering construction are available in technical libraries and frequently in the libraries of the corporation. The periodical indexes should be consulted to secure periodical references covering the latest practices. A few hours spent in such reading will give viewpoints which cannot be readily acquired in the field except by long experience. It is this viewpoint which distinguishes the professional or administrative engineer from the vocational surveyor. Consultation with officers of the company or with other experienced men is desirable. All maps and data available in the territory under consideration must be found and used where helpful.

It is difficult properly to emphasize the necessity of this preliminary study, particularly where the survey to be undertaken covers unfamiliar or progressive types of projects. It is frequently helpful to make tentative specifications and progress schedules for the field and office work.

258. Organization. Two general methods of organization are possible.

(1) The quick thinking and quick acting engineer connected with a similar type of private corporation is inclined to put a small survey party or parties into the field immediately. During their early work and from observation of their results and activities, he crystallizes his plans for the entire survey and expands this nucleus to full size. This procedure sometimes involves some lost work in the earlier stages, but because of the advantage that can be taken of actual field conditions on the project, it is apt to develop surveys which are satisfactory and rapid. The engineer in charge must be a man who can and will adopt changes quickly, and who is willing to assume much responsibility.

(2) The slower and more conservative engineer connected with a similar type of corporation, including most governmental divisions, who is not inclined quickly to adopt innova-

tions or changes, is apt, after careful and long pre-planning, to put a fairly large organization into the field and to proceed with few changes. The more nearly the work in hand is similar to work previously executed by the engineer in charge, the better this procedure. It is less apt to involve any work or methods which must be abandoned, but it is quite possible that the methods adopted from previous experience will not work so well here. In general it tends to make the surveys slower, surer, and more expensive. Of course, it best fits persons and organizations accustomed to routine procedures.

259. Factors Affecting Costs. Surveys made during the summer months, when brush and trees are in full leaf and when crops, such as corn, are high enough to interfere with the view (necessitating axes, machetes, brush hooks, saws, and pruning shears), will be more expensive than those made during the early spring, winter, or late fall, when less clearing is necessary. Severe winter weather increases the cost of surveys, as does also rainy weather. The character of the topography and the amount of clearing of course materially affect costs. Wage rates and salaries vary considerably. Finally, the competence and morale of the survey organization will influence costs.

Organization of personnel, and progress to be expected, for various "unit" types of survey operations are given in following sections of this chapter. If estimates of costs are to be made, current wage rates must be ascertained, and allowances made for supplies, instruments, and overhead.

260. Transit-Tape Traversing. The minimum party would consist of a transitman and two tapemen. This, however, is not an economical group as an added stakeman will always pay. An automobile is usually needed for transportation. From one to six axmen must be added for clearing the line where considerable timber or brush is encountered. For a large party it will frequently be advisable to add a chief of party and a notekeeper, particularly if the notes are extensive.

A typical field party would therefore include one transitman, two tapemen, one stakeman, one axman, one automobile and chauffeur.

Using a 100-foot tape in open, rolling, or level country, such a party should average about 3 miles of traverse per day; in light timber or underbrush about 2 miles; and perhaps half a mile in heavy timber or underbrush. If curves must be staked, as in railway and highway location, then this progress will be much reduced, being frequently cut in half.

Using 300-foot or 500-foot tapes, the average progress away from towns where traffic interferes with long tapes should be 50 per cent or more greater than with the 100-foot tape. In rough country, progress is considerably slower, especially if the 100-foot tape is used. The longer tapes are particularly helpful here. The axman would be omitted in open country, but extra axmen would be needed in timber and underbrush.

261. Transit-Stadia Traversing. The minimum party consists of an instrumentman and rodman, but for economy it is desirable to add another rodman and a notekeeper or recorder, with one or more axmen if clearing is necessary. It is rarely necessary to use more than one axman, since the stadia method is not well adapted to timbered country, as mentioned on page 171. In fairly open country, the minimum party would comprise one transitman, one rodman, one axman, and one automobile and chauffeur. Such a party should run 4 or 5 miles per day in open country whether flat or quite hilly, and considerably less in brush or timber.

It is not contemplated that any side shots for topography will be taken, but approximate elevations of transit stations may be carried by vertical angle with little loss of time. Azimuths would be carried to the nearest minute, and forward and backward rod intervals and vertical angles would serve as a check.

The transit-stadia method becomes even more rapid if the transit is set up only at alternate stations and only needle readings are taken for direction, no azimuths being carried on the transit plates, as explained on page 237. With the foregoing party, six to ten miles per day may be accomplished. A considerable advantage in using needle readings only is the case of passing trees or obstacles. It is not necessary to turn angles around the obstacle, but simply to set up the transit or compass beyond the obstacle and to proceed by needle reading. Such lines are not very precise, although on long lines the errors tend to balance or compensate. Where the instrument is set up on every other point no check on the stadia distance or vertical angle is obtained.

262. Sketching Board Traversing. A party comprising one sketcher, and one automobile and chauffeur should cover ten to thirty miles of traverse along roads in one day, including a few sketched adjoining contours at perhaps 20-foot intervals. This does not give elevations, except perhaps by aneroid barometer. Distances are secured from the automobile speedometer. This method is not precise but serves well for reconnoissance, and is largely a military usage.

263. Levels. The level party would comprise one levelman, one rodman, and one automobile and chauffeur. In fairly level open country, such a party will run 3 or 4 miles of differential levels or 2 or 3 miles of profile levels in a day. As the country becomes rougher and timbered or brushy, progress will decrease – in extreme cases to half a mile per day.

It should be borne in mind that approximate levels can be carried by transit and stadia or by long tapes on the slope in connection with traversing at small expense since most of the work is necessary for traversing and only a few added data are necessary to secure rough elevations.

264. Area Surveys. It is more difficult to organize or estimate costs of area surveys. The best procedure in estimating for extensive areas is first to outline methods and to compute the number of miles of traverse and levels to be run, as well as the number of triangulation stations to be occupied. These, multiplied by the cost per mile and the cost per triangulation station occupied, will give the cost of the skeleton or control surveys. The organization for filling in detail is covered in the next section.

265. Filling in Details. Transit and stadia or plane table may be used with a party comprising one instrumentman, one recorder, one draftsman, two rodmen, one axman, and one automobile and chauffeur.

Such a party will average from 100 to 300 side shots per day, more in open easy country and less in difficult brushy country. The number of shots required per square mile or per acre will, of course, depend upon the scale of the map, the contour interval, the required precision, and the nature of the country. In the above figures it is contemplated that the map will be penciled in the field as the work progresses. If checked elevations from levels previously run are available, the contours may be definitely drawn in the field, although this will slightly diminish the speed of the party. If no elevations, or only stadia elevations are available, the contours are sketched for shape only, and are drawn later in the office after satisfactory elevations are available.

If, on large scale stadia-transit maps, the field draftsman is omitted and the map is made in the office from transit notes, many more points must be taken in the field. While more shots are necessary, they can be taken more quickly. The perennial discussion of which is the cheaper method is still unsettled. If the map is made in the office, it should be taken into the field for checking.

The plane-table method is probably the best, provided the weather is suitable and an experienced plane-table man is available.

266. Filling in Details by Rectangles. It is frequently necessary to make an area survey of lands wholly or largely covered with timber, in which case the transit-stadia or plane table is impractical. The area is divided into rectangles, along the sides of which are run rough traverses, usually with compass and 300-foot tape, as described on page 265. Elevations are taken with the barometer or with the engineers' level. Occasionally elevations are computed as vertical components of slope taping, the vertical angles having been measured with a clinometer. Such maps are frequently drawn to a small scale, such as 1″ = 1000′, and with a wide contour interval, such as 25 or 50 feet. It is customary to use the section lines if the country has been sectionalized and to run the traverses perhaps 1/2 or 1/4 mile apart between section lines. The estimated costs are obtained from the number of miles of traversing.

267. Triangulation Surveys. It is difficult to estimate the progress of this type of survey. The topography, weather, precision, and so on all affect progress. The following example will indicate ordinary engineering (not geodetic) triangulation. A party may comprise one instrumentman, one recorder, one computer, one flagman, and one automobile and chauffeur.

Such a party will set and occupy perhaps 2 to 4 triangulation points per day.

268. Costs. Examples of costs on specific surveys under modern conditions may be found in current periodicals, located through periodical indexes. The figures will vary widely and it must be borne in mind that probably only the better organized and cheaper surveys are quoted and given publicity. Inefficient work may cost several times as much, particularly where the surveys must be repeated, or where construction work and/or estimates have been based on erroneous surveys.

PROBLEMS

1. Outline the survey under these heads: maps available, organization, control, filling in detail, administrative reports on progress and costs, and mapping for (a) 16,000 acres of gently rolling country about half covered with heavy underbrush and light timber. Property lines are fairly well established but about half of the section and quarter-section corners are missing. Maps are to be on a scale of 1 inch equals 400 feet with a 5-foot contour interval, and are to show existing property lines and corners, although no search is to be made for corners. The work is to be completed with finished maps in six months.

(b) 5000 acres of hilly country to be surveyed as a reservoir site. It is largely covered with light underbrush and heavy timber. The country is not sectionalized, and only easily recognized property lines are to be shown on the map, which is to be drawn to a scale of 1 inch equals 1000 feet, with a 10-foot contour interval. The maps are to be completed in nine months.

(c) 20,000 acres of very rough and timbered country. Logging maps to a scale of 1 inch equals 1000 feet with a 25-foot contour interval are to be completed in three months. Section lines are to be approximately reestablished; about half of them are available on the ground.

2. What would be the cost of making the property survey of Problem 3 (page 234), if we assume open rolling country?

3. The organization may be planned and the costs estimated, for various types of surveys.

ENGINEERING SURVEYS: APPLIED

CHAPTER XIX

INTRODUCTION TO APPLIED ENGINEERING SURVEYS

269. Applications. In elementary surveying, the engineer learns to use types of surveys which are adequate for simple projects and for pioneering development. With the increasing complexity of our modern social and industrial order, however, more precise results and more elaborate survey procedures are frequently demanded of him, and these are described in the following chapters.

From the inception of a modern engineering project to its completion and thereafter throughout its operation, surveys are needed in order to guide those who are responsible for the design and for the efficient management of the enterprise. To this end, surveys are made continuously throughout the life of the project. They must always be accurate and reliable, and frequently demand a degree of accuracy and judgment seldom found in the untrained engineer.

Greater dependence for accurate data and maps is being placed increasingly upon the engineer. His knowledge of proper methods of procedure and his ability to produce consistently dependable results place him in an advantageous professional and consulting position in industry and public service. In order to maintain and to increase that advantage, the engineer must strive continually for better methods both for himself and for his organization.

In the performance of these professional duties, the engineer has a social responsibility of a public or quasi-public nature. The results of his work extend far beyond the immediate future and involve much more than mere technical performance. For example, the design and establishment of a new real estate subdivision influence the social and economic development of the community. Insufficient street widths create traffic and health hazards. Poorly designed lot arrangements and building restrictions may result in slums, while proper design on the same site might yield a highly satisfactory and relatively permanent residential district. Proper boundary surveys also minimize legal difficulties.

Surveying techniques and the nature of the projects for which they are used are changing so rapidly that the young engineer has many opportunities to develop new procedures which may be more effective than traditional practice. Among recent developments might be mentioned those in photogrammetry which have changed the methods of mapping (Chapter XXIV), systems of state plane coordinates (Chapters XXV and XXVII), electronic surveying (Chapter XXVI), modern city planning practices (Chapter XXVIII), applications of legal knowledge in connection with boundary surveys (Chapter XXVII), revival of the use of the solar attachment in public land surveys (§ 321), the Simplex Shield for direct solar observation (§ 320), and the like.

270. Instruments. Manufacturers are alert constantly to improve their instruments to meet the greater precision demanded for both general and specific surveying purposes. For example, the thirty-second vernier is now standard equipment on the transit in contrast with the one-minute vernier of only a few years ago.

Although foreign makes of instruments of radically different design are quite precise, they are expensive and have rarely been accepted by American engineers. It is difficult or impossible to have them repaired in this country. Domestic makes of the foreign designs are in only occasional use in this country.

The necessity for careful selection, use, and adjustment of improved models cannot be over emphasized. Contact and consultation with manufacturers is frequently desirable, especially in such rapidly developing fields as photogrammetry.

271. Organization of the Survey. Extensive or precise surveys must be carefully planned and organized before they are undertaken. The first step is to decide the limits of error which will be acceptable. It is then possible to select the personnel and equipment and to adopt methods of procedure which will be most feasible and economical from among the various possible alternatives. That this is not an academic abstraction is evidenced by the numerous changes and innovations that occur on most large survey projects for which early planning was inadequate. Planning at the early stages of the operation will yield greater returns than at any subsequent time.

Professional societies are continually attempting to improve surveying and mapping procedures and their publications should be examined. Among these may be mentioned the American Congress on Surveying and Mapping, and the Committee of Surveying and Mapping Division on Topographical Surveys whose Progress Report appears in the Proceedings of the American Society of Civil Engineers, Volume 72, Numbers 4, 8, and 9, for April, October, and November, 1946.

272. Surveys by Contract. Many extensive or special types of surveys, such as photogrammetry and high precision surveys, require special equipment and special personnel, and it may therefore be necessary or desirable to do the work by contract under definite survey specifications. These should be drawn up by the client's engineer in consultation with the survey contractor. Such specifications usually cover —

1. Scale of final map
2. Contour interval
3. Traverse, triangulation, and elevation closures
4. Fidelity of topography and detail
5. Methods to be used in checking
6. Orientation
7. Finishing and reproduction of the final map
8. Special features.

Survey contracts may be let on a lump-sum, unit-price, or cost-plus basis. Adherence to the specifications is determined by inspection in the field and in the office. In topographic mapping, for example, the check most commonly used is that in which a test profile as surveyed on the ground is compared with a profile along the same line taken from the contract map.

CHAPTER XX

HORIZONTAL CONTROL

TRIANGULATION *

273. Fundamentals. Horizontal control over an area by means of triangulation is based upon the mathematical fact that if one side and the three angles of a triangle are measured the remaining two sides can be calculated If either of these lines is also a side of an adjoining triangle, the measurement of the three angles in the second figure is sufficient for the determination of its remaining two sides

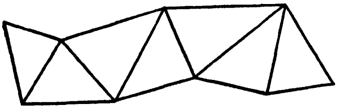

FIG. 274 CHAIN OF TRIANGLES

274. Systems. A chain of triangles, Fig 274, is the simplest form of triangulation system since it involves a minimum of angular measurement and calculation for the amount of area covered Although this form is satisfactory in many cases, obviously it is not the best method because it lacks certain checks and opportunities to balance small errors which are certain to occur even in the most careful work

The best system is that of a chain of quadrilaterals, Fig 274a, in which each figure is composed of two pairs of overlapping triangles This arrangement gives the maximum opportunity for the distribution of angular errors; furthermore it provides a double calculation for the line which is to become the entering side of the next figure

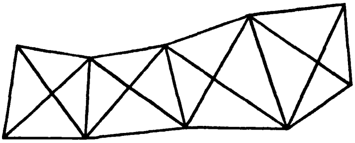

FIG. 274a CHAIN OF QUADRILATERALS

A third system, commonly used in establishing control for a city, where the choice of locations for the triangulation points is much more restricted than in open country, is that of a system of polygons or central point figures, Fig. 274b. In desirability, this scheme probably lies between the first two, but if it is carefully planned it can be very acceptable from the standpoints of checks and accuracy.

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* See also Chapter XI

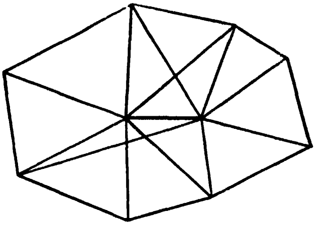


Fig. 274b. Central Point Figure.

275. Classification and Uses. Methods of triangulation are classified according to the precision obtained from the field measurements, proper adjustments, and calculations. The following abbreviated table gives certain of the requirements for the *four orders of triangulation.*

	Discrepancy between Any Computed and Measured Distance	Angular Error in Triangle		Precision of Base Line Measurement
		Ave	Max	
First Order	1 part in 25,000	1″	3″	1/300,000
Second Order	1 part in 10,000	3″	8″	1/150,000
Third Order	1 part in 5,000	6″	12″	1/75,000
Fourth Order	1 part in 2,500	12″	30″	1/25,000

First and second orders are used principally to establish primary control along national boundaries or nation-wide chains to which all lesser units can be tied.

The third and fourth orders are of ordinary or, in rare instances, of low precision and are used principally to control smaller areas for the large scale detail mapping needed in connection with such problems as flood control, water supply, power developments, large bridges, general topographic mapping, and the like.

276. Reconnaissance. A careful reconnaissance of the area to be controlled is of great importance not only from the standpoint of minimizing time and cost but also to insure final accuracy. All available maps should be studied, and this should be followed by actual inspection of the area. The reconnaissance party should be equipped with pocket compasses, binoculars, and an aneroid barometer (see § 11 Rubey's *Route*

Surveys) for determining relative elevations of triangulation stations.

The important points to be considered in the selection of the triangulation stations are size of figures, accessibility and permanence of the stations, intervisibility with a minimum of clearing of lines or construction of towers, and selection of the best location for the base line. In addition, and probably most important from the standpoint of accuracy of results, the shapes of the triangles should be taken into consideration in order that the *strength of figure* may be the best obtainable.

277. Strength of Figure. Since the computed distances are obtained by use of the law of sines, it is essential that small angles be avoided wherever possible, since they give the most uncertain results. Even with the figure established there is, except in the case of a chain of triangles, a choice of equations by which the length of an unknown side can be obtained from the known side. For example, in a quadrilateral with one side known, the opposite side may be calculated in four different ways. An investigation of these possible computation routes results in the determination of the *strength of figure*.

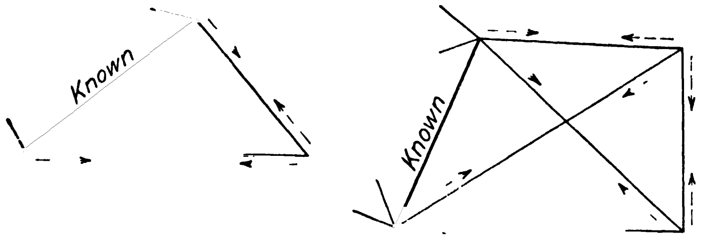


FIG. 277. STRENGTH OF FIGURE.

This term is usually stated in equation form as

$$R = \frac{D - C}{D}(\delta_A^2 + \delta_A\delta_B + \delta_B^2) \text{ in which}$$

D = the number of *new* directions in the figure, i.e., four in a triangle, 10 in a quadrilateral, etc. See Fig. 277.

C = the number of geometric conditions, angle and side, in the figure. Its value may be obtained from the equation

$C = 2L - L' - 3S + S' + 4$ in which

L = total number of lines in the figure

L' = number of lines observed in but one direction

S = total number of △ stations in the figure

S' = number of △ stations unoccupied.

δ_A, δ_B = the difference for 1″ in the sixth decimal place of the log sines of the distance angles A and B. The distance angles in any triangle are those opposite the known side and the side being computed. Table 277 gives values for the term $\delta_A^2 + \delta_A\delta_B + \delta_B^2$ for practically every combination of distance angles.

Figure 277a is a representative quadrilateral in which the strength of figure is to be investigated.

$$C = 12 - 0 - 12 + 0 + 4 = 4$$

$$\frac{D - C}{D} = \frac{10 - 4}{10} = 0.6$$

The side CD may be calculated from the known side AB by four different pairs of angles. The $(\delta_1^2 + \delta_A\delta_B + \delta_A^2)$ values from Table 277 are

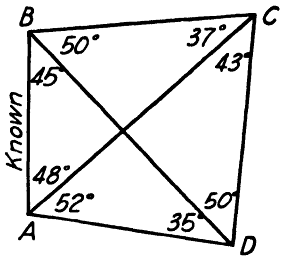


FIG. 277a. STRENGTH OF FIGURE.

Route	*Distance Angles*	$\delta_A^2+\delta_A\delta_B+\delta_B^2$	Σ
1. AB to BD to CD =	$\frac{35°}{100°} + \frac{80°}{50°}$	= 8 + 4	= 12
2. AB to AD to CD =	$\frac{35°}{45°} + \frac{43°}{52°}$	= 20 + 12	= 32
3. AB to AC to CD =	$\frac{37°}{95°} + \frac{85°}{52°}$	= 7 + 2	= 9
4. AB to BC to CD =	$\frac{37°}{48°} + \frac{50°}{50°}$	= 18 + 9	= 27

Multiplying each tabular summation by the factor of the figure, 0.6, the relative strengths of figure are found to be 7.2, 19.2, 5.4, and 16.2 respectively. The route giving the smallest number is the best for the calculation of the side with which the next figure is to begin. This investigation will result in eliminating, to a great extent, the errors inherent in the use

TABLE 277. VALUES FOR $(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$ IN THE 6TH DECIMAL OF THE LOG SINES

°	10°	12°	14°	16	18	20	22°	24°	26°	28°	30	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

of logarithms of functions. It should be noted that strength of figure is concerned with the relative sizes of the angles and is independent of the precision with which they are measured.

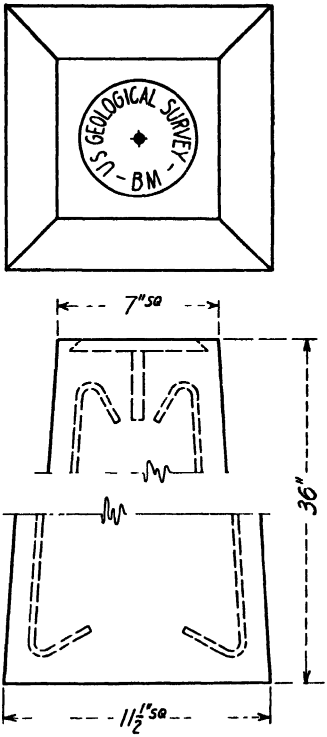


FIG. 278. SURVEY MONUMENT.

278. Establishment of Stations. Triangulation systems of all orders are usually designed to establish control over an area for many years. It is therefore necessary to construct the station markers so that they will remain in place permanently. They should be referenced to nearby permanent objects or monuments in order that accurate replacement may be made, and so that the stations may be found and identified.

Ordinarily, a point on a bar or plate of some enduring metal cemented into masonry, solid rock, or the top of a pre-cast concrete monument will be a satisfactory station mark. If pre-cast monuments are to be used, they should be of the type shown in Fig. 278, smaller at the top than at the bottom and long enough to extend well below frost line — at least 36″.

The United States Coast and Geodetic Survey specifications call for the standard triangulation disk, Fig. 278a, to be set in concrete at the surface of the ground directly over another disk several feet below.

279. Signals and Towers. The stations of a triangulation figure must be intervisible for determination of angular values. Intervening trees or high ground quite often necessitate the erection of signals, platforms, or towers.

It is frequently possible to sight the telescope at some part of the transit occupying another station, such as the plumb bob cord, the vertical axis between a pair of leveling screws, or the intersection of the telescope barrel and the horizontal axis. If these cannot be seen, a large tripod can be made from 2″ by 4″ lumber with a mast extending upward to a considerable height.

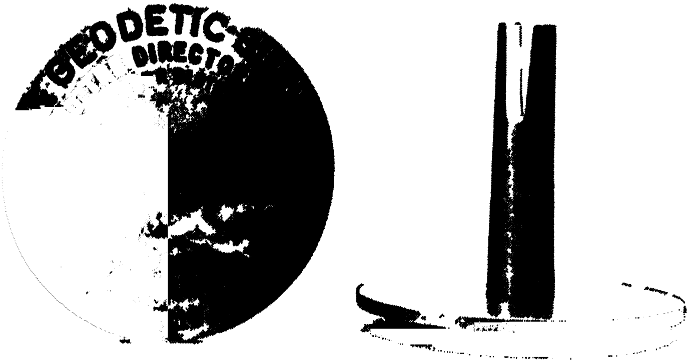


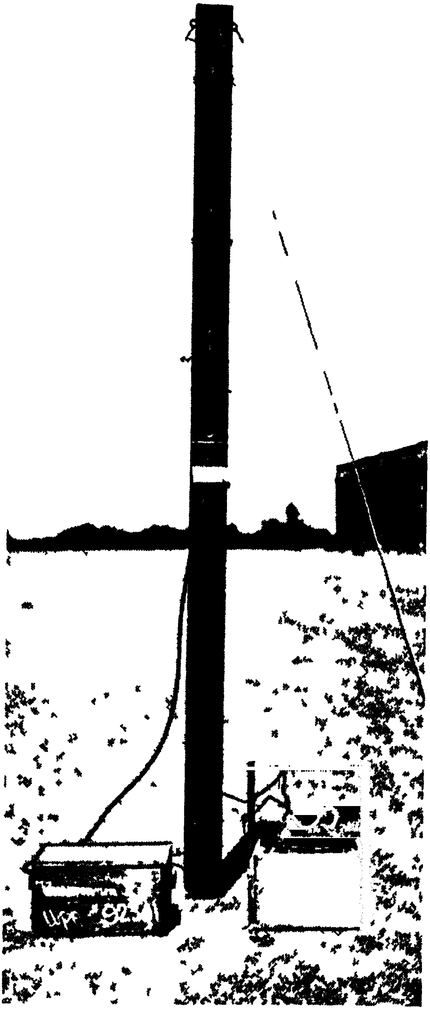
FIG. 278e TRIANGULATION STATION DISK.

The mast should be plumbed over the station by means of two transits and securely wired in place. The signal tripod should be of sufficient size to permit the regular operation of a transit beneath it.

A red neon tube 4 or 5 feet long, Fig. 279, mounted on a rod and plumbed over a point, has been found to give excellent results in the measuring of angles at night.

Occasionally, the transit may be elevated to advantage by the construction of a sturdy triangular steel or timber platform which supports the instrument. The instrument man should work on an entirely separate platform which is not in contact with the one supporting the instrument.

For long distance sights in day time work the *heliotrope* can be used to advantage. The principle of this device is merely that of reflecting the sun's rays from the surface of a mirror toward the instrument station. Figure 279a shows one type of heliotrope.



Courtesy of Cleveland Regional Geodetic Survey

FIG 279 NEON SIGHT

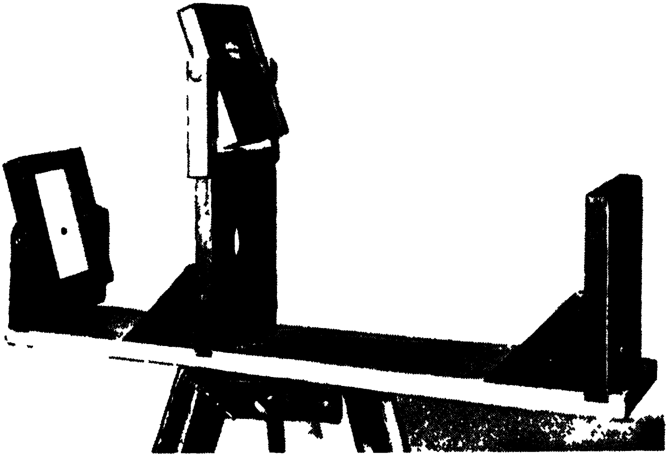


FIG. 279a. HELIOTROPE.

Angular measurements of first or second order usually are made at night when atmospheric conditions are best, by sighting on a battery-operated light. The lengths of sights are usually such as to necessitate the use of high towers to clear intervening obstacles and overcome the curvature of the earth. The United States Coast and Geodetic Survey uses the Bilby steel tower shown in Fig. 279b. This tower consists of one tripod support for the instrument outside of which is another for the observers' platform. It is made in sections so that any height, in multiples of about 7 feet, can be obtained up to a maximum of 126 feet. The Bilby tower can be erected or dismantled by trained crews in a surprisingly short time. Its development has added much to both safety and speed over the old improvised timber tower.

280. Curvature and Refraction. Although many triangulation control systems are not of sufficient size to make necessary the consideration of the curvature of the earth, nevertheless this rapidly becomes a factor as distances increase. The question of intervisibility of stations involves two factors: that of the earth's curvature, and that of refraction in the vertical plane of the

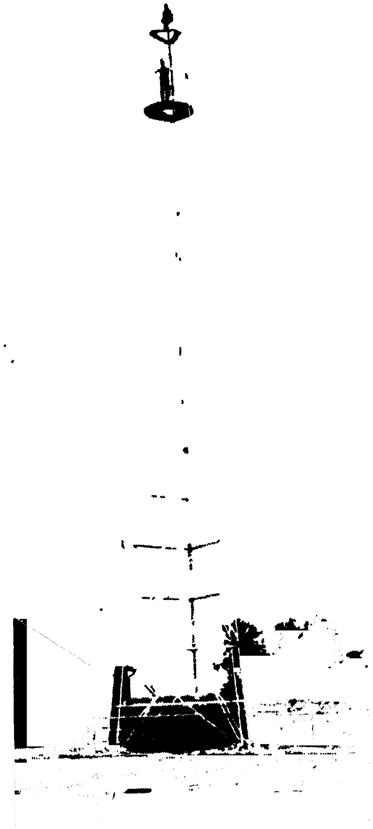


FIG. 279b. BILBY TOWER.

rays of light from a distant object. The following equations and Fig. 280 develop these corrections.

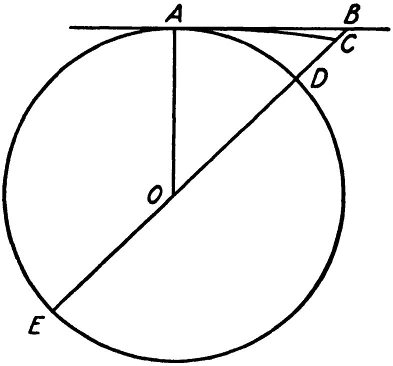


FIG. 280. CURVATURE AND REFRACTION.

CURVATURE (BD)

$AD = D$, the distance between stations

$$AB^2 = BD \times BE$$

$$BD = \frac{AB^2}{BE}$$

$AB = AD$ approximately,

$BE = 2R$ approximately,

(280) $\therefore\ BD = \dfrac{D^2}{2R}$ approximately.

REFRACTION (BC)

Due to the effect of refraction an object apparently at B is actually at C. Thus the refraction correction tends to diminish the curvature correction BD by the amount BC.

The angle of refraction $= BAC = m \times AOB = m \times 2\,BAD$ in which m is the *coefficient of refraction* (usually taken as 0.07) by which the central angle AOB must be multiplied to obtain BAC.

$\dfrac{BC}{BD} = \dfrac{BAC}{BAD}$ approximately,

$BC = 2m \times BD$ approximately.

The combined curvature and refraction correction is $h = BD - BC =$

$$\frac{D^2}{2\,R} - 2\,m\,\frac{D^2}{2\,R} = (1 - 2\,m)\,\frac{D^2}{2\,R} \text{ approximately.}$$

Stating h in feet and the distance, D, in miles we have

$$h = 0.574\ D^2 \qquad \text{approximately.} \tag{280a}$$

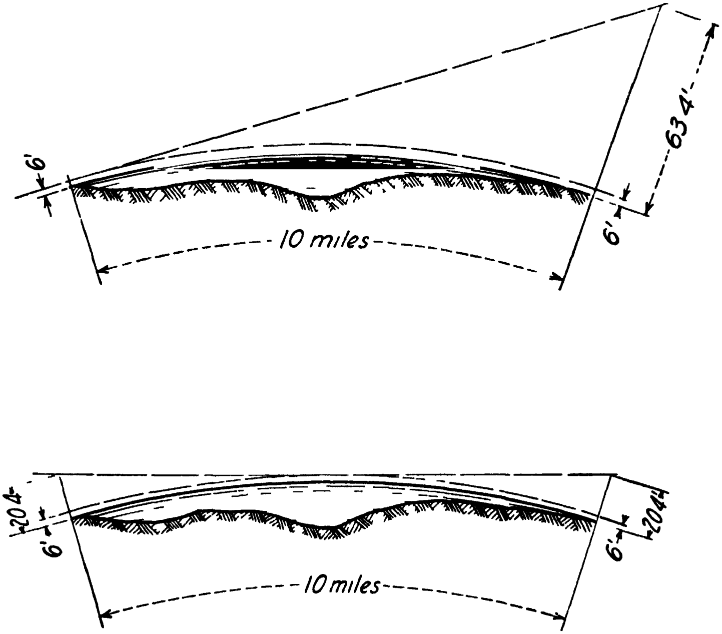


FIG. 280a. INTERVISIBILITY.

EXAMPLE: Two stations are at the water's edge on opposite sides of a lake 10 miles wide, Fig. 280a. (a) If the telescope at one station is 6 feet above the water level what is the height of a tower at the other station that will be just visible without the line of sight approaching nearer than 6 feet to the water? (b) What would equal heights of instrument at the two stations need be in order to be intervisible?

(a) $h = 6 + 0.574(10)^2 = 63.4$ ft.
(b) $h = 6 + 0.574(5)^2 = 20.4$ ft.

281. Angle Measurement in Triangulation. The values of the angles of a triangulation system are obtained either by means of a *direction instrument* or a *repeating instrument.*

The direction instrument (Fig. 281) usually is used only on work of the highest type such as the first order work of the United States Coast and Geodetic Survey. Descriptions of this instrument and of its use may be found in texts on geodesy and in publications of the United States Coast and Geodetic Survey.

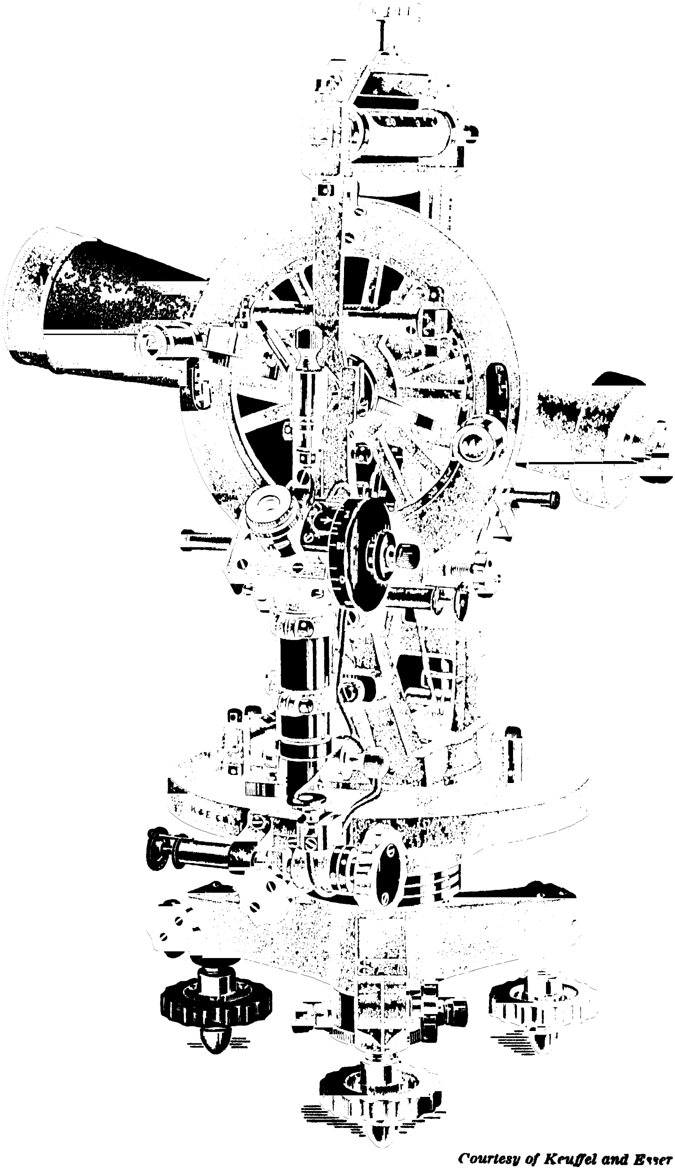
The repeating instrument, not shown, is used in triangulation work of all orders. It may be considered simply to be a very high grade transit. The least count of the verniers may be from 1′ to 5″. For angular readings to less than 5″ of arc the vernier must be abandoned in favor of the micrometer microscope and a direction instrument must be used.

However, the multiple repetition of an angle and the determination of its most probable value constitutes the best method for the elimination of practically all instrumental and personal errors. Such a procedure will give angular values correct to a very small part of the least count of the instrument used. The specifications for repeated angles in second order traverse of the United States Coast and Geodetic Survey state that the average closure of the triangles shall not exceed 3″ and the maximum shall but seldom exceed 6″.

The transit theodolite used in measuring the angles of a triangulation system should be in the best possible adjustment. These adjustments are the same as those given for the transit (see Chapter VII) with the addition of that of the striding level. The axis of this level should be parallel to the horizontal axis of the instrument. The test is made by centering the striding level bubble by means of the leveling screws, then reversing it end for end upon the horizontal axis. If an error appears, the bubble is brought half way back by adjusting the tube itself, the instrument is releveled, and the process repeated until the condition is satisfied.

Use of a properly adjusted striding level gives a much better guarantee that the horizontal axis is horizontal than does the smaller plate bubble. It can be used also to determine whether the vertical axis is truly vertical.

Many different procedures and forms of notes are used in



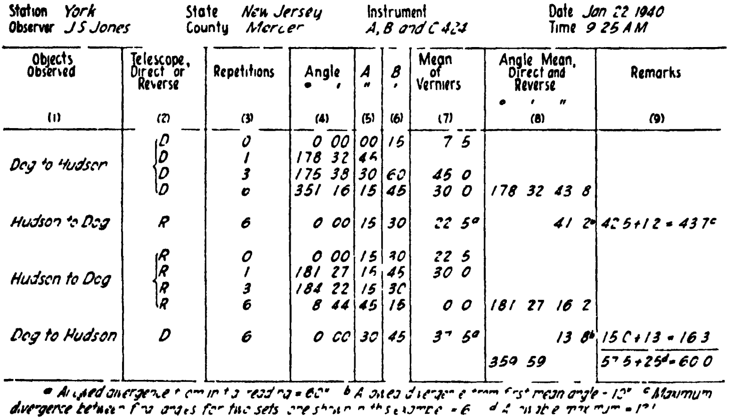
Courtesy of Keuffel and Esser Co.

FIG. 281. PRECISE THEODOLITE.

determining angular values by repetition.* However, they all contain the same three essential points; namely, half of the pointings with the telescope direct, *D*, and half with it reversed, *R;* readings of both *A* and *B* verniers; and readings taken at places entirely around the horizontal plate. Such procedure will assure that existing errors of the horizontal axis, graduation of plates, eccentricity of centers, or failure of the verniers to be 180° apart will be eliminated for the most part.

The number of repetitions to be made depends upon the instrument used and the precision desired. It is common practice to regard six or twelve turns of an angle as a set. Experience seems to prove that the precision cannot be increased indefinitely by continued repetition. This probably is due to instrumental errors caused by excessive clamping and unclamping of the motions or personal errors arising from many pointings of the telescope.

The best results in angle measurement are obtained by a deliberate and systematic handling of the transit without rushing the work or making unnecessary pointings or permitting anything to interrupt its continuity. The instrument should be protected from wind and sun by a board or canvas shelter.



Station *York* State *New Jersey* Instrument Date *Jan 22 1940*
Observer *J S Jones* County *Mercer* *A, B and C 424* Time *9 25 A M*

Objects Observed (1)	Telescope, Direct or Reverse (2)	Repetitions (3)	Angle ° ′ (4)	A ″ (5)	B ″ (6)	Mean of Verniers (7)	Angle Mean, Direct and Reverse ° ′ ″ (8)	Remarks (9)
Dog to Hudson	*D*	*0*	*0 00*	*00*	*15*	*7 5*		
	D	*1*	*178 32*	*45*				
	D	*3*	*175 38*	*30*	*60*	*45 0*		
	D	*6*	*351 16*	*15*	*45*	*30 0*	*178 32 43 8*	
Hudson to Dog	*R*	*6*	*0 00*	*15*	*30*	*22 5*[a]	*41 2*[a]	*42 5+1 2 = 43 7*[c]
Hudson to Dog	*R*	*0*	*0 00*	*15*	*30*	*22 5*		
	R	*1*	*181 27*	*15*	*45*	*30 0*		
	R	*3*	*184 22*	*15*	*30*			
	R	*6*	*8 44*	*45*	*15*	*0 0*	*181 27 16 2*	
Dog to Hudson	*D*	*6*	*0 00*	*30*	*45*	*37 5*[a]	*13 8*[b]	*15 0+13 = 16 3*
							359 59	*57 5+25*[d] *= 60 0*

[a] *Allowed divergence from first reading = 60″* [b] *Allowed divergence from first mean angle = 10″* [c] *Maximum divergence between final angles for two sets (as shown in this example) = 6* [d] *Allowable maximum = 10″*

Courtesy of American Society of Civil Engineers

FIG. 281a. HORIZONTAL ANGLE NOTES — TRANSIT TRAVERSE.

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* For details see Special Publication No. 145, *United States Coast and Geodetic Survey*, 1929, and Am. Soc. C. E. *Manual of Engineering Practice* No. 20, 1940.

Systematic handling of the transit should include such niceties as always turning the instrument in the same direction, checking on both verniers at each reading, and pointing by means of the positive tangent screw rather than by means of the opposing coil spring.

Figure 281a is a sample note used in triangulation.

282. Adjustment of Angles. The most common triangulation figure is the quadrilateral with the two diagonals sighted in both directions. This figure may be considered as consisting of four triangles in each of which the three independent angles have been measured.

Figure 282 gives the following angle conditions that need to be satisfied by mathematical adjustment.

Station Adjustment. Angle 1 in triangle 1, 2, 3, 4 plus angle 8 in triangle 5, 6, 7, 8 must equal the angle 1 + 8 as measured in triangle 1, 2, 7, 8. This adjustment is made ordinarily by giving to each of angles 1 and 8 one-third of the difference in order to make their sum equal angle 1 + 8, to which the other third, with opposite sign, has been applied. Similar adjustments are made at the other three stations.

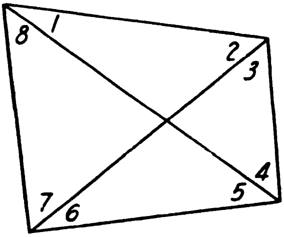


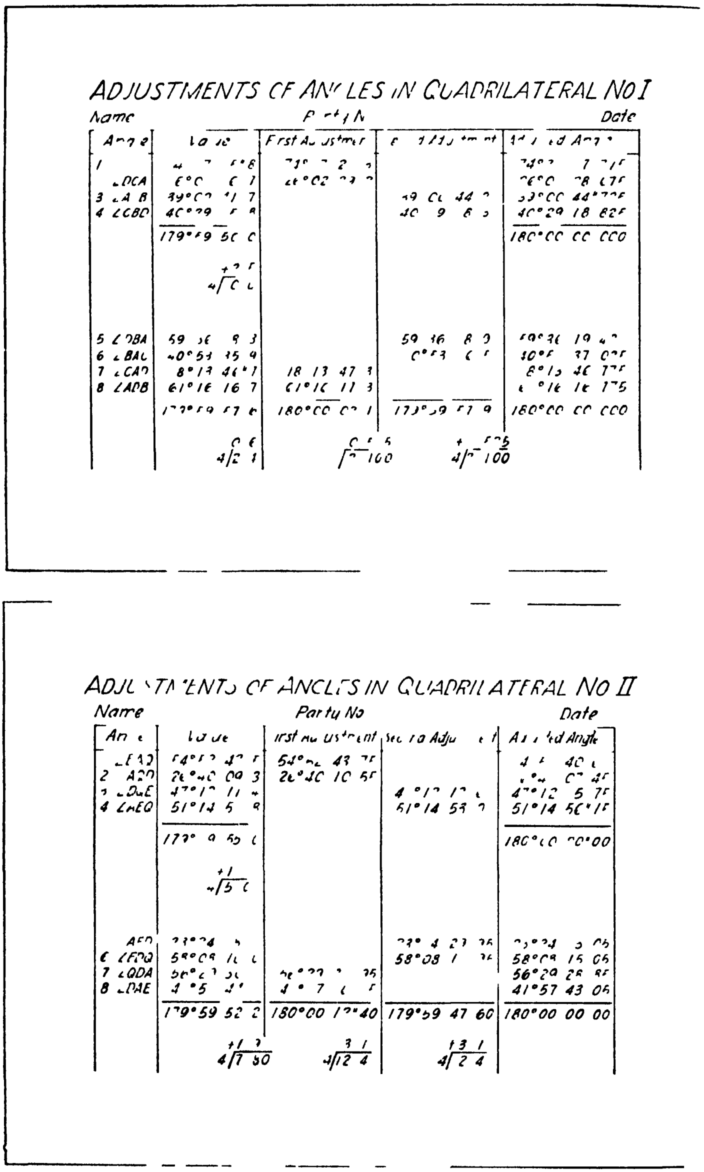
FIG. 282. ANGLE ADJUSTMENT.

283. Quadrilateral Adjustments. An inspection of Fig. 282 reveals that the following conditions must be satisfied:

$$\begin{array}{l}
1+2+3+4+5+6+7+8 = 360^\circ \\
1+2+3+4 \qquad\qquad\qquad\quad\;\; = 180^\circ \\
\qquad\qquad\qquad\;\; 5+6+7+8 = 180^\circ \\
1+2 \qquad\qquad\qquad\qquad\;\; +7+8 = 180^\circ \\
\qquad\;\; 3+4+5+6 \qquad\qquad\;\; = 180^\circ
\end{array}$$

The following example shows a way to meet these adjustments.

This procedure gives probable values for the angles well ithin the limits set for any ordinary control system. If greater recision is obtained by use of higher type instruments and more laborate field procedure, the adjustments should be made in ccordance with the mathematical laws of probability. This



ADJUSTMENTS OF ANGLES IN QUADRILATERAL NO I

Name [illegible] Party No Date

Angle	Value	First Adjustment	Second Adjustment	Adjusted Angle
1	[illegible]	[illegible]		[illegible]
∠DCA	[illegible]	[illegible]		[illegible]
3 ∠A[illegible]B	[illegible]		[illegible]	[illegible]
4 ∠CBD	40°29 [illegible]		40 9 [illegible]	40°29 18 82[illegible]
	179°59 50 0			180°00 00 000
5 ∠DBA	59 [illegible]		59 [illegible]	[illegible]
6 ∠BAC	40°5[illegible]		[illegible]	[illegible]
7 ∠CAD	[illegible]8°13 46.1	18 13 47 3		[illegible]
8 ∠ADB	61°16 16 7	61°16 11 3		[illegible]°16 16 175
	[illegible]	180°00 0[illegible] 1	179°59 [illegible] 9	180°00 00 000

ADJUSTMENTS OF ANGLES IN QUADRILATERAL NO II

Name Party No Date

Angle	Value	First Adjustment	Second Adjustment	Adjusted Angle
∠[illegible]	[illegible]	54°[illegible] 43 [illegible]		[illegible] 40 [illegible]
2 ∠A[illegible]	26°40 09 3	26°40 10 5[illegible]		[illegible]
[illegible] ∠[illegible]	47°1[illegible] 11 [illegible]		4[illegible]°1[illegible]	47°12 5 7[illegible]
4 ∠[illegible]EQ	51°14 5 [illegible]		51°14 53 [illegible]	51°14 56.1[illegible]
	179° [illegible]			180°00 00.00
[illegible]	[illegible]		[illegible]	[illegible]
6 ∠[illegible]	[illegible]		58°08 1 [illegible]	[illegible] 15 06
7 ∠QDA	[illegible]	[illegible]		56°2[illegible]
8 ∠DAE	[illegible]	[illegible]		41°57 43 06
	179°59 52 2	180°00 1[illegible]"40	179°59 47 60	180°00 00 00

FIG. 283 ADJUSTMENT OF ANGLES

calls for use of the method of least squares (see § 31) and the inclusion of the *side equations* computation. When control points (towers, etc.) cannot be occupied by the transit, the *reduction to center* calculation is also necessary. These subjects are discussed in books on geodesy.

284. Spherical Excess. The foregoing considerations of triangulation figures are based upon the assumption that all the triangles are plane triangles. This is true unless the control system contains triangles of sufficient size to necessitate inclusion of a correction for the curvature of the earth. This correction is directly proportional to the area of the earth's surface contained in the triangle and is to be added to 180° to obtain the proper angular summation.

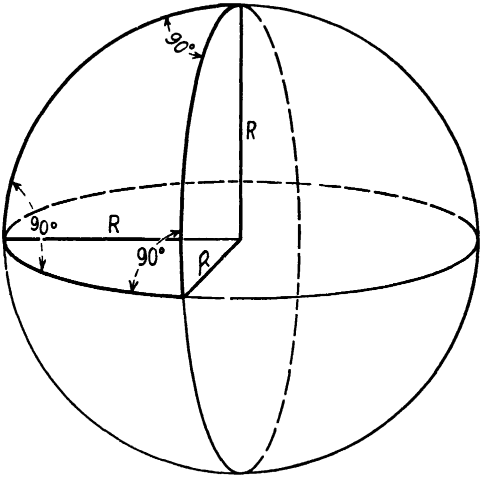


FIG. 284. TRI-RECTANGULAR TRIANGLE.

The following equations and Fig. 284 show the principles involved.

Given: Tri-rectangular triangle with area A on sphere of radius R. The surface area of the sphere $= 4\,\pi R^2$. The spherical excess $= e = 90° = \frac{\pi}{2}$.

$$\frac{e}{\frac{\pi}{2}} = \frac{A}{\frac{4\,\pi R^2}{8}}$$

or

$$e = \frac{A}{R^2}$$

Stating the area of a triangle as $1/2\,ab \sin C$ and dividing by arc 1″ we find

$$e'' = \frac{ab \sin C}{2\,R^2 \text{ arc } 1''}$$

It can then be found that for a spherical excess of 1″ the triangle must have an area of approximately 75 square miles. Consequently, spherical excess does not become a factor except in very large figures.

285. Calculation of Sides. When the most probable values of the base line and angles are known the calculation of the unknown distance is merely an application of the law of sines. It should be remembered, however, that usually there is a choice of equation which should be made in accordance with the rules governing the strength of the figure as given in § 277.

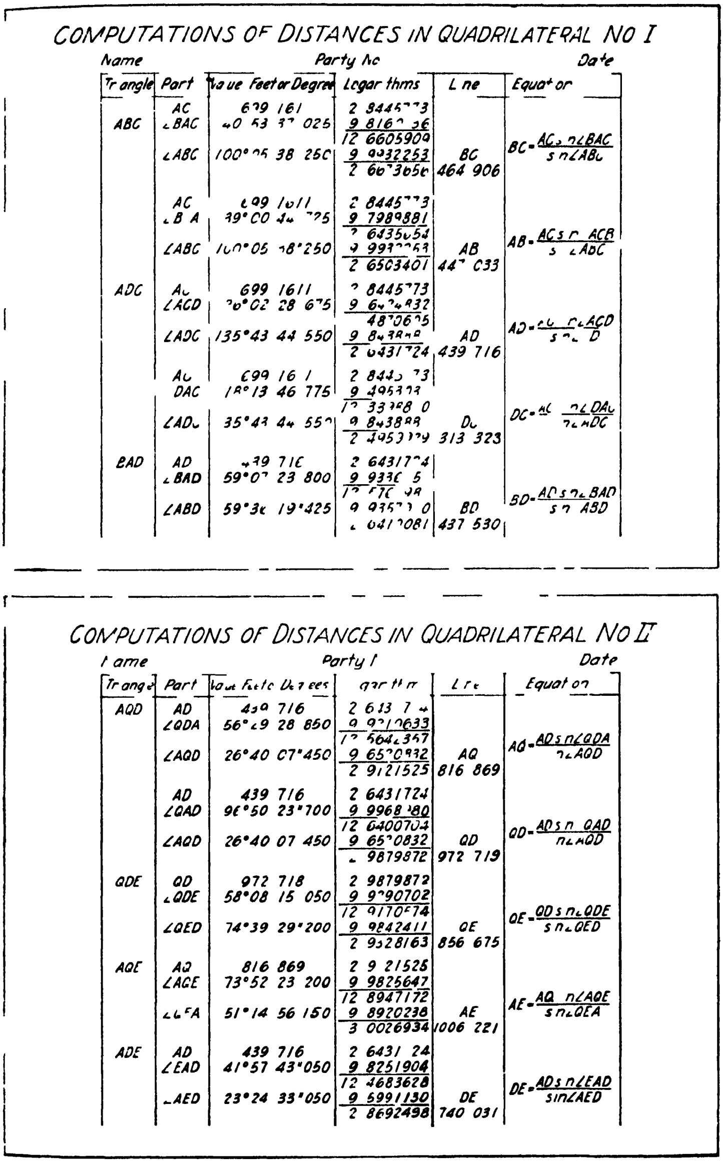
Figure 285 shows a systematic tabulation using logarithms. Natural sines and a calculating machine are preferable, if available.

286. Base Line Measurement. The accuracy with which it is possible to determine the lengths of the lines in a control system depends equally upon the measurements of the angles and of the line selected as the *base line.* In order to be consistent, and considering all the precautions that are taken in the determinations of the angles, great care must be exercised in the base line measurement to insure control of the factors that cause errors.

Lengths of lines of ordinary surveys can be obtained with a precision of the order of 1/2500 to 1/5000 by means of a good steel tape and procedure which includes estimating horizontal position and using a plumb bob on slopes, increasing the tension somewhat to offset the shortening due to sag, and neglecting temperature except in extreme cases.

However, in the measurement of a base line, the effects of an error extend far beyond the line itself into all the calculated lengths of the triangulation system. It is necessary, therefore, even in work of the lower orders of accuracy, that the base line be measured in such a manner as to reduce to a minimum each possible error in the triangulation system.

The tape used must have been standardized (see page 23) in order to have a basis from which to make corrections, since the



COMPUTATIONS OF DISTANCES IN QUADRILATERAL NO I

Name Party No Date

Triangle	Part	Value Feet or Degrees	Logarithms	Line	Equation
	AC	699 161	2 8445773		
ABC	∠BAC	40 53 37 025	9 8160136		
			12 6605909		$BC = \frac{AC \sin\angle BAC}{\sin\angle ABC}$
	∠ABC	100°05 38 250	9 9932253	BC	
			2 6673656	464 906	
	AC	699 1611	2 8445773		
	∠BCA	39°00 44 775	9 7989881		
			2 6435654		$AB = \frac{AC \sin\angle ACB}{\sin\angle ABC}$
	∠ABC	100°05 38"250	9 9932253	AB	
			2 6503401	447 033	
ADC	AC	699 1611	2 8445773		
	∠ACD	26°02 28 675	9 6424832		
			4870605		$AD = \frac{AC \sin\angle ACD}{\sin\angle ADC}$
	∠ADC	135°43 44 550	9 8438881	AD	
			2 6431724	439 716	
	AC	699 161	2 8445773		
	DAC	18°13 46 775	9 4953[illegible]		
			12 33[illegible]8 0		$DC = \frac{AC \sin\angle DAC}{\sin\angle ADC}$
	∠ADC	35°43 44 550	9 8438881	DC	
			2 495[illegible]9	313 323	
BAD	AD	439 716	2 6431724		
	∠BAD	59°07 23 800	9 9336 5		
			12 576 [illegible]		$BD = \frac{AD \sin\angle BAD}{\sin\angle ABD}$
	∠ABD	59°36 19"425	9 935[illegible]0	BD	
			2 6410081	437 530	

COMPUTATIONS OF DISTANCES IN QUADRILATERAL NO II

Name Party No Date

Triangle	Part	Value Feet or Degrees	Logarithms	Line	Equation
AQD	AD	439 716	2 6431724		
	∠QDA	56°29 28 850	9 9210633		
			12 5642357		$AQ = \frac{AD \sin\angle QDA}{\sin\angle AQD}$
	∠AQD	26°40 07"450	9 6520832	AQ	
			2 9121525	816 869	
	AD	439 716	2 6431724		
	∠QAD	96°50 23"700	9 9968980		
			12 6400704		$QD = \frac{AD \sin\angle QAD}{\sin\angle AQD}$
	∠AQD	26°40 07 450	9 6520832	QD	
			2 9879872	972 719	
QDE	QD	972 718	2 9879872		
	∠QDE	58°08 15 050	9 9290702		
			12 9170574		$QE = \frac{QD \sin\angle QDE}{\sin\angle QED}$
	∠QED	74°39 29"200	9 9842411	QE	
			2 9328163	856 675	
AQE	AQ	816 869	2 9121525		
	∠AQE	73°52 23 200	9 9825647		
			12 8947172		$AE = \frac{AQ \sin\angle AQE}{\sin\angle QEA}$
	∠QEA	51°14 56 150	9 8920238	AE	
			3 0026934	1006 221	
ADE	AD	439 716	2 6431724		
	∠EAD	41°57 43"050	9 8251904		
			12 4683628		$DE = \frac{AD \sin\angle EAD}{\sin\angle AED}$
	∠AED	23°24 33"050	9 5991130	DE	
			2 8692498	740 031	

Fig. 285. Quadrilateral Calculations

Fig 286 Base Line Measurement on Stakes

standardization certificate states the length of the tape under certain conditions.

The measurement of the line must be carried forward upon supports provided for the purpose such as stakes, as in Fig. 286, or upon taping "bucks," as in Fig. 286a. This is necessary in order that the difference in elevation of the ends of each tape

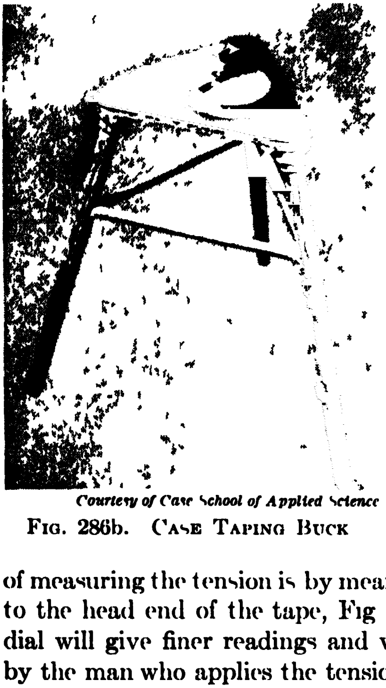
Courtesy of Cleveland Regional Geodetic Survey

FIG. 286a. PRECISE MEASUREMENT ON TAPING BUCKS.

span can be obtained for use in making the slope correction. If intermediate supports are used, they must be set at definite positions in the span and on a straight line between the end supports. This can be done by making the intermediate support a nail driven horizontally into the side of a stake set just off the line. If the support were at the mid-point of the total span the nail should be driven at the mean of the end elevations. When taping bucks are used there usually are no intermediate supports and the leveling is carried along as the measuring progresses.

If stake supports are used, they should be covered with copper, tin, or zinc strips upon which the forward end of each tape length can be marked with a knife point or sharp awl. This corresponds to the setting of the taping pins in ordinary work and gives a fine line upon which to hold the rear mark of the tape for the next span. After marking, the tension should be released and then applied a second time or until two measurements result in the same mark.

The forward end of the tape may occasionally miss the stake top, in which case a "set-forward" or a "set-back" will be necessary. This can be taken with a machinist's scale with great accuracy. The distance will always be very short and no corrections will be needed.



Courtesy of Case School of Applied Science

FIG. 286b. CASE TAPING BUCK

Certain makes of taping bucks, Fig 286b, have a line etched upon a movable plate which is adjusted into position at the head end of the tape, thus eliminating the scratching of a new line for each span.

Measuring must be done with the tape under a uniform tension. This tension may vary from 10 to 40 pounds depending upon the kind of tape being used, its length, and the method of support. The usual method of measuring the tension is by means of a spring balance attached to the head end of the tape, Fig 286c. A balance with a large dial will give finer readings and will be much more easily seen by the man who applies the tension than will the common type of spring balance.

The temperature of the tape during the measurement is obtained by attaching a small specially designed thermometer near each end. These are read and the reading recorded for each span; the mean is used in making the temperature correction.

The correction for temperature probably is the least certain of all the corrections made in the reduction of the observed length of base line. This results from various causes: one portion of the tape may be in shade with the remainder in the sun; the tape may not adjust itself to a rapid change in temperature at the same rate as does a thermometer; and at best the attached thermometer reads only the temperature of the air near the tape, not the temperature of the tape itself.

Courtesy of Cleveland Regional Geodetic Survey

FIG. 286c. TENSION MEASUREMENT.

Courtesy of American Society of Civil Engineering

FIG. 286d. WOODEN TAPING BUCK. NOTE USE OF PIN.

The uncertainties of the temperature corrections make the use of an invar tape very desirable. Since this nickel-steel alloy has a coefficient of thermal expansion approximately 1/25 that of tape steel (0.00000028 foot per foot per 1° Fahrenheit as compared with 0.00000645), it is obvious that errors in obtaining the temperature of an invar tape would give an actual error in the correction but 4 per cent of the amount resulting from the use of a steel tape.

A constant temperature is highly desirable for base line measuring. Hence it is good practice to do this work on cloudy days or at night. Times at which wind of appreciable velocity is blowing should be avoided.

Since the slope and temperature probably will be different for each span, it is well to consider each span as a separate line for which all corrections are to be made. The length of the base line will be the summation of the corrected horizontal distances.

287. Correction for Slope. The slope in base line measure usually is given as the difference in elevation of the ends of the tape rather than by means of the slope angle. In Fig. 287, if V is the difference in elevation of the ends of a span S, the correction for slope

$$C_h = S - h = S - \sqrt{S^2 - V^2} = S - S\left(1 - \frac{V^2}{S^2}\right)^{\frac{1}{2}}$$

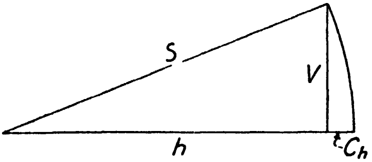


FIG. 287.

However,

$$\left(1 - \frac{V^2}{S^2}\right)^{\frac{1}{2}} = 1 - \frac{V^2}{2S^2} - \frac{V^4}{8S^4} - \cdots \qquad \text{whence}$$

$$C_h = \frac{V^2}{2S} + \frac{V^4}{8S^3} + \cdots \tag{287}$$

Except in rare instances of very steep slopes, the first term $\frac{V^2}{2S}$ will give results of sufficient accuracy.

288. Correction for Temperature. The correction for temperature for any span is directly proportional to the length of the span, the difference between the standardization temperature and that of the measurement, and the coefficient of thermal expansion of the tape.

The coefficient of expansion k for steel tapes is commonly taken as 0.00000645 and for invar tapes 0.00000028 foot per foot per 1° Fahrenheit.

Thus the temperature correction

$$(288) \qquad C_t \text{ (in decimals of a foot)} = Lk(t - t_s)$$

in which L is the length of the span in feet, t the temperature of the tape, and t_s the temperature of standardization.

289. Correction for Tension or Pull. The correction for tension is the deformation caused by a tension differing from that used during the standardization. It is directly proportional to the length of the span and the change in tension and inversely proportional to the cross-sectional area and the modulus of elasticity.

$$(289) \qquad C_p \text{ (in decimals of a foot)} = \frac{L(p - p_s)}{EA}$$

in which L is the span length in feet, p is the tension in pounds used in the measurement, p_s is the standardization tension in pounds, E is the modulus of elasticity in pounds per square inch, and A is the cross-sectional area in square inches.

The modulus of elasticity for steel tapes is usually taken as 28,000,000 and for invar tapes 23,000,000 pounds per square inch.*

Since the cross-sectional area A is difficult to measure directly, it can be expressed as a function of the weight of the tape (see § 23).

The best method, if the necessary equipment is available, for determining the value of the product EA is actually to measure the change in the length of the tape caused by an increase of 5 pounds or more in the tension being applied. The temperature should be constant and the tape should be supported throughout its entire length. This gives:

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* From National Bureau of Standards.

(289a) $$EA = \frac{(p_2 - p_1)L}{\Delta L}$$

in which ΔL is the change in length L when the tension is changed from p_1 pounds to p_2 pounds.

290. Correction for Sag. The correction for sag (for any span of length l between adjacent supports) is the difference between the length of the arc of the tape and that of the chord between the end supports. The arc is that of a catenary but is usually considered as that of a parabola the equation of which is

$$x^2 = \left(\frac{l}{4\,v}\right)y$$

The length of the arc is

$$A = l + \frac{8\,v^2}{3\,l} - \frac{32\,v^4}{5\,l^3} + \cdots$$

Thus the correction for sag is approximately

(290) $$C_s = A - l = \frac{8\,v^2}{3\,l}.$$

If the tape weighs w pounds per foot, moments about point B (Fig. 290) give

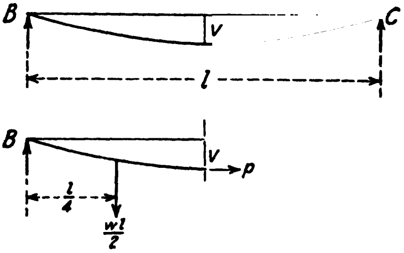

FIG. 290. SAG CORRECTION.

$$\left(\frac{wl}{2}\right)\frac{l}{4} = pv$$

and $$v = \frac{wl^2}{8\,p}.$$

Substituting in (290),

(290a) $$C_s = \frac{w^2l^3}{24\,p^2} = -\frac{l}{24}\left(\frac{wl}{p}\right)^2$$

in which l is the span length between intermediate supports and p is the tension being applied to the tape.

If there are no intermediate supports, the length of span equals the length of tape L, the total weight is W, and the correction for sag between the ends of the tape (the total span) would be

$$C_s = -\left(\frac{L}{24}\right)\left(\frac{W}{p}\right)^2 \tag{290b}$$

291. Reduction to Datum. Surveys of an area should show all of the lines reduced to some common datum of reference. For maps of the United States the datum is mean sea level and all distances measured at other elevations are reduced to their lengths on that datum. Surveys of smaller areas not a part of the national system may be made to datums at any chosen elevation. Often the datum used is that of an average elevation of the area, and no corrections are applied. The correction for sea level C_{sl}, to be subtracted from the actual length, is found by

$$C_{sl} = \frac{Lh}{R} \tag{291}$$

in which L is the length of line, h the average elevation of the line above sea level, and R the mean radius of the earth, commonly taken as 20,890,000 feet (log = 7.3199384).

292. General Statement on Corrections. It should be noted that

(1) if thermometers are attached to the tape, their weights should be taken into account in computing sag or they should be mounted in position when the tape is standardized;

(2) if the measurement is not carried upon previously established stakes, a transit must be used for alignment;

(3) if a heavy tape is being used and/or the spans are long, it becomes increasingly important to obtain the correct weight and pull since the sag correction varies as w^2, l^3, and $\frac{1}{p^2}$.

A sample tabulation of the reduction of a short base line is shown in the following example.

BASE LINE REDUCTION

Run 3 24 Lbs. Tension June 23, 1949 Hot, clear
Computed by W.J. Kay

Span	Dist in Feet	Ave. Temp.	Diff El.	C_t	C_p	C_s	C_h	Tape Corr.
0-1	100.000	88.0	0.17	.0129000	.0075666	.0291961	.0001445	Zero for all spans
1-2	100.000	88.0	1.52	.0129000	0075666	.0291961	.0115520	
2-3	100.000	87.0	0.20	.0122550	.0075666	.0291961	.0002000	
3-4	100.000	87.5	0.85	0125775	0075666	.0291961	.0036125	
4-5	99.900	87.0	0.24	.0122427	.0075590	.0291087	.0002880	
5-6	100.000	88.0	0.73	.0129000	.0075666	.0291961	.0026645	
6-7	100.000	88.5	1.10	.0132225	.0075666	.0291961	.0060500	
7-8	47.378	87.0	0.41	.0058062	.0035850	.0031049	.0017740	
	747.278			+.0948039	+.0565436	-.2073902	-.0262855	
	-.082			+ 0565436			-.2073902	
	747.196 = True length of base line			+.1513475			-.2336757	
							+.1513475	
							-.0823282 = Total correction to be applied	

Standard tension = 11.5 pounds
Standard temperature = 68°F
Weight = 0.02009 pounds per ft.
Cross section 0.0059 sq. in.
E = 28 000 000 pounds per sq. in.
k = 0.000 00645 ft per ft. per 1°Fah.

FIG. 292. BASE LINE CALCULATIONS.

TRAVERSE

293. Fundamentals. Horizontal control over an area by means of traverse consists of a series of straight surveyed lines connecting established transit stations. All of the lines are measured in the field and the angular relation at each corner is determined with a transit. Only closed traverses, those that bound areas in themselves or close on known monuments, should be considered. Open traverses do not afford sufficient checks upon the field work to permit their use except as short spurs to control small isolated areas or to establish a control distance for the scale determination of aerial photographs.

294. Principal Use. Traverse control for topographic mapping is not usually used alone except for areas of limited extent. Its most important use is to supplement triangulation control. Traversing necessitates the direct measurement of all lines while triangulation requires only the measurement of one or two base lines. This gives triangulation a very distinct advantage over traverse control. However, a triangulation system does not ordinarily provide a sufficient number of established instrument stations from which to cover the area for topographic detailing. Therefore, the ideal arrangement is a general triangulation system interlaced and augmented by traverse lines. Thus a side of a triangulation figure often serves as the closing line of a traverse as well; that is, the traverse is run from one triangulation station to another.

Each triangulation station should have a permanent azimuth mark established approximately 1/4 mile distant. With the traverse control in mind, it usually is a simple matter to establish the azimuth mark in such a place that it can become a transit station in a traverse beginning or terminating at the triangulation point. This greatly facilitates the adjustment of such a traverse.

The location of a triangulation station on the top of a building or other inaccessible place would prevent such a connection with the traverse line. In such cases, a small well-shaped auxiliary triangle with the other vertices established as two adjacent traverse stations will suffice.*

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* For other cases see Am. Soc. C. E. *Manual of Engineering Practice No.* 20, 1940.

295. Precision. A traverse line, perhaps 5 or 10 miles long, beginning and ending in the triangulation net is considered a primary loop. Often shorter loops or connections are required to give additional transit stations needed for detailing. These secondary loops require less accuracy than the more important circuits since their errors cannot affect more than the mapping of a small area.

No general rule can be given for specifications of precision in traversing. The stations and lines established may be used for many purposes other than control for topographic mapping, such as a coordinate system for property boundaries or control for surveys that are to follow.

Considering only the topographic map of an area, the criterion of accuracy is that the probable errors of field procedure will be smaller than can be plotted at the scale chosen for the map. For example, assuming that the precision with which a point can be plotted is ±0.02 inch and that the adjustment of the traverse loop would account for an equal amount, an order of precision in which the discrepancy does not exceed that represented by 0.04 inch on the map would be acceptable. If a scale of 1 in. = 200 ft. were being used, this would represent an allowable difference of 8 feet which would require work of the order of 1/6600 in a ten-mile traverse. If the scale of the map is increased, the accuracy of the field work must be increased. Conversely, if the traverse loop is shorter, the order of accuracy could be lowered somewhat, which seems to imply that all traverse loops should be short. The fallacy of this would appear in the adjustment of a long circuit consisting of parts of several secondary traverses, in which case the order of accuracy would be far from acceptable.

296. Establishment. A traverse should be laid out to connect the control points by as nearly a direct line as ground conditions will permit. In general, the routes are limited to public rights of way such as park strips along streets or highways where the monuments probably will be free from disturbance and intervisibility can be maintained. Monuments should not be established on private property if it can be avoided.

The transit stations should be set as far apart as possible to avoid short sights in the azimuth control. In residential areas with many curvilinear streets, this is often impossible and the traverse must, of necessity, be a series of very short lines. In such cases a long sight, cutting off several of the short ones, should be arranged for azimuth control of a considerable section of the whole traverse. This cut-off sight forms, with the series of short traverse lines, a small secondary loop in which angle adjustment can be made. Such azimuth control sights normally are between two traverse stations set at high points in the area. Figure 296 illustrates such an arrangement of traverse and azimuth angles.

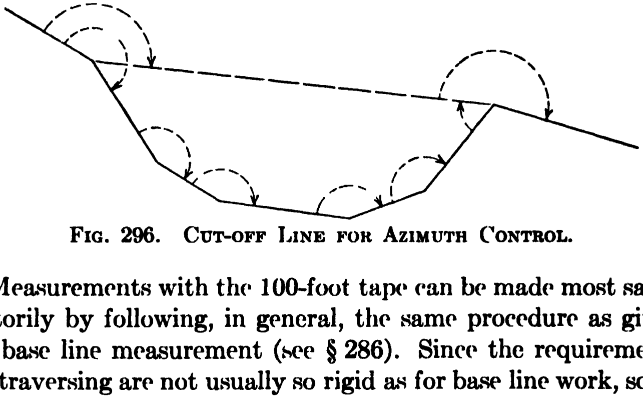

FIG. 296. CUT-OFF LINE FOR AZIMUTH CONTROL.

Measurements with the 100-foot tape can be made most satisfactorily by following, in general, the same procedure as given for base line measurement (see § 286). Since the requirements for traversing are not usually so rigid as for base line work, some of the refinements may be modified; alignment may be done by eye instead of by the transit; mean temperature may be read from a thermometer mounted in a block of wood and kept lying on the ground at a transit station; difference in elevation between supports may be obtained by use of a hand level; heavy pins stuck in wooden top of taping buck may be used to mark the point instead of etching line on metal plate; etc. A constant tension should be maintained by use of a tested spring balance. The tension should be that under which the tape was standardized, usually 10 pounds if supported its entire length and 20 or more pounds if supported at the ends only.

If the *normal tension* is used (see § 25), i.e., that tension at which the elongation of the tape due to pull exactly equals the shortening of the tape due to sag, the tension and sag corrections will be eliminated from the calculations. For an ordinary

100-foot tape, supported at the ends only, this normal tension will be between 20 and 25 pounds.

Except for extensive, open-country traverses in which long tapes, taping pins, and plumb bobs are employed, the use of some form of support, or taping buck, is recommended. This practice insures definite, elevated end supports, control of the slope factor, and a uniform method of marking the end point. The first and last spans will probably end at points on the ground. If these spans cannot be measured without the tape touching along its length, a support can be set under the transit and the point placed on it by means of a plumb bob. Tapes should always be carried along the line since dragging on pavements soon reduces the cross-sectional area thus causing the tension correction to increase and the sag correction to decrease by undetermined amounts.

Each angle point should be monumented if possible although this is not absolutely necessary in all cases. For example, an angle point lying between two monumented points could well be a drill hole in a concrete pavement if properly referenced to nearby permanent objects. Should a very long sight be encountered, the establishment of one or more intermediate points at definite plus distances along the line is recommended. These will be of value in the mapping program and make the control system more readily available to nearby property. In general, the more control points that are set, the more valuable becomes the control system for all purposes.

297. Traverse Taping. A 100-foot steel or invar tape is most often used in traverse work especially over rough or wooded territory or in cities where traffic is a serious problem. However, in open country, the 301-foot tape, as used by the United States Geological Survey, greatly facilitates the work. Whatever tape is used, it must be standardized carefully for each traverse or at regular intervals.

A convenient testing base for tapes is very desirable and since its construction is not difficult, it is recommended wherever much tape measurement is to be done. All tapes should be tagged for identification and records kept of their standardization data. Figure 297 shows one end of a tape testing base.

Courtesy of Cleveland Regional Geodetic Survey

FIG. 297. TAPE TESTING BASE.

As in base line measurement, best results will be obtained by working at night or on cloudy days. If the work is to be done on bright days an invar tape should be used. The only check that can be made on the observed length of a line is by repeating the measurement. A second measurement is recommended whenever difficulties of any nature are encountered.

298. Traverse Angles. At each transit station the value of the angle to the right from the station at the rear should be determined. The procedure and the form of notes are the same as those that are used in triangulation work, see § 281, except that a set may consist of fewer repetitions or fewer sets need be taken on an angle. A 10″ repeating instrument is most desirable, but satisfactory results can be obtained with a 30″ or even a 1′ transit by increasing the number of sets per angle. Use of less precise instruments requires more time and increases the cost of the work. The standard precautions with regard to solid set up, transit adjustments, protection of instrument, and systematic handling should be observed.

299. Signals. *Signals used in traversing may be of whatever* kind available or best suited to the work: rods, tripods, lights, or heliotropes. Of greater importance than the type of signal is the exact plumbing of it over the station. Traverse sight distances usually are much shorter than those of triangulation, and consequently a signal which is off the point even a small amount will introduce serious errors.

300. Traverse Adjustment. Adjustment of traverse circuits cannot be considered as definite a procedure as that of triangulation figures in which the consideration of any quadrilateral follows the completion of the one preceding it. A long traverse loop of primary nature may have several cut-off lines forming smaller loops. The smaller ones may be further complicated by azimuth control lines cutting off sections comprising a series of short traverse lines. These are all inter-related and theoretically all should be considered in an adjustment of the entire system. However, such a procedure is impractical on account of the very large amount of data to be handled and because the results of portions of the work should be made available for other purposes before the whole is completed.

Since traverse control follows no definite pattern it is good practice to plan the work so that traverses most closely connected to the triangulation system will be run first and adjusted first. These are held fixed and additional traverse loops are adjusted to them through whatever portion they have in common. This scheme obviously demands that the accuracy of the field work in practically all the traversing be held to a common high standard.

The correct summation of the angles in any traverse loop, dependent upon the number of sides in the figure, is always a known value. The angular error of closure is ordinarily distributed equally among all the angles unless there is some good reason for doing otherwise. Specifications for angle measurement usually state that the average correction shall not exceed 3″ or 5″ per angle, the amount allowed being in accordance with the field procedure and the equipment used.

The observed lengths of all lines are reduced to grid lengths by the application of all corrections made necessary by the tape itself, the conditions of measurement, and the average elevation of the lines with reference to sea level or other reference datum. Grid bearings of all lines are computed from the adjusted angles and the latitudes and departures are calculated. Errors of closure in latitude and departure are distributed in proportion to the lengths of the courses.

Geographic positions are computed by starting with a fixed position and grid azimuth (see § 425) at one end of the traverse and carrying through by means of the adjusted angles to a check on another fixed point and line. The grid azimuth should be checked with the astronomic azimuth wherever possible, taking into consideration the convergence of meridians. At latitude 40° N, astronomic azimuths less than 180° will decrease about 45″ for each mile of west departure and azimuths over 180° will increase at the same rate.

The length of each cut-off line in the azimuth control will need to be computed from the auxiliary traverse which it closes. This is done by assuming the closing line to have an azimuth of 0° 00′ 00″ and by computing the projections or latitudes of the several courses against it using the corrected lengths and azimuths figured from adjusted angles. An additional calculation of the perpendiculars or departures of the lines will give an excellent check on the field work and calculations. Any considerable difference in the summation of the plus and minus departures indicates an error in some part of the work that should be corrected, since it might appreciably affect the computed length of the closing line.

301. General Statements. The foregoing statements regarding traversing have to do with work of comparatively high order. Many acceptable traverses are run by merely doubling the angle to the right or double sighting the deflection angle, accompanied by good steel tape measurements along the ground. The best example is that of a boundary survey of a piece of property. Such a traverse should have an angle closure and an accuracy ratio resulting from its latitudes and departures which accord with the value of the property controlled. It also would be assumed in such a survey that the traverse would be a complete unit, the results of which are not to be used in extending control over adjacent areas. In other words, each job must be studied and field and adjustment procedures decided upon which are in keeping with the accuracy desired.

Most of the traversing done by surveyors is work of second or third order accuracy. Acceptable closures may range from 1 part in 5000 to 1 part in 25,000 depending upon the purposes for which the traverses are run and the value of the property involved. For further considerations regarding specifications and procedures than are presented in this chapter special reference is made to "Horizontal Control Surveys to Supplement the Fundamental Net," American Society of Civil Engineers Manual of Engineering Practice No. 20, 1940.

PROBLEMS

1. Compute the $\frac{D-C}{D}$ value for the following triangulation figures: (a) a completed quadrilateral, (b) a four-sided, central-point figure, (c) a four-sided, central-point figure with one corner not on the fixed line occupied, (d) a six-sided central-point figure, (e) a five-sided, central-point figure with the central station not on the fixed line unoccupied.

2. The angles in a quadrilateral numbered consecutively as in Fig. 282 are 44°, 116°, 9°, 11°, 82°, 78°, 12°, and 8°. Compute the strength of this figure. Is this a recommended figure? Give reasons.

3. Same as 2 with angles 9°, 11°, 82°, 78°, 12°, 8°, 44°, and 116°.

4. Three hills A, B, and C are in a straight line. $AB = 10$ miles and $BC = 15$ miles. The elevations are 725, 675, and 775 feet respectively. When the line of sight clears B by 25 feet, calculate the following: (1) the height of tower at C to make A visible, (2) the height of tower at A to make C visible, (3) the height of two equal towers at A and C to be intervisible.

5. Calculate the spherical excess in a triangle two sides of which are 30,000 feet and 45,000 feet with an included angle of 48°.

6. The standardization certificate for a steel tape gives 99.992 feet as the length of the tape when the temperature is 68° Fahrenheit, supported on a horizontal surface throughout its entire length and under a tension of 10 pounds. The tape weighs 1 pound.

A base line is measured with this tape and the recorded results are:

Observed length = 3518.148 feet
Average temperature = 43° F.
Tension = 20 pounds
Support at 0 and 100 foot points
Grade of supports = 2 per cent
Sum of set-backs = 0.246 foot
Sum of set-forwards = 0.135 foot.

Compute the corrected length of the base.

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CHAPTER XXI

ENGINEERING ASTRONOMY

302. Relative Accuracy. No attempt was made in Chapter VIII to develop the theory of engineering astronomy because an understanding of that theory is not essential when the allowable error in azimuth or latitude is more than one minute. However, since specifications and instructions * for many traverse, location, and topographic surveys set thirty seconds or less as the allowable error it is necessary that the surveyor comprehend clearly and develop that ability to visualize the relative motions of celestial bodies which is the foundation upon which the theory of engineering astronomy is built. That theory is developed in the first part of this chapter. Its application is later illustrated by typical field and office problems, the data for which were assumed as having been taken at the Purdue University Civil Engineering Camp.

Methods of observation and calculations used by engineers of the United States Coast and Geodetic Survey, which yield precisions on the order of one second of angle, are not included herein since that work is highly specialized and requires trained observers and computers in addition to special field equipment not usually available.

GENERAL THEORY

303. Celestial Bodies. An observation on a clear night of the hemispherical dome, popularly called the heavens, reveals groups of celestial bodies of varying degrees of brilliance. These groups, called *constellations*, form vague outlines of objects, animals, and mythical beings which were visualized and named by early star gazers and astronomers.

The science of the geography of the heavens is called Uranography. A study of Uranography aided by star maps and

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* See, for example, "*Instructions for Transit Traverse,*" United States Geological Survey, 1940; also "*Horizontal Control Surveys,*" Am. Soc. C. E., *Manual of Engineering Practice No. 20*, 1940.

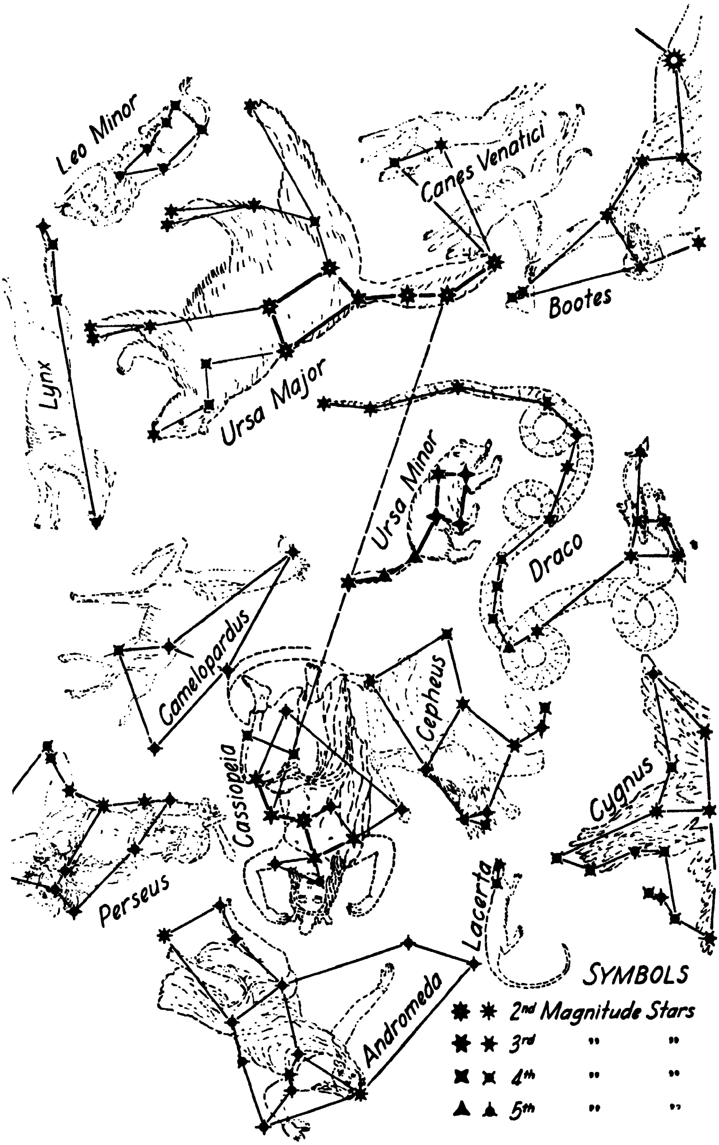

FIG. 303. URANOGRAPHIC CHART.

planispheres is recommended. It is a useful, interesting, and absorbing hobby.

To the observer, all celestial bodies which are scattered over the hemispherical dome seem to be equidistant from his position. The astronomer, however, knows that great distances separate these bodies. These distances are so great that to make them comprehensible, they are measured in light years, one light year being the distance light travels in one year.*

Because these light-year distances from Earth and between celestial bodies are immense and even though they are rapidly changing positions, the apparent relative positions of most of the bodies remain sensibly unchanged over long periods of time. Such celestial bodies are called stars.

Our sun is a star,† but it is relatively so close to Earth that its apparent motion with reference to Earth is easily observed. It is the central body of our solar system and the point about which all planets revolve in their elliptical orbits. Earth is but one of nine known members of the sun's planetary family. The other members are Mercury, Venus, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto.

TABLE 303. THE SOLAR SYSTEM

Name of Celestial Body	Diameter in Miles[a]	Revolution around Sun in Years[a]	Approximate Distance from Sun in Millions of Miles
Sun	864,392	0	0
Mercury	3,008	0.24	36
Venus	7,575	0.62	67
Earth	7,927	1.00	93
Mars	4,216	1.88	141
Jupiter	88,698	11.86	483
Saturn	75,060	29.46	886
Uranus	30,878	84.02	1782
Neptune	32,932	164.79	2793
Pluto	[b]	247.70	3680

[a] As given in American Ephemeris.

[b] Not listed in American Ephemeris. It is supposed to be about 5000.

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* Light travels at the rate of 186,324 miles per second (American Nautical Almanac). Thus the distance from New York to San Francisco is negotiated by light in slightly less than 0.014 second.

† The sun is about 93,000,000 miles from Earth and light from it reaches us in about 8 minutes of time. Beyond the sun and the planets the nearest star is more than 4 light years from Earth.

Through an ordinary field glass or small telescope, planets appear as disks of light, while all stars are merely points of light. Planets also change their positions with reference to stars and are consequently not considered as a part of any constellation. Planets are non-luminous; they reflect the light of the sun. Stars are self-luminous and intensely hot.

The moon is a celestial body revolving as a satellite around Earth at a distance of about 239,000 miles. Its diameter is 2160 miles. Several other planets in our solar system have from one to nine such satellite bodies.

304. The Celestial Sphere. In making observations for longitude, latitude, time, or azimuth, the engineer is not greatly concerned with distances between celestial bodies. His object in practically every observation is the determination of angular relations, as measured on Earth, between celestial bodies or between some point on Earth and the celestial body which is being observed. For such purposes the universe is considered as being spherical in shape with infinite radius. The celestial bodies are considered as points on this *Celestial Sphere.*

305. Earth's Motions. If, after dark in the northern hemisphere, a camera were pointed toward the north and the film or plate exposed for several hours, the result would be a picture showing concentric circular streaks of star light. The center of the circles would be above the horizon and would indicate the position of the north celestial pole. The height of this pole above the horizontal plane would vary with the terrestrial latitude of the photographer's camera. Such an experiment would demonstrate the fact that Earth revolves about its own axis, the axial ends being the true poles, North and South. A careful study of the picture or an hour by hour observation of the stars would show that this apparent motion of Earth, as viewed from Earth, is counter-clockwise and that the real motion is therefore clockwise or from west to east. Further proof of this lies in the fact that stars, planets, and sun rise in the east and set in the west.

Careful study and observation of stellar bodies made day by day and at the same hour each day show that these bodies apparently change positions on the celestial sphere, which indicates that Earth's location in space and its relation to other celestial bodies is continually changing. This is caused by Earth's annual journey along its elliptical path around the sun, the sun being at one of the foci of the ellipse.

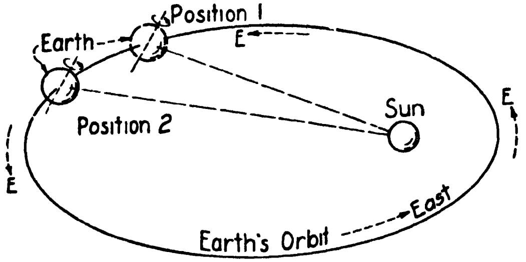

FIG 305. EARTH'S MOTIONS.

Thus the two independent motions of Earth: daily (diurnal motion) around its own axis, and annually (orbital motion) around the sun, cause continual apparent changes in the positions of celestial bodies as viewed from Earth.

306. Great and Small Circles. The intersection of a plane passing through the center of a sphere with the surface of that sphere is called a *Great Circle*. A *Small Circle* on the sphere results when the intersecting plane does not pass through the center. A spherical triangle is bounded by arcs of great circles. These arcs are usually measured in degrees, minutes, and seconds.

307. Azimuth and Altitude — The Horizon System. If at any instant all celestial motions were considered as static, then every celestial body at that instant would be in a definitely fixed location on the celestial sphere.

When a perfectly adjusted transit is set up on Earth with plate bubbles and telescope bubble leveled, the vertical axis, extended an infinite distance, would pierce the celestial sphere overhead at *Zenith* and underfoot at *Nadir*. If the telescope were revolved around the vertical axis and if the horizontal plane thus generated were produced an infinite distance, the intersection of this plane and the celestial sphere would be a great circle on the celestial sphere called the *Horizon*. If the telescope were revolved around the horizontal axis, the line of sight would generate a vertical plane which, when projected to an intersection with the celestial sphere, would be a great circle on the celestial sphere and would pass through zenith and nadir. Such great circles are called *Vertical Circles.* If this second revolution of the line of sight (around the horizontal axis) were made when the telescope was pointed toward true north or south, the particular vertical circle on the celestial sphere would be the *Meridian* of the place of observation.

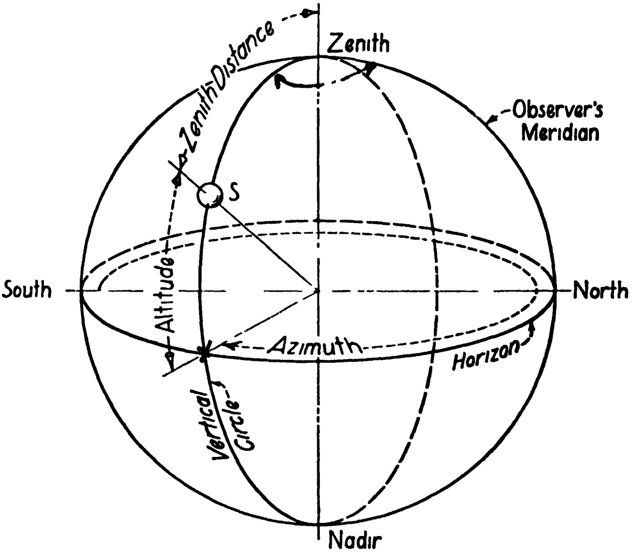

FIG. 307. THE HORIZON SYSTEM.

At any single set up of the transit there is an infinite number of vertical circles but only one horizon and one observer's meridian.

The vertical angle between the horizon and any celestial body is called the *Altitude* of the body. The *Zenith Distance* of a celestial body is the angular distance from the body to zenith measured along the vertical circle passing through the body.

Zenith distance = 90° − altitude.

The horizontal angle measured from true south in a clockwise direction to the vertical circle through the celestial body is the *Azimuth* of the body. The spherical angle at zenith between the vertical circles through the celestial body and true south equals the azimuth angle.

Thus any celestial body may be located on the celestial sphere by using the horizon system of great circles, the angular (spherical) coordinates of the body being azimuth and altitude.

308. Right Ascension and Declination — The Equatorial System. The terrestrial equator is a great circle on Earth midway between the terrestrial poles. The projection of the plane of the terrestrial equator to an intersection with the celestial sphere results in a great circle on the celestial sphere called the *Celestial Equator.*

The plane in which Earth travels in its annual revolution around the Sun projected to an intersection with the celestial sphere results in a great circle on the celestial sphere called the *Ecliptic.* The ecliptic, therefore, is the trace of the sun's apparent annual journey around the celestial sphere.

In the northern hemisphere, Earth's axis of rotation is inclined toward the sun in the summer season and away from the sun in the winter season. The ecliptic, therefore, does not coincide with the celestial equator, the angle between the two great circles being slightly greater than 23° 27′.* It is evident also that because of this fact, the ecliptic and celestial equator intersect at two points and that half of the ecliptic is in the northern celestial hemisphere and the other half in the southern.

These two points of intersection of the ecliptic and equator are important. That intersection of the sun's apparent path (the ecliptic) with the equator which occurs in the spring season (about March 20) † is called the *Vernal Equinox.* At that time of year the sun apparently is moving from the southern into the northern celestial hemisphere. Six months later (about September 21) the sun's apparent motion is from the northern into the southern hemisphere and the intersection is called the *Autumnal Equinox.* A great circle passing through the celestial poles and the vernal equinox is called the *Equinoctial Colure.*

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* This angular relation changes slightly each year and is different for each planet of our solar system.

† These dates vary slightly from year to year and may be determined accurately by consulting a solar ephemeris.

The vernal equinox is therefore an imaginary point on the celestial sphere which has an apparent motion similar to that of the stars.

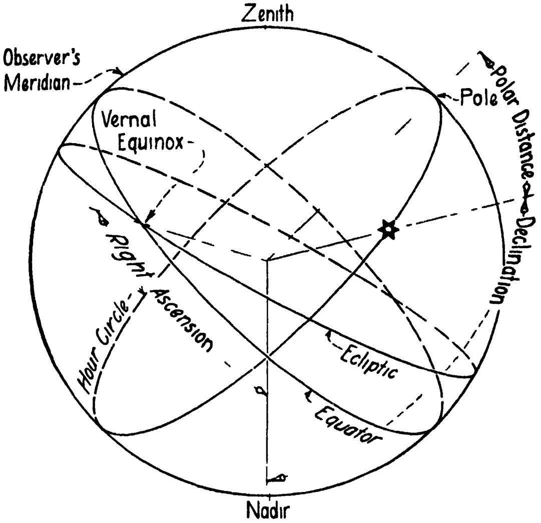

FIG. 308. THE EQUATORIAL SYSTEM.

On Earth's surface great circles which pass through the poles and are therefore perpendicular to the terrestrial equator are called meridians or longitudinal circles. Positions east and west of the reference meridian through Greenwich, England (0° longitude), are measured in degrees (and fractions thereof) east or west of Greenwich up to 180°.

Projections of these longitudinal circles in infinite number to intersections with the celestial sphere result in great circles on the celestial sphere called *Hour Circles.* Any hour circle passes through the celestial poles and is perpendicular to the celestial equator. The equinoctial colure is a particular hour circle. The meridian of the observer is also a particular hour circle. The *Hour Angle* of any celestial body is the spherical angle between two hour circles, one of which passes through the body, the other being either the meridian of the observer or the equinoctial colure (see § 315).

Right Ascension of a celestial body is its hour angular relation to the vernal equinox measured eastwardly along the equator from the vernal equinox or equinoctial colure to the hour circle passing through the body. It is also equal to the spherical angle at the pole between the equinoctial colure and the hour circle through the celestial body.

Small circles on Earth's surface parallel to the terrestrial equator are used to fix the location of points on Earth. These small circles are called parallels of latitude. The terrestrial equator is at 0° latitude and all points on Earth are either south or north latitude, the distance from the equator being expressed in degrees and fractions thereof. The terrestrial poles are at 90° latitude north and south.

Projections of parallels of latitude to intersections with the celestial sphere permit a similar measure of angular distance north and south of the celestial equator. Such a measure of distance or position on the celestial sphere is called *Declination*. Declination is north or south. The declination of any celestial body is its angular relation north or south to the celestial equator as measured at the center of the celestial sphere or along an hour circle from the celestial equator to the body.

Polar Distance is the angular relation between the celestial body and the pole measured along the hour circle through the body.

Polar distance = 90° ± declination.

Any celestial body may therefore be located on the celestial sphere by the spherical coordinates right ascension and declination.

Star tables showing apparent places of stars (right ascension and declination) for different days of the year are given in the *American Ephemeris and Nautical Almanac.* A study of these tables reveals the fact that almost all stars are relatively in the same position throughout the year. The variations in right ascension and declination as shown in the tables are so slight (less than 1 minute in a year) that they may be neglected in azimuth or latitude calculations. They indicate the fact, however, that the direction of Earth's axis is changing slowly and consequently that the vernal equinox is not quite a fixed point. This slight shifting is called *Precession.*

309. Hour Angle and Declination — The Hour Angle System. As explained on page 367, Earth's daily rotation about its own axis produces an apparent daily rotation of all celestial bodies around Earth. Each such body must therefore cross the meridian of the observer twice each day. To an observer at the equator, the celestial poles lie on the horizon and all celestial bodies appear above the horizon for half of the day and disappear below the horizon for the other half day. To an observer at the north pole, the pole coincides with zenith and celestial bodies appear above the horizon all day. Their apparent motions are in small circles parallel to the celestial equator which, in this extreme case, coincides with the observer's horizon.

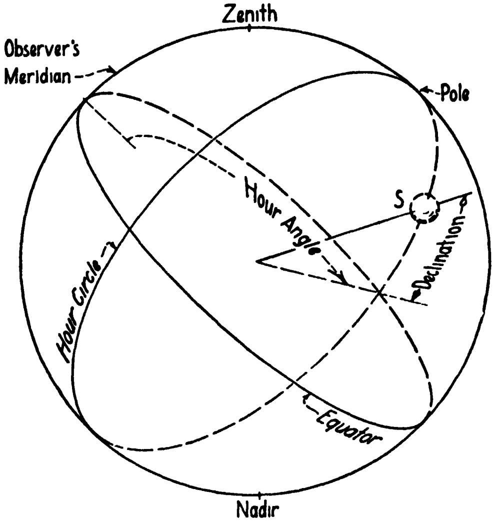

FIG. 309. THE HOUR ANGLE SYSTEM.

At any place of observation on Earth between the equator and pole, certain celestial bodies appear above the horizon for part of the day and disappear below the horizon for the rest of the 24-hour period. Other bodies such as stars near the pole remain above the horizon throughout the day. Such stars are called *Circumpolar Stars* for that particular place on Earth.

For every position on Earth, every celestial body reaches its maximum altitude for that day at the instant it crosses the meridian of the observer. The instant of meridian passage when the celestial body is above the pole is called *Upper Culmination;* when below the pole, *Lower Culmination.*

Since the observer's meridian is the only great circle on the celestial sphere which is both an hour circle and also a vertical circle, the instant of meridian passage of any celestial body is at 0 hour angle for that particular body and place (see also § 312).

An apparent westerly motion of a celestial body through 15° of hour angle changes its hour angular relation to the meridian by exactly 1 hour of time. For a star or the vernal equinox, the hour angle is expressed in sidereal units and for the sun, in solar units (see also § 314 and § 315). Hour angular relations, measured either along the equator or at the pole, are generally measured from the meridian westwardly for the full 24 hour period. The sun's hour angle, however, is measured westwardly from 0^h (noon) to 12^h (midnight). This is P.M. (Post Meridian) time. A.M. (Ante Meridian) time is measured from 0^h (midnight) to 12^h (noon). Thus at 10 A.M. the sun is two hours east of the meridian.

Any celestial body may therefore be located on the celestial sphere by hour angle and declination.

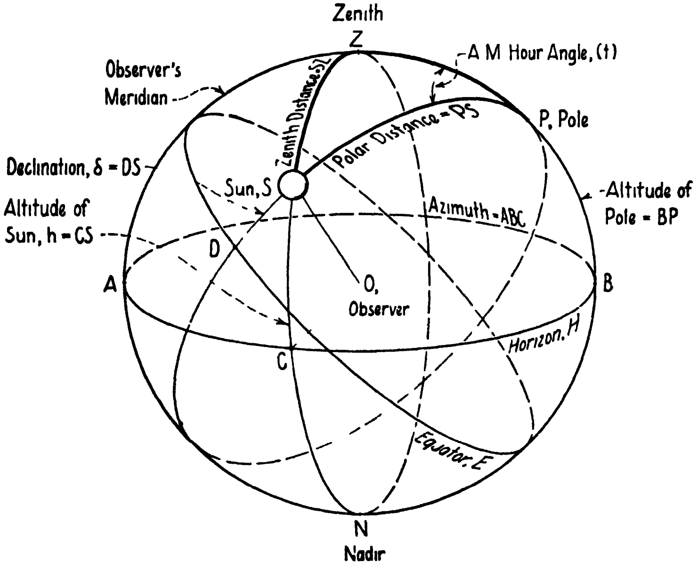

FIG. 310. AN A.M. SUN OBSERVATION.

310. The Astronomical Triangle. In the analysis of field data taken for the purpose of determining position on Earth, use is made of all three of the foregoing methods of fixing the position of points on the celestial sphere.

In Fig. 310, for example, the observer *O* is on Earth, but since the radius of the celestial sphere is infinite, *O* is assumed to be at Earth's center. The sun *S* is the celestial body. The figure illustrates conditions for a morning observation during that season of the year when the sun's apparent position is above the equator. Declination δ is north.

The altitude *h* of the sun is its height above the horizon and is measured along a vertical circle. The hour angle *t* represents the angular distance (measured at the pole or along the equator) through which the sun apparently must travel before it reaches upper culmination on the observer's meridian.

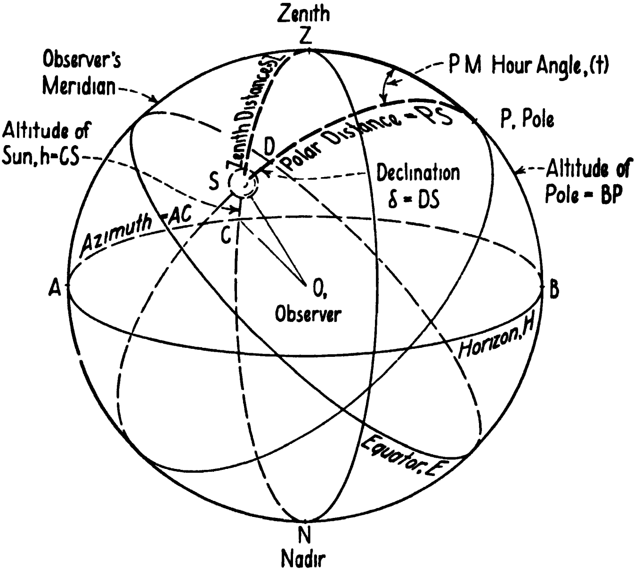

FIG. 310a. A P.M. SUN OBSERVATION.

Fig. 310a represents conditions at the instant of an afternoon sun observation during that season of the year when the sun's declination is south.

In both Fig. 310 and Fig. 310a the angular relation between the sun and true south is indicated by the azimuth angle.

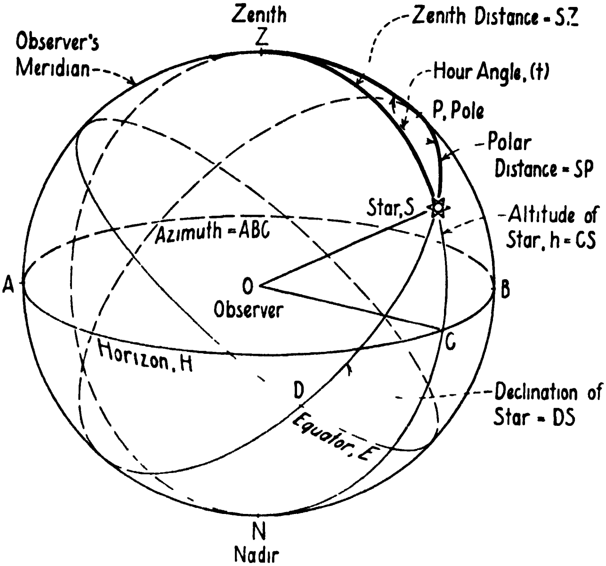

FIG. 310b. AN OBSERVATION ON POLARIS.

Figure 310b represents conditions existing for an observation on a star *S* which is *h*° in altitude above the horizon; *δ*° in declination above the equator; *t* hours before upper culmination and angle *ABC* in azimuth as measured clockwise from true south.

At any instant of observation therefore the three points, *P*, the pole; *Z*, the zenith; and *S*, the celestial body, form a spherical triangle, the solution of which (see § 324) yields the latitude, azimuth, or time (hour angle) for which the observation was made.

311. Terrestrial and Celestial Latitude. In Fig. 311, the smaller circle represents Earth and the larger circle the celestial sphere. The transit setup is at *ϕ*° angle above the equator, i.e., at *ϕ*° north latitude. On the celestial sphere this angle is measured along the meridian from *E* to *Z*.

The latitude of the place of observation is therefore equal to the declination of the observer's zenith and the polar distance of zenith is equal to the co-latitude of the place of observation

In Fig. 311, the altitude of the pole is HP (angle HOP) and the zenith distance of the pole is PZ.

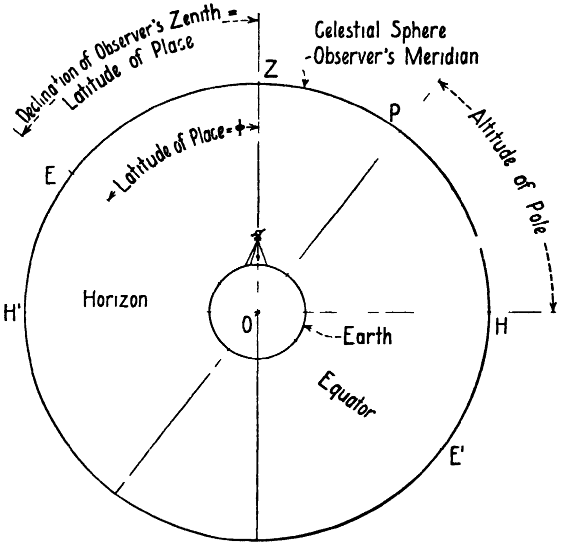

FIG. 311. RELATION BETWEEN ALTITUDE OF POLE AND LATITUDE OF PLACE.

The latitude of the place (EZ) is therefore equal to the altitude of the pole (HP) at that place.

312. Special Conditions of Observation. When an observation is made on a celestial body at the instant of meridian passage (culmination) of that body, the three points P, Z, and S all lie on the same great circle, the observer's meridian. Latitude determinations under these special conditions are easily and accurately made.

For example, Fig. 312 shows the relative positions of P, Z, and S, t hours after the instant shown in Fig. 310. In Fig. 312,

HS = altitude of the sun; ES = declination, and the hour angle of S is 0^h.

$$EZ = \text{latitude of the place}$$
$$H'P = \text{altitude of the pole}$$
$$H'P = EZ$$
312) $$EZ = 90^\circ - HS + ES.$$

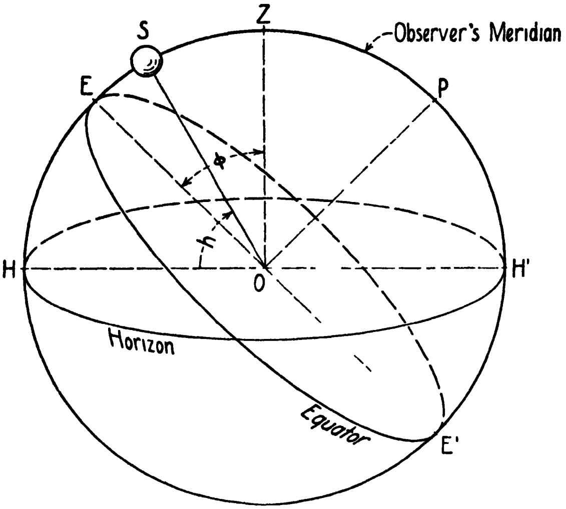

FIG. 312. SUN OBSERVATION AT CULMINATION.

When the sun has south declination, formula 312 becomes

(312a) $$EZ = 90° - HS - ES.$$

In Fig. 312a

(312b) $$EZ = H'P = H'S - SP.$$

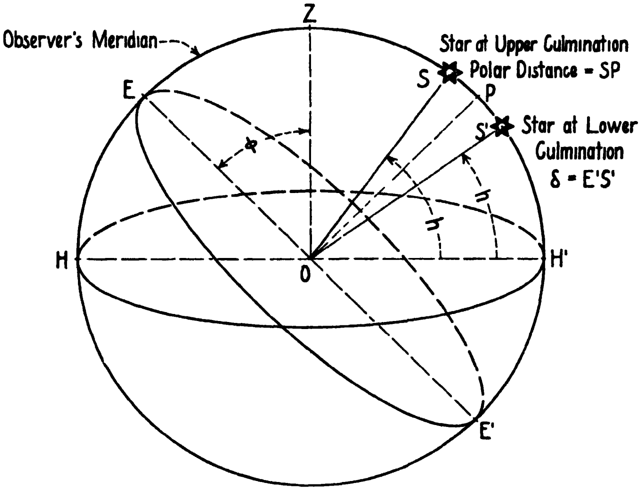

FIG. 312a. STAR OBSERVATION AT CULMINATION.

When the star, Fig. 312a, is at lower culmination, formula 312b becomes

$$EZ = H'P = H'S' + S'P \tag{312c}$$

313. Time Measurement. Measurement of time is based on the apparent motions of celestial bodies. These apparent motions are caused by the two actual motions of Earth, one about its own axis and the other in its orbit around the sun. These actual motions are both eastward, resulting in apparent westward motions as viewed from Earth.

The sun being at one of the foci of the ellipse of the orbital path of Earth, the actual angular speed of Earth in its path is variable. Figure 313 illustrates these relations at different seasons of the year. It should be noted that the radius vector of the ellipse covers equal areas in equal intervals of time. This accounts for the variable speed of Earth. If, therefore, the sun's apparent variable motion were used as a method of measuring time, the construction of mechanical devices such as clocks would be rather difficult.

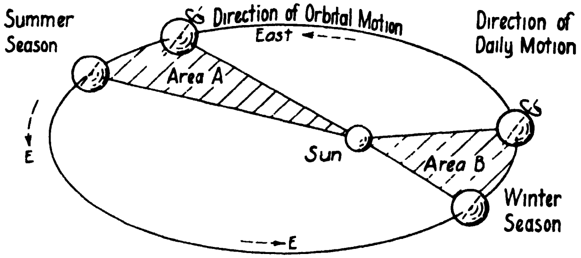

FIG. 313. REASONS FOR VARIABLE SPEED OF EARTH.

Irregularities in the apparent motion of the sun are also caused by the fact that the axis of Earth is inclined to the plane of orbital motion. This causes variable hour angles as measured at the pole (or along the equator) for equal distances along the ecliptic.

314. Sidereal Time. The motion of Earth about its own axis is uniform. Therefore, any object infinitely distant from Earth, such as a star or the vernal equinox, would be on the observer's meridian A and A' (Fig. 314) at the beginning and ending of one complete revolution of Earth. Stars, therefore, make possible a uniform and accurate method of measuring time. Mechanical devices such as sidereal clocks measure these apparent uniform movements of stellar bodies. Although it is an imaginary star, the vernal equinox is used to measure star, or *Sidereal Time.*

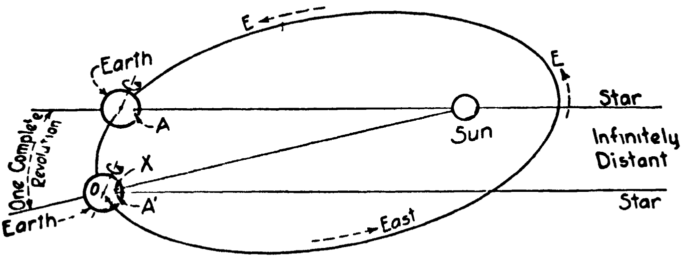

Fig. 314. Relation between a Sidereal Day and an Apparent Solar Day.

At any place on Earth's surface, a sidereal day begins (0 hours, sidereal time) when the vernal equinox is on the observer's meridian and above his horizon. Twenty-four hours (sidereal time) later, the vernal equinox is again at meridian passage and one sidereal day has elapsed. Sidereal time is measured by clocks regulated so as to gain 24 hours a year over ordinary clocks. Sidereal time at any place at any instant of time is the hour angle of the vernal equinox referred to the meridian of that place.

315. Solar Time. Referring to Fig. 314, if the apparent (actual) sun were on the meridian of the observer at point A at the beginning of the day, Earth must revolve through an additional angle $A'OX$ in order to complete one apparent solar day. This additional angle amounts to about four minutes (time measure) for each day, or 24 hours in a year. Apparent solar time is measured by a sun dial. An apparent solar day is the time elapsing between two successive culminations (meridian transits) of the apparent sun. Apparent solar time at any instant of time is the hour angle of the apparent sun.

To Earth's inhabitants, the sun, giver of light and heat, is the most important celestial body, but because of its apparent variable motion it cannot be used conveniently as the time measuring object. However, a method has been devised for sun time measurement which employs the average daily motion of the apparent sun during the year. The result of that average is an imaginary sun called the *Mean Sun.* The mean sun moves at a uniform rate eastwardly along the celestial equator making one complete revolution in 365.2422 ± mean solar days. This interval of time is called a *Tropical Year.*

Mean time is measured by watches and clocks. It is the hour angle of the mean sun. A mean solar day is the mean time hours which elapse between two successive transits of the mean sun. *Mean Noon* is the instant of time when the mean sun is on the meridian. A mean or *Civil Day* begins at midnight (0^h) and is measured A.M. and P.M. *Local Mean Time* is the hour angle of the mean sun referred to the local meridian.

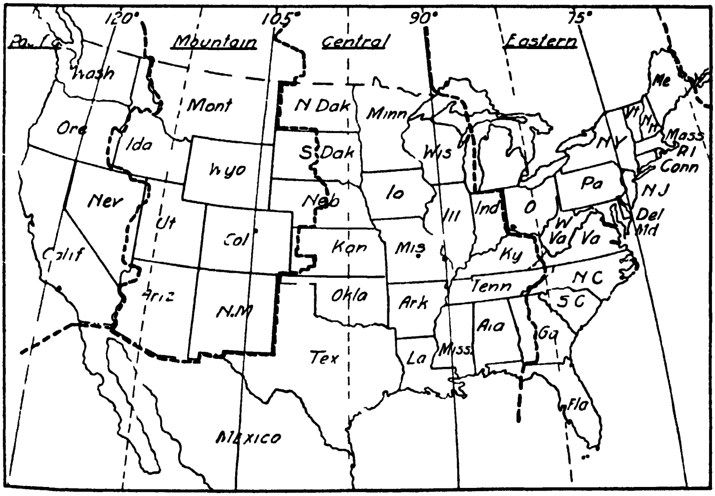

FIG. 315. TIME BELTS

The two suns, apparent and mean, one moving along the ecliptic at a variable rate, the other along the equator at a uniform rate, are at all times separated by a constantly changing hour angle. This angle is called the *Equation of Time* and is the factor needed to convert mean time into apparent time or vice versa.

Standard Time is the hour angle of the mean sun referred to a standard meridian. The adoption of standard time as a system of time measurement was merely a convenience. Theoretically, time belts are about one hour (15° longitude) wide and are bounded by irregular lines spaced approximately 30 minutes (7½°) on each side of the whole hour meridians (15°, 30°, 45°, etc.) east and west of Greenwich, England. Variations in the widths and boundaries of these belts (see Fig. 315) are due to locations of important business and railway centers. *Daylight Saving Time*, as adopted locally during the summer season in many communities, is the Standard Time of the time belt east of the belt in which the community is located.

316. Conversion of Time Intervals. As mentioned in § 315, the tropical year is the time elapsing during one complete revolution of the mean sun around the equator. One tropical year = 365.2422 ± mean solar days. During that same time interval, the apparent sun also makes one complete revolution around the ecliptic.

As shown in Fig. 314 the difference in time between a mean solar day and a sidereal day is about four minutes, the sidereal clock gaining this amount over the mean time clock each day so that at the end of a tropical year the sidereal clock is precisely one full day (24 hours) ahead of the mean time clock.

One tropical year = 366.2422 ± sidereal days. The American Ephemeris gives the following:

One sidereal day = $23^h\ 56^m\ 04^s.091$ of mean solar time.

One mean solar day = $24^h\ 03^m\ 56^s.555$ of sidereal time. Tables 316 and 316a make possible easy and rapid conversions of solar and sidereal time intervals.

317. Longitude and Time. Since all three time measuring systems are based upon the apparent motions of celestial bodies, it follows that

One apparent solar day = 24 hours of apparent solar time = 360 degrees of longitude.

One mean solar day = 24 hours of mean solar time = 360 degrees of longitude.

TABLE 316. TO CONVERT A SIDEREAL TIME INTERVAL INTO AN EQUIVALENT MEAN SOLAR TIME INTERVAL

Sidereal time interval − C = Mean time interval

Sidereal Hours	C	Sidereal Minutes	C	Sidereal Minutes	C	Sidereal Seconds	C	Sidereal Seconds	C
	m s		s		s		s		s
1	0 09.830	1	0.164	31	5.079	1	0.003	31	0.085
2	0 19.659	2	0.328	32	5.242	2	0.005	32	0.087
3	0 29.489	3	0.491	33	5.406	3	0.008	33	0.090
4	0 39.318	4	0.655	34	5.570	4	0.011	34	0.093
5	0 49.148	5	0.819	35	5.734	5	0.014	35	0.096
6	0 58.977	6	0.983	36	5.898	6	0.016	36	0.098
7	1 08.807	7	1.147	37	6.062	7	0.019	37	0.101
8	1 18.636	8	1.311	38	6.225	8	0.022	38	0.104
9	1 28.466	9	1.474	39	6.389	9	0.025	39	0.106
10	1 38.296	10	1.638	40	6.553	10	0.027	40	0.109
11	1 48.125	11	1.802	41	6.717	11	0.030	41	0.112
12	1 57.955	12	1.966	42	6.881	12	0.033	42	0.115
13	2 07.784	13	2.130	43	7.045	13	0.035	43	0.117
14	2 17.614	14	2.294	44	7.208	14	0.038	44	0.120
15	2 27.443	15	2.457	45	7.372	15	0.041	45	0.123
16	2 37.273	16	2.621	46	7.536	16	0.044	46	0.126
17	2 47.102	17	2.785	47	7.700	17	0.046	47	0.128
18	2 56.932	18	2.949	48	7.864	18	0.049	48	0.131
19	3 06.762	19	3.113	49	8.027	19	0.052	49	0.134
20	3 16.591	20	3.277	50	8.191	20	0.055	50	0.137
21	3 26.421	21	3.440	51	8.355	21	0.057	51	0.139
22	3 36.250	22	3.604	52	8.519	22	0.060	52	0.142
23	3 46.080	23	3.768	53	8.683	23	0.063	53	0.145
24	3 55.909	24	3.932	54	8.847	24	0.066	54	0.147
		25	4.096	55	9.010	25	0.068	55	0.150
		26	4.259	56	9.174	26	0.071	56	0.153
		27	4.423	57	9.338	27	0.074	57	0.156
		28	4.587	58	9.502	28	0.076	58	0.158
		29	4.751	59	9.666	29	0.079	59	0.161
		30	4.915	60	9.830	30	0.082	60	0.164

One sidereal day = 24 hours of sidereal time = 360 degrees of longitude.

Expressed in another way, the apparent sun moves through 15°, or one hour of longitude, in one hour of apparent solar time; the mean sun moves through 15°, or one hour of longitude, in one hour of mean solar time; and the vernal equinox moves through 15°, or one hour of longitude, in one hour of sidereal time.

TABLE 316a. TO CONVERT A MEAN SOLAR TIME INTERVAL INTO AN EQUIVALENT SIDEREAL TIME INTERVAL

Mean time interval + C = Sidereal time interval

Mean Hours	C	Mean Minutes	C	Mean Minutes	C	Mean Seconds	C	Mean Seconds	C
	m s		s		s		s		s
1	0 09.856	**1**	0.164	**31**	5.093	**1**	0.003	**31**	0.085
2	0 19.713	**2**	0.329	**32**	5.257	**2**	0.005	**32**	0.088
3	0 29.569	**3**	0.493	**33**	5.421	**3**	0.008	**33**	0.090
4	0 39.426	**4**	0.657	**34**	5.585	**4**	0.011	**34**	0.093
5	0 49.282	**5**	0.821	**35**	5.750	**5**	0.014	**35**	0.096
6	0 59.139	**6**	0.986	**36**	5.914	**6**	0.016	**36**	0.099
7	1 08.995	**7**	1.150	**37**	6.078	**7**	0.019	**37**	0.101
8	1 18.852	**8**	1.314	**38**	6.242	**8**	0.022	**38**	0.104
9	1 28.708	**9**	1.478	**39**	6.407	**9**	0.025	**39**	0.107
10	1 38.565	**10**	1.643	**40**	6.571	**10**	0.027	**40**	0.110
11	1 48.421	**11**	1.807	**41**	6.735	**11**	0.030	**41**	0.112
12	1 58.278	**12**	1.971	**42**	6.900	**12**	0.033	**42**	0.115
13	2 08.134	**13**	2.136	**43**	7.064	**13**	0.036	**43**	0.118
14	2 17.991	**14**	2.300	**44**	7.228	**14**	0.038	**44**	0.120
15	2 27.847	**15**	2.464	**45**	7.392	**15**	0.041	**45**	0.123
16	2 37.704	**16**	2.628	**46**	7.557	**16**	0.044	**46**	0.126
17	2 47.560	**17**	2.793	**47**	7.721	**17**	0.047	**47**	0.129
18	2 57.417	**18**	2.957	**48**	7.885	**18**	0.049	**48**	0.131
19	3 07.273	**19**	3.121	**49**	8.049	**19**	0.052	**49**	0.134
20	3 17.129	**20**	3.285	**50**	8.214	**20**	0.055	**50**	0.137
21	3 26.986	**21**	3.450	**51**	8.378	**21**	0.057	**51**	0.140
22	3 36.842	**22**	3.614	**52**	8.542	**22**	0.060	**52**	0.142
23	3 46.699	**23**	3.778	**53**	8.707	**23**	0.063	**53**	0.145
24	3 56.555	**24**	3.943	**54**	8.871	**24**	0 066	**54**	0.148
		25	4.107	**55**	9.035	**25**	0.068	**55**	0.151
		26	4.271	**56**	9.199	**26**	0.071	**56**	0.153
		27	4.435	**57**	9.364	**27**	0.074	**57**	0.156
		28	4.600	**58**	9.528	**28**	0.077	**58**	0.160
		29	4.764	**59**	9.692	**29**	0.079	**59**	0.162
		30	4.928	**60**	9.856	**30**	0.082	**60**	0.164

The difference in longitude between two points on Earth's surface is therefore a measure of the difference in time between those places. However, it must be remembered that conversion factors must be applied to these differences in longitude when they are to be expressed in different time units. For example

Mean time ± equation of time = apparent time.
Sidereal time = mean time + correction (see Table 316a).
Mean time = sidereal time − correction (see Table 316).

Regardless of whether mean solar, apparent solar, or sidereal time is used, the following relationships are true:

One hour of time = 15 degrees of longitude.
One minute of time = 15 minutes of longitude.
One second of time = 15 seconds of longitude.

318. Sidereal Time and Right Ascension. In each annual edition of the American Ephemeris and Nautical Almanac the apparent places of all important stars are tabulated. From these tabulations the right ascension of any listed star may be obtained.

This tabulated value plus or minus the hour angle of the star at any instant of time is the sidereal time at that place and at that instant. This relation is shown in Figs. 318 and 318a.

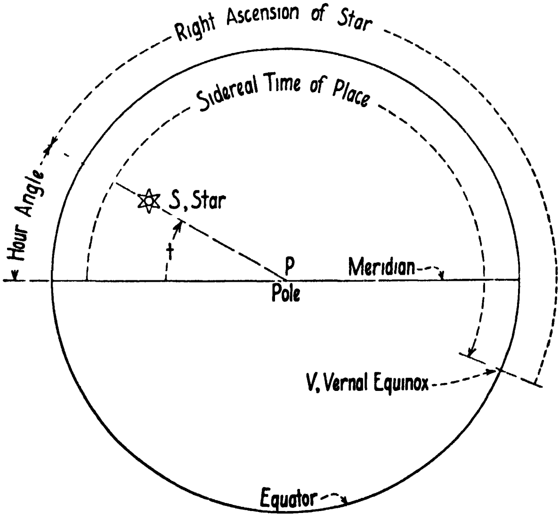

FIG. 318. SIDEREAL TIME = RIGHT ASCENSION PLUS HOUR ANGLE.

319. Corrections to Observed Altitudes. The measured vertical angle (altitude) of a celestial body is not its true altitude. Two corrections to this measured value must be made before using it in the solution of the *PZS* triangle. One of these corrections, *Refraction,* is necessary because of the bending of light rays as they pass through Earth's atmosphere. This is illustrated in Fig. 319, in which the corrected altitude of the celestial

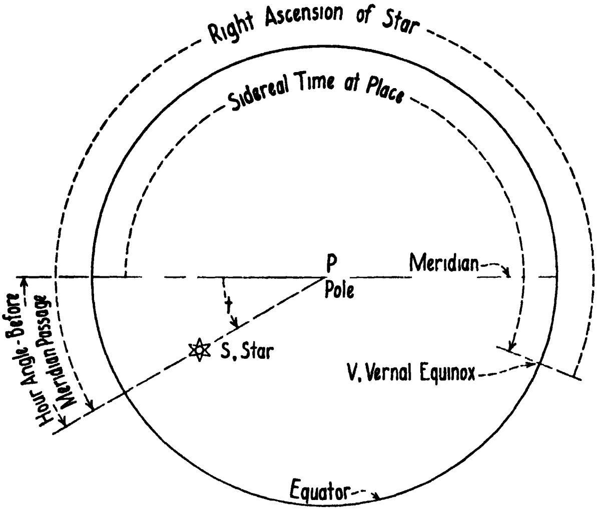

FIG. 318a. SIDEREAL TIME = RIGHT ASCENSION MINUS HOUR ANGLE.

body is angle *SOH* and the observed altitude is *S'OH*, the apparent direction *OS'* of the body from Earth's surface being tangent to the curve *OS* of the refracted ray at the position of the observer *O*. Angle *S'OS* is the refraction correction and is always subtracted from the measured (observed) altitude to obtain the corrected altitude of any celestial body. The value of this correction angle varies from 0 minutes when the celestial

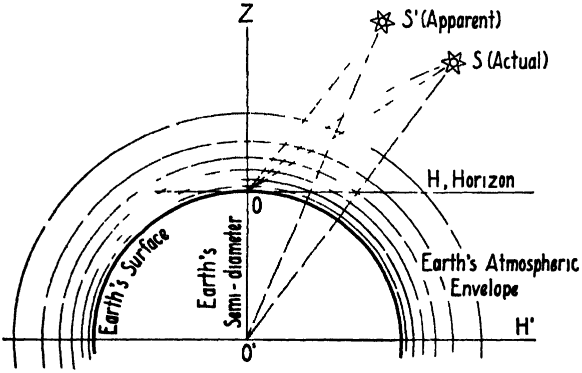

FIG. 319. PARALLAX AND REFRACTION.

body is at the zenith to about 11 minutes when the body is a few degrees above the horizon. It also decreases slightly as the temperature increases. The apparent flattening of the sun's disk at sunset illustrates the effect of refraction.

When an observation is made upon a celestial body which is a finite distance from Earth, such as the sun or moon, the observed altitude of such body must also be corrected for *Parallax*. Parallax is the apparent displacement of the body caused by the fact that the observer is on Earth's surface and not at Earth's center.

Parallax corrections to observed altitudes of celestial bodies which are infinitely distant from Earth are practically zero.

From Fig. 319, angle $S'OH$ is the observed altitude of the celestial body as measured on Earth's surface and angle $S'O'H'$ is the observed altitude of that same body at Earth's center. It is obvious that $S'O'H'$ is greater than $S'OH$. Therefore, the measured (observed) altitude of the celestial body must be increased by adding the parallax correction to the measured altitude in order to get the corrected altitude of that body. Parallax corrections are very small in comparison with refraction corrections; therefore, the net correction for parallax and refraction must be subtracted from the observed altitude to get corrected altitude.

Table 319 gives values of these corrections for values of altitude between 5° and 90°.

When an observation is made on the upper or lower edge of the sun the observed altitude must also be corrected by the addition or subtraction of the sun's semi-diameter. Values of apparent semi-diameter for various dates throughout the year may be obtained from a Solar Ephemeris or from the Sun Ephemeris in the American Ephemeris and Nautical Almanac.

INSTRUMENTAL EQUIPMENT

320. Transits. Detailed descriptions and illustrations of transit instruments with which field parties are usually equipped may be found in Chapters VI and XX.

Precision of results of astronomical observations beyond that obtainable with ordinary one-minute or thirty-second transits is possible only with equipment designed for that

TABLE 319. CORRECTION FOR PARALLAX AND REFRACTION TO BE SUBTRACTED FROM THE OBSERVED ALTITUDE OF THE SUN

For Polaris Observations add 0.′2 for altitudes to 15°, inclusive; and 0.′1 for altitudes to 70°, inclusive.

Apparent Altitude (Degrees)	Temperature. Fahrenheit					
	0°	+ 20°	+ 40°	+ 60°	+ 80°	+ 100°
	′	′	′	′	′	′
5	10.9	10.4	9.9	9.5	9.1	8.8
6	9.3	8.9	8.5	8.1	7.8	7.5
7	8.1	7.7	7.4	7.1	6.8	6.6
8	7.1	6.8	6.5	6.3	6.0	5.8
9	6.4	6.1	5.9	5.6	5.4	5.2
10	5.8	5.5	5.3	5.1	4.9	4.7
11	5.2	5.0	4.8	4.6	4.4	4.3
12	4.8	4.6	4.4	4.2	4.1	3.9
13	4.4	4.2	4.1	3.9	3.7	3.6
14	4.1	3.9	3.8	3.6	3.5	3.4
15	3.8	3.6	3.5	3.4	3.2	3.1
16	3.6	3.4	3.3	3.2	3.0	2.9
17	3.3	3.2	3.1	3.0	2.8	2.7
18	3.2	3.0	2.9	2.8	2.7	2.6
19	3.0	2.8	2.7	2.6	2.5	2.4
20	2.8	2.7	2.6	2.5	2.4	2.3
21	2.6	2.5	2.4	2.3	2.3	2.2
22	2.5	2.4	2.3	2.2	2.1	2.1
23	2.4	2.3	2.2	2.1	2.0	1.9
24	2.3	2.2	2.1	2.0	1.9	1.9
25	2.2	2.1	2.0	1.9	1.8	1.8
26	2.1	2.0	1.9	1.8	1.8	1.7
27	2.0	1.9	1.8	1.7	1.7	1.6
28	1.9	1.8	1.7	1.7	1.6	1.5
29	1.8	1.7	1.7	1.6	1.5	1.5
30	1.7	1.7	1.6	1.5	1.5	1.4
32	1.6	1.5	1.5	1.4	1.3	1.3
34	1.5	1.4	1.3	1.3	1.2	1.2
36	1.4	1.3	1.2	1.2	1.2	1.1
38	1.3	1.2	1.2	1.1	1.1	1.0
40	1.2	1.1	1.1	1.0	1.0	1.0
42	1.1	1.0	1.0	1.0	0.9	0.9
44	1.0	1.0	0.9	0.9	0.8	0.8
46	0.9	0.9	0.9	0.8	0.8	0.8
48	0.9	0.8	0.8	0.8	0.7	0.7
50	0.8	0.8	0.7	0.7	0.7	0.7
52	0.7	0.7	0.7	0.7	0.6	0.6
54	0.7	0.7	0.6	0.6	0.6	0.6
56	0.6	0.6	0.6	0.6	0.5	0.5
58	0.6	0.6	0.6	0.5	0.5	0.5
60	0.6	0.5	0.5	0.5	0.5	0.5
70	0.3	0.3	0.3	0.3	0.3	0.3
80	0.2	0.2	0.2	0.2	0 1	0.1
90	0.0	0.0	0.0	0.0	0.0	0.0

purpose. Special transit instruments, such as zenith telescopes, one-second theodolites, or even ten-second transits are not usually available.

Values of azimuth or latitude, sufficiently precise for most purposes, may be obtained with the ordinary transit by using care in the field and by checking results by means of a series of similar observations. It is decidedly important, for example, that the vertical axis of the transit be made truly vertical. On ordinary instruments, the plate bubbles are not sensitive enough to produce this result precisely and the instrumentman must therefore rely upon the longer and more sensitive telescope bubble for increased accuracy. The detailed procedure is given on page 136.

In addition to the transit instrument, a field party should be equipped with a high-grade watch or chronometer which should be checked repeatedly against radio time signals. For night observations, flash lights are needed to illuminate the cross-hairs by directing light into the telescope through the objective lens. Special reflectors for this purpose are listed in instrument manufacturers' catalogues and are a decided convenience but not a necessity. If reflectors are not available, care must be exercised by the assistant instrumentman when projecting light into the telescope that the flash light beam be directed into the telescope and not into the instrumentman's eyes. White paper shades substituted for the regular telescope sun shade or rolled snugly inside of the sunshade may be helpful.

For observations on the sun at high altitudes a prismatic eyepiece is needed. This should be fitted with a dark colored cover glass in order to protect the eye of the observer. Sun observations are also made by focusing the sun's disk back of the eyepiece as shown in Fig. 330. This is best performed by two operators, one manipulating the transit and the other the card upon which the sun's image is focused.

A new device called The Simplex Solar Shield has been invented by Professor C. H. Wall of the Ohio State University. This attachment to the transit permits the instrumentman to "secure accurate pointing on the center of the sun and to guarantee accurate time intervals between successive pointings. . . . A small shield of symmetrical design has been devised to be placed in front of a plate on which the sun's image and the cross-wires of the transit may be focused, . . . The shape and size of this shield are such as to match the sun's image and facilitate successive timing as the image moves." * Either

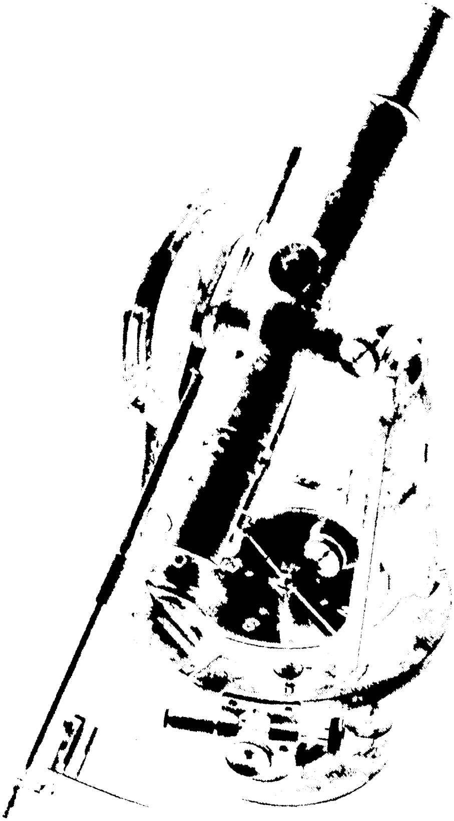

FIG. 320. SIMPLEX SOLAR SHIELD.

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* See "Civil Engineering" magazine, June 1940, page 363.

azimuth or latitude observations may be made with the Simplex Shield.

321. Solar Attachment. "The solar attachment to the engineers' transit has been designed for instrumentally setting off the sides of the pole-zenith-sun triangle in agreement with their angular values at the station and time of observation. The sun's image may be brought into the line of collimation of an auxiliary telescope by orientation of the transit to the position where the instrumental parts are made parallel to the respective sides of the celestial triangle, whereupon the vertical plane of the pole-zenith arc of the solar attachment will coincide with the true meridian. Skillfully handled, the solar attachment will give at once close approximations to the true meridian comparing favorably for accuracy with direct observations." *

Although several types of solar attachments are offered by instrument makers, only the Smith is used by the engineers of the General Land Office. This type, according to Mr. Arthur D. Kidder,† District Engineer in charge of field operations in the Eastern district, gives results well within limits of accuracy set by the Manual. It is particularly useful in mountainous regions or in heavily wooded areas. Use of this instrument is definitely specialized and the reader is therefore referred to the Manual issued by the Bureau of Land Management.

322. The Sextant. The sextant is an angle measuring instrument used generally by navigators for latitude determinations and occasionally by engineers on land for measurements of angles in any plane. It is particularly useful in hydrographic surveys in the measurement of angles to shore stations from boats where soundings are being made. Convenient size and the rapidity with which an angle can be measured make the sextant a useful instrument for many other surveying operations.

In Fig. 322a, the index glass, a mirror, is rigidly attached to the index arm *IV* which is free to rotate about *I*, the axis of the instrument. The *V* end of the index arm carries a vernier which rides the arc from *B* to *A*, the arc reading zero at *O*.

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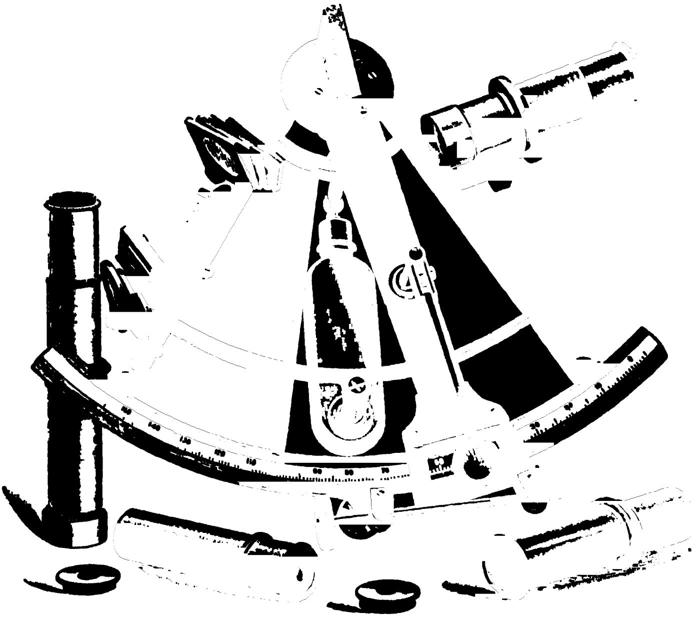
* "Manual of Instructions for the Survey of the Public Lands of the United States," 1930; Reprint 1934. A new edition of this manual is dated 1947.

† "The Solar Transit as Constructed for the General Land Office" — a paper presented by Arthur D. Kidder at the 1940 National Surveying Teachers' Conference sponsored by the Society for the Promotion of Engineering Education.



Courtesy of Keuffel & Esser Co

FIG. 322. THE SEXTANT.

The horizon glass, *H*, is rigidly attached to the frame of the sextant so that its surface is parallel to *IO*, a line through the axis and the zero point on the arc. It is silvered on the half next to the sextant frame and clear on the other half.

When the eye of the observer is at *E* and the index arm reads 0° 00′ on the arc, the reflected image of a star *S* to *I* to *H* to *E* coincides with the direct image of the same star *S′* as seen through the clear half of the horizon glass, *H*.

In determining the angle in any plane between two objects, a direct sight *ES′* toward one object is taken through the clear glass portion of the horizon glass. The index arm is then moved along the graduated arc until the reflected image of the second object following the path *SIHE* coincides with the direct image of the first object. The reading on the arc is then a measure of the angle between the two objects.

In Fig. 322a, while *OIV* represents the reading of arc when the two objects sighted are *S* and *S′*, angle *IEH* is the actual angle at *E* between the two objects. A knowledge of the true relation between these two angles is essential to an understanding of the principle of the sextant.

In Fig. 322a, the normals to the index glass and the horizon glass intersect at a point *P*, *IO* being perpendicular to *HP* and *IV* perpendicular to *NP*.

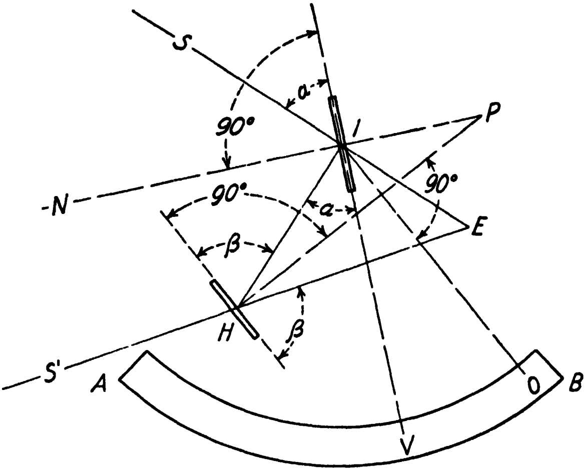


FIG. 322a. PRINCIPLE OF THE SEXTANT.

Therefore angle OIV = angle IPH.

In triangle IPH

$$
\begin{aligned}
\text{angle } IPH &= 180^\circ - (\text{angle } PHI + \text{angle } HIP)\\
&= 180^\circ - [(90^\circ - \beta) + (90^\circ + \alpha)]\\
&= \beta - \alpha.
\end{aligned}
$$

In triangle IEH

$$
\begin{aligned}
\text{angle } IEH &= 180^\circ - (\text{angle } EHI + \text{angle } HIE)\\
&= 180^\circ - [(180^\circ - 2\beta) + 2\alpha]\\
&= 2\beta - 2\alpha\\
&= 2(\beta - \alpha).
\end{aligned}
$$

Therefore the actual angle between the two objects is twice as large as the angle through which the index arm moves. In order that the true value of the angle may be read directly on the arc, each half degree subdivision is valued at one degree and is so marked.

The navigator measures the altitude of celestial bodies with the sextant by bringing the reflected image coincident with (or tangent to) the actual horizon. On land, altitudes must be measured with the help of an artificial horizon, usually a surface of mercury. Fig. 322b and Fig. 322c show how such measurements are made.

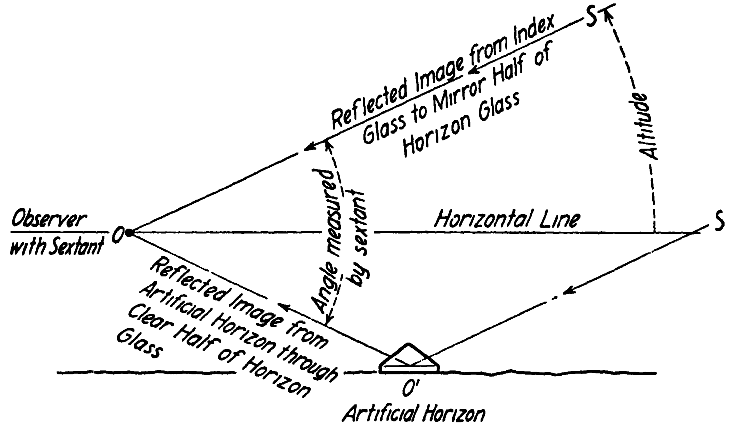

Reflected Image from Index Glass to Mirror Half of Horizon Glass
S
Altitude
Observer with Sextant
O
Horizontal Line
S
Angle measured by sextant
Reflected Image from Artificial Horizon through Clear Half of Horizon Glass
O'
Artificial Horizon

Fig. 322b. Altitude Using Artificial Horizon.

In Fig. 322b the observer sighting directly through the clear half of the horizon glass finds the image of the celestial body as reflected by the mercury surface of the artificial horizon. The image of the same body as reflected into the eyepiece of the sextant by way of the index glass mirror and the horizon glass mirror is then brought into coincidence with the first image by manipulating the index arm. When the two images coincide, the angle recorded on the arc is twice the altitude of the celestial body.

323. Adjustments of the Sextant. When a sextant is in adjustment, both mirrors are at right angles to the plane of the instrument and if a single infinitely distant object is observed, the index arm cuts the arc at 0° 00′.

To check the relation between the index glass and the plane of the sextant, the instrument is held horizontally with the observer's eye close to the index glass. The index arm should be clamped at 40° or 50°. The reflected image of the arc and the arc itself should appear as a continuous straight line. If this is not the case, the adjusting screws which permit tilting the index glass (usually located at the back) should be turned until the test for the relation is satisfied.

FIG. 322c. USING ARTIFICIAL HORIZON.

To check the relation between the horizon glass and the plane of the sextant the instrument is held vertically and a sight is taken on a distant point, preferably a star or the horizon as defined by a large lake or ocean. If the star is used, the reflected image of the star should cut across the direct image as the index arm is moved back and forth at the 0° end of the arc. If the horizon is available its reflected and direct images should be a continuous line as the instrument is tilted to the left and right of a vertical position. The horizon glass is adjusted until the proper relation between the images is attained.

In sighting toward a point infinitely distant, the vernier should read 0° 00′ when the direct and reflected images coincide. If such is the case, the horizon glass and index glass are parallel to each other. In many instruments, the horizon glass may be made parallel to the index glass by means of adjusting screws. This adjustment, however, may be omitted if the index error is measured. Care must be exercised in determining whether this correction to the measured angle is on or off the scale. If it is on the scale, the correction must be subtracted from the vernier reading.

324. Spherical Trigonometry. The derivation of the various spherical trigonometric formulae used in solving the astronomical triangle should be understood by the student. Their applications to the solution of the *PZS* triangle are as follows:

(a) In the oblique spherical triangle *PZS* (see Fig. 327c):

Given the three sides *PS*, *ZS*, and *PZ*

$$s = 1/2(PS + ZS + PZ) \tag{324}$$

$$\sin\frac{Z}{2} = \sqrt{\frac{\sin(s - ZS) \times \sin(s - PZ)}{\sin ZS \times \sin PZ}} \tag{324a}$$

$$\cos\frac{Z}{2} = \sqrt{\frac{\sin s \times \sin(s - PS)}{\sin ZS \times \sin PZ}} \tag{324b}$$

$$\tan\frac{Z}{2} = \sqrt{\frac{\sin(s - ZS) \times \sin(s - PZ)}{\sin s \times \sin(s - PS)}} \tag{324c}$$

(b) In the oblique spherical triangle *PZS*

Given sides *PS*, *ZS*, and angle *P*

$$\sin Z = \frac{\sin P \times \sin PS}{\sin ZS} \tag{324d}$$

which gives two possible values for *Z*. The time of observation or the hour angle of the body indicates the relation between the body and the meridian and which one of the two values is to be chosen. The selection of the proper value of *Z* is therefore simplified by visualizing the relative positions of *P*, *Z*, and *S* at the instant of observation. With this *Z* angle

$$\tan 1/2\ PZ = \frac{\cos 1/2(Z + P)}{\cos 1/2(Z - P)} \tan 1/2(PS + ZS) \tag{324e}$$

or

$$\tan 1/2\,PZ = \frac{\cos 1/2(P + Z)}{\cos 1/2(P - Z)} \tan 1/2(PS + ZS). \tag{324f}$$

FIELD WORK AND CALCULATIONS
AZIMUTH OBSERVATIONS

325. True North. To the engineer, the most important operation in the field of engineering astronomy is the determination of true north. Various methods may be used, all of them founded on the general theory developed in this chapter or in Chapter VIII. Observational methods and resulting calculations for true azimuth will be discussed in the following order:

1. Observation on Polaris at elongation
2. Observation on Polaris at any hour angle
3. Equal altitude observation on any star
4. Observation on the sun at any hour angle.

Other methods are used occasionally by engineers, but it is believed that with an understanding of the theory involved in these four methods most field engineers will be sufficiently equipped to determine true azimuth within one or two minutes or even less if the observation is made on Polaris.

326. Polaris at Elongation.* Data are needed from tables "Azimuth of Polaris at Elongation" and "Time of Upper Culmination and Elongation East or West of Polaris." These tables are printed annually in The American Ephemeris and Nautical Almanac or in abridgements published by instrument companies.

The tables make possible the calculation of the watch time (standard time) of elongation of Polaris at any place of observation where the star can be seen.† They also give the horizontal angular relation between the star and the true meridian at the instant of elongation.

Elongation instants are best used for azimuth observations because at those points in the apparent path of the star around the pole, the rate of change in its horizontal angle is a minimum.

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* See also pages 151 and 154.

† Polaris is not visible at points on Earth which lie between about 1° South latitude and the south pole; it is a circumpolar star for all points on Earth between about 1° north latitude and the north pole.

For example, an observation for the purpose of determining the true azimuth of a line is to be made on Polaris at elongation on October 17, 1949, at a place longitude $5^h\ 42^m$ West and latitude 40° North. The following steps should be taken:

Step 1. Find standard time of elongation. From Keuffel and Esser Company *Ephemeris*, 1949, Tables 8 and 9, p. 81.

$6^h\ 08^m.3$ A.M. = local mean time (civil time) of western elongation at Greenwich on October 16
$-\ 3.9$ = $(1 \times 3^m.93)$ = change due to date
$6^h\ 04^m.4$ A.M. = local mean time of western elongation at Greenwich on October 17
$-\ 0.9$ = $(.16^m \times 5.7)$ = change due to longitude
$6^h\ 03^m.5$ A.M = local mean time of western elongation at $5^h\ 42^m$ West on October 17
00.0 = change due to latitude
$6^h\ 03^m.5$ A.M. = local mean time of western elongation at $5^h\ 42^m$ West and 40° North on October 17
$-\ 18.0$ = difference between local mean time and standard time at $5^h\ 42^m$ West
$5^h\ 45^m.5$ A.M. = standard time of western elongation at $5^h\ 42^m$ West and 40° North on October 17

Or the same calculation, using Table VII, p. 548 of the *American Ephemeris and Nautical Almanac*, 1949, is

$0^h\ 08^m\ 38^s.0$ = civil time (local mean time) of upper culmination of Polaris at Greenwich on October 17
$0^m\ 00^s.0$ = change due to date
$0^h\ 08^m\ 38^s.0$ A M = local mean time of upper culmination at Greenwich on October 17
$-\ 56^s.0$ = $(5.7 \times 9^s.82)$ = change due to longitude
$0^h\ 07^m\ 42^s.0$ A.M = local mean time of upper culmination at $5^h\ 42^m$ West on October 17
$+\ 5^h\ 55^m\ 48^s.0$ = mean time interval between upper culmination and western elongation
$6^h\ 03^m\ 30^s.0$ A M = local mean time of western elongation at $5^h\ 42^m$ West on October 17
$18^m\ 00^s.0$ = difference between local mean time and standard time at $5^h\ 42^m$ West
$5^h\ 45^m\ 30^s.0$ A.M = standard time of western elongation at $5^h\ 42^m$ West on October 17

The standard time of elongation obtained from the American Ephemeris is probably more precise than the one from the abridged table in the Keuffel and Esser Ephemeris. Either time, however, is sufficiently accurate for this observation.

Step 2 (*a*). About thirty minutes before time of elongation, set up the transit at one end of the line the azimuth of which is to be determined. The vertical axis should be made precisely vertical with the telescope bubble or with a striding bubble. About five minutes before elongation, the star should be sighted and its direction transferred to a stake set under the star at a point at least 300 feet from the transit. Illumination of the sight at the stake is necessary. A series of double-sightings (telescope normal and reversed) should be made as rapidly as possible until about five minutes after the instant of elongation. The mean of each double-sighting should fall at the same point on the stake.

Step 3 (*a*). From the table "Azimuth of Polaris at Elongation" which may be found in the American Ephemeris and Nautical Almanac or in one of the abridgements of the Almanac published by instrument manufacturers, find the horizontal angle from true north to Polaris. Note that azimuth in this instance is east or west of north instead of clockwise from south and also that the value varies slightly for different latitudes and declinations. For example, Table 3 in the 1949 Keuffel and Esser Ephemeris gives the declination of Polaris for Oct. 16 as 89° 01′.69. In Table 7 the azimuth of Polaris is 1° 16′.2 when the latitude is 40° N and the declination is 89° 01′.69.

For an observation at western elongation, true north may be located by turning 1° 16′.2 from the point as established in step 2 (*a*). In this case the meridian would lie east (to the right) of that established point. The angle should be turned by the method described in § 95.

Alternative Procedure. ***Step 2*** (***b***). About thirty minutes before time of elongation set up at one end of the line and about five minutes before elongation sight toward a point at the other end of the line with horizontal verniers set at 0° 00′. Then double-sight the horizontal angle to Polaris as many times as possible for a period of ten minutes. The mean horizontal angle is the angle between the line and the star.

Step 3 **(*b*).** Determine the azimuth of Polaris at elongation, as in step 3 (*a*). Combine this with the horizontal angle as measured in 2 (*b*). The result is the true azimuth of the line.

For example, if the average horizontal angle from the point to the star were 93° 22′ 12″ to the right as in step 2 (*b*), the azimuth of the line measured from south in a clockwise direction would be:

$$180^\circ - (93^\circ\,22'\,12'' + 1^\circ\,16'\,12'') = 85^\circ\,21'\,36''$$

The use of either of these two methods should give results within 30 seconds of the correct azimuth angle. Some difficulty may be encountered at certain times of the year due to the fact that both eastern and western elongation instants occur during the early evening and morning hours. Although Polaris may be observed during the daytime, it is not common practice to attempt a daylight observation with an ordinary transit.

327. Polaris at Any Hour Angle. Excellent results of observations for true azimuth of a line may be obtained by sighting on Polaris at any instant of time. The observation involves:

(1) Determination of longitude of the place of observation from a reliable map or by making an observation for longitude (see § 340)
(2) An accurate determination of the correct standard time of observation
(3) Accurate measurement of the horizontal angle from a point to the star
(4) Accurate measurement of the vertical angle to the star
(5) Analysis of the data with the help of tables in the American Ephemeris
(6) Trigonometric solution of the *PZS* triangle.

Steps (3) and (4) are possible only when the vertical axis is truly vertical.

EXAMPLE. An observation was made on Polaris at a place longitude $5^h\,48^m\,18^s.7$ West on July 2, 1949. The temperature at the time of observation was 60° Fahrenheit. The data from the field notebook were:

BUBBLE	WATCH TIME	HORIZONTAL ANGLE TO THE LEFT	VERTICAL ANGLE
Down	$9^h\ 31^m\ 57^s$ P.M.	145° 13′ 30″	39° 40′
Up	$9^h\ 36^m\ 48^s$ P.M.	325° 13′ 00″	39° 40′
Mean Values	$9^h\ 34^m\ 22^s.5$ P.M.	145° 13′ 15″	39° 40′

The chronometer* (checked by radio time signals) was $13^s.0$ slow at 11 : 00 A.M. on July 2, 1949, and $15^s.5$ slow at 11 : 00 A.M. on July 3. The watch was $18^s.0$ faster than the chronometer at 7 : 26 P.M., July 2 and $14^s.0$ faster at 10 : 45 P.M., July 2.

Analysis:

(1) The chronometer rate of change was 2.5 seconds in 24 hours. From $11^h : 00^m$ A.M. July 2 to $9^h : 31^m$ P.M. = $10^h\ 34^m$ = 10.57 hours

$$\text{Chronometer correction*} = \frac{2.5}{24} \times 10.57 = 1.1 \text{ seconds}$$

The chronometer, therefore, was $13^s.0 + 1^s.1 = 14^s.1$ slow at the instant of observation.

(2) The watch rate of change was $4^s.0$ in $3^h\ 19^m$ = 3.32 hours. From $7^h\ 26^m$ P.M., July 2 to $9^h\ 34^m$ P.M., July 2 = $2^h\ 08^m$ = 2.13 hours

$$\text{Correction} = \frac{4.0}{3.32} \times 2.13 = 2.6 \text{ seconds}$$

The watch, therefore, was $18^s.0 - 2^s.6 = 15.4$ seconds faster than the chronometer at the instant of observation.

(3) $9^h\ 34^m\ 22^s.5$ P.M. = watch reading (mean)
$-\ 15^s.4$ = watch faster than chronometer
$9^h\ 34^m\ 07^s.1$ = chronometer time of observation
$+\ 14^s.1$ = chronometer slower than standard time
$9^h\ 34^m\ 21^s.2$ P.M.* = correct central standard time of observation.

(4) From The American Ephemeris and Nautical Almanac for 1949.

$1^h\ 47^m\ 32^s.34$ = right ascension of Polaris
$+\ 89°\ 01'\ 16''.94$ = declination of Polaris

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* Determination of exact time is unnecessary to calculate declination. It is needed for an accurate value of the hour angle t. In practice the field watch is usually the chronometer thus eliminating the calculation of a chronometer correction.

Both values are taken from the table of Apparent Place of Polaris, and for the date July 2.

From the same book from the Sun Ephemeris for Washington Apparent Noon.

$18^h\,39^m\,38^s.785$ = sidereal time of 0^h civil time at Washington July 2
$18^h\,43^m\,35^s.337$ = sidereal time of 0^h civil time at Washington July 3
$18^h\,41^m\,37^s.061$ = average
$-\ 12^h\,00^m\,00^s$
$6^h\,41^m\,37^s.061$ = sidereal time of Washington mean noon on July 2

These relations are shown in Fig. 327.

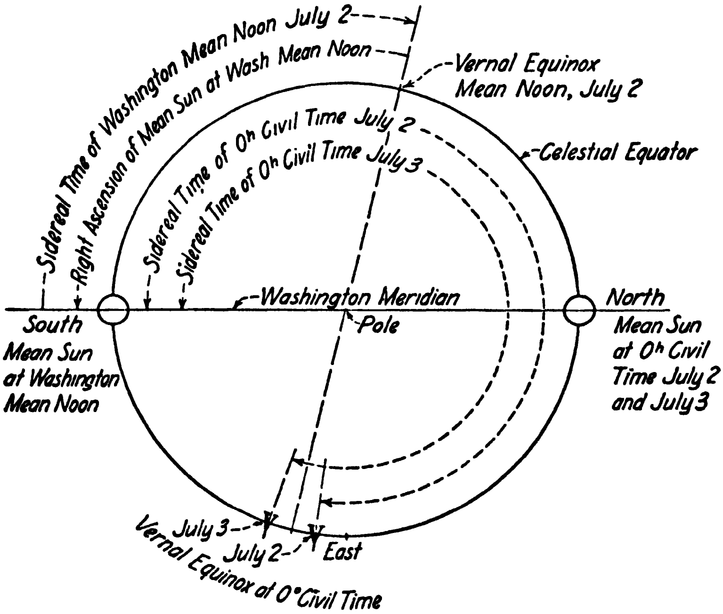
Sidereal Time of Washington Mean Noon July 2
Right Ascension of Mean Sun at Wash Mean Noon
Vernal Equinox Mean Noon, July 2
Sidereal Time of 0h Civil Time July 2
Sidereal Time of 0h Civil Time July 3
Celestial Equator
Washington Meridian
Pole
South
Mean Sun at Washington Mean Noon
North
Mean Sun at 0h Civil Time July 2 and July 3
July 3
July 2
East
Vernal Equinox at 0° Civil Time

Fig. 327. Mean Time and Sidereal Time Relations.

(5) From the table of Active Observatories of The American Ephemeris and Nautical Almanac, the longitude of Washington $= 5^h\,08^m\,15^s.78$ West.

(6) Figure 327a represents relations at the instant Washington Mean Noon, July 2.

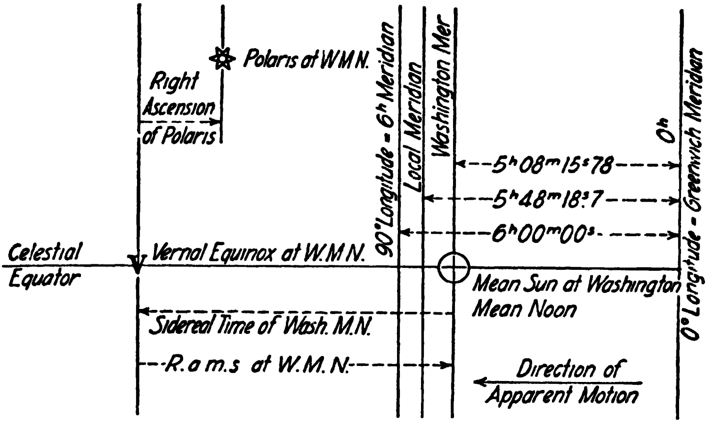
Polaris at W.M.N.
Right Ascension of Polaris
90° Longitude - 6h Meridian
Local Meridian
Washington Mer.
0h
0° Longitude - Greenwich Meridian
5h08m15s.78
5h48m18s.7
6h00m00s
Celestial Equator
Vernal Equinox at W.M.N.
Mean Sun at Washington Mean Noon
Sidereal Time of Wash. M.N.
R.a.m.s. at W.M.N.
Direction of Apparent Motion

FIG. 327a. AT THE EPHEMERIS INSTANT.

(7) Figure 327b represents relations at the instant of observation. This instant is $9^h\ 34^m\ 21^s.2$ P.M. and the place is longitude $5^h\ 48^m\ 18^s.7$ West.

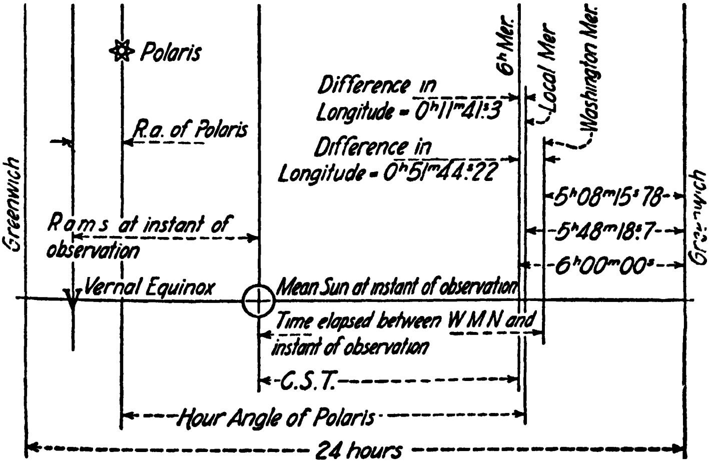
Polaris
R.a. of Polaris
Greenwich
6h Mer.
Local Mer.
Washington Mer.
Difference in Longitude = 0h11m41s.3
Difference in Longitude = 0h51m44s.22
5h08m15s.78
5h48m18s.7
6h00m00s
Greenwich
R.a.m.s. at instant of observation
Vernal Equinox
Mean Sun at instant of observation
Time elapsed between W.M.N. and instant of observation
C.S.T.
Hour Angle of Polaris
24 hours

FIG. 327b. AT THE INSTANT OF OBSERVATION.

At the instant of Washington mean noon the mean sun was on Washington meridian and at the instant of observation the mean sun was $9^h\ 34^m\ 21^s.2$ west of the 6 hour meridian (central standard time).

$9^h\ 34^m\ 21^s.2$ = central standard time of observation
$+\ \underline{0^h\ 51^m\ 44^s.22}$ = difference in longitude Washington and 6^h meridian
$10^h\ 26^m\ 05^s.42$ = time elapsed = 10.435 hours

During these 10.435 hours of elapsed time both the mean sun and vernal equinox were apparently moving along the equator at uniform rates of speed but not at the same rate. The vernal equinox was moving at a faster rate than the mean sun. The right ascension of the mean sun was therefore increasing at a uniform rate. This increase, which amounts to about four minutes a day (more exactly, 9.856 seconds per hour), is the difference between a sidereal day and a solar day (see § 316).

Therefore, the increase in right ascension of the mean sun since the ephemeris instant, Washington mean noon, was

$$10.435 \times 9^s.856 = 1^m\ 42^s.85,$$

and therefore

$6^h\ 41^m\ 37^s.061$ = right ascension of the mean sun at Washington mean noon
$+\ 1^m\ 42^s.85$
$6^h\ 43^m\ 19^s.911$ = right ascension of the mean sun at instant of observation.

The change in the right ascension of Polaris during these 10.435 hours of elapsed time is so small that it may be neglected.

(8) With the relative positions of the vernal equinox, Polaris, the mean sun, and the local meridian determined (Fig. 327b), it is possible to compute the hour angle at the pole between the local meridian and the hour circle through Polaris.

$0^h 11^m 41^s.3$ = difference in longitude between local meridian and 6^h meridian
$+\ 9^h\ 34^m\ 21^s.2$ = hour angle of mean sun referred to standard (6^h) meridian
$\underline{+\ 6^h\ 43^m\ 19^s.911}$ = hour angle between mean sun and vernal equinox

$16^h\,29^m\,22^s.41$ = local sidereal time of observation
$-\ 1^h\,47^m\,32^s.34$ = right ascension of Polaris
$14^h\,41^m\,50^s.07$ = hour angle of Polaris at place of observation.

This value for the hour angle of the star indicates that the position of the star in its path around the pole is somewhere between lower culmination and eastern elongation (see Fig. 310b) and that the hour angle inside the *PZS* triangle is

$$24^h - 14^h\,41^m\,50^s.07 = 9^h\,18^m\,09^s.93.$$

Expressing this in arc units,

$15 \times 9^h\,18^m\,09^s.93 = 139°\,32'\,28''.95$ = angle at P.

(9) The measured altitude of the star was 39° 40′. This must be corrected for refraction (parallax negligible).

$39°\,40'$ measured altitude
$-\ 01'.1$ refraction correction from Table 319
$39°\,38'.9$ = corrected altitude

$90° - 39°\,38'.9 = 50°\,21'.1 = 50°\,21'\,06''$ = zenith distance = ZS.

(10) The declination of Polaris, from the American Ephemeris, is $+\ 89°\,01'\,16''.94$

$90° - 89°\,01'\,16''.94 = 0°\,58'\,43''.06$ = polar distance = PS

(11) Solve the *PZS* triangle for the angle *Z*

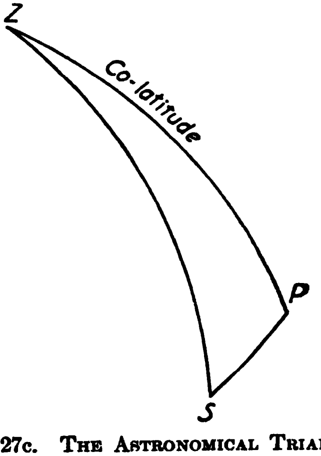

FIG. 327c. THE ASTRONOMICAL TRIANGLE.

$$\sin Z = \frac{\sin P \times \sin PS}{\sin ZS}$$

$$\sin Z = \frac{\sin 139^\circ\, 32'\, 28''.95 \times \sin 0^\circ\, 58'\, 43''.06}{\sin 50^\circ\, 21'\, 06''}$$

9.8121770 = log sin P
8.2324738 = log sin PS
8.0446508
9.8864768 = log sin ZS
8.1581740 = log sin Z
$Z = 0^\circ\, 49'\, 28''.7$.

(12) Figure 327d shows the relation between the parts of the PZS triangle and the line, the azimuth of which is to be found

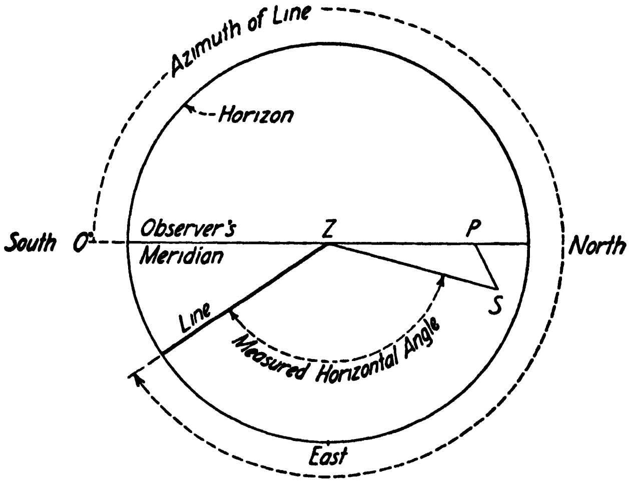

FIG. 327d. AZIMUTH RELATIONS.

$145^\circ\, 13'\, 15''$ = measured horizontal angle
$+\ 0^\circ\, 49'\, 28''.7$ = calculated Z angle
$+\ 180^\circ$
$326^\circ\, 02'\, 43''.7$ = azimuth of line.

Alternative Method. Another method for determining true north by observing Polaris at any hour angle is given in the Ephemeris tables published by Keuffel and Esser, C. L. Berger and Sons, and W. and L. E. Gurley. It involves the use of a table "Azimuth of Polaris at All Hour Angles." Results by this method are of the same order of accuracy as those obtained by the above method.

328. Latitude and Longitude from a Map. If latitude and longitude are determined from a reliable map or by observation, both field work and resulting calculations for azimuth are simplified.

Example. Assuming a scaled latitude for the place of observation to be 40° 24′ North,* PZ (Fig. 310b) $= 90° - 40° 24' = 49° 36'$ and

$$\sin\frac{Z}{2} = \sqrt{\frac{\sin(s - ZS) \times \sin(s - PZ)}{\sin ZS \times \sin PZ}}$$

where s is $1/2(PS + ZS + PZ)$.

Solution:

$$\begin{array}{rll} 0°\,58' & 43''.06 = PS & \qquad s - ZS = 0°\,06'\,48''.53 \\ 50°\,21' & 06''.00 = ZS & \\ 49°\,36' & 00''.00 = PZ & \qquad s - PZ = 0°\,51'\,54''.53 \\ 100°\,55' & 49''.06 & \\ 2\,|\,\underline{50°\,27'} & \underline{54''.53} = s & \end{array}$$

$$\sin\frac{Z}{2} = \sqrt{\frac{\sin 0°\,06'\,48''.53 \times \sin 0°\,51'\,54''.53}{\sin 50°\,21'\,06'' \times \sin 49°\,36'}}$$

$$\begin{array}{rlrl} 7.2967982 & = \log\sin 0°\,06'\,48''.53 & & \\ 8.1789509 & = \log\sin 0°\,51'\,54''.53 & & \\ 15.4757491 & & 9.8864767 & = \log\sin 50°\,21'\,06'' \\ 9.7681685 & & 9.8816918 & = \log\sin 49°\,36' \\ 2\,|\,\underline{15.7075806} & & 9.7681685 & \end{array}$$

$$7.8537903 = \log\sin\frac{Z}{2}$$

$$\frac{Z}{2} = 0°\,24'\,33''.06$$

$$Z = 0°\,49'\,06''.12$$

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* This value of latitude and the value for longitude of the place were scaled m a United States Engineer's map of the Wabash River and Valley.

and the resulting azimuth of the line (see Fig. 327d) is

$$\begin{array}{rl} 145^\circ\,13'\,15'' & = \text{the measured horizontal angle}\\ +\quad 0^\circ\,49'\,06''.12 & = \text{the calculated } Z \text{ angle}\\ +\ 180^\circ & \\ \hline 326^\circ\,02'\,21''.12 & = \text{the azimuth of line.} \end{array}$$

Thus an azimuth may be obtained, if latitude and longitude are known, by measuring the altitude of the star and by the use of ephemeris tables for declination.

329. Equal Altitude Observation on Any Star. Reasonably accurate results for azimuth may also be obtained by selecting a certain star or several stars which can be easily identified The star or stars selected should be to the south of the observer and at altitudes between 20 and 50 degrees.

Procedure: At any point of set-up of the transit and at instants of time when the stars are east of the meridian, each star is sighted separately, vertical angles are read, and tacked stakes are set under each star by depressing the telescope. Each stake should be properly identified.

After these same stars have passed the meridian and are approaching an altitude equal to that of the first sighting, the vertical circle vernier should be set to read that altitude, the telescope pointed toward the star, and at the instant of the star's traverse over the intersection of the cross-hairs a point is again established on the ground by means of a tacked stake.

The bisector of the angle from the first stake to the second stake is due south.

The mean value of several nights' observations or of observations made on several stars will yield a more accurate result. This method is commonly used in tropical countries and gives values of azimuth within one minute.

330. Azimuth from Solar Observations. Although observations on the sun do not give results as accurate as those computed from star observations, such observations are made and used. This is due probably to the facts that daytime is more convenient for field work and that the results are sufficiently accurate for the purpose of the survey.

Figure 330 illustrates the field method of obtaining the sun's position by using a card held back of the eyepiece upon which the cross-hairs and the sun's disk are focused simultaneously.

FIG. 330. SUN OBSERVATION.

The preparations preliminary to the taking of field data for a un observation and the analysis of the data are similar to those escribed for a Polaris observation in § 327. In the following xample, a 1949 Keuffel and Esser Company Solar Ephemeris is used in obtaining the ephemeris data.

Example. An observation was made on the sun at a place longitude $5^h\,48^m\,18^s.7$ West on July 1, 1949. The temperature at the time of observation was 80° Fahrenheit. The data from the field notebook were:

Image	Bubble	Watch Time	Horizontal Angle to the Left	Vertical Angle
${}^{\circ}+$	Down	$9^h\,04^m\,18^s$ A.M.	43° 13′ 30″	51° 14′
$+_{\circ}$	Up	$9^h\,05^m\,49^s$ A.M.	221° 59′ 30″	51° 01′
	Mean Values	$9^h\,05^m\,03^s.5$ A.M.	42° 36′ 30″	51° 07′.5

The chronometer* was 7.5 seconds slow at 11:00 A.M. on June 30 and 10 seconds slow at 11:00 A.M. on July 1. The watch was 28 seconds slower than the chronometer at 7:00 A.M on July 1 and $28^s.5$ slower at 9:30 A.M.

Analysis:

(1) The chronometer* rate of change was 2.5 seconds in 24 hours.
From 11^h00^m A.M. June 30 to 9^h05^m A.M. July 1 = 22^h05^m = $22^h.1$

$$\text{Chronometer correction} = \frac{2.5}{24} \times 22.1 = 2.3 \text{ seconds}$$

The chronometer, therefore, was $7^s.5 + 2^s.3 = 9^s.8$ slow at the instant of observation.

(2) The watch rate of change was $0^s.5$ in 2.5 hours. From 7:00 A.M., July 1 to 9:05 A.M., July 1 = $2^h\,05^m$ = 2.1 hours.

$$\text{Watch correction} = \frac{0.5}{2.5} \times 2.1 = 0.4 \text{ seconds}$$

The watch therefore was $28^s + 0^s.4 = 28.4$ seconds slower than the chronometer at the instant of observation.

(3) $9^h\,05^m\,03^s.5$ A.M. = watch reading (mean)
$+\ 28^s.4$ = correction, watch to chronometer
$+\ 09^s.8$ = correction, chronometer to central standard time
$9^h\,05^m\,41^s.7$ A.M.* = corrected central standard time.

(4) The following data are computed from solar ephemeris

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* See footnote, page 401.

tables. The ephemeris data are for instant 0^h Greenwich mean time.

Date	Declination	Difference for 1 Hour	Equation of Time to Be Subtracted from Mean Time	Variation per Hour
July 1	N 23° 07′.3	0′.17	$3^m\ 38^s.8$	$0^s.49$
July 2	N 23° 03′.1		$3^m\ 50^s.3$	

These data are for the instant Greenwich mean noon.

(5) Figure 330a represents relations existing between places and celestial bodies at Greenwich mean noon.

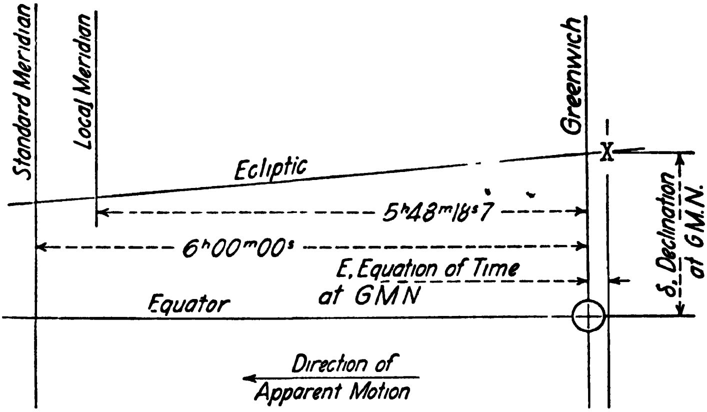

FIG. 330a. GREENWICH MEAN NOON.

In Fig. 330a the apparent sun, X, is shown above the equator and on the ecliptic because declination, δ, is north. The line representing the ecliptic slants downward to the left because at this season the sun's declination is decreasing.

The mean sun, O, is shown on the equator and west of the apparent sun, X, because on July 1 the equation of time E must be subtracted from mean time to get apparent time.

(6) Figure 330b represents relations at the instant of observa-on. For this observation the instant is $9^h\ 05^m\ 41^s.7$ A.M. The lace of observation is $5^h\ 48^m\ 18^s.7$ west longitude.

At the instant Greenwich mean noon, the apparent sun was

$0^h\ 03^m\ 38^s.8$ east of Greenwich meridian. At the instant of observation the mean sun was $12^h - 9^h\ 05^m\ 41^s.7 = 2^h\ 54^m\ 18^s.3$ east of the standard meridian.

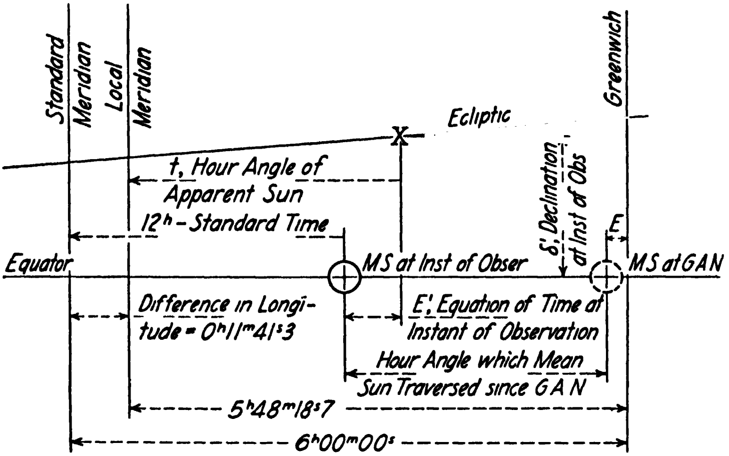
Standard Meridian
Local Meridian
Greenwich
Ecliptic
t, Hour Angle of Apparent Sun
12h - Standard Time
Equator
MS at Inst of Obser
δ', Declination at Inst of Obs
E
MS at GAN
Difference in Longitude = 0h11m41s3
E', Equation of Time at Instant of Observation
Hour Angle which Mean Sun Traversed since G A N
5h48m18s7
6h00m00s

FIG. 330b. AT INSTANT OF OBSERVATION.

Therefore $6^h - 2^h\ 54^m\ 18^s.3 = 3^h\ 05^m\ 41^s.7$ represents the hour angle through which the mean sun traveled between the two instants of time represented by Figs. 330a and 330b. During these time elapsed hours, the declination of the apparent sun decreased from δ to δ' and the equation of time increased from E to E'.

$$3^h\ 05^m\ 41^s.7 = 3.09 \text{ hours}$$

The rates of changes for both E and δ as given in the solar ephemeris make possible the calculation of values for δ' and E',

$$3.09 \times 0'.17 = 0'.5 = \text{change in } \delta$$
$$3.09 \times 0^s.49 = 1^s.5 = \text{change in } E,$$

and therefore

$$23^\circ\ 07'.3 - 0'.5 = 23^\circ\ 06'.8 = \delta'$$
$$90^\circ - 23^\circ\ 06'.8 = 66^\circ\ 53'.2 = \text{polar distance of sun} = PS \text{ side of the } PZS \text{ triangle.}$$
$$0^h\ 03^m\ 38^s.8 + 1^s.5 = 0^h\ 03^m\ 40^s.3 = E'.$$

At the place of observation (local meridian) the hour angle of the apparent sun is (see Fig. 330b)

$2^h\,54^m\,18^s.3$ = hours until noon

$+\,0^h\,03^m\,40^s.3 = E'$

$2^h\,57^m\,58^s.6$ = hour angle between apparent sun and standard meridian.

$-\,0^h\,11^m\,41^s.3$ = difference in longitude between local meridian and standard meridian.

$2^h\,46^m\,17^s.3$ = hour angle of apparent sun at place of observation.

$2^h\,46^m\,17^s.3 \times 15 = 41^\circ\,34'\,19''.5$ = angle P in the PZS triangle.

(7) The measured altitude of the sun's center must be corrected for parallax and refraction. From Table 319 this correction is found to be 00′.6.

$$\begin{array}{rl} 51^\circ\,07'.5 & = \text{measured altitude} \\ -\quad 00'.6 & = \text{parallax and refraction correction} \\ \hline 51^\circ\,06'.9 & = \text{true altitude of sun} \end{array}$$

and therefore

$90^\circ - 51^\circ\,06'.9 = 38^\circ\,53'.1$ = zenith distance of sun = ZS side of the PZS triangle.

(8) Figure 330c represents relations for these data and calculations.

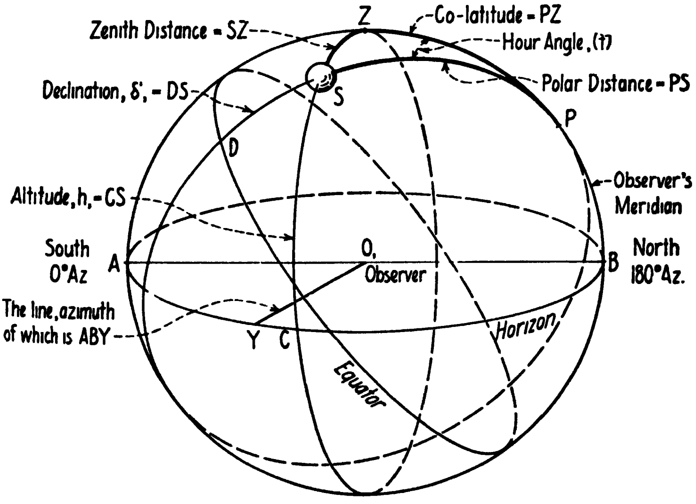

FIG. 330c. AN A.M. SUN OBSERVATION.

(9) This PZS triangle (Fig. 330d) must now be solved for the angle at Z.

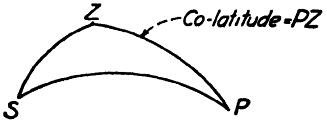

FIG. 330d.

$$\sin Z = \frac{\sin P \times \sin PS}{\sin ZS}$$

$$= \frac{\sin 41^\circ 34' 19''.5 \times \sin 66^\circ 53'.2}{\sin 38^\circ 53'.1}$$

$9.8218813 = \log \sin P$
$9.9636604 = \log \sin PS$
9.7855417
$9.7977932 = \log \sin ZS$
$9.9877485 = \log \sin Z$
$Z = 76^\circ 27' 16''$ or $103^\circ 32' 44''$.

Since the observation was made when the sun was approaching the meridian and obviously south and east of the observer, the larger of the two Z angles is inside the PZS triangle and the azimuth of the line is determined as in Fig. 330e.

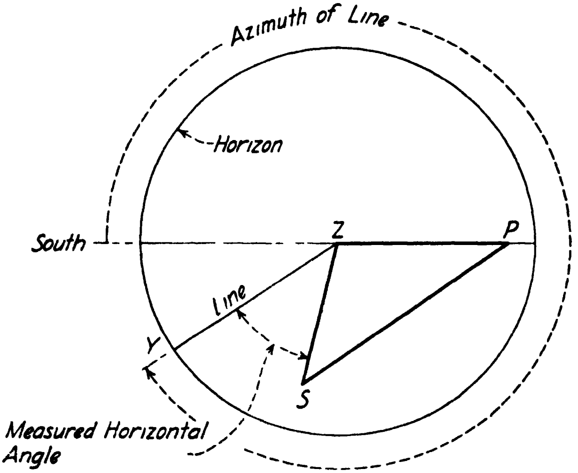

FIG. 330e. AZIMUTH RELATIONS.

42° 36′ 30″ = measured horizontal angle
103° 32′ 44″ = angle Z in PZS triangle
180°
326° 09′ 14″ = azimuth of line.

331. Latitude and Longitude Known. As shown in § 328 when both latitude and longitude are scaled from a reliable map or obtained by observation, the azimuth of the line may be easily computed. In this case it is necessary to measure horizontal and vertical angles to the sun. Correcting the vertical angle for parallax and refraction makes possible the calculation of the side ZS. With PZ (co-latitude) known, angle Z is found by substitution in formula 324a. This angle, combined with the measured horizontal angle, gives the required azimuth.

Latitude Observations

332. Observations at Culmination. Any celestial body reaches its maximum altitude when it is on the meridian of the observer. At that instant of time, as explained in § 312 and shown in Figs. 312 and 312a, the latitude of the place of observation may be determined quickly, accurately, and without involved computations.

However, in order to make such an observation, it is necessary to have either an established meridian or the watch time of culmination. This latter is determined by calculation before the observation is attempted.

333. Latitude on an Established Meridian. The determination of latitude of any point of set-up on the meridian is made by measuring the altitude of the celestial body at meridian passage, correcting that measured altitude to true altitude and then combining certain ephemeris data with the true altitude.

For example, in Fig. 312a which shows a Polaris observation at lower culmination,

$H'S'$ = measured altitude of Polaris corrected for refraction.
$S'P$ = polar distance of Polaris = 90° − declination.
$H'P$ = altitude of pole = latitude of place.
$H'P = H'S' + S'P$ (formula 312c).

At upper culmination
$H'P = H'S - SP$ (formula 312b).

Figure 312 illustrates conditions when the observed celestial body is the sun.

In this case the horizontal cross-hair is made tangent to the sun's disk at either the lower or upper edge. In Fig. 312

HS = true altitude of sun's center.
HS = measured altitude of the sun's upper or lower edge, corrected for parallax, refraction and semi-diameter. Values of these correction factors may be found in the solar ephemeris.
ES = declination of the sun at local apparent noon.
EZ = latitude of the place of observation.
$EZ = 90° - HS + ES$ (formula 312).

When the sun's declination is south (sun below equator)

$$EZ = 90° - HS - ES \text{ (formula 312a)}.$$

When observations such as these are made with a transit, it is usually necessary because of the sun's altitude that the instrument be equipped with a prismatic eyepiece. The altitude should be double-sighted, one reading taken on the upper edge of the sun and the other on the lower edge. The mean of the two readings eliminates the necessity for the semi-diameter correction to the measured altitude.

334. Time of Culmination. Without an established meridian, altitudes at culmination may be determined by rapidly measuring a series of altitudes, beginning the measurements before meridian passage of the celestial body and discontinuing operations when the vertical angles begin to decrease. The maximum value is obviously the altitude at culmination. Either the transit or sextant may be used for this purpose.

In Polaris observations, it is possible to sight on the star with the transit a few minutes before culmination, and when the star remains on the horizontal cross-hair for at least one full minute, quickly double-sight the vertical angle.

For Polaris observations, the approximate time of either culmination or elongation of the star may be determined by reference to Fig. 127.

In this figure, the dashed line which connects the second star in Cassoopcia (δ Cassoopcia) with the second star in the handle of the Big Dipper (ζ Ursa Majoris) passes approximately through Polaris and the Pole. When this line is vertical, the star (Polaris) is very near culmination. When the line is parallel to the horizon, the star is near its elongation position.

For a more accurate determination of time of elongation or culmination, ephemeris tables must be used. Such tables give the civil date and civil time of culmination or elongation for some standard meridian such as either Greenwich or the 6^h meridian. These tabular values of time must be corrected for date and place, the final correction being for the difference between standard time and local mean time at the place of observation. For example, if an observation were to be made on Polaris at upper culmination on December 18, 1949, the place being Ross Camp (longitude $5^h\ 48^m\ 18^s.7$ West), the calculations involved in determining the time of culmination are as follows:*

$8^h\ 08^m.4$ P.M. = civil time of upper culmination at Greenwich, latitude 40°, Dec. 16
$-\ \ 7^m.86$ = $2 \times 3^m.93$ = decrease per day
$8^h\ 00^m.5\bar{4}$ P.M. = civil time of upper culmination at Greenwich, Dec. 18
$-\ \ 0^m.90$ = $5.8 \times .16^m$ = correction for local longitude
$7^h\ 59^m.6\bar{4}$ P.M. = local civil time of upper culmination at Ross Camp, Dec. 18
$-\ 11^m.69$ = $6^h - 5^h\ 48^m\ 18^s.7$ = difference in time between Ross Camp and standard (6^h) meridian.
$7^h\ 47^m.95$ P.M. = central standard time at Ross Camp when Polaris is at upper culmination on Ross Camp meridian

Although calculations such as these give instants of culmination or elongation accurately, the exact values of these instants are not required for most observations. At culmination the apparent movement of Polaris vertically is a minimum; at elongation changes in horizontal angle are difficult to detect. If, therefore, the time of culmination were determined with an accuracy of ± five minutes, the resulting calculated latitude would be precise to less than one minute. If an elongation observation were made within five minutes of the exact time, the resulting azimuth would in all probability be within one minute of the true azimuth.

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* In this calculation, the correction for latitude is neglected. See Table 8 the Keuffel and Esser Ephemeris

335. Latitude from Polaris at Any Hour Angle. When an observation is made on Polaris as described and analyzed in § 327 but one additional step is necessary in order to determine the latitude of the place of observation.

Referring to Fig. 310b,

PZ = co-latitude of the place and, therefore,

$$\tan 1/2\,PZ = \frac{\cos 1/2(P+Z)}{\cos 1/2(P-Z)}\tan 1/2(PS+ZS)$$

$$\begin{array}{rl@{\qquad}rl}
 & 139^\circ\,32'\,28''.95 = P & & 139^\circ\,32'\,28''.95 = P \\
 & 0^\circ\,49'\,28''.70 = Z & & 0^\circ\,49'\,28''.70 = Z \\
2\,| & 140^\circ\,21'\,57''.65 = P+Z & 2\,| & 138^\circ\,43'\,00''.25 = P-Z \\
 & 70^\circ\,10'\,58''.82 = 1/2(P+Z) & & 69^\circ\,21'\,30''.\bar{1}2 = 1/2(P-Z)
\end{array}$$

$$\begin{array}{rl}
 & 0^\circ\,58'\,43''.06 = PS \\
 & 50^\circ\,21'\,06''.0\ \ = ZS \\
2\,| & 51^\circ\,19'\,49''.06 = PS+ZS \\
 & 25^\circ\,39'\,54''.5\bar{3} = 1/2(PS+ZS)
\end{array}$$

$$\tan 1/2\,PZ = \frac{\cos 70^\circ\,10'\,58''.82}{\cos 69^\circ\,21'\,30''.12}\tan 25^\circ\,39'\,54''.53$$

$$\begin{array}{l}
9.5302216 = \log\cos 1/2(P+Z) \\
9.5471860 = \log\cos 1/2(P-Z) \\
9.9830356 \\
9.6817102 = \log\tan 1/2(PS+ZS) \\
9.6647458 = \log\tan 1/2\,PZ
\end{array}$$

$$\begin{array}{l}
24^\circ\,48'\,08'' = 1/2\,PZ \\
\qquad\times 2 \\
49^\circ\,36'\,16'' = PZ = \text{co-latitude}
\end{array}$$

$$90^\circ - 49^\circ\,36'\,16'' = 40^\circ\,23'\,44'' = \text{latitude of place.}$$

336. Latitude from the Sun at Any Hour Angle. In the PZS triangle (Fig. 330d), the side PZ is the co-latitude of the place. The completion of the analysis of the data follows.

$$\tan 1/2\,PZ = \frac{\cos 1/2(Z+P)}{\cos 1/2(Z-P)}\tan 1/2(PS+ZS)$$

$$\begin{array}{rlrl}
 & 103^\circ 32' 44''.0 = Z & & 103^\circ 32' 44''.0 = Z \\
 & 41^\circ 34' 19''.5 = P & & 41^\circ 34' 19''.5 = P \\
2\,| & 145^\circ 07' 03''.5 = Z+P & 2\,| & 61^\circ 58' 24''.5 = Z-P \\
 & 72^\circ 33' 31''.8 = 1/2(Z+P) & & 30^\circ 59' 12''.2 = 1/2(Z-P)
\end{array}$$

$$\begin{array}{rl}
 & 66^\circ 53' 12''.0 = PS \\
 & 38^\circ 53' 06''.0 = ZS \\
2\,| & 105^\circ 46' 18''.0 = PS + ZS \\
 & 52^\circ 53' 09''.0 = 1/2(PS + ZS)
\end{array}$$

$$\tan 1/2\,PZ = \frac{\cos 72^\circ 33' 31''.8}{\cos 30^\circ 59' 12''.2} \tan 52^\circ 53' 09''.0$$

$$\begin{array}{l}
9.4767249 = \log \cos 1/2(Z + P) \\
9.9331260 = \log \cos 1/2(Z - P) \\
9.5435989 \\
0.1210861 = \log \tan 1/2(PS + ZS) \\
9.6646850 = \log \tan 1/2\,PZ
\end{array}$$

$$\begin{array}{l}
24^\circ 47' 57'' = 1/2\,PZ \\
\qquad\quad \times 2 \\
\hline
49^\circ 35' 54'' = PZ
\end{array}$$

$90^\circ - 49^\circ 35' 54'' = 40^\circ 24' 06''$ = latitude of place.

LONGITUDE AND TIME

337. Principles of Observations. Determination of the longitude of the place of observation or of the watch correction at the instant of observation are required infrequently in field surveying operations.

The general theory upon which such observations are based has been developed in the preceding pages of this chapter. The particular theories involved are that all celestial bodies reach their maximum altitude at meridian passage and that, since relations between the different bodies may be computed with the aid of ephemeris tables, observations for these purposes are taken on established meridians.

It should be remembered that local noon, mean or apparent, is that instant of time when the mean or apparent sun is on the local meridian. Another important principle is that a definite relation exists between longitude and time, also between sidereal time and right ascension. These relations are explained in § 317 and § 318.

338. Time at Local Apparent Noon. With the longitude of the place known and a meridian established, the transit is oriented in the meridian and pointed toward the path of the sun. The watch time instant when the preceding edge of the sun's disk is tangent to the vertical cross-hair is noted. The telescope is then quickly double-sighted and the same operation repeated. The second watch reading is taken when the following edge of the sun's disk is tangent to the vertical cross-hair. The mean of the two watch readings is the watch instant of meridian passage of the sun's center.

To determine the error in the watch time, it is necessary to analyze the relations as shown in Fig. 338.

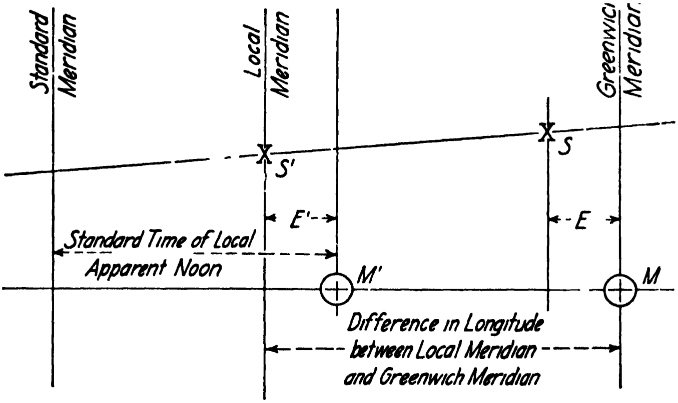
Standard Meridian
Local Meridian
Greenwich Meridian
S'
S
E'
E
Standard Time of Local Apparent Noon
M'
M
Difference in Longitude between Local Meridian and Greenwich Meridian

FIG. 338 SUN OBSERVATION FOR TIME

S represents the position of the apparent sun at Greenwich mean noon; *S'* at local apparent noon.

M represents the position of the mean sun at Greenwich mean noon; *M'* at local apparent noon.

E is the equation of time at Greenwich mean noon for the date of observation. The relative positions of the mean sun and apparent sun at Greenwich mean noon are determined from the ephemeris. In Fig. 338 *E* is to be added to mean time to get apparent time. The apparent sun is therefore ahead of (west of) the mean sun at Greenwich mean noon.

E' represents the equation of time at local apparent noon. *E'* = *E* ± difference in longitude between Greenwich and the local meridian (expressed in hours) multiplied by the variation in E per hour.

At the instant of local meridian passage of the apparent sun, the standard time watch should read the hour angle represented by the difference in longitude between the local and standard meridian ± E'. The actual watch reading at observation compared with the calculated reading is the watch correction.

For example, a time observation made at the place $5^h\ 48^m\ 18^s.7$ West on December 2, 1949, gave a mean watch reading of $11^h\ 39^m\ 02^s$ A.M.

The calculation of the watch correction is:

$0^h\ 10^m\ 36^s.2$ = E, from the solar ephemeris, Greenwich mean noon
$0^s.96$ = variation of E per hour (ephemeris)
$5^h\ 48^m\ 18^s.7$ = $5^h.8$ = difference in longitude, Greenwich to local meridian
$5.8 \times 0^s.96 = 5^s.6$ = change in E

E is decreasing between December 2 and December 3, therefore

$$0^h\ 10^m\ 36^s.2 - 5^s.6 = 0^h\ 10^m\ 30^s.6 = E'$$

$6^h - 5^h\ 48^m\ 18^s.7 = 0^h\ 11^m\ 41^s.3$ = difference in longitude between standard meridian and local meridian
$0^h\ 11^m\ 41^s.3 + 0^h\ 10^m\ 30^s.6 = 0^h\ 22^m\ 11^s.9$ A.M. = mean time hour angle
$12^h - 0^h\ 22^m\ 11^s.9 = 11^h\ 37^m\ 48^s.1$ = correct standard time of local apparent noon
$11^h\ 39^m\ 02^s.0 - 11^h\ 37^m\ 48^s.1 = 1^m\ 13^s.9$ = watch ahead of correct standard time

339. Time from a Star Observation. An inspection of either Fig. 318 or Fig. 318a shows that when a star is in the meridian of the observer (hour angle = 0^h), the local sidereal time at that instant is equal to the right ascension of the star.

Since values of right ascensions of all important stars are tabulated in the American Ephemeris and Nautical Almanac, the relation mentioned above is a useful one in determining time at meridian transit.

A set-up is made on an established meridian, the longitude of which is known. A prominent star, easily identified and preferably near the celestial equator, is sighted as it crosses the meridian. The watch time of meridian passage is recorded.

Two values are obtained from the American Ephemeris: the right ascension of the star for the date of observation and the civil time of 0^h sidereal time at Greenwich.

EXAMPLE. An observation made on Rigel (β Orionis) at Ross Camp ($5^h\,48^m\,18^s.7$ West) on Feb. 10, 1949 resulted in a watch time (central standard time) of $7^h\,38^m\,40^s$ P.M.

The American Ephemeris gave $5^h\,12^m\,05^s.70$ as the right ascension of the star and $14^h\,38^m\,38^s.45$ as the civil time of 0^h sidereal time at Greenwich.

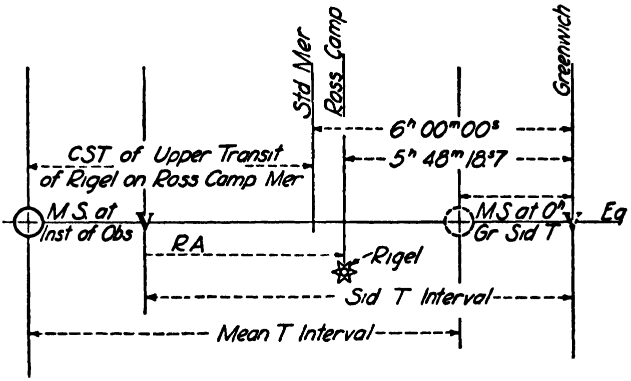

FIG 339 TIME FROM A STAR OBSERVATION.

Analysis:

$5^h\,48^m\,18^s.7$ = local longitude
$5^h\,12^m\,05^s.70$ = right ascension of star (from ephemeris)
$11^h\,00^m\,24^s.40$ = hour angle traversed by vernal equinox since 0^h sidereal time at Greenwich (sidereal time units)
$-\ 1^m\,48^s.19$ = conversion table (C)
$10^h\,58^m\,36^s.21$ = equivalent solar interval = hour angle traversed by mean sun (solar units)
$6^h\,00^m\,00^s$ = longitude of standard meridian
$2^h\,38^m\,38^s.45$ = civil time of 0^h sidereal time at Greenwich
$3^h\,21^m\,21^s.55$ = until noon (central standard time) at Greenwich 0^h (sidereal time)

$10^{h}\ 58^{m}\ 36^{s}.21$ = mean time hours since 0^{h} Greenwich, sidereal time
$-3^{h}\ 21^{m}\ 21^{s}.55$ = mean time hours until noon (C.S.T.)
$7^{h}\ 37^{m}\ 14^{s}.66$ P.M. = correct central standard time of upper transit of star
$7^{h}\ 38^{m}\ 40$ P.M. = watch time at instant of transit
$1^{m}\ 25^{s}.34$ = watch fast

340. Observations for Longitude. The difference in longitude between two places on Earth may be determined by observing the difference in time of meridian passage of the same celestial body over established meridians at each of the two places. Radio or telegraphic communication between the two places must be established so that interchange of signals and the synchronization of time pieces is as accurate as possible.

In order to determine the longitude of an established meridian, the approximate longitude may be obtained from a map. The standard time of meridian passage of a celestial body is then computed for the assumed longitude as in § 339. Comparing this calculated time with the actual watch time of observation gives the error in the assumption of longitude. A good watch or chronometer and close checking of radio time signals is necessary. Several trials, reducing the error in each successive estimation of longitude, should yield satisfactory results.

341. Precision. A comparison of results of astronomical observations, especially those made for azimuth or latitude, leads definitely to the conclusion that star observations yield better values. The United States Geological Survey instructions * to field parties, for example, clearly state that sun observations for azimuth "should be used only when authority is obtained in advance from the Chief, Section of Computing," and that Polaris observations "are more accurate than solar observations, are easier to compute, and under favorable weather conditions are easier to make." Such observations yield values within 30 seconds of true azimuth.

On the other hand, the Bureau of Land Management's 'Manual of Instructions for the Survey of the Public Lands of the United States" describes in detail the solar transit, and Mr. Arthur D. Kidder, District Cadastral Engineer, recommended the use of the Smith attachment in a paper presented at the 1940 Conference of Surveying Teachers. True meridians may be established with this attachment within one and one-half minutes.

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* *Instructions for Transit Traverse*, United States Geological Survey, 1940.

Under any circumstances it must be remembered that accuracy of results of astronomical observations depends largely upon the type of instrumental equipment, its adjustment, and upon the care exercised in its manipulation. The instant of observation must be read from a good watch rated by radio time signals or by Western Union Telegraph Company time signals.

OUTLINE FOR CONSTRUCTION OF STRAIGHT LINE DIAGRAM SUN OBSERVATION AT ANY HOUR ANGLE

1. From Ephemeris data calculate values for declination and equation of time at Greenwich Mean Noon for the date of observation and for the following date. These values for any date are averages of the tabular values for the preceding and following Greenwich 0 hour instants.

   EXAMPLE: Date of observation Dec. 6, 1949

DATE	DECLINATION			EQUATION OF TIME		
	0 Hours	Mean Noon	Rate	0 Hours	Mean Noon	Rate
Dec. 6	S 22° 26'.0	S 22° 29'.6	0'.31	$9^m\ 12^s.2$	$8^m\ 59^s.6$	$1^s.04$
Dec. 7	S 22° 33'.2	S 22° 36'.6		$8^m\ 47^s.0$	$8^m\ 34^s.1$	
Dec. 8	S 22° 40'.0			$8^m\ 21^s.2$		

2. Draw a horizontal line representing the equator. Draw a vertical line representing Greenwich. Draw vertical lines representing local and standard meridians.
3. Is declination south or north? (See the Ephemeris.) Is declination increasing or decreasing? The apparent sun moves along the ecliptic. Its distance from the equator is the declination of the sun. Draw a line representing the ecliptic.
4. Where is the mean sun at Greenwich mean noon? Show by 0.
5. Is the apparent sun ahead of or behind the mean sun? (See Ephemeris.) Draw a vertical line and show the apparent sun by X at Greenwich mean noon. Show by E the Equation of Time at Greenwich mean noon.

6. What is the standard time of observation? Show the mean sun's relation to the standard meridian at instant of observation. Is this Standard Time? Show the apparent sun's position at the same instant. Indicate by letter E' the equation of time at instant of observation. Is E' equal to E?
7. Show by d the declination at Greenwich mean noon. Show by d' the declination at instant of observation. Is $d = d'$?
8. Show on the diagram by letters *T. E.* the interval of mean time elapsed between the two instants: (1) Greenwich mean noon and (2) Instant of observation.
9. Show the hour angle of the apparent sun at instant of observation. Mark it t.

To Calculate E′ and d′

10. In the Ephemeris find the rate of change of E per hour. Using *T. E.*, find the change in E. Is E increasing or decreasing? Calculate E'.
11. In the Ephemeris find the rate of change of d per hour. Using *T. E.*, find change in d. Is d decreasing or increasing? Calculate d'.
12. Compute value of t and express it in arc units.

OUTLINE FOR CONSTRUCTION OF STRAIGHT LINE DIAGRAMS POLARIS AT ANY HOUR ANGLE

1. Draw two figures representing: (1) conditions at Washington mean noon, (2) at instant of observation. Washington longitude $= 5^h\ 08^m\ 15^s.78$ W.
2. In the American Ephemeris find right ascension and declination of Polaris for the date and place, also Sidereal Time of Washington Mean Noon. This second value is found by interpolating in the table (Sun for Washington apparent noon) which gives values for Sidereal Time of 0^h Civil Time at Washington.

To Construct Figure (1)

3. Draw a horizontal line representing the equator.
4. Draw a vertical line and label it Washington meridian.
5. Where is the mean sun at Washington mean noon? Show on the figure by 0.
6. Show by vertical lines the Standard Meridian and Local Meridian.
7. Show on the diagram difference in longitude between Washington meridian and standard meridian, also between Washington meridian and local meridian.
8. Show the position of the vernal equinox on the diagram. Mark it V.
9. Draw a vertical line representing the hour circle through Polaris. Show Polaris. What is the right ascension of Polaris from the Ephemeris? Show the right ascension of Polaris. Mark it *R. A.*
10. Show the right ascension of the Mean Sun by *R.A.M.S.* What relation does this bear to the Sidereal Time of Washington Mean Noon?

To Construct Figure (2)

11. Draw a horizontal line representing the equator.
12. Draw a vertical line representing Washington meridian.
13. Draw vertical lines representing the local and standard meridians.
14. Show difference in longitude as above.
15. Show the position of the Mean Sun at the instant of observation.
16. Mark the distance representing the Standard Time of observation.
17. Show by *V* the position of Vernal equinox at instant of observation.
18. Draw a vertical line representing the hour circle through Polaris at the instant of observation.
19. Show the right ascension of Polaris and of the Mean Sun at the instant of observation.
20. Right ascension of Mean Sun increases at the rate of 9.856 seconds per hour
21. How many hours have elapsed since Washington Mean Noon?
22. Mark this by *T. E.* on diagram.
23. Calculate *T. E.*
24. Calculate change in *R.A.M.S.* since Washington mean noon.
25. Calculate *R.A.M.S.* at instant of observation.
26. Show by *t* the hour angle of Polaris at instant of observation.
27. Calculate value of *t* and express it in arc units.

PROBLEMS

1. What is the zenith distance of the north pole at a place, the latitude of which is (a) 40° North? (b) 25° North? (c) 90° North? (d) 0° (e) 42° 13′ North? (f) 16° 28′ North? *Ans.* (a) 50

2. The latitude of Washington, D. C., is 38° 55′ 14″.0 North; of Cincinnati is 39° 08′ 19″.8 North; of Denver is 39° 40′ 36″.4 North; of St Louis is 38° 38′ 03″.0 North; of Mt. Wilson, Calif., is 34° 12′ 59″.5 North and of Columbia, Mo., is 38° 56′ 51″.7 North.

(a) What is the declination of a star which culminates at the zenith of each of these places? Show this by means of a sketch.

Ans. At Washington 38° 55′ 14″.0 North

(b) What is the zenith distance of Sirius at each place when the star is on the meridian of that place? The declination of Sirius is 16° 38′ South Sketch the relations. *Ans.* At Washington 55° 33′ 14″.0

(c) What is the altitude of Polaris at upper culmination and at lower culmination at each place? The declination of Polaris is 89° 01′ 50″ North Sketch the relations. *Ans.* At Washington at u.c. 39° 53′ 24″.0
at l.c. 37° 57′ 04″.0

3. Convert the following intervals of sidereal time into equivalent mean solar time intervals.

(a) $6^h 20^m 15^s.6$, (b) $12^h 56^m 37^s.0$, (c) $2^h 17^m 18^s.7$, (d) $18^h 20^m 37^s.6$, (e) $23^h 46^m 26^s.9$, (f) $9^h 59^m 16^s.9$. *Ans.* (a) $6^h 19^m 13^s.3$

4. Convert the following intervals of mean solar time into equivalent sidereal time intervals.

(a) $12^h\ 43^m\ 16^s.4$, (b) $6^h\ 12^m\ 19^s.8$, (c) $23^h\ 01^m\ 17^s.7$, (d) $2^h\ 37^m\ 44^s.4$, (e) $9^h\ 56^m\ 16^s.8$, (f) $15^h\ 08^m\ 06^s.4$ *Ans.* (a) $12^h\ 45^m\ 21^s.8$

5. The longitude of Washington, D. C., is $5^h\ 08^m\ 15^s.78$ West; of Cincinnati is $5^h\ 37^m\ 41^s.40$ West; of Denver is $6^h\ 59^m\ 47^s.72$ West; of St. Louis is $6^h\ 00^m\ 49^s.26$ West; of Mt. Wilson, Calif., is $7^h\ 52^m\ 14^s.33$ West; and of Columbia, Mo., is $6^h\ 09^m\ 18^s.33$ West. What is the longitude of each place expressed in degrees, minutes and seconds?

Ans. Washington 77° 03′ 56″.7 West

6. (a) If the local mean time at St. Louis is $2^h\ 45^m$ P.M. what is the local mean time at that same instant at all other places listed in problem 5? Sketch the relations. *Ans.* At Washington $3^h\ 37^m\ 33^s.48$ P.M l.m.t.

(b) The same as (a) except local mean time is $9^h\ 32^m\ 15^s$ A.M.

Ans. At Washington $10^h\ 24^m\ 48^s.48$ A.M. l.m.t.

(c) The same as (a) except substitute the words "sidereal time" for "local mean time." *Ans.* At Washington $3^h\ 37^m\ 33^s.48$ sid. time.

(d) The same as (a) except substitute the words "standard time" for local mean time." *Ans.* At Washington $3^h\ 45^m$ P.M. E.S.T.

(e) If the local mean time at St. Louis is $2^h\ 45^m$ P.M. and the date July 4, what is the local apparent time at that same instant at all other places listed in problem 5? Sketch the relations. Use ephemeris data.

Ans. On July 4, 1949, the equation of time at Gr. M. Noon was $4^m\ 12^s.4$ to be subtracted from mean time, variation $0^s.45$ per hour. The local app. time at Wash. was $3^h\ 33^m\ 17^s.18$ P.M.

(f) The same as (e) except local mean time is $9^h\ 32^m\ 15^s$ A.M. and the date Nov. 11. *Ans.* *E* on Nov. 11, 1949, at Gr. M. Noon was $15^m\ 57^s.6$ to be added to mean time, variation $0^s.28$ per hour. The local app. time at Wash. was $10^h\ 40^m\ 45^s.09$ A.M.

(g) Find the local apparent times at the places 8 mean time hours after the instant of time of problem (e).

Ans. On July 4, 1949, *E* was *increasing* at $0^s.45$ per hr. The local app. time at Wash. was $11^h\ 33^m\ 13^s.54$ P.M.

(h) The same as (g) except substitute the words "problem (f)" for problem (e)." *Ans.* On Nov. 11, 1949, as in problem (f) *E* was *decreasing* at $0^s.27$ per hour. The l.a.t. at Wash. was $6^h\ 40^m\ 42^s.96$ P.M.

7. The measured altitude of a star is (a) 23° 15′ 10″, (b) 46° 28′ 40″, c) 56° 28′ 15″, (d) 19° 10′ 30″, (e) 65° 19′ 50″, (f) 32° 12′ 00″. The emperature is 50° Fahrenheit. Compute the true altitude of the star.

Ans. (a) 23° 12′ 55″

8. The measured altitude of the sun's center is (a) 15° 20′ 30″, (b) 24°

08′ 40″, (c) 65° 42′ 30″, (d) 36° 18′ 10″, (e) 53° 19′ 30″, (f) 45° 08′ 10″. The temperature is 77° Fahrenheit. Compute the true altitude of the sun.

Ans. (a) 15° 17′ 18″

9. Observations are made on stars at culmination. The measured altitudes and temperature are as given in problem 7. From an ephemeris, the declinations of the stars are (a) 10° 12′ 06″ N., (b) 6° 07′ 12″ S., (c) 24° 19′ 26″ N., (d) 53° 08′ 12″ N., (e) 17° 18′ 22″ S., (f) 48° 09′ 16″ N. What are the latitudes of these places of observation? Sketch the relations.

Ans. (a) 76° 59′ 11″ N.

10. Observations are made on the sun at local apparent noon. The measured altitudes and temperature are as given in problem 8. From an ephemeris, the declinations are (a) 10° 12′.3 N., (b) 6° 07′.1 S., (c) 22 19′.4 S., (d) 23° 08′.5 N., (e) 17° 16′.4 N., (f) 12° 08′.7 S. What are the latitudes of these places of observation? Sketch the relations.

Ans. (a) 84° 55′ N.

11. Using an ephemeris, construct diagrams showing conditions at the ephemeris instant and at the instant of observation (see Figs. 330a and 330b) for a sun observation.

	Standard Time of Observation	Place	Date
(a)	10^h A.M.	Washington	January 10
(b)	3^h P.M.	Cincinnati	March 11
(c)	9^h A.M.	Denver	April 10
(d)	2^h P.M.	Mt. Wilson	July 16
(e)	$10^h\ 30^m$ A.M.	Columbia, Mo.	September 10

12. Using the American Ephemeris and Nautical Almanac, construct diagrams showing conditions at the ephemeris instant and at the instant of observation (see Figs. 327a and 327b) for a Polaris observation.

	Standard Time of Observation	Place	Date
(a)	8^h P.M.	Washington	January 10
(b)	$9^h\ 30^m$ P.M.	Cincinnati	March 11
(c)	$12^h\ 30^m$ A.M.	Denver	April 10
(d)	1^h A.M.	Mt. Wilson	July 16
(e)	$3^h\ 30^m$ A.M.	Columbia, Mo.	September 10

13. Find the Standard Time of eastern elongation of Polaris at Denver for the following dates:

(a) Feb. 6, (b) June 9, (c) Oct. 26, (d) Apr. 4, (e) Dec. 1.

Ans. See § 326

14. Find values for angle *Z* and side *PZ* in the following spherical triangles:

	Side ZS	Side PS	Angle P
(a)	43° 52′	77° 28′	$2^h\ 48^m\ 00^s$
(b)	35° 08′	18° 15′	$2^h\ 01^m\ 00^s$
(c)	42° 30′ 20″	100° 31′ 25″	$1^h\ 59^m\ 40^s$
(d)	59° 50′ 14″	19° 47′ 50″	$8^h\ 12^m\ 05^s$
(e)	52° 10′ 22″	62° 03′ 49″	$4^h\ 04^m\ 09^s$
(f)	69° 48′ 47″	112° 02′ 12″	$3^h\ 23^m\ 10^s$

Ans. $Z = 79° 29′ 16″$ or $100° 30′ 44″$

The larger Z angle is inside and $PZ = 55° 32′ 59″$.

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CHAPTER XXII

VERTICAL CONTROL

342. General Statement. A planimetric map of an area consists of the representation to a chosen scale of all physical features, natural or man made, existing within the area. The horizontal control, of all orders, established over the area furnishes the framework by which each topographic detail is placed in its proper position on the map.

Similarly, if it is desired to show by some artificial means, such as hachures or contours, the relative differences of elevation over the area it becomes necessary to establish a system of vertical control. This is analogous to horizontal control in that it comprises a general level net of a high order of precision interlaced with connecting lines of somewhat lower order. These correspond to the principal triangulation or traverse control, and the supplementary traverses, respectively.

343. Establishment of Control. The vertical control system should be planned according to the needs of the job and after careful reconnaissance over the area. Each level line should be a loop ending at its place of beginning or closing upon another loop of equal or higher precision. Permanent bench marks should be established at reasonable intervals. There is no hard and fast rule for the spacing of bench marks, but they should be set wherever they will be most convenient for future level work. Bench marks near street or road intersections afford maximum opportunity for starting or ending connecting level lines

Permanent points established for the horizontal control can serve a dual use by being made bench marks. This is particularly true of traverse control monuments which are nearly always accessible for rod readings.

The sea level datum established for use in the United States * should be the datum of reference for all vertical control unless there is some very good reason for using another. The deplorable condition existing in some cities in which each department doing level work has its own datum will never be improved by continuing the same practice.

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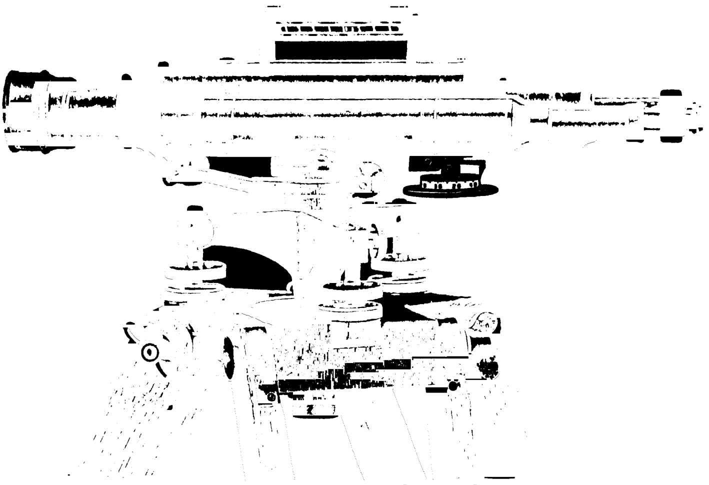
* See references to this subject at end of chapter.

344. Bench Marks. The essential specifications for a bench mark are that it be permanent, identifiable, and accessible. Pre-cast concrete monuments with identifying cap similar to those used as traverse monuments are very satisfactory. These ordinarily are set flush with the ground. A better practice would be to set them lower and enclose them within a heavy cast-iron cylinder with a removable cover. The most common location along streets is between the sidewalk and curb and along roads near the property fences. A standard tablet, similar to Fig. 278a, set in the top of a bridge abutment or heavy masonry wall is an excellent bench mark.

345. Specifications. The relative precision of leveling is usually stated as an allowable tolerance in a certain distance. In first order leveling, the allowable error of closure is commonly taken as about ± 0.02 ft. $\sqrt{M}$ in which M is length of the circuit in miles. For second order leveling the figure is ± 0.05 ft. $\sqrt{M}$ and for ordinary leveling, ± 0.10 ft. $\sqrt{M}$.

346. Equipment. The very small tolerance given in first order leveling necessitates use of the most precise levels and rods.

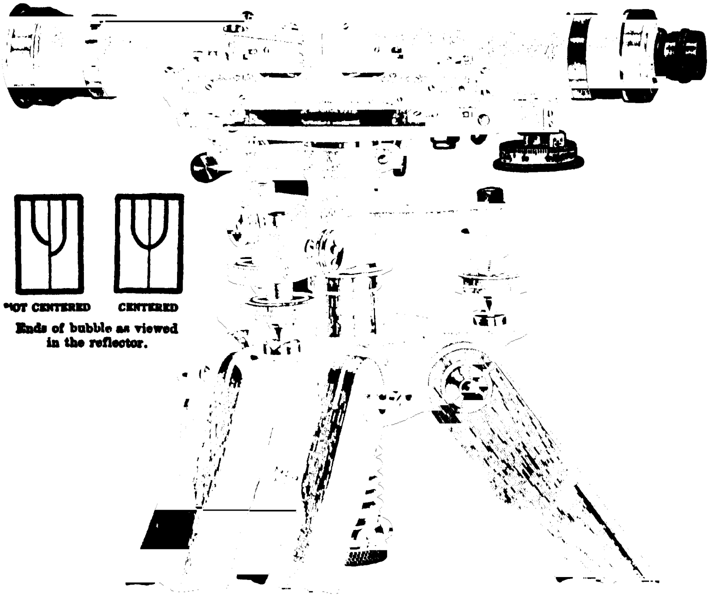


Courtesy of Keuffel and Esser Co

FIG. 346. PRECISE LEVEL. U. S. C. & G. S.

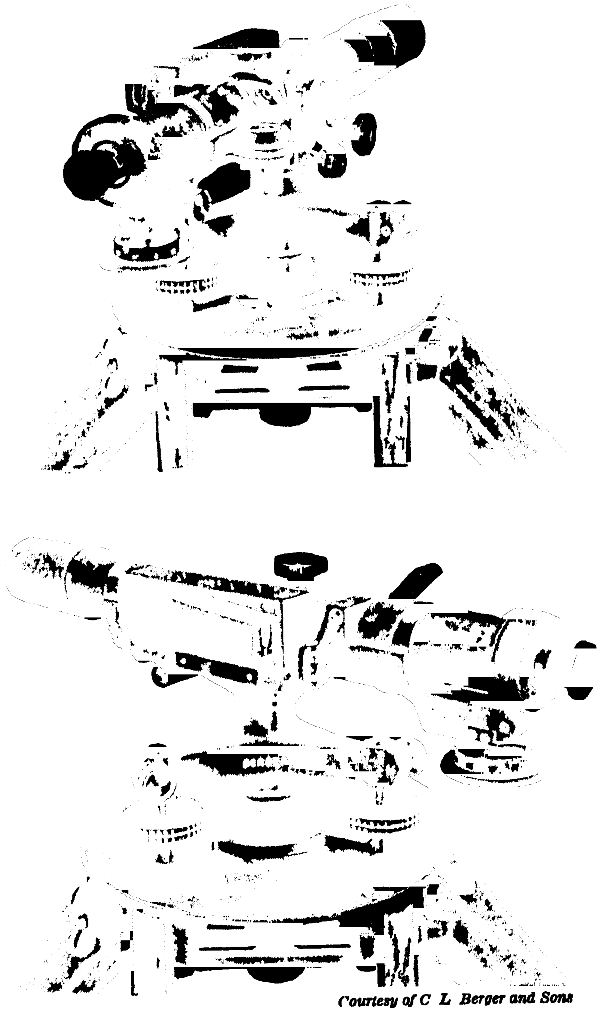
Figure 346 shows the United States Coast and Geodetic Survey type of precise level. This instrument is built of the best materials to rigid specifications. The bubble is very sensitive; a movement of 2 mm. equals a vertical angle of 2 seconds. Approximate leveling of the instrument is obtained by means of a circular bubble through manipulation of the leveling screws. Final leveling is accomplished by means of a slow motion screw beneath the eyepiece, the upper part of the instrument being hinged at the front. The telescope bubble is viewed through the left barrel of the instrument and a double prism permits the matching of the two ends of the bubble into a continuous curve at the instant of centering, as in Fig. 346a. The telescope is equipped with two additional horizontal hairs similar to stadia hairs in a transit by means of which errors in reading the rods can be minimized and sight distances balanced by the stadia principle.

Another type of precise level, known as the United States Geological Survey level, is shown in Fig. 346a. This instrument,



Courtesy of Keuffel and Esser Co.

Fig. 346a. Precise Level. U.S.G.S.



Courtesy of C L Berger and Sons

Fig. 346b.

though much less expensive, embodies all of the principles of the level described above less certain refinements of construction. It has a less sensitive bubble, 2 mm. equaling about 10 seconds.

Figure 346b shows two views of a 15-inch precise tilting dumpy level recently perfected by C. L. Berger and Sons. It is equipped with a prismatic reading device which enables the levelman to read the rod and bubble position simultaneously.

Second order precision can be obtained by means of a first class engineer's level if careful consideration is given to all the sources of errors in level work. In all probability more loops must be re-run to obtain satisfactory closures and therefore they should not be made too long. Two miles in hilly country or five miles over level ground are reasonable loop distances.

Second order and ordinary precision can sometimes be obtained more quickly and more cheaply with a precise level since longer sights and longer loops are permissible, and fewer re-runs are necessary.

Rods used with the precise levels are of the type shown in Fig. 346c. The United States Coast and Geodetic Survey uses the metric system of linear units and the United States Geological Survey uses the yard and decimals thereof. These rods are very well constructed of selected wood and made in the form of a

Fig. 346c. Precise Rod.

T to prevent warping. The larger graduations are painted directly upon the wood face and the smaller ones marked on an invar strip securely fastened at the bottom and held in place by a tension spring at the top. This arrangement permits the wooden rod to vary in length somewhat due to moisture or temperature changes without in any way affecting the length of the invar strip which has a much lower coefficient of thermal expansion.

Precise level rods are not equipped with targets, all readings being taken directly to 0.001 meter or 0.001 yard. A steel shoe slightly rounded on the bottom forms the base of the rod. This insures that a turning point taken on any solid object will actually be a point and not a small area with a possible difference in the rod readings taken on it. A circular bubble, for use in plumbing, and a thermometer are attached to the back of precise level rods.

Other equipment for a precise level party should include a large umbrella and a portable windbreak for protection of the instrument, steel pins for use as turning points, and a hand axe.

347. Field Procedure. A precise level party consists of five men: an instrument man who usually is the chief of party, two rodmen, a recorder who also does field computations, and a man to take care of the umbrella and windbreak. Under favorable weather conditions, the recorder can handle the umbrella and thus reduce the party to four.

The following general rules for field procedure in first order leveling are the recommendations of a Committee on City Surveys of the Surveying and Mapping Division of the American Society of Civil Engineers: *

"1. All lines shall be leveled forward and backward independently and preferably under different atmospheric conditions, that is, on different days or in the forenoon and in the afternoon of the same day.

"2. The forward and backward runnings along any line shall begin and end on permanent bench marks. No temporary bench marks may be used to hold the line, not even over night.

"3. Forward and backward levelings should be repeated again and again, if necessary, until the difference between the two is not greater than 0.017 ft.$\sqrt{M}$, in which, M = length of section, in miles. (Judgment should be exercised in the application of this rule.) If any measure over a section gives a result differing by more than 0.020 ft. from the mean of the other measures over the section, this measure shall be rejected.

---

* Am. Soc. C. E. *Manual of Engineering Practice*, No 10 1934.

Courtesy of Cleveland Regional Geodetic Survey

FIG. 347. PRECISE LEVEL PARTY.

"4. The observation program at each station should be as follows: Set up and level the instrument. Read the three wires of the diaphragm, as seen projected against one of the rods, to the nearest 0.001 yd., while the bubble is held continuously in the middle of the tube. Immediately after this, read the three lines of the diaphragm as seen projected against the other rod. The abstract sum of the three wire readings (equalling the mean reading, in feet) may differ from the middle wire reading multiplied by three, by not more than 0.002 ft. It is desirable that the rod interval between the middle and upper wires shall not differ from the one read between the middle and lower wires by more than 0.002 yd. The recorder tests the results immediately after the observation, and the instrument should not be moved unless and until the tests satisfy these rules.

"5. In the record, all instrument stations are to be numbered consecutively assigning the number, 1, to the first station of the day. At odd stations the back-sight is taken before the fore-sight, while at even stations the fore-sight is taken before the back-sight. Since the same rod is held on a station for both fore-sight and back-sight, the effect of this practice is that at each set-up, the same rod, that is, the rod used for the back-sight at the first instrument station, is read first.

"6. At each rod station the thermometer is read to the nearest degree and the temperature is recorded.

"7. The difference in length between a fore-sight and a corresponding back-sight must not exceed 10 yd.; in fact, it should be made as small as feasible without extra expenditure of time. The recorder shall record the stadia distances, in yards, for both back-sight and fore-sight, together with their continuous sums for each section. The sums of the back-sight distances and the fore-sight distances shall be kept as nearly equal as feasible without extra expenditure of time, and shall never differ by more than 20 yd.

"8. One daily observation of the collimation error of the level should be made in the regular course of leveling and should be recorded in a separate opening of the record book, as follows: Make the last fore-sight reading at an instrument station a part of the collimation check; have the back rod placed about 10 yd. back of the instrument; read the rod and move the instrument to a position about 10 yd. behind the fore-sight rod; and read the fore-sight and the back-sight rods. The required constant, C, to be determined, namely, the ratio of the correction for any rod reading to the corresponding subtended rod interval, is

$$C = \frac{I_n - I_d}{i_d - i_n} \tag{347}$$

in which, I_n = sum of the two near rod readings, I_d = sum of the two distant rod readings, i_n = sum of the two near rod intervals between upper and lower stadia hairs and i_d = sum of the two distant rod intervals, respectively. The total correction for curvature and refraction must be applied to the sum of the distant rod readings before substituting it in equation 347. The level should not be adjusted if C is less than 0.005. If C is between 0.005 and 0.010, adjustment is not advisable, but if C is greater than 0.010 the level must be adjusted, and C should be re-determined at once. The adjustment for reducing C must be made by moving the level vial, not the reticule.

"9. The instrument shall be shaded from the direct rays of the sun, both during observation and when moving from station to station.

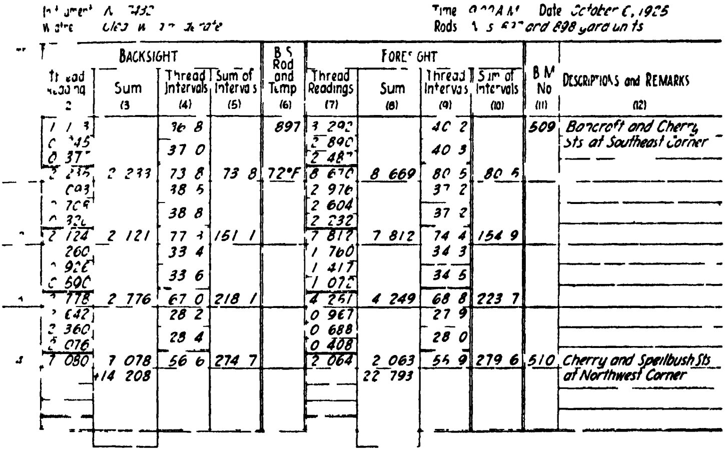
"10. The maximum length of sight shall be 150 yd., and this maximum shall be permitted only under the most favorable conditions.

"The rod station or turning point may be any substantial object having a definite high point, on which the rod can be supported and turned for both fore-sight and back-sight. Bricks in pavements, corners of expansion joints of concrete curbs or sidewalks, stones or boulders firmly embedded in the ground, tops of tie-spikes along railroad tracks, and similar objects, make excellent rod stations. Where such natural rod stations are lacking, portable turning points should be used. When using tie-spikes along a railroad track, after the passage of trains sufficient time must be allowed for the rails and ties to return to their normal elevation.

"First-Order Level Notes. A sample page of first-order level notes is reproduced in Fig. 347a. In this case only the general arrangement and the essential details are indicated. Actually, such forms should provide for records of personnel, file numbers, miscellaneous remarks, etc.

"Columns (2) and (7) contain the three thread readings in their proper order. The quantity below a set of three thread readings is the middle thread reading, multiplied by 3. It is used only as a check on the sum or mean of the three readings. If rods graduated in feet or in meters are used, this value would be the sum of three readings, while Columns (3) and (8) would represent this quantity divided by 3.

"Columns (3) and (8) contain the sum of the three thread readings, which is numerically equal to the mean reading, in feet. If rods graduated in feet or in meters are used, this column should be headed "Mean," and would contain sums of the three thread readings, divided by 3.



Instrument [illegible] Time [illegible] A.M. Date October 6, 1925
Weather [illegible] Rods [illegible] and 898 yard units

Thread Reading (2)	Backsight Sum (3)	Backsight Thread Intervals (4)	Backsight Sum of Intervals (5)	B.S. Rod and Temp (6)	Foresight Thread Readings (7)	Foresight Sum (8)	Foresight Thread Intervals (9)	Foresight Sum of Intervals (10)	B.M. No. (11)	Descriptions and Remarks (12)
1 113		36 8		897	3 292		40 2		509	Bancroft and Cherry
0 745		37 0			2 890		40 3			Sts at Southeast Corner
0 377					2 487					
2 235	2 233	73 8	73 8	72°F	8 670	8 669	80 5	80 5		
1 093		38 5			2 976		37 2			
0 709		38 8			2 604		37 2			
0 322					2 232					
2 124	2 121	77 3	151 1		7 812	7 812	74 4	154 9		
1 260		33 4			1 760		34 3			
0 928		33 6			1 417		34 5			
0 590					1 072					
2 778	2 776	67 0	218 1		4 251	4 249	68 8	223 7		
2 642		28 2			0 967		27 9			
2 360		28 4			0 688		28 0			
2 076					0 408					
7 080	7 078	56 6	274 7		2 064	2 063	55 9	279 6	510	Cherry and Spellbush Sts
	+14 208					22 793				at Northwest Corner

Courtesy of American Society of Civil Engineers

FIG. 347a. FIRST-ORDER LEVEL NOTES.

"Columns (4) and (9) show the rod intervals intercepted on the rod between the middle and the upper threads, and between the lower and the middle threads. The two intervals, of course, should be to each other in the same proportion as the actual intervals in the diaphragm are to each other. For the sake of convenience, the thread intervals are multiplied by 100, the threads in this instrument being set at a ratio of 1 : 100. The sum of the two intervals is shown below them.

"Columns (5) and (10) show the cumulative sum of the intervals. With this value shown for each instrument station, the total back-sight and fore-sight distances may be kept nearly equal.

"Column (6) shows the number and the temperature of the rod used in the back-sight at each bench-mark.

"From these notes, of course, the forward-run difference of elevation between B.M. 509 and B.M. 510 is determined as follows:

$$
\begin{aligned}
\text{Sum of fore-sights} &= -\ 22.793\\
\text{Sum of back-sights} &= \underline{+\ 14.208}\\
\text{Difference} &= -\quad 8.585."
\end{aligned}
$$

Second order leveling usually requires but two men, an instrumentman and a rodman. Two rodmen will increase the speed of the work somewhat but without decreasing cost.

The principal rules of field procedure are as follows:

"1. The level shall be tested for error of bubble once each day, and shall be adjusted if found to be in error by more than 0.01 ft. per 100 ft. of distance.

"2. The difference in length between a fore-sight and a corresponding back-sight for any set-up shall not exceed approximately 50 ft. (Distances in second-order leveling are usually paced, and the number of paces on each sight is noted in the record book.)

"3. The cumulative back-sight distances shall never differ from the cumulative fore-sight distances on any section by more than about 100 ft. (A continuous total of the number of paces for both back-sight and fore-sight distances should be kept in the record book.)

"Only one wire is read in this class of leveling, and to avoid the possibility of extensive re-running due to reading the bottom or top wire instead of the middle wire, it is advisable to use a level having only one wire."

348. Computation and Adjustment. Computations begin with a careful check of the field notes to make certain there are no errors in the summation of the three hair readings and that the upper and lower stadia intervals agree within satisfactory limits. These were checked by the recorder in the field but occasionally an error is found. The notes are then abstracted to determine the mean difference in elevation of each section from the outward and the return runs. Ordinarily these values receive no correction unless the rods used are known to have an error in absolute length or such extreme temperatures are encountered as might appreciably affect the length of the invar strip. The differences in elevation of each section are used to compute the closures of the loops.

The next computation is the adjustment of the difference in elevation of each loop side to eliminate the loop closures. In first order levels, this adjustment is made by the method of least squares (see reference 3 on page 442).

The precision of the best level work done by surveyors other than those connected with governmental agencies will, in all probability, fall between first and second orders. The level used will be a good engineers' level or one of the Geological Survey type using one-hair or three-hair readings. The results of this work could be adjusted by least squares, but the following method is much less cumbersome and quite satisfactory for any but the very highest type of work.

The error of closure of a level circuit is an accumulation of errors that have occurred all along the line. It is evident, therefore, that all intermediate bench mark elevations are probably in error by a fraction of the total error of closure. Since the principal errors in level work are accidental in nature the mathematics of probability shows that the probable error varies as the square root of the number of chances for error. This could be taken to mean the square root of the number of rod readings, set-ups, or miles traveled, assuming the same number of set-ups per mile.

The laws of probability state also that the *weights*, or measure of reliability, that should be given to any values are inversely proportional to the squares of the probable errors in them.

Thus the reliability of a bench mark elevation should be inversely proportional to its distance from the beginning of the circuit. Conversely, the correction to be applied to any elevation should be directly proportional to its distance from the initial bench mark.

EXAMPLE:

STATION	DISTANCE MILES	OBSERVED ELEVATION	CORRECTION	CORRECTED ELEVATION
BM_1	0	615.49	0.00	615.49
BM_2	2	648.23	0.04	648.27
BM_3	3	703.91	0.06	703.97
BM_1	6	615.37	0.12	615.49

$$\text{Error of Closure} = 0.12$$
$$C_2 = 2/6 \times 0.12 = 0.04$$
$$C_3 = 3/6 \times 0.12 = 0.06$$
$$C_1 = 6/6 \times 0.12 = 0.12.$$

This scheme of adjustment can be used equally well upon a line of levels that does not close at the starting bench mark but upon another the elevation of which has been determined previously.

Establishment of vertical control over a considerable area will necessitate the running of several level loops. These will have certain bench marks in common for which different elevations will be found from several circuits. In such cases the separate adjustment of each loop, explained above, is considered a preliminary adjustment since no inter-relation with the level net as a whole has been established. The elevations for any one junction point should be assigned weights in inverse ratio to the distances from accepted bench marks from which each elevation was obtained. The weighted mean obtained will then be taken as the most probable value of the elevation. Secondary adjustments of other elevations in each loop can then be made by working backwards from the junction point elevations with due regard to the sign of the new error of closure.

It is obvious that this process could be repeated for other junction points but if the field work has been done properly the changes in elevations become negligible. If the field work has not been done well, accurate results cannot be obtained by any amount of mathematical adjustment.

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CHAPTER XXIII

TOPOGRAPHIC DETAILS

FIELD METHODS

349. Stadia Measurements. The theory of stadia measurements is explained in Chapter IX. The application of that theory to topographic mapping provides a rapid, economical method of locating details. It is discussed in Chapter XIV.

The accuracy (§ 143) with which a point may be located by stadia is well within the range of accuracy possible in locating that point on the map. For example, when the map scale is 100 feet to the inch, stadia measurements on the order of 1 in 500 result in displacements of points on the map of only ± 0.002 inch for each hundred feet of measured distance.

Stadia measurements are made with either the transit or the plane table alidade. When the transit is used, rather elaborate field notes (see Fig. 352) are taken and field sketches are made. The plane table method, which is primarily a field mapping operation, necessitates few such notes. In either case the stadia multiplying constant (§ 137) for the instrument must be carefully determined.

THE TRANSIT METHOD

350. Equipment. The transit best suited for topographic mapping is equipped with a full vertical circle * with attached vernier reading to one minute or thirty seconds, preferably the latter. The least count of the horizontal vernier should be the same as that of the vertical circle vernier. The stadia hairs should be fixed to the cross-hair ring and the stadia multiplying constant should be 100.000. An instrument such as is illustrated on page 100 serves admirably as a topographic survey transit. For ordinary surveys, the usual level rod may be used as stadia rod. Where the survey is extensive or long sights are

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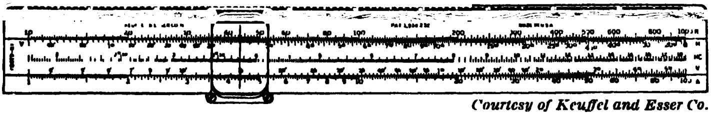
*** The ordinary transit is not equipped with a vernier bubble. This addition would be helpful for transit-stadia surveys.**

taken, special stadia rods such as are shown on page 169 will be needed.

The field party must also be supplied with equipment which will make possible the rapid and accurate reduction of inclined sights to horizontal and vertical components. Stadia circles or arcs attached to the transit or stadia reduction tables, diagrams, or slide rules serve such purposes, the latter being favored by most engineers. Directions for operating them are found on the backs of most rules or are given in instruction books supplied by the manufacturer.



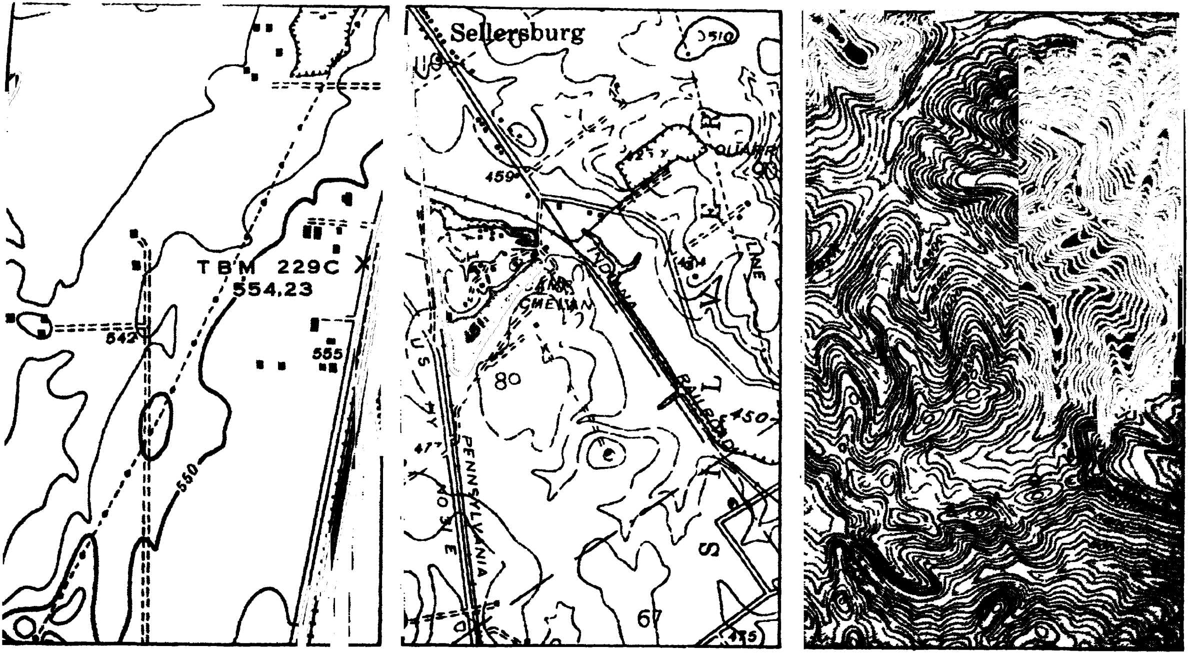
Courtesy of Keuffel and Esser Co.

FIG. 350. THE STADIA SLIDE RULE.

351. Control. Control of the survey is necessary in both horizontal and vertical planes. The best practice is to establish such control completely before any attempt is made to delineate any part of the map. It must be remembered that the accuracy of the final map depends primarily upon the accuracy with which the control points are established on the map.

Any form of control (Chapters XX and XXII) may be used, the selection depending upon the extent of the survey, scale of the map, contour interval, purpose of the map, and roughness of the area being surveyed. For example, a topographic map made to serve as the basis for the landscape design of a park or private estate is best controlled by a series of uniformly-spaced, parallel, straight lines, staked at regular intervals, usually 50 or 100 feet. This forms a checkerboard control system over the area. Details of topography can then be tied to this system with the tape, level, and rod. Such maps are plotted to scales of 50, 100, or 200 feet to the inch with contour intervals of from 1 to 5 feet. Grading estimates (see § 209 and § 102, Rubey's *Route Surveys*) may be made from such maps and the architect's designated locations of structures, drives, trees, and shrubs may be transferred from the map to the ground by means of scaled ties to the checkerboard corners.

The selection of scale and contour interval for the quadrangle



Scale 1/12,000

Scale 1/24,000

Scale 1/125,000

FIG. 351. COMPARISON OF TOPOGRAPHIC MAP SCALES.

surveys of the United States Geological Survey are based upon the importance of the area to the general public. For example, if the area to be surveyed is one in which there are problems of great public importance such as flood control, irrigation, reclamation, or mineral development, the scale used is either 2000 feet to the inch (1/24,000) or about one-half mile to the inch (1/31,680) and the contour interval 1, 5, or 10 feet. If the area includes problems of average public importance, the scale used is about one mile to the inch (1/62,500), with a contour interval of from 10 to 50 feet. Areas of lesser importance have been mapped to a scale of about two miles to the inch (1/125,000), and with a contour interval of from 20 to 100 feet.

Control for these surveys is established generally by transit and tape traverses supplementing whatever triangulation data are available in the area. Level circuits are run through the area to establish vertical control. Details of topography are filled in by plane table and aerial photographs.

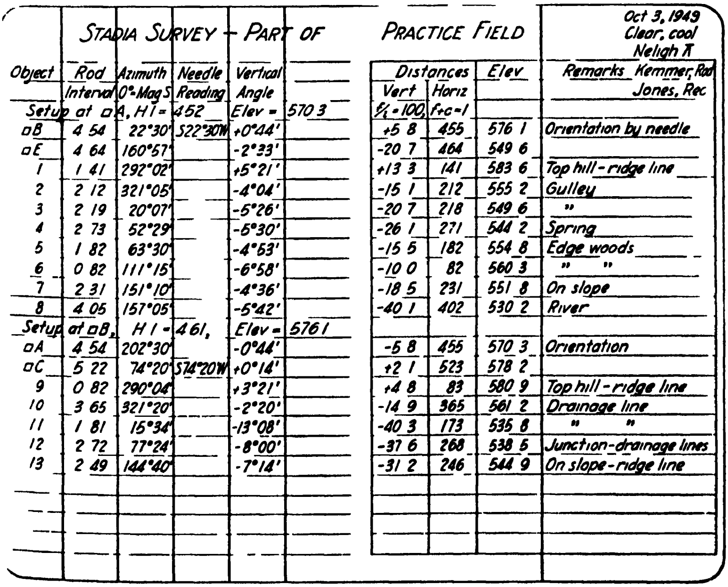
The survey of the Wabash River and Valley, directed by the United States Engineer Office, Louisville, Ky., was controlled by means of transit traverses and precise levels. Aerial photographs were used to fill in topographic details. These maps are printed to a scale of 1 inch = 1000 feet (1/12,000) and with a contour interval of 10 feet.

352. Transit Survey Details. With the control system established and calculated, the transit is set up over one of the control points and oriented by sighting toward another control point. Orientation (direction control) may be accomplished by using either the magnetic azimuth or the true azimuth of the line connecting the two control points. True azimuth is the better control especially if local attraction exists. Magnetic azimuth is more convenient, since it does not involve determination of the true meridian and because a needle check on the accuracy of orientation can be quickly made at any point of set-up. Having determined the azimuth, the vernier on the horizontal circle is set to the calculated azimuth angle, using that set of graduations which read in a clockwise direction around the graduated circle. With the upper motion clamped and lower motion free, the sight is taken to the control point.

After clamping the lower motion, the procedure may be checked by setting the vernier to read 0° 00′ and then releasing the magnetic needle. If the azimuth used were referred to the magnetic meridian (0° azimuth at the South point), the needle should read 0° 00′ and the telescope, with attached bubble down, should be pointing toward magnetic south. Some engineers prefer to use magnetic north as 0° azimuth. If true azimuth were used in orienting, the needle should read the magnetic declination unless this angle had been set off on the variation plate of the compass. Here, too, the needle would read 0° 00′

After the transit has been oriented and checked for orientation, the direction of any point can be determined merely by sighting toward that point and reading the horizontal angle (in the clockwise direction). The lower motion must be tightly clamped at all times while these side shots are being taken from the point of set-up.

The height of instrument above the control point and the elevation of the control point, previously determined with the level, are recorded in the stadia field notes, as in Fig. 352.



STADIA SURVEY — PART OF PRACTICE FIELD

Oct 3, 1949
Clear, cool
Neligh ⊼

Object	Rod Interval	Azimuth 0°-Mag S	Needle Reading	Vertical Angle		Distances Vert $f/i = 100$,	Distances Horiz $f+c = 1$	Elev	Remarks Kemmer, Rod Jones, Rec
Setup	at □	A, HI =	4 52	Elev =	570 3				
□B	4 54	22°30′	S22°30′W	+0°44′		+5 8	455	576 1	Orientation by needle
□E	4 64	160°57′		-2°33′		-20 7	464	549 6	
1	1 41	292°02′		+5°21′		+13 3	141	583 6	Top hill-ridge line
2	2 12	321°05′		-4°04′		-15 1	212	555 2	Gulley
3	2 19	20°07′		-5°26′		-20 7	218	549 6	"
4	2 73	52°29′		-5°30′		-26 1	271	544 2	Spring
5	1 82	63°30′		-4°53′		-15 5	182	554 8	Edge woods
6	0 82	111°15′		-6°58′		-10 0	82	560 3	" "
7	2 31	151°10′		-4°36′		-18 5	231	551 8	On slope
8	4 05	157°05′		-5°42′		-40 1	402	530 2	River
Setup	at □B,	H I -	4 61,	Elev =	576 1				
□A	4 54	202°30′		-0°44′		-5 8	455	570 3	Orientation
□C	5 22	74°20′	S74°20′W	+0°14′		+2 1	523	578 2	
9	0 82	290°04′		+3°21′		+4 8	83	580 9	Top hill-ridge line
10	3 65	321°20′		-2°20′		-14 9	365	561 2	Drainage line
11	1 81	15°34′		-13°08′		-40 3	173	535 8	" "
12	2 72	77°24′		-8°00′		-37 6	268	538 5	Junction-drainage lines
13	2 49	144°40′		-7°14′		-31 2	246	544 9	On slope-ridge line

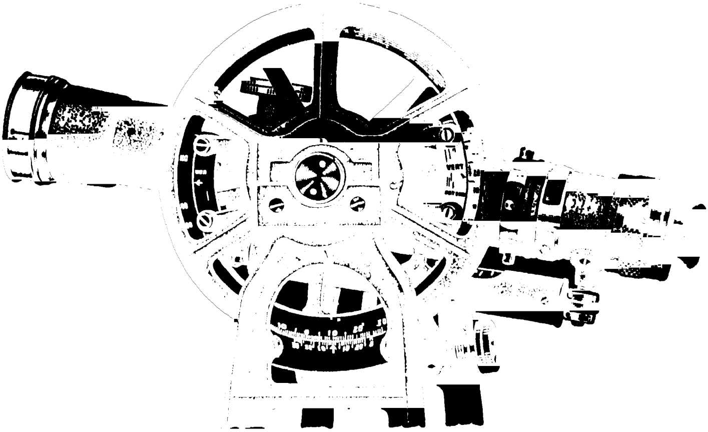
FIG 352. STADIA SURVEY NOTES.

The distance to and elevation of a side shot are found by sighting toward the stadia rod held at the point to be located. The lower stadia hair is first set on an even foot mark, as 2.00 or 3.00. The upper hair reading minus the lower hair reading gives the rod intercept. The instrumentman should call these two readings aloud so that the recorder has a chance to check the mental subtraction made by the instrumentman and recorded in the field notebook. The middle hair is then set to the H.I. point on the rod and the horizontal and vertical motions of the instrument are clamped. Both angular readings are taken and recorded as the rodman moves to the next point.

The horizontal distance to the point and the difference in elevation between the foot of the rod and the point over which the instrument is set are found by substitution in the formula for inclined sights (§ 133) if exact values are required or by the use of the stadia slide rule, diagram, reduction table, or approximate formulas. This reduction may be made either in the field or in the office. The horizontal distance and azimuth angle are used to plot the location of the point and the difference in elevation added to or subtracted from the elevation of the control point gives the elevation of the point at which the rod was held.

It is necessary, occasionally, to set up the transit at points outside of the established control system. These points, called stadia stations, are located by stadia. Their positions and elevations with reference to the plotted triangulation or traverse stations should be checked by reading all angle and rod intercept values in both directions, i.e., from triangulation station (△) to stadia station (⊡) and then, after occupying the stadia station, from stadia station back to the triangulation station. A series of stadia stations (a stadia traverse) must close and check on one of the principal control points.

353. The Stadia Circle. Figure 353 illustrates a special form of stadia reduction device which is manufactured by the Keuffel and Esser Company. In the figure, the lower scale is the standard vertical angle scale and vernier. The other two sets of graduations (Hor. and Vert. in Fig. 353) give multiplying factors which when applied to the rod intercept result in horizontal and vertical components of the stadia distance. Thus an ordinary slide rule instead of the special stadia rule may be used to reduce the notes with a resulting accuracy comparable to that usually attained with the stadia rule and sufficient for practically all mapping needs. When the instrument is in adjustment, the vertical index reads 0 and the horizontal index reads 100, the line of sight being truly horizontal.



Courtesy of Keuffel and Esser Co

FIG. 353. STADIA CIRCLE.

EXAMPLE. In Fig. 353, the vertical index reads + 17.8 and the horizontal, 96.7. On the assumption that the middle horizontal hair was set on the H.I. point on the rod (at 4.83) and that the lower stadia hair then read 1.92 and the upper hair 7.74, the rod intercept was 5.82. Therefore, 5.82 × 17.8 = + 103.6 feet is the difference in elevation, and 5.82 × 96.7 = 563 feet is the horizontal distance.

If the telescope is of the conventional focusing type, 0.01 foot, the $f + c$ correction, should be added to the rod intercept before applying the multiplying factor. This addition is not necessary when the telescope has an internal focusing lens.

354. Plotting Stadia Notes. The reproduction of the field work in map form is a most important part of the topographic survey. Many engineers prefer to do a large part of this work in the field and, when weather conditions permit, such procedure usually results in a map of superior quality. With topographer, recorder, transitman, and rodman working as a team, points and objects on the ground can be selected and mapped with greater rapidity and efficiency than when all of the mapping operation is confined to the office drafting table. Contour lines are best shaped when the ground being contoured is viewed by the "topographic eye" of the field draftsman and when the "topography soaks up through the soles of his shoes."

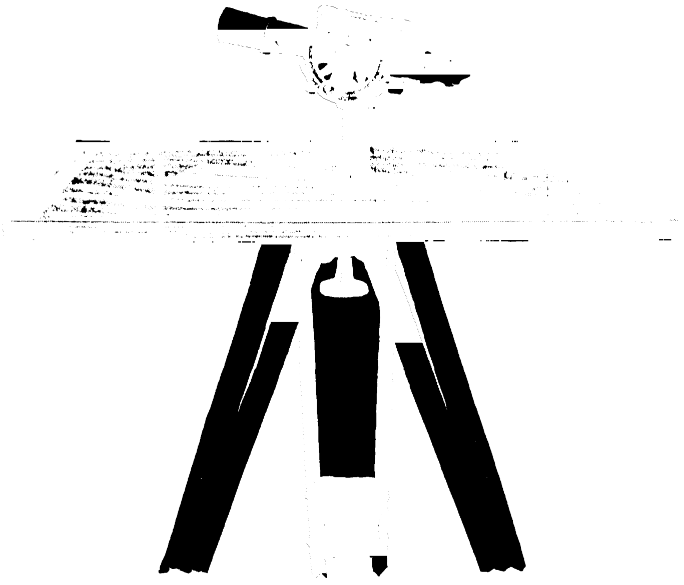
The equipment needed by the topographer depends upon the form of control and the method used to fill in details. For example, in the simple checkerboard control system the squares may be plotted on the field sheet before the detailed survey begins. Triangles, scale, compass, and drawing pencils constitute the topographer's field equipment. Points and elevations are plotted as rapidly as they are determined in the field and contour interpolation is completed square by square (see page 178).

If triangulation and traverse control has been established, such control points are accurately plotted on the field sheet and the details are plotted with reference to these points by protractor and scale. Where and when weather conditions permit its use, tracing linen, preferably waterproofed, may be used in the field. When the control system has been inked on the linen, a full circle paper protractor may be oriented by azimuth angle and fastened to the field drawing board, the center of the protractor being directly under the plotted position of the particular control point occupied by the transit. The paper protractor then represents the horizontal circle of the transit and the azimuth of and distance to a located point may be plotted. A small needle driven through the plotted triangulation point and the center of the protractor and a very small notch cut into the zero point of the scale facilitate this plotting process. As with checkerboard control, this field mapping should keep pace with the work of the instrumentman and recorder. Contours are interpolated between plotted elevation points when the ground surface is sloping uniformly between such points. With practice, rodmen soon learn to select critical points on drainage and ridge lines and thus aid materially in making accurate map reproductions of ground surfaces.

Map drafting in the office instead of in the field does not depend upon weather conditions. On the other hand, field notes must be complete and reinforced by field sketches and observations. Kodak views of situations particularly difficult to map are also useful in helping the office draftsman to interpret field notes.

THE PLANE TABLE METHOD

355. The Plane Table. The plane table is an instrument in which the manufacturer has combined the telescope of the transit with the straight edge and drafting board of the draftsman. When some form of stadia reduction arc is added to the vertical circle, the topographic engineer has an excellent field mapping instrument at his disposal. Figure 355 illustrates this instrument. Note the comparatively heavy tripod supporting the board on which the plane table alidade rests. The lower part of the leveling head may be seen under the board. The telescope to which a vertical arc is attached is connected to the straightedge by the pedestal. This combination of telescope,



Courtesy of C L Berger and Sons

FIG. 355. THE PLANE TABLE.

pedestal, and straightedge is called a telescopic alidade. Note also the circular bubble and trough compass or declinatoire on the straight edge.

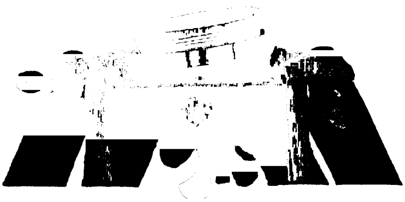


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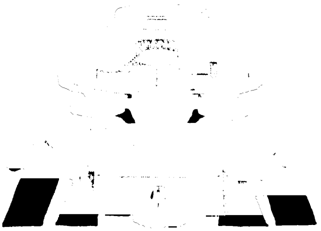
Fig. 355a. Johnson Leveling Head.

Two types of leveling head are available. The Johnson head (Fig. 355a) which is used by the United States Geological Survey, relies upon cup-shaped friction clamps for control of the horizontal movement of the plane table and also for leveling the table. Each clamp is operated by a wing nut at the lower end of the leveling head assembly.

The standard leveling head (Fig. 355b) is equipped with conventional leveling screws (usually three) and with horizontal motion clamp and slow-motion screw. This arrangement results in a heavier leveling head and larger tripod.

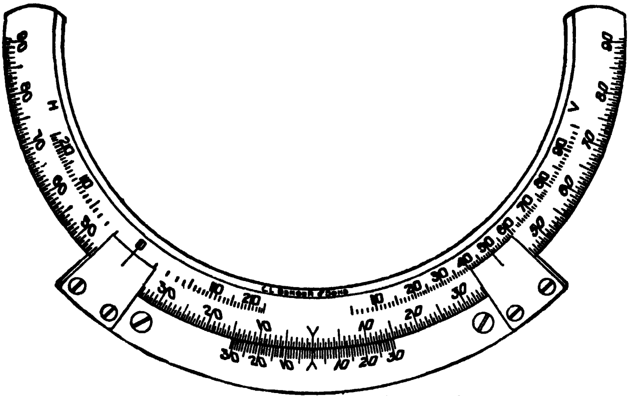


Courtesy of C. L. Berger and Sons

Fig. 355b. Standard Leveling Head.

The field sheet is fastened to the table top with U-shaped clamps or by means of thumb screws the heads of which are countersunk in order not to interfere with movement of the alidade. When it is necessary to set a point on the map exactly over that point on the ground, a collapsible hinged arm is used, the plumb bob being suspended from a hook at the lower end of the arm.

356. Beaman's Arc. This arc is another stadia reduction device (see also § 353) which is attached to the vertical circle of all United States Geological Survey plane table alidades and can be attached to transits or plane tables using either style as shown in Figs. 356 and 356a. The device was invented by William Beaman of the United States Geological Survey. Its design is based on the principle that for certain vertical angles the difference in elevation is a whole-number multiple of the rod

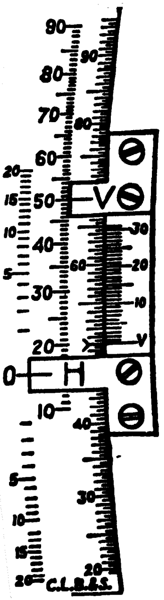


Courtesy of C L Berger and Sons

FIG. 356. BEAMAN ARC.

intercept and that the correction to be subtracted from the stadia (slant) distance to obtain the horizontal component is related directly to the vertical angle. For example, 10 is the exact multiple when the vertical angle is 5° 46′ (see also Table V) and at that angle the horizontal component (H) is equal to the stadia distance minus 1 per cent of that distance.

When the attachment is used, the rod intercept is determined by setting the lower stadia hair at an even foot mark and then reading the upper stadia hair. The difference between the two readings multiplied by $\frac{f}{i}$ (usually 100), to which $f + c$ (usually 1 foot) must be added if the telescope is of the conventional type, gives the stadia distance. The middle cross-hair is then set to the H.I. point on the rod and the H scale reading taken from the H index on the Beaman arc. This reading is the correction, expressed in per cent of stadia distance, which must be subtracted from the stadia distance. The vertical motion is then moved by means of the vertical tangent screw until the V index on the Beaman arc is coincident with a gradu-



Courtesy of C L. Berger and Sons

FIG. 356a. BEAMAN ARC.

ation line on that arc. The middle hair reading on the rod is then taken. The V index reading minus 50 gives the multiple to be used in determining the difference in elevation between the line of sight and the middle hair rod reading. In Figs. 356 and 356a note that the V index is at 50 when the telescope is level. By this method of marking the graduations, direct scale readings determine whether the angle is plus or minus, since the arc scale reading *minus* 50 is the multiplying factor to be used.

For example, an instrument which is set 4.6 (H.I.) feet above a control point, the elevation of which is 610.7, is at elevation 615.3. Assume that when sighted toward a stadia rod the lower hair is set at 3.00 feet and the upper hair reads 8.64 feet. The rod intercept is therefore 5.64 to which 0.01 must be added for the $f + c$ correction if the telescope is not of the internal focusing type. When the middle hair is then sighted to the 4.60 foot point on the rod, the H index reads 1.3. On the assumption that $\frac{f}{i} = 100$, the horizontal distance is therefore:

$$5.65 \times 100 - 5.65 \times 1.3 = 558 \text{ feet}$$

At this setting of the arc, the V index reads between 38 and 39

Shifting the arc so that the V index reads exactly 39 results in a middle hair reading on the rod of 4.75.

The multiple is therefore $39 - 50 = -11$ and the elevation of the ground at the foot of the rod is

$$615.3 - [(11 \times 5.65) + 4.75] = 548.4$$

357. Plane Table Adjustments. Although the necessity for perfect adjustment of the plane table is not as great as in the transit or level, the relations between the elementary parts of the instrument should be tested periodically and corrections made if necessary. The procedure is as follows:

1. To make the vertical hair truly vertical, set the alidade on a perfectly level surface and compare the vertical hair with a true plumb line such as the corner of a building. Make the full correction by loosening the reticule screws and shifting the cross-hair ring until the vertical hair coincides with the plumb line.

2. To make the line of sight coincide with the axis of the

sleeve in which the telescope revolves, focus the intersection of the cross-hairs on a distant point and revolve the telescope in the sleeve through 180 degrees. If the intersection of the hairs moves off the point, correct the cross-hairs for one-half the apparent error. Repeat the test until the intersection stays on the sighted point.

3. To adjust the level bubbles attached to the straightedge, set the alidade on a smooth, level surface. Lift the alidade and turn it end for end. Correct the bubbles for one-half the apparent error. It is necessary, in this end for end reversal, to mark the position of the straightedge at the first setting and reverse it end for end to that same marked position. This is especially important if the smooth surface used for testing is the top of the plane table.

4. To adjust the striding level bubble, set the alidade on a firm level surface and center the bubble with the vertical circle clamp and tangent screw. Then lift the striding level and turn it end for end. One-half the apparent error is corrected by means of the vertical adjusting screws at one end of the bubble tube. To bring the striding level bubble line into the same vertical plane as the axis of the sleeve, level the bubble, then rotate the telescope in the sleeve through an angle of about 30 degrees. Any displacement of the bubble must be corrected by means of the lateral adjusting screws at the end of the bubble tube. The full correction is made.

5. If the alidade is equipped with a bubble on the vernier, level the telescope by means of the striding level, set the vernier index to read 0° 00′, and bring vernier bubble to the center by means of the adjusting screws on the bubble tube. If there is no striding level, the vernier bubble must be tested by the two-peg method (see page 139).

6. To test the straightedge, set the alidade on the table top, draw a fine line (6H pencil with chiseled point) along the full length of the straightedge. Reverse the straightedge end for end, placing the same edge along the line. If the edge is not true, it must be corrected by the manufacturer.

It is not necessary that the line of sight be parallel to the straightedge for ordinary use of the plane table. The relation between the two lines must, however, be constant.

358. Locating Points by Radiation. Several methods of mapping locations of points with the plane table are used. One of the most common of these methods is by radiation. The instrument is set up at a control point, oriented by sighting toward another control point, and checked on a third point or on some prominent land mark which is easily visible from several control stations. Points on the ground, selected by the rodman, are then located on the map by turning the alidade toward the rodman, lightly penciling the ray along the straightedge, and, after reading and reducing the stadia distance, plotting the distance to and elevation of the point. Figure 358 illustrates this method. The set-up is over △ *C* which is marked on the map by a pin or needle at *c*. The fiducial edge of the alidade along the ray *cx* marks the direction of the line of sight toward the rodman at *X*.

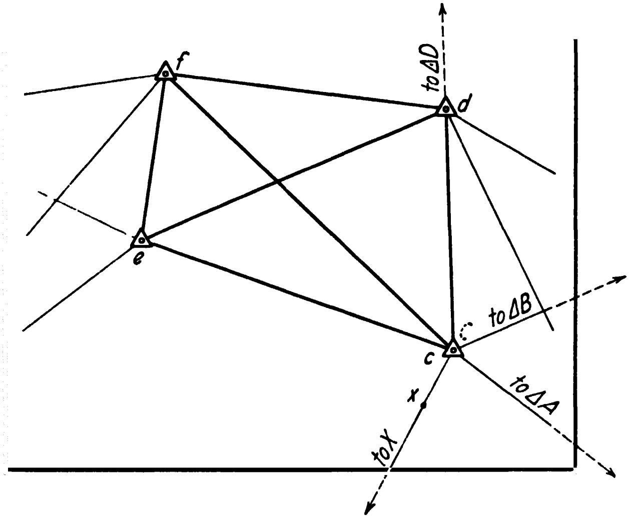


Fig. 358. Radiation.

359. Locating Points by Intersection. Points on the map may be located from plotted control points by intersecting rays. This method is illustrated by Figs. 359 and 359a.

The instrument is first set up at △ *C*, oriented and checked on △ *A*, △ *E*, or △ *D* and a ray drawn on the map toward rodman at X. The instrument is then set up at △ E and oriented. A second ray from e toward X intersects the first ray cx at point x on the map. If x is a point of major importance its position with relation to the plotted control system may be checked as often

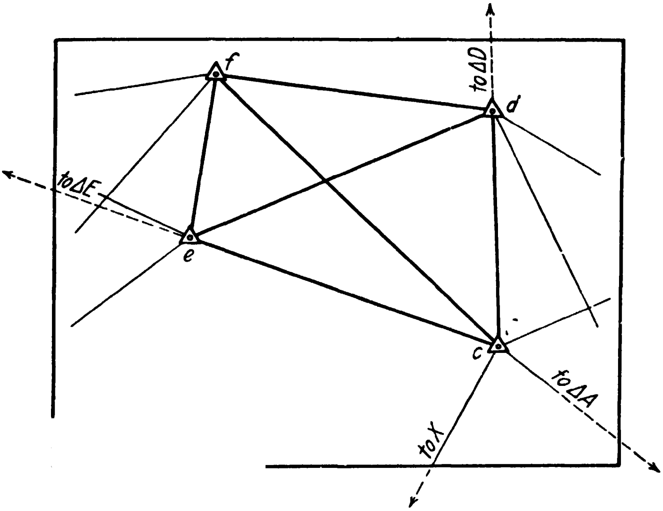


FIG. 359. INTERSECTION METHOD. SET-UP AT △ C.

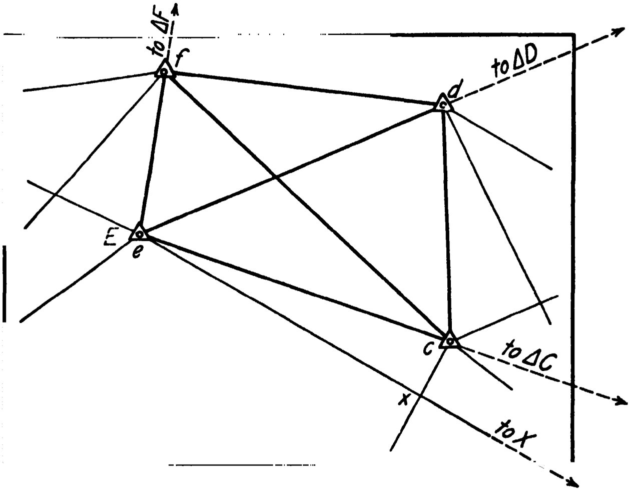


FIG. 359a. INTERSECTION METHOD. SET-UP AT △ E.

as it is visible from any point of set-up of the plane table as, for example, at △ *G* or △ *A*.

The intersection method is satisfactory for locating a few important points on the map. When, however, an attempt is made to map many points from the same set-ups, interpretation of the direction rays may prove difficult. The elevations at the points must be determined either by leveling or by stadia.

360. Resection. It is necessary occasionally to locate the position of the plane table on the map when only one of two control stations can be occupied. For example, in Fig. 360. assume that △ *C* can be occupied and that △ *E* cannot and that a set-up at point *X* is necessary. In order to locate *x* on the map, set-up at △ *C*, orient, and draw ray *cx* of indefinite length.

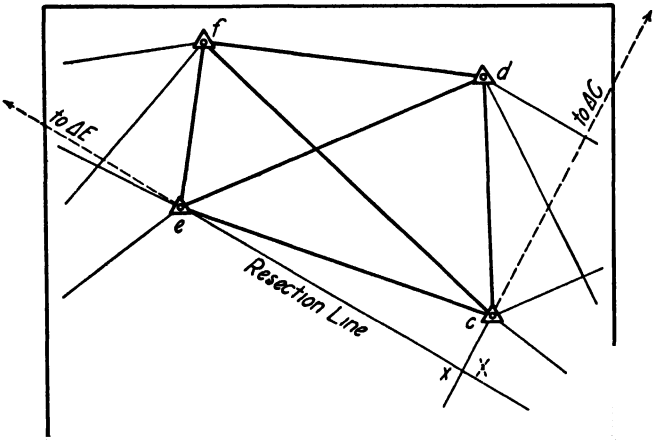


FIG. 360. RESECTION.

Move the plane table to *X* and, by roughly estimating the position of *x* on ray *cx* on the map, set-up so that the estimated map point *x* is over ground point *X*. Orient by setting the alidade straightedge along *xc;* loosen the horizontal motion and turn the table top until the line of sight passes through △ *C*. Clamp the horizontal motion. Then with the forward end of the straightedge at *e*, pivot the rear end around *e* as a center until the line of sight passes through △ *E*. Draw the resection line from *e* to an intersection with ray *cx*. The location of *x* on the map may then be checked by sighting toward other visible control points or land marks. Location by resection usually gives more accurate results than location by solving the three point problem. It is obvious that best results are obtained when the resection line is at right angles to the ray from the control station.

361. The Three Point Problem. The three point method of locating the point of set-up of the plane table may be used when three or more previously located points are visible. The location of the point on the map and the orientation of the plane table are both determined by the solution.

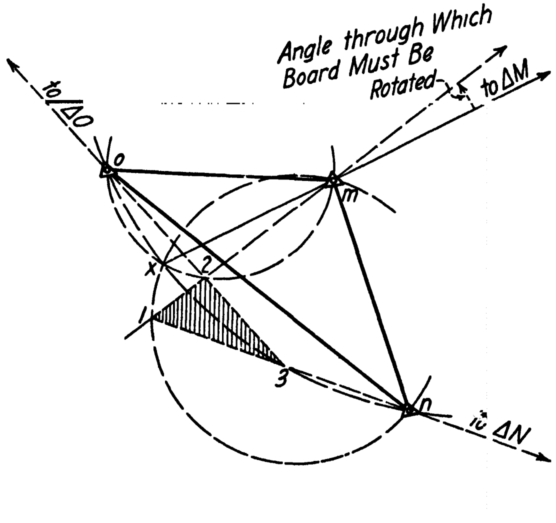


FIG. 361. THREE POINT PROBLEM.

One method, called Lehmann's method, is illustrated by Fig. 361. The three points △ O, △ M, and △ N are visible from point X. Plotted points o, m, and n represent the three triangulation stations. In this case X is outside of the triangle OMN. Its projected position on the map, x, may be estimated and an approximate orientation of the table top made by using the compass needle. If it were possible to make these estimations of position and orientation exactly, resection lines drawn on the map through *o*, *m*, and *n* would intersect at a common point *x*. The usual result of this resectioning, however, is a triangle (*1*, *2*, *3* in Fig. 361), called the triangle of error. The solution of the problem is to reduce this triangle, by successive approximations and re-orientations, to a point. It is obvious that if *X* is inside of the triangle *OMN*, *x* should be found inside of the triangle of error. In order to arrive at the solution (point *x*) quickly, find the intersection of the arcs of the circles which pass through *n*, *m*, and the intersection (point *1*) of the resection lines through *n* and *m*; *n*, *o*, and point *3*; *o*, *m*, and point *2*. This intersection may be approximately determined by roughly sketching the arcs on the field sheet. With the newly determined point *x*, re-set and re-orient the plane table and repeat the operation until the resection lines meet at *x*.

When *O*, *M*, *N*, and *X* are points on the same circle, no solution is possible.

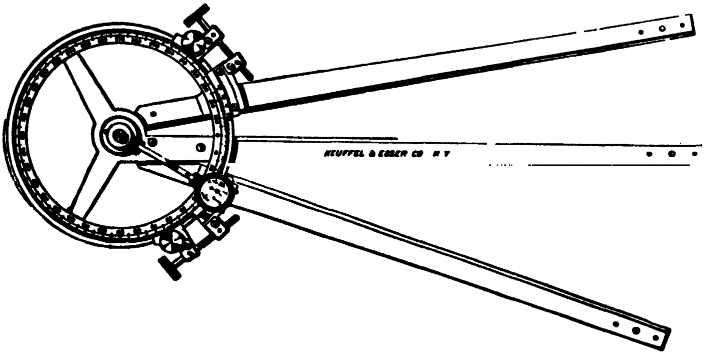


FIG. 361a. THREE ARM PROTRACTOR.

Another solution, called the tracing paper method, gives results quickly but not quite as accurately as does Lehmann's Method. To locate *x* by this method, it is necessary to tack an auxiliary sheet of tracing paper on the table top and at an assumed point *x* draw rays *xo*, *xm*, and *xn*. Lift the tracing paper and shift the plotted rays until these pass simultaneously through the plotted points *o*, *m*, and *n*. Prick point *x* through the tracing paper to the field sheet, thus locating the point of set-up on the map. Check by orienting on one of the triangulation stations and then sighting toward the other two.

If a transit were set up at point X and the horizontal angles to the triangulation stations measured, the position of x can be located by using a three arm protractor (Fig. 361a).

362. Plane Table Traverse and Triangulation. Although the best practice in topographic mapping is to establish and plot the control system before details are surveyed, control may be carried forward simultaneously with the detailed mapping operations. When such procedure is adopted, however, it is very important that checks on the locations of plane table stations should be made at every opportunity. Quadrilateral closures and direction rays to prominent land marks, such as distant spires, stacks, etc., are two examples of such checks. Any method or combination of several methods of locating points, as described in the preceding pages, may be used in establishing the control points on the map.

363. Plane Table Operators. The plane table is being used increasingly as a mapping instrument. The demand for its use would be accelerated if more engineers appreciated its advantages and recognized the fact that the accuracy of its results is dependent in large measure upon the operator of the instrument. Field engineers as a general rule are not skilled in the manipulation of the plane table and cannot become good topographers without going through a comparatively long training period. Nor can all field men acquire facility in the use of the instrument because many do not have the inherent ability to visualize map forms from an inspection of the terrain being mapped. It is also true that plane table outfits are not considered as essential equipment in the offices of practicing engineers although they are frequently available in government offices.

In the Topographic Manual published by the United States Coast and Geodetic Survey, "important points to be remembered in using the plane table for topographic surveying" are listed as instructions to field parties. Some of these reminders apply particularly to the student engineer, such as:

"1. In orienting, see that alidade edge cuts both station points equally

2. Orient on distant signals and resect on nearby ones

3. Check orientation of table at completion of work at a station and before drawing important cuts (rays)
4. See that rodmen hold stadia rods vertical by standing squarely on both feet directly behind the rod
5. Keep pencil sharp and, when drawing direction lines, hold it close to edge of alidade at constant angle
6. See that alidade rule makes close contact with the paper
7. Draw magnetic meridian on sheet
8. Keep sheet clean
9. Keep bottom of alidade clean
10. Do not rub alidade over sheet more than necessary. Lift it
11. Do not let body or arms rest on the table
12. Watch sheet for distortion errors
13. Check up before leaving a station to see that everything desired at the station has been obtained and
14. Keep notes for descriptive report."

OFFICE PRACTICE

364. Map Making. Maps are so varied in nature and are made for so many different purposes that a perfect classification of them is almost impossible. Each map is designed to give the reader of it certain definite information. This idea governs everything with regard to the actual construction of the map in order that the important information may be represented clearly and that other features may be omitted or at least subordinated. Brief considerations of the important things common to all map construction follow. See also Chapter XVI.

365. Scales. The scale to be used in drawing a map is very important. It must be large enough to show the desired information clearly. A larger scale would increase the cost of the map and the size of the sheet to no advantage while a smaller scale would be unsatisfactory for proper reading.

In general, scales are classed as large if 1 inch = 100 feet or less; intermediate, between 1 inch = 100 feet and 1 inch = 1000 feet; and small if 1 inch = 1000 feet or more.

Mapping from survey data may vary in scale from 1 inch = 10 feet for planting plans or accident surveys to 1 inch = 2000 feet, one-half mile, one mile, or two miles in the quadrangle sheets of the United States Geological Survey. The range of scale for most mapping lies between 1 inch = 20 feet and 1 inch = 400 feet. A scale of 1 inch = 40 feet is best for use of those who employ both engineers' and architects' scales in their work.

Every map should show the original scale to which it was drawn. In addition, coordinate lines or a graphical scale as shown on Fig. 368 should be indicated. This is necessary in order to give the scales of photographic enlargements or reductions which are made from many maps and to make allowance for shrinkage.

366. Materials. The paper or cloth upon which a map is made represents but a small fraction of the cost of the map. It is therefore most economical to use the best materials available for all maps of importance. Use of inferior drafting or printing papers which distort with slight atmospheric changes results in scale variations that seriously affect the accuracy of the map. All inks, crayons, and color washes should be chosen for ease of handling and lasting qualities.

367. Plotting. The specifications for field measurements are usually such that the errors must not be plottable on the map. These conditions are generally met and therefore the draftsman has no tolerance but must produce as nearly perfect work as is possible. To do this he must first have as good equipment as the field parties had, and second, he must use proper methods in all phases of his work to obtain the necessary precision. The plane table method, of course, combines the surveying and drafting operations.

The plotting of the control system requires the greatest precision in drafting, corresponding to precise field work. A straight line can be drawn to correct length within the limits of accuracy of the scale and the draftsman's ability to read it and to mark or prick the end points. The method of plotting points depends upon the data available. If coordinates are available, they afford an excellent method of precise plotting with straight edge and triangles, preferably of steel. For traverse data with deflection angles or angles to the right given, some scheme involving a trigonometric function multiplied by a constant, usually 10, gives a good sized triangle with which to work. Of the three schemes shown in Fig. 367, probably the tangent method is most used. In this method one leg of the angle is extended beyond *A* for a distance of 10 inches at which point a perpendicular, 10 tan α long, is erected, thus locating *B*. *AB* is the other leg of the deflection angle.

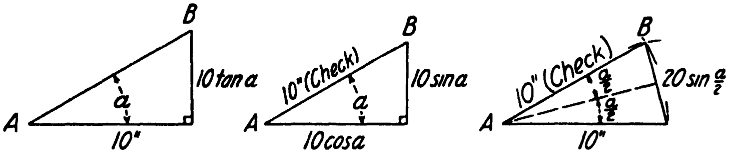


FIG. 367. PLOTTING ANGLES BY TANGENT, SINE-COSINE, AND INTERSECTING ARCS.

The figure also illustrates a similar method which involves the use of two trigonometric functions and the measuring of two odd lengths. It has a good check, however, in that the hypotenuse must always be exactly 10 inches long.

For plotting triangles all sides of which are known, as in the case of triangulation control, the best method is by use of intersecting arcs. The same principle can be used for any angle, Fig. 367, in which the angle is formed at the vertex of an isosceles triangle the base of which is computed. This method eliminates the erection of perpendiculars, but with good drafting equipment that operation should not be a source of error.

The plotting of topographic details is a repetition on the drafting board of the field operations; angle and distance (stadia, see § 352), right angle offset and distance from a control line, or intersecting arcs locate a point from two control stations.

368. Title. The title of a topographic map should be placed on the sheet where it will best balance the sheet as a whole. The lower right corner is preferred, as in the case of mechanical drawings, for standard maps such as state highway plans and the like. However, for any unstandardized map, this should not be given much weight.

A title should not be made so large as to be conspicuous yet it is an important part of the map. The name of the area mapped should stand out in the title by means of larger or heavier letters. The location of the area, organization or persons doing the work, date, original scale, and graphic scale should be included. In addition, on standard sheets, spaces are provided for file number and initials of checkers and approvers.

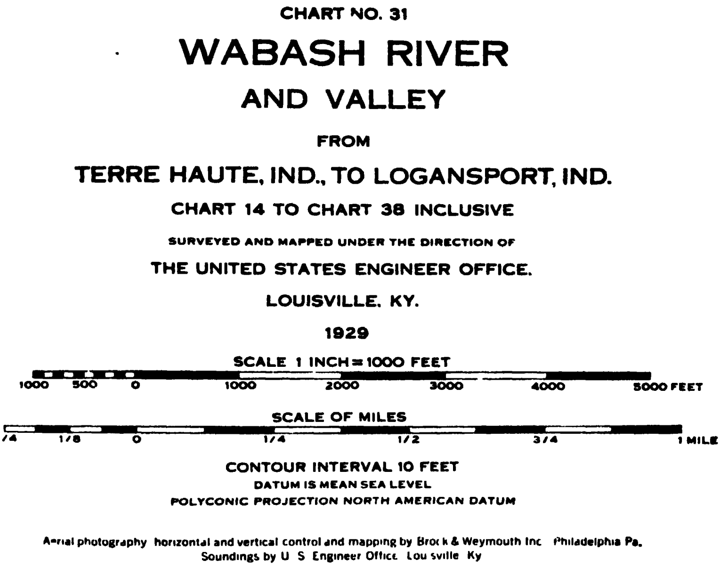


CHART NO. 31

WABASH RIVER

AND VALLEY

FROM

TERRE HAUTE, IND., TO LOGANSPORT, IND.

CHART 14 TO CHART 38 INCLUSIVE

SURVEYED AND MAPPED UNDER THE DIRECTION OF

THE UNITED STATES ENGINEER OFFICE.

LOUISVILLE, KY.

1929

SCALE 1 INCH = 1000 FEET

1000
500
0
1000
2000
3000
4000
5000 FEET

SCALE OF MILES

1/4
1/8
0
1/4
1/2
3/4
1 MILE

CONTOUR INTERVAL 10 FEET

DATUM IS MEAN SEA LEVEL

POLYCONIC PROJECTION NORTH AMERICAN DATUM

Aerial photography horizontal and vertical control and mapping by Brock & Weymouth Inc Philadelphia Pa.
Soundings by U S Engineer Office Louisville Ky

FIG. 368. SAMPLE TITLE.

The entire title should be symmetrical, proportioned to the sheet as a whole, and of a uniform type of lettering. The increased use of lettering guides usually results in the various parts of a title differing only in size of letter and weight of line.

369. Lettering. Certain kinds of maps require a considerable amount of lettering. This is placed at all angles and frequently in quite small spaces requiring very careful work. Mechanical lettering guides cannot be used for all of such work and it is therefore recommended that they be used for none of it. No one thing adds as much to the general appearance of a map as the excellence of its lettering. Good freehand lettering (see page 108, Table X) gives character and individuality to a map that mechanical lettering can never supplant. The man who can produce excellent maps is an artist in every sense of the word.

370. North Point. Every map should have an arrow or arrows indicating the true and magnetic north directions. These should be simple drawings, long enough so that a parallel can be taken from them with accuracy, and placed wherever space is available If both arrows are shown, a statement of the amount of magnetic declination is helpful. When only one arrow is drawn, it would be well to designate which north is represented. Figure 370 is an example showing all that is needed. Orientation of the map so that the north arrow points approximately toward the top of the sheet is desirable.

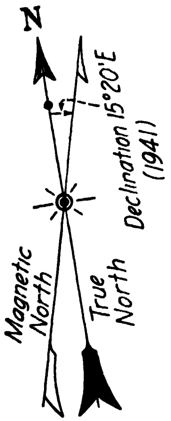


Fig. 370.

371. Symbols. The use of conventional symbols to represent certain types of physical features facilitates the reading of the map. Symbols should always be made proportional to the scale of the map. For example, a pine tree on a very large scale map would be shown by the conventional symbol representing the actual spread of the tree while on a smaller scale map a pine forest would be shown by smaller symbols well drawn and uniformly spaced over the map area. Effort has been made by those agencies that do much mapping to standardize the symbols used for many of the common things that appear on maps. Figure 371 gives a representation of some of the symbols prepared by the United States Geological Survey and adopted by the Federal Board of Surveys and Maps. These symbols when properly shown, especially those showing crops or natural vegetation, add much to the map, but if badly drawn result in a map of poorer general appearance than if no symbols had been used.

372. Legend. Each map should show a legend explaining the meanings of the symbols used. This is needed because many people who may use the map will not be acquainted with the standard symbols. In addition, other symbols may be used that are not standard. An excellent example is that of the much used state highway maps with their own symbols representing roadside parks, fish hatcheries, police posts, and points of scenic or historic interest.

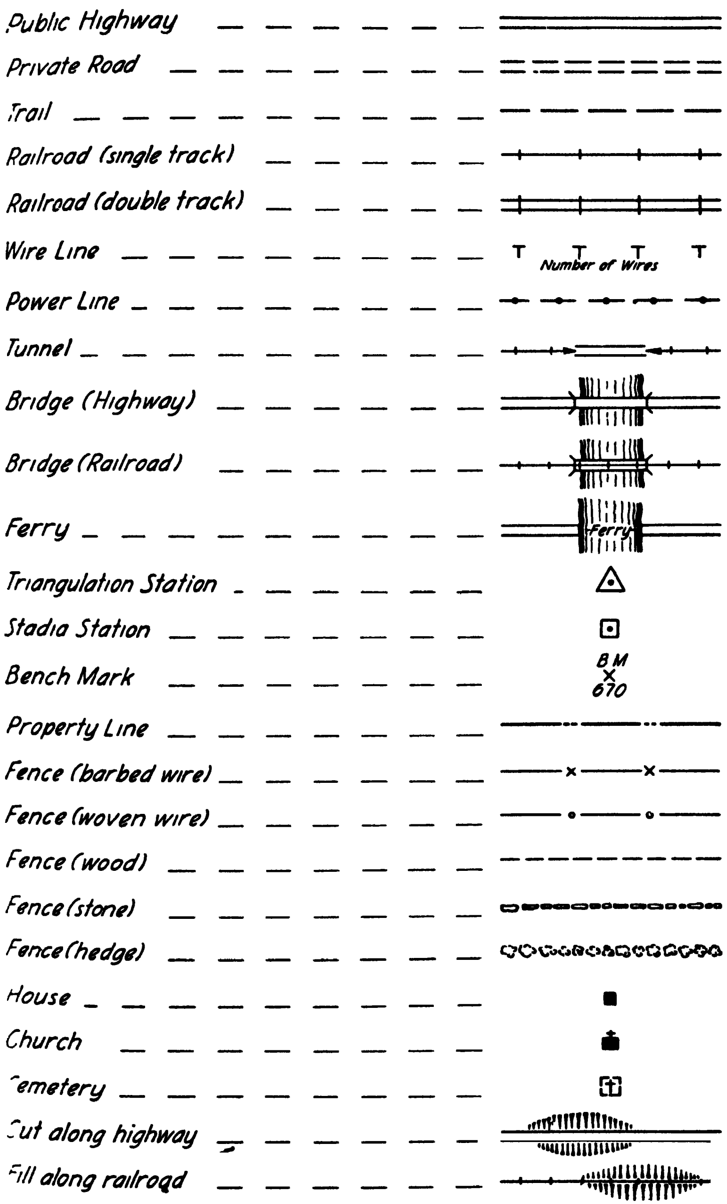


FIG. 371. CONVENTIONAL SIGNS AND SYMBOLS.

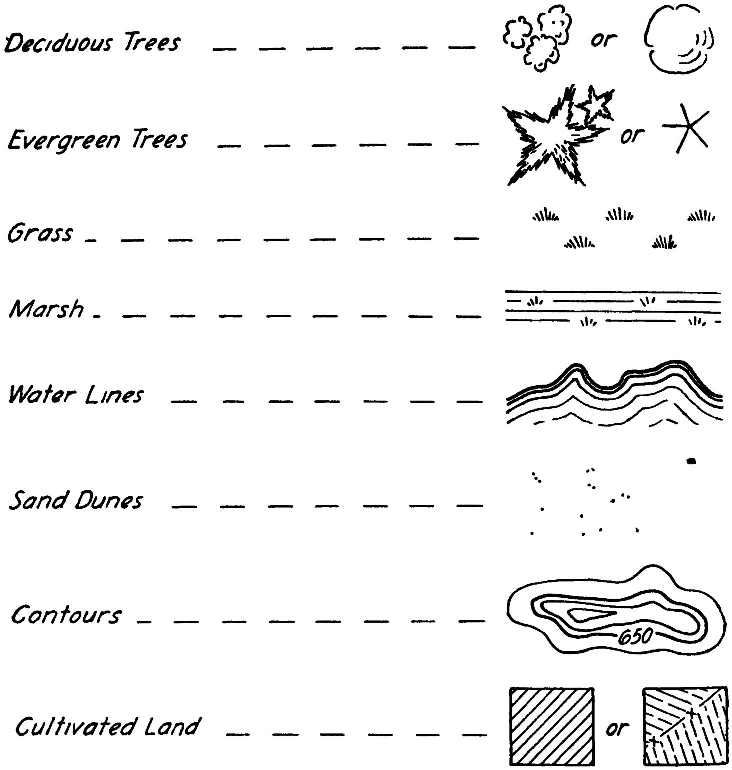


FIG. 371 (*continued*).

373. Color. Colors may be used to great advantage on many kinds of maps to emphasize certain features. Most maps seen in color are the result of chromo-lithographing processes rather than work done by a draftsman. This may vary from merely showing Federal highways and political boundaries in color to the very extensive use of color in the United States Airway Maps in which different elevations are tinted in various shades.

The draftsman is called upon occasionally to prepare a map in color. Display maps of parks, country club grounds, or sales maps for new subdivisions are examples. This work is done on paper by means of water color washes or the use of crayons. Such work, especially water coloring, calls for considerable skill and practice but can produce excellent results. Perhaps the scheme requiring the least skill is that of covering an area with light colored-pencil lines and then rubbing it out to the desired tint by means of a soft eraser. The tendency among beginners is always to use too much color. If a pale green is used for grassed areas the trees may be gone over with a darker green, water may be shown in blue, streets in brown, buildings in grey, and so on, to produce a much different effect than does a plain line drawing.

Similar effects can be obtained on tracing cloth by means of colored inks. Different types of cross-hatching should be used rather than complete covering in order to prevent distortion of the cloth. The map legend should show a key explaining the various colors. Sepia ink is frequently used for contour lines. This ink, being less dense than black, will cause the contours to appear somewhat subdued on all direct reproductions.

PROBLEMS

1. A transit is set up at △ X, the elevation of which is 721.3. The top of the triangulation station is at ground elevation and on a ridge line. The distance from this top to the horizontal axis of the transit (H.I.) is 4.76 feet. $\frac{f}{i} = 100.00$, $f + c = 1.00$ foot.

Stadia readings as follows were taken at ground points in the vicinity of △ X:

Reading No.	Rod Intercept	True Azimuth 0° = North	Vertical Angle	Description
1	4.68	352° 12	− 3° 10′	Bottom slope
2	5.47	24° 05′	− 2° 41′	Bottom slope
3	6.31	38° 41′	− 2° 28′	Bottom valley
4	3.06	58° 16′	− 3° 38′	Point on valley line
5	1.66	104° 22′	− 4° 37′	Point on valley line
6	2.61	95° 27′	− 1° 10′	On ridge line
7	3.67	79° 28′	− 0° 56′	On ridge line
8	5.52	69° 17′	− 1° 30′	On ridge line
9	5.00	92° 51′	− 2° 06′	On slope
10	3.09	152° 48′	− 1° 10′	On slope
11	2.81	164° 25′	− 0° 46′	On valley line
12	2.49	195° 45′	+ 0° 09′	On ridge line
13	4.20	240° 47′	− 2° 38′	Bottom slope
14	3.53	287° 36′	− 4° 07′	Bottom slope

(a) With these data, make up a set of stadia field notes using the form as shown in Fig. 352.

(b) Compute the horizontal distances and elevations using a stadia slide rule.

(c) Compute same values as in (b) using Table V.

(d) Plot these notes to a scale of 100 feet to the inch and interpolate contours using a 5-foot contour interval.

NOTE: The configuration of the ground is such that interpolation of contours may be made between the following points: △ X–1, △ X–2, △ X–14, 12–13, 13–14, △ X–12, 3–4, 4–5, △ X–5, 5–11, 11–12, 8–7, 7–6, 4–7, 6–9, and 11–10.

2. The following stadia measurements have been made in the field:

READING NO.	ROD INTERCEPT	VERTICAL ANGLE
1	8.46	+ 22° 14′
2	0.54	− 0° 34′
3	5.62	− 5° 42′
4	2.12	+ 0° 08′
5	1.18	− 7° 15′

(a) Compute horizontal and vertical distances by Table V, with the stadia slide rule and by interpolation in the stadia formulas when $\frac{f}{i}$ is 100.00 and $f + c$ is 1.00 foot.

(b) The same as (a) except that the instrument used is of the internal focusing type.

(c) The same as (a) except that $\frac{f}{i}$ is 99.81 and $f + c$ is 0.75 foot.

REFERENCES

1. BIRDSEYE, C. H., *Topographic Instructions of the United States Geological Survey, Bulletin 788.* Government Printing Office, Washington, 1928

This is a bulletin of 432 pages in which detailed instructions for control and mapping are given. The transit, plane table, aner barometer, and aerial photographic methods are discussed. Ten pages of standard map symbols are included.

Mimeographed instructions to field parties, supplementing the Manual, have been issued recently.

2. BREED and HOSMER, *The Principles and Practices of Surveying, Vol. II* John Wiley and Sons, New York, 1938.

One of the best known text-books on the subject. In addition to the standard treatment of the subject of topographic surveying, an excellent discussion of the relation of geology to topography is given in Chapter VI.

3. Davis and Foote, *Surveying*. McGraw-Hill Book Company, New York, 1940.

More than 1,000 pages of text material, three chapters of which deal with the subject of topographic engineering.

4 Rubey, *Route Surveys*. The Macmillan Company, New York, 1938.

Contains detailed descriptions, illustrated, of the use of the aneroid barometer and aerial surveys as applied to reconnaissance surveys.

5. Swainson, O. W., *Topographic Manual of the United States Coast and Geodetic Survey, Special Publication No. 144*. Government Printing Office, Washington, 1928.

A bulletin of 120 pages on instruments and equipment, field work, and office work. Ten pages of topographic symbols and seven useful tables are included.

6. United States Geological Survey, Color-Sound films "Topographic Mapping by Photogrammetric Methods" may be borrowed from this agency.

CHAPTER XXIV

PHOTOGRAMMETRY

By Professor H. O. Sharp

Rensselaer Polytechnic Institute

374. History. Photogrammetry is the science of using photographs in the compilation of mosaics and maps. Research in this field has been carried out for many years, in fact long before the camera was invented. The fundamentals are over two centuries old. As early as 1715, more than a century before the invention of photography, Brook Taylor, of Taylor's theorem fame, published a book on linear perspective but it was not until 1759 that J. H. Lambert of Zurich suggested the value of perspective as an aid to mapping. Early experimenters had to depend on horizon sketches and the camera lucida.[*] These experiments were carried on for many years but no practical use was made of them until about 1840 when a French engineer, Aimé Laussedat, applied the principles of perspective to map making. His first work was done with a camera lucida but improvements in photographic methods and equipment soon enabled him to substitute photographs for sketches.

The work of Laussedat marked the beginning of photographic surveying and continued to be its basis until about 1909, when Dr. Pulfrich of Germany began experiments in the use of stereoscopic pairs of photographs for mapping purposes.

Photographic surveying entered a new phase through the studies made by Dr. Pulfrich and the results of his studies have been the basis for most of the modern methods involving the use of stereoscopic work with both terrestrial and aerial photographs.

Surveying by ground photography was first introduced in America in 1886 by Deville, then Surveyor General of the

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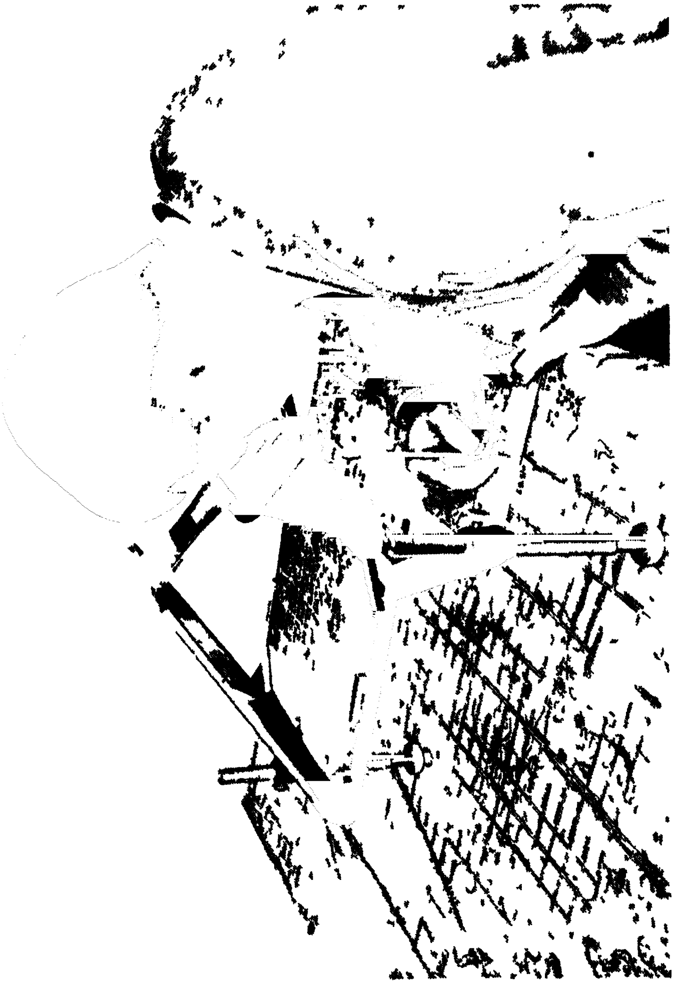
[*] The camera lucida might be likened to a box camera in outside appearance but it was not designed to take pictures. The rays of light which entered the box through a lens were deflected 90 degrees to a transparent medium on top of the box where the sketching was done. It might be compared to a reflex camera except that no pictures were taken.

Dominion of Canada. It was of particular value in surveying the rugged mountain regions of western Canada. So successful was the work of the Canadians that in 1894 the United States Coast and Geodetic Survey adopted the use of the photograph for mapping along the international boundary between Alaska and Canada. In 1904, this organization began experiments in photographic surveying in Alaska.

No important developments were made, however, until 1917. A sudden development of the art resulted from the First World War, and aerial photographs were used extensively in the United States and abroad. The United States Corps of Engineers and the United States Geological Survey cooperated in the development of aerial camera equipment and in methods for reducing the photographic information to maps. Photogrammetry has continued in its development since that time and many improvements in planes, photographic equipment, plotting devices, and methods have resulted. In fact, hardly a map of any size is attempted today without the use of the photograph in some form.

375. Camera. The camera is one of the great developments of modern civilization. All cameras have certain fundamental characteristics and are designed to record images on a sensitized medium. It might be said that a camera is similar to the eye of a normal person where the person is constantly recording multitudes of images on the retina of the eye. However, in this case no permanent record is made on a medium such as the negative in a camera. The eye is a more perfect camera than its counterpart, the commercial camera, as adjustments of the eye for intensity of light, focusing, and directing are all quite automatic whereas the camera requires the incorporation of adjustable mechanisms to produce a flexibility which anywhere approaches the human eye.

Furthermore, the normal person takes his eye photographs in pairs which gives him the added advantage of depth perception. There are a few stereoscopic cameras on the market which take photographs in stereoscopic pairs but this may also be accomplished with a single lens camera, as will be explained later in this chapter.



Courtesy Abrams Aerial Survey Corp

FIG. 275. [illegible]

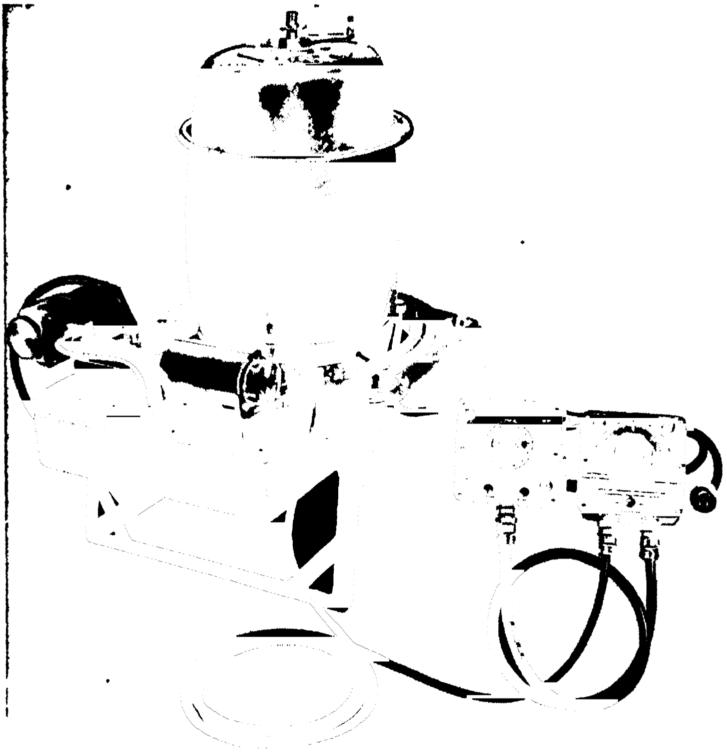
Commercial cameras consist of a miniature dark room or box which is built to exclude all light. A lens is placed at the front of this box with a shutter to act as a window through which light may be admitted. The amount of light is important; therefore, the size of opening must be regulated. This is done by means of an iris diaphragm usually composed of thin opaque plates which may be closed in to form a small opening or expanded to form a large opening. The only light admitted to the dark-room box during the exposure is that which is admitted through the lens and iris diaphragm.

Courtesy Fairchild Camera and Instrument Corporation

FIG. 375a. FAIRCHILD AERIAL CAMERA F-51.

The light which is admitted is directed to the rear of the box where a holder is provided with a sensitized film or sensitized glass plate. The plane in which this sensitized medium is placed is called the *focal plane*.

The distance between the lens and focal plane may be varied in most cameras for focusing purposes, but in some cases this distance is fixed; in the latter case the camera is called a *fixed focus camera.*



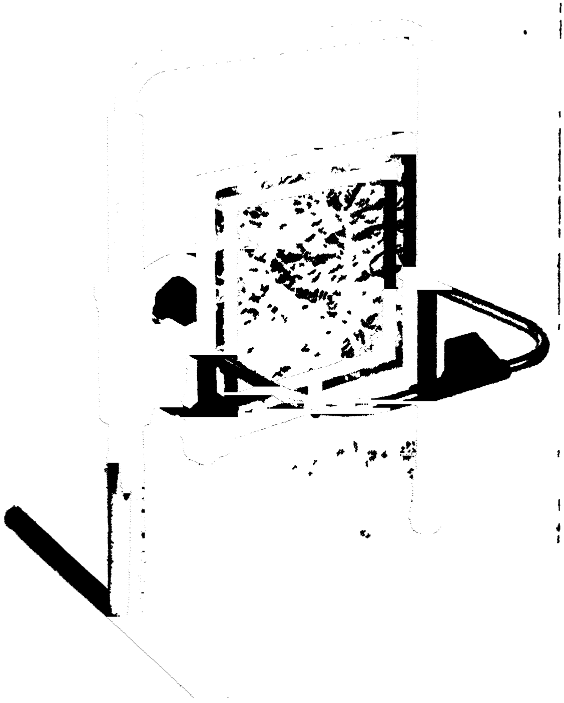
Courtesy Abrams Aerial Survey Corp

FIG. 375b. ABRAMS AUTOMATIC CAMERA MOUNT AND ABRAMS PRECISION EXPLORER CAMERA.

Some cameras are equipped with a so-called focal plane shutter. In this case, the light is admitted through a narrow slit in a curtain or screen which passes across the sensitized medium during exposure. This type of camera is not as common as the between-the-lens shutter type but modification of the principle has recently been applied to a method of strip photography which will be discussed later.

Certain characteristics of lenses are important from the standpoint of the quality of the resulting photographs and their dimensional characteristics.

376. The Camera Lens. The lens is the seeing eye of the camera and the quality and precision of the resulting photographs depends to a large extent on the lens.



rtesy Fairc
Instrument Corporation

FIG. 376. FAIRCHILD RECTOPLANIGRAPH.

LENS TESTS AND CAMERA CALIBRATIONS. With an ever-increasing use of stereoscopic plotting instruments, it is important that a camera of high precision be used and that it be tested and calibrated by an organization fully equipped to do this work. The Bureau of Standards is best equipped with instruments and personnel to handle these tests.

Photogrammetrists should appreciate the importance of precise camera equipment. In the following paragraphs an attempt is made to give some of the fundamental requirements necessary for a good camera.

Elementary Optical Principles. Chromatic aberrations are produced by the different wave lengths of various colors as projected through the lens. To illustrate, assume that two colors *A* and *B* enter a lens where they are deflected and come to focus at different points, such as *A′* and *B′* (Fig. 376a). Under normal conditions, there may be many colors entering a lens and all of them coming to focus at different points. This distortion is known as chromatic aberrations.

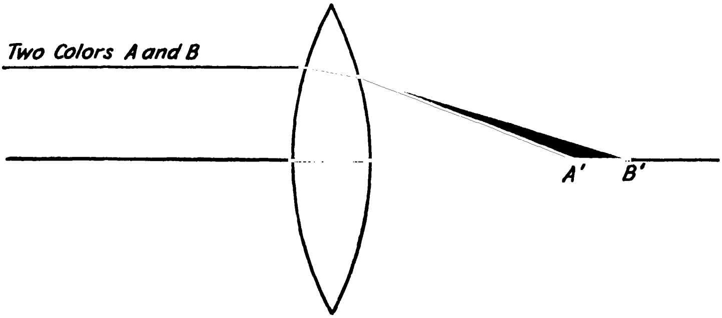


FIG. 376a. CHROMATIC ABERRATIONS.

Spherical aberrations are produced by slight imperfections in the grinding of a lens. The rays of light entering a lens may not come to focus in the same plane. To illustrate, assume that four rays of light enter a lens such as *A*, *B*, *C*, *D*. They are deflected and come to focus at *A′*, *B′*, *C′*, *D′*. (See Fig. 376b.)

If the lenses shown are used in a camera, it can be seen that not all the images will come to focus in the plane of the negative. The distortions are greater at the outer edges of a lens. Nearly every one has taken pictures at some time and realizes that better results are obtainable by using a relatively small aperture opening. In higher-priced cameras, most of the distortions have been removed, which is one reason they cost more.

The F value, such as F-8, F-11, etc., is the ratio of the focal length of the lens to the diameter of aperture. Since a larger aperture opening admits a greater amount of light, there must be a corresponding reduction in the time of exposure. F-1.9 is a relatively fast lens because the size of opening (aperture) is larger with respect to the focal length and therefore admits a large amount of light.

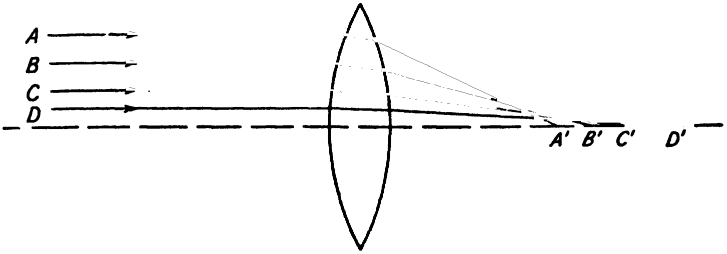


Fig. 376b. Spherical Aberrations.

Do not confuse the F values just mentioned with the focal length of a lens, which is usually designated by *f*.

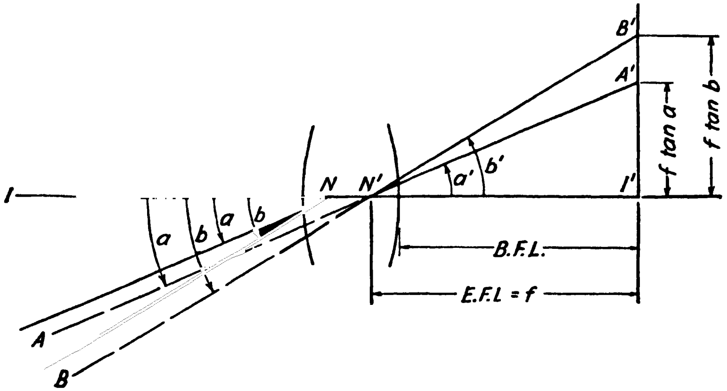


Fig. 376c. Metrical Characteristics (Infinite Distance).

The metrical characteristics of an image formed by an ideal lens, free from distortions, are determined by the equivalent focal length and the location of the two nodal points. The nodal points of a camera lens are the points of unit angular magnification. To illustrate, assume a ray entering a lens from a near point at the left and making an angle a with the horizontal axis at N. It will leave the lens with the angle a at N' N and N' are the nodal points (Fig. 376c).

When the points A and B are at infinite distance, the rays pass through the rear nodal point and the images formed in the focal plane are at A' and B'. The dotted lines are parallel to the heavy lines. When a lens is tested for infinite focus, the point N need not be considered. When a camera is used in an airplane, the surface to be photographed is at a great, but finite, distance from the lens. The conditions shown in Fig. 376c can, however, be used as the distance is so great that we may assume that a sharp image will be formed in the focal plane.

The plane of the film can fail to coincide with the focal plane by 0.05 mm. without harmful results. It can be shown that satisfactory results may be obtained when this condition is satisfied:

$$\frac{D}{f} \geqq \frac{f}{0.05}$$

where D = distance to object photographed.
f = E.F.L. (Equivalent Focal Length).

Example. When $f = 150$ mm. (6 in.) D must equal or exceed 450 meters (1500 ft.) to satisfy this condition.

If the distance between nodal points is 3 mm., the angle between the lines drawn from the two nodal points to an object 30 degrees from the axis will not exceed one-half second. These values are typical and indicate the distance to the object. They may be considered as infinite for practically all aerial photography.

If f is known, the angle a corresponding to any object point may be found by measuring the length $I'A'$ on the negative and using the equation, $\tan a = I'A'/f$.

Object at Finite Distance. If the object and image are at a finite distance, the metrical relations are as shown in Fig. 376d.

This corresponds to a lens used for copying or rectification.

The magnification M is the ratio of a length in the image plane to its conjugate length in the object plane.

If points A and A', B and B', or any other pair of conjugate points are joined by straight lines, these lines will intersect at a point lying between N and N' and divide the distance II' into segments, one of which is M times the other. This relation is shown by the dotted lines in the figure.

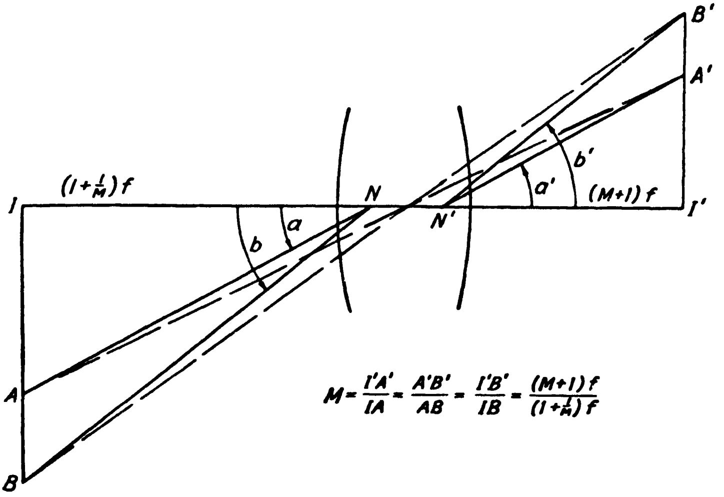

B'
A'
b'
a'
$(M+1)f$
I'
$(1+\frac{1}{M})f$
I
N
N'
a
b
A
B
$M = \frac{I'A'}{IA} = \frac{A'B'}{AB} = \frac{I'B'}{IB} = \frac{(M+1)f}{(1+\frac{1}{M})f}$

FIG. 376d. METRICAL CHARACTERISTICS (FINITE DISTANCE).

N and N' do not necessarily lie in the order shown in Fig. 376d. It sometimes happens that N' precedes N, that is, the vertices of the projections systems overlap instead of being separated.

DETERMINATION OF FOCAL LENGTH AND LOCATION OF NODAL POINTS

We may write the following relation

$$f = \frac{I'A'}{\tan a} \text{ (from Fig. 376c)}$$

and this suggests a method for the determination of the focal length of a lens.

1. Two distant targets are provided, one on the optical axis and the other at a known angle a from it.

2. These targets are photographed by the lens to be tested and the distance $I'A'$ in Fig. 376c measured on the resulting negative.
3. Substitute these values in the above equations and obtain f
4. The back focal length (B.F.L.) is measured from the rear vertex of the lens to the position occupied by the photographic plate.

All lenses having the same equivalent focal length are equivalent in that they will produce equal images.

Lenses of the same equivalent focal length may have back focal lengths that differ greatly.

A telephoto lens may have a back focal length one-half or one-fourth that of a lens of the more usual type of the same equivalent focal length.

From Fig. 376c, it may be seen that the distance from the vertex of the lens to the back nodal point = E.F.L. − B.F.L

The front focal length (F.F.L.) may be determined by reversing the lens and repeating the process to determine the distance from the front surface of the lens to the plane $I'B'$. The distance from the front of the lens to the front nodal point may now be determined.

Distortions Due to Aberrations. The statements and equations mentioned in the preceding paragraphs are all based upon the assumption that a lens is free from aberrations. Unfortunately, most lenses have these characteristics and therefore give a distorted image. It is impossible to eliminate aberrations entirely, but it is the function of the lens designer to so choose the glasses to be used and to so arrange the surfaces as to give an image substantially free from harmful aberrations when the lens is properly used. Each lens, therefore, is a compromise, and the preferred compromise is determined by the purpose of the lens. This compromise gives rise to the different types of lenses.

Most military cameras require a fast lens as many pictures must be taken under adverse conditions. In this case, it is necessary to limit the angle of view to avoid distortions at the edges of the photographs.

On the other hand, for ordinary photographic surveying, it is economical to use a lens of wider angle and less speed. In this case, the photographer may wait for favorable weather and lighting conditions.

Aberrations which interest the photogrammetrist are of two kinds:

(*a*) Those which introduce systematic errors of distortion in the metrical characteristics.

(*b*) Those producing lack of definition.

If a lens is accurately constructed so that all points are symmetrical about the axis, the distortions will also be symmetrical about the same axis. Thus, an image will be displaced radially toward or from the center of the field.

If two determinations of the focal length are made by measuring angles a and b and distances $I'A'$ and $I'B'$ of Fig. 376c and the focal lengths are determined by

$$f_1 = \frac{I'A'}{\tan a}$$

$$f_2 = \frac{I'B'}{\tan b}$$

it will be found generally that the two values of f are different. In cases like this, it is usual to define the equivalent focal length f as the limiting value f that is obtained as the angle a approaches zero as a limit.

With this definition of f and distortion present, we may write

$$I'A' = f \tan a + d_A$$

$$I'B' = f \tan b + d_B$$

where d is the distortion and will have different values for points at different distances from the axis.

The distortion will be zero for points near the center of the field of view and will increase directly with the distance from the center outward.

The National Bureau of Standards is equipped with a precision camera and with it the distortion may be measured at five-degree intervals from 0 to 30 degrees.

The displacement is designated as + if away from the center; therefore the correction will be negative.

Since the photogrammetrist is interested in f for use in the scale fraction, arbitrary corrections may be used where distortions exist. By arbitrarily choosing a slightly larger value of f, the corrections may be so distributed that those near the center of the field are negative and those away from the center are positive. To do this, it is necessary to have a list of the distortions at five-degree intervals. The distortion of modern cameras is usually less than 0.1 mm. even near the edge of the plate.

The focal length furnished by the Bureau of Standards is the limiting value of *f* at the center of the field and has not been altered to distribute the errors from the presence of distortion

From the preceding discussion, it will be seen that the distortion of a lens used for infinite distance will be different from one used at finite distance. Therefore, when a lens is to be tested, this fact should be remembered and stated in the request for testing. A more complete description of camera calibrations may be found in *Photogrammetric Engineering*, Vol. III, Number 1.*

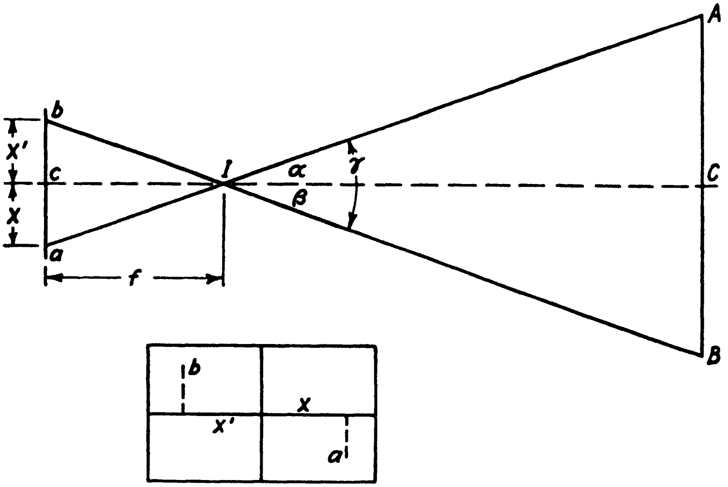


FIG. 377. FOCAL LENGTHS DETERMINED.

377. Determining the Value of *f*. It will be noted from the foregoing discussion that the focal length of the camera must be known. This may be furnished by the maker of the camera, but often the photographer must determine the focal length of his camera.

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* The title of the article is "The Interpretation of Lens Tests and Camera Calibrations," by Irvine C. Gardner.

1. In Fig. 377, assume I to be the position of a camera lens and that two points A and B are photographed, giving the images a and b on the negative. Cc is the axis of the camera lens; f is the focal length of the camera lens as indicated.
2. Measure carefully the angle γ. This can be done with a transit.
3. Measure the distances x and x' on the negative along the horizon line. The problem then is to find f knowing x, x', and γ.

Proof. From Fig. 377

$$\tan \alpha = \frac{x}{f} \quad \text{and} \quad \tan \beta = \frac{x'}{f}$$

and

$$\tan \alpha \tan \beta = \frac{xx'}{f^2}$$

From trigonometry we have the following transformation equation:

$$\tan (\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Using this transformation in our problem, we have

$$\tan (\alpha + \beta) = \tan \gamma = \frac{\frac{x}{f} + \frac{x'}{f}}{1 - \frac{xx'}{f^2}}$$

Remembering that f is our unknown, we have

$$\tan \gamma \left(1 - \frac{xx'}{f^2}\right) = \frac{x}{f} + \frac{x'}{f}$$

or

$$\tan \gamma \left(\frac{f^2 - xx'}{f^2}\right) = \frac{x}{f} + \frac{x'}{f}$$

$$\tan \gamma \, (f^2 - xx') = fx + fx'$$

or

$$f^2 - xx' = \frac{f(x + x')}{\tan \gamma}$$

or

$$f^2 - \left(\frac{x + x'}{\tan \gamma}\right) f = xx'$$

Completing the square of the left-hand member of this equation by adding to both sides the square of one-half the coefficient of f, we have

$$f^2 - \left(\frac{x + x'}{\tan \gamma}\right) f + \left[\frac{x + x'}{2 \tan \gamma}\right]^2 = xx' + \left[\frac{x + x'}{2 \tan \gamma}\right]^2$$

Next, take the square root of both sides:

$$f - \frac{x + x'}{2 \tan \gamma} = \sqrt{xx' + \left(\frac{x + x'}{2 \tan \gamma}\right)^2}$$

$$f = \frac{x + x'}{2 \tan \gamma} \pm \sqrt{xx' + \left(\frac{x + x'}{2 \tan \gamma}\right)^2}$$

In this equation we have x, x', and γ in the right-hand member which are known and therefore f can be found.

378. Terrestrial Cameras. Much emphasis has been placed upon Aerial Photographic Surveying and it is one of the great developments in recent years. However, no text would be complete without some mention of Terrestrial Photographic Surveying. The terrestrial photograph has been used in survey work for many years and many of the fundamentals of metrical photography were developed before the invention of the airplane.

A revival of interest in terrestrial photographic surveying has taken place in recent years and new instruments and cameras have been developed. Certain phases of small photographic surveys may be done more economically on the ground. A calibrated camera is about the only extra piece of equipment over and above the normal equipment needed by all surveyors This camera may be a remodeled plate camera with methods of leveling and the calibration for the metrical characteristics.

379. Principal Point. The principal point of a camera is defined as "the point at the foot of the perpendicular dropped from the rear nodal point of the lens." All planimetric and topographic mapping assumes a correctly located principal point and depends upon this assumption for accuracy. Any deviation from the correct position will cause errors both in elevation and horizontal position.

380. Fiducial Marks. These marks (also known as the *collimating marks*) are placed at the center of each edge of the negative opening and are placed so that they record images on each photograph. The principal point of a photograph may then be located by the intersection of lines drawn through the fiducial marks See Fig. 381 in which *a* represents the fiducial marks and *c* the principal point. On precise photographic cameras, the fiducial marks are adjusted so that the intersection will lie within 0.005 of an inch of the center of the focal plane opening. Furthermore, the fiducial marks are so adjusted that the projected lines intersect at an angle of 90° ± 1′.

381. Focal Plane Flatness. Errors caused by failure to maintain the emulsion surface flat in the focal plane can be large, non-uniform, and consequently serious. Not only does failure of the film to lie flat in the focal plane cause metrical errors but it also may impair the photographic quality. Provision must be made to keep the film flat as in the Fairchild Precision Camera in which a locating film back is provided with vacuum holes which hold the film tightly against an accurately finished plate.

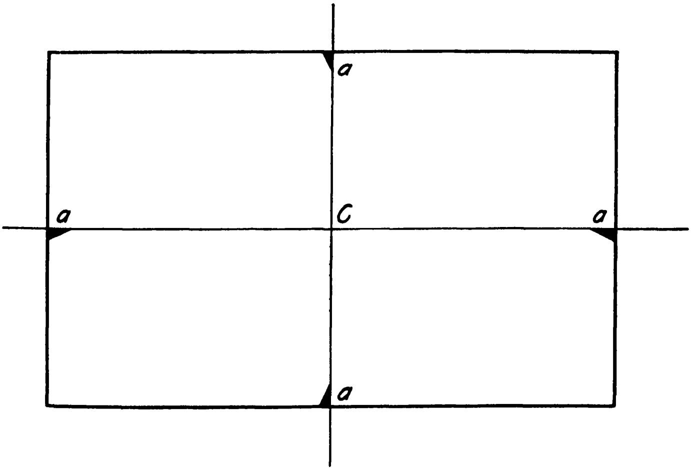


FIG. 381. FIDUCIAL MARKS.

382. Shutter. Focal plane shutters under normal conditions of use can cause a distortion of the metrical characteristics of a negative approximately four times greater than the combined distortions from other sources. The "between the lens" shutter is recommended for precision cameras as its simultaneous exposure over the entire negative eliminates metrical distortions in the negative. Speeds usually range from 1/50 to 1/300 of a second.

383. Filters. Filters are used for practically all aerial photographic work. Great care is used in selecting optically flat colored glass for this purpose, as much of the accuracy of a camera may be offset by the use of poor filters.

384. The Negative. Film or glass plates may be used in survey cameras. Film is used more often than glass plates because of its compactness and weight per photograph. Film, however, may introduce distortions if it is not held flat in the focal plane at the instant of exposure. It is quite common practice to use glass plates in a terrestrial camera and film in an aerial camera. However, one of the leading commercial companies in this country is using a plate camera for aerial work with excellent results.

385. Perspective. All photographs are perspective views so that a photogrammetrist should have a thorough understanding of the methods used in the construction of perspectives. The same principles of perspective are used in both terrestrial photographic surveying and aerial photographic surveying. In the normal construction of a perspective, a person usually starts out with the plan view and an elevation view. From these views he proceeds to imagine the object placed at some point behind a picture plane on which the perspective is to be drawn.

As a preliminary we shall define some of the common terms used in developing our fundamental concept of a perspective drawing.

PERSPECTIVE DEFINITIONS. The fact that the same principles of perspective are used in both terrestrial photographic surveying and aerial photographic surveying necessitates a few definitions in perspective before entering the theoretical discussion of photogrammetry.

All photographs are perspective views, and it is the duty of the photographic mapper to change these photographs into plan or orthographic views. In other words, he reverses the process which is ordinarily followed in making a perspective view, because in this case he starts with the orthographic views and constructs the perspective. In photographic surveying the perspective view is furnished and it is required to construct the orthographic plan view. This latter method is sometimes called iconometry.

These terms are in common use and should be thoroughly understood before further discussion:

Ground Plane — A horizontal plane usually selected below the lowest point in the territory to be mapped.

I — The position of the eye or the optical center of the camera lens.

SP — The station point or the horizontal projection of *I* on the ground plane.

C — The center of vision.

Picture Plane — A vertical plane located between the eye or camera lens and the object, and perpendicular to the optical axis of the camera lens.

Horizon Plane — A horizontal plane containing the eye or the camera lens.

Principal Plane — A vertical plane perpendicular to the horizon plane and also the picture plane and containing the point *I*.

Horizon Line — In Fig. 385 it is the line *HH'*. It is the line found by the intersection of the horizon plane and the picture plane.

Principal Line — Line *PL*, Fig. 385, the line formed by the intersection of the picture plane with the principal plane.

Ground Line — Line *GL*, Fig. 385, the line formed by the intersection of the picture plane with the ground plane.

Principal Point — Point *C*, Fig. 385, the point of intersection of the principal line with the horizon line.

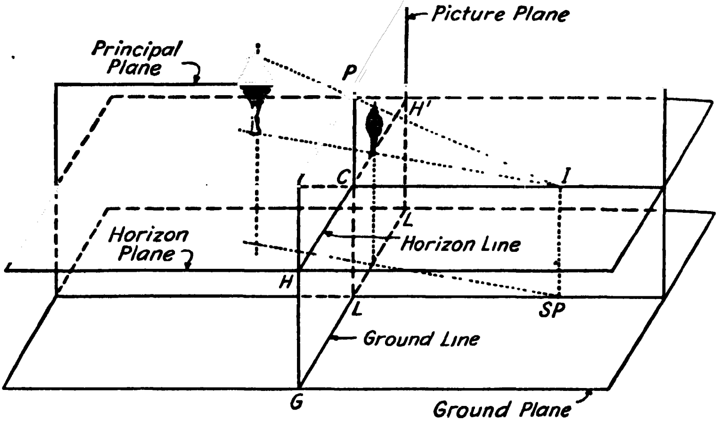


FIG. 385.

NEGATIVES AND POSITIVES. If we consider the point I to be a camera station and the optical center of the camera lens, the actual photograph taken will be on a plate or film in back of the lens and at a distance f, the focal length of the lens. This photograph will be inverted. To simplify the use of this photograph we will consider the photograph in front of the camera, erect and at the same distance f as the plate or film was from

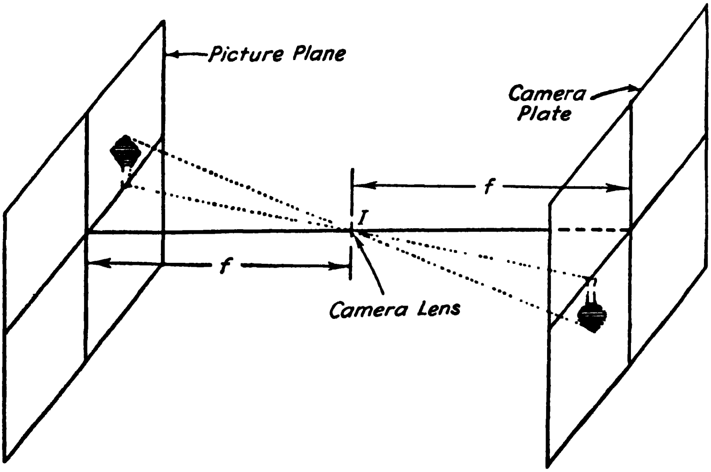


FIG. 385a. NEGATIVE AND POSITIVE.

the lens. The transposition is shown in Fig. 385a. The camera plate shown in Fig. 385a is called the negative and the picture plane is called the positive. A paper print made from a negative is called a photograph; a positive print made on a glass plate is called a diapositive.

PERSPECTIVE CONSTRUCTION. We will now construct the perspective view of a simple solid from the plan and elevation views and then attempt to show how the reverse construction would be made, that is, make the orthographic plan view from the perspective view, since in photogrammetry it is the perspective view that is furnished by the photographs.

In Fig. 385b is shown the plan and elevation views of a solid *abcdefgh*. Let us imagine that this is photographed from a camera station and the position of the camera lens is indicated at the point *I* and the horizontal projection of this point on the ground plane is at *SP*. The picture plane as shown is at a distance *f* from the point *I*. In making a perspective drawing the plan and elevation views are drawn from the known dimensions and placed at some given angle with the picture plane, *i.e.*, the position of the point *I* is known with respect to both the plan and elevation views and the picture plane.

Steps in Making the Perspective

1. Assume the ground line as shown, which, it will be remembered, is the line formed by the intersection of the picture plane with the ground plane. This line can be assumed at any convenient distance. The picture plane is simply brought forward so as not to complicate the drawing.
2. Draw the horizon line at a distance of *I–SP*, Fig. 385b, part *b*, above the ground line and parallel to it.
3. From *I*, part *a*, draw lines parallel to sides *ac* and *ab*, respectively. These two lines will intersect the picture plane at VP_1 and VP_2, which are the vanishing points of the two lines and all other lines parallel to them.
4. Project the vanishing point straight down to the horizon line at VP_1' and VP_2'.
5. From *I*, part *a*, draw the visual rays to *ah*, *be*, *cg*, and *df*, and where these lines intersect the picture plane project

straight down. Points *a* and *h* in the perspective view will fall on the line projected down from *ah*, etc.

6. Since *a* and *h*, part *c*, may fall anywhere along the projected line from *ah* in the plan view it is necessary to fix these points on this line.

7. Project the side *ah* into the picture plane by extending the side *ah-cg*, part *a*, until it intersects the picture plane at *k*.

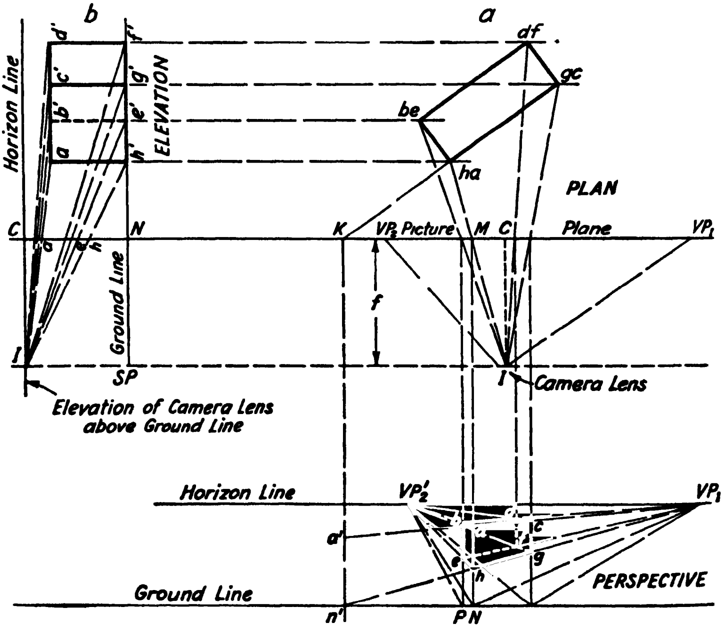


FIG. 385b. PERSPECTIVE CONSTRUCTION.

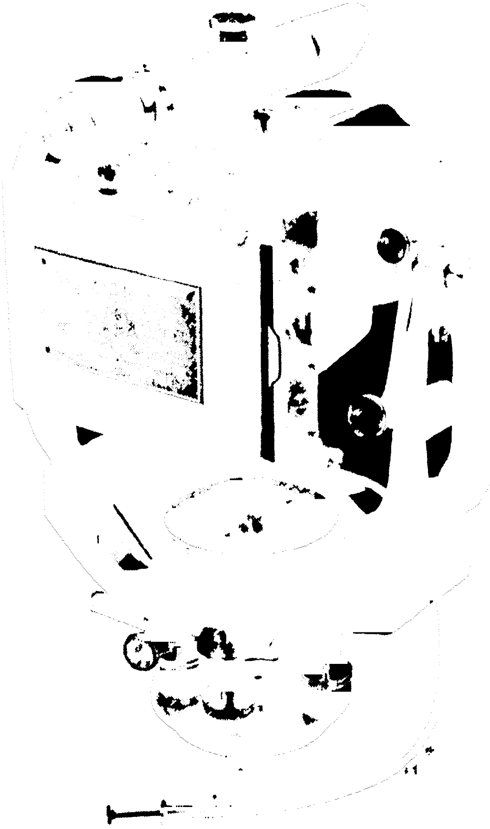
8. Project a line from *k* straight down until it intersects the ground line at *h'*. This line will be in the picture plane and true distances may be laid off on it.

9. From *h'* measure the distance *h'a'* equal to *h'a'* from part *b*. This is the height of the solid as measured in the elevation view.

10. From *a'* and *h'* of part *c*, draw lines to VP_1'. These lines intersect the line dropped down from *M* at *a* and *h* in part *c*, which are the perspective views of these points.

11. Other points of the solid are found in like manner.

12. It is to be noted that sides *ab*, *he*, *dc*, and *fg*, if extended, will meet at VP_2', and sides *bd*, *ac*, *ef*, and *hg* will meet at VP_1'. All vertical lines of the solid will appear as parallel vertical lines in the perspective view. Horizontal lines of the solid parallel with the picture plane will appear as horizontal lines in the perspective.



Courtesy Fairchild Camera and Instrument Corporation

FIG. 386. TRANSIT CAMERA

386. Terrestrial Photographic Surveying. Terrestrial photographic surveying furnishes rapid, economical, and accurate procedures which may be used on small jobs. It is not necessary to have a large amount of expensive equipment to accomplish good results. In fact, about the only extra piece of equipment is the phototheodolite or the transit camera.

387. Iconometry. As has been stated, photographs are perspective views. Therefore, some method must be used to change from a perspective to a plan view of orthographic projection. The process used is sometimes called *iconometry*. A person must have an understanding of perspective in order that a reversal of the perspective theories may be carried out. Some of the general procedures will now be considered. Refer again to Fig. 385b; we will start with part *c* known. The required information to locate an object is:

(a) The focal length of the camera lens.
(b) The scale of the map.
(c) The plotted position of the camera station.
(d) The direction to an object shown in the photograph.
(e) The elevation of the camera station.
(f) If one photograph is used, the elevation of the photographed object must be known.

Let us suppose that the position of I is known and, knowing f, we can draw the picture plane a distance of f from the point I. From I the direction of the visual ray to points a and h are known so that the line IM can be drawn indefinite in length. Point a will lie somewhere along this line. The point C can be located on the photograph because a perpendicular from I to the picture plane will give this point (see Fig. 385). The point C is called the principal point. The horizon line is drawn through this point and parallel to the picture plane. Having located C, we can draw the ground line parallel to the horizon line and at a distance of I-SP below, Fig. 385b(*b*). This is the elevation of the point I. On the photograph (perspective view) measure the distance Na and lay this off to scale on the line cN (part *b*). Next draw an indefinite line Ia (part *b*). If the elevation of a above the ground line is known, the indefinite line Ia is extended until $a'h'$ (part *b*) is equal to the elevation of point a. Point a is then fixed in elevation. Extend a line from this point horizontally across to meet the indefinite line Ia in part a, and the point a is located in the plan. In a similar manner other points can be located.

TWO PHOTOGRAPHS. It will be noted that in the last paragraph only one photograph was used and that in order to use only one photograph it was necessary to know the elevation

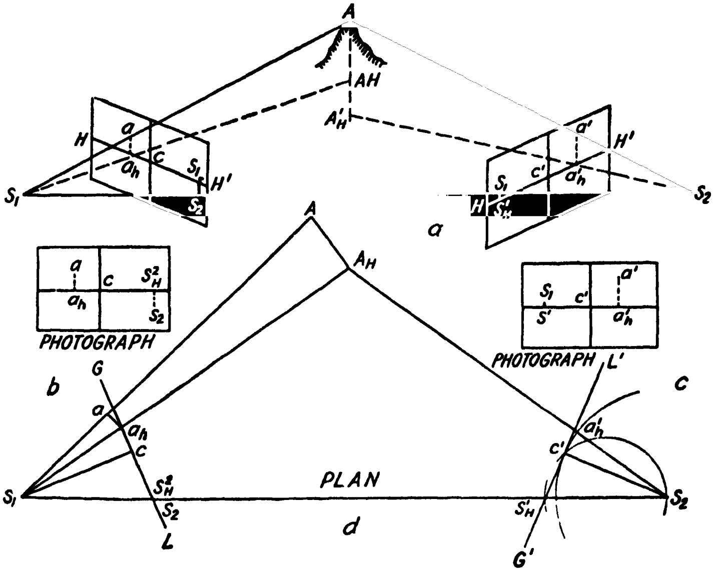


FIG. 387. GRAPHICAL PLOTTING.

of the object to be plotted. This can be used only in special cases because usually the elevations of a number of points in a photograph are not known. To eliminate this difficulty two photographs can be used. The following discussion will show how this is accomplished.

In Fig. 387(a) let us assume that photographs of a point are taken from stations S_1 and S_2. Let this point be A. Each photograph must include the station at the other end of the base line. The length of this base line (S_1S_2) is known and the elevation of each station is known. The two plate positives are shown in Fig. 387(a) with the photographed image of the point A and the stations shown, designated by a and a' for the point and by s_1 and s_2 for the stations; c and c' have the same meaning as before — the principal point of the respective photographs. Figures 387(b) and 387(c) represent the photographs as viewed by an observer when held directly in front of him. Coordinate axes are drawn on all the representations of the photographs. These enable the observer to measure x and y coordinates of points shown on the photographs. Cameras used for mapping purposes have some means of indicating the points through which these axes must be drawn. Some points appear on the edge of the photograph as small notches, others have very small circular dots. Figures 387(b) and 387(c) are the photographs from which the measurements should be made. If Fig. 387(a) were projected into the ground plane, Fig. 387(d) would result. The two picture traces are shown as GL and $G'L'$ If these two traces are properly oriented with respect to each other, a line drawn from S_1 through a_h will intersect a line through S_2 and a_h' and A_H (Fig. 387[d]). This is the location of the point A in plan. Other points can be located in a similar manner. The location of a_h in Fig. 387(d) can be accomplished by measuring the distance ca_h on the photograph (Fig. 387[b]) and making ca_h equal this in Fig. 387(d). The same process can be used on the other photograph to locate a_h'. These are the x coordinates of the photographed point.

ELEVATION OF POINTS. In Fig. 387(a) the lines S_1A, S_1A_H, and AA_H form a vertical right triangle in which AA_H is the height of A above the station S_1 since S_1A_H is a horizontal line. Let us now imagine this triangle laid over on its side so that it will appear in the plane of the paper and can be plotted as shown in Fig. 387(d) as S_1AA_H. To construct this triangle, measure a_ha (Fig. 387[b]) and lay this distance off on a perpendicular to the line S_1A_H at a_h, Fig. 387(d). Now draw the line S_1a (Fig. 387[d]) to meet a perpendicular erected at A_H. The line AA_H of Fig. 387(d) is equal to AA_H of Fig. 387(a) and is the elevation of the point above S_1.

ORIENTING THE PICTURE TRACE. As stated in the last article, it is necessary to have the picture traces of the two photographs properly oriented. The following is one common method of accomplishing this without any great difficulty. Referring to the lower right-hand part of Fig. 387(*d*), we see that the distance $s_H'c'$ is measured from the photograph, the distance $c'S_2$ is f, the focal length of the photograph. From the triangle $c'S_2s_H'$ the hypotenuse $s_H'S_2$ is given by the equation

$$s_H'S_2 = \sqrt{(s_H'c')^2 + f^2}$$

1. Lay off this distance from S_2 to s_H' and, with this as a diameter, construct a semicircle $s_H'c'S_2$.

2. Next with f as a radius and a center at S_2, describe an arc cutting the above semicircle at c'.

3. Points s_H' and c' are two points on the picture trace and therefore $G'L'$ can be drawn.

4. The same process can be used in plotting the picture trace of the other photograph and will be shown at GL.

The picture traces are now oriented with respect to S_1 and S_2 and with respect to each other. Since the locations of S_1 and S_2 are known and the elevation of each is known, we are ready to determine the location and elevation of points appearing in the photographs as explained in the preceding article. If contour lines are to be drawn by this method it is necessary to find the location and elevation of numerous points and then interpolate as in any stadia or plane-table survey.

PROCEDURE IN MAKING A TERRESTRIAL PHOTOGRAPHIC SURVEY.

Field Work

1. Establish a number of triangulation stations by the usual transit methods. The selection of these stations is important in order to get good intersections. The highest points do not make the best, as is usually the case in a triangulation survey, because the photograph will contain too much sky and clouds.

2. Occupy the triangulation stations with the camera, taking as many photographs as will cover the territory to be mapped. A general sketch and numbering the photographs will assist in the office work when compiling the finished map.

3. All points to be located should appear in at least two photographs, preferably three or more for checking.

4. If a transit camera (see Fig. 386) is used, angles between the camera axis and a known base line are measured.

5. If no angles are measured, each photograph should contain the photographed image of one or more adjacent triangulation stations.

6. Negatives should overlap so that the points to be mapped are common to two adjacent pictures.

Plotting.

1. Plot all triangulation stations by coordinates as for any triangulation system.

2. Draw on the map the picture traces as previously explained

3. Draw coordinate axes on the photographs so that x and y coordinates of points to be located can be measured.

4. Horizontal positions of points are then found.

5. Elevations of points are found by methods previously explained.

6. If contours are to be drawn, the elevations of a number of points are determined and the contours interpolated for, as in transit-stadia or plane-table work.

SKETCHING FROM A GRID SYSTEM. This method is particularly adapted to making sketches of the shore line of lakes, because in this case the elevation of the water surface can easily be determined and used to advantage in making the map

In Fig. 387a assume that we have photographed a lake from the camera station *I* and that we know the elevation of this camera station above the water level of the lake.

1. On a piece of mapping paper draw a grid system to any desired scale as in part *a*. This grid is composed of a block of squares of equal size. Take the lower line as the intersection of the picture plane with the plane of the paper.

2. On the center line extended, plot *I* a distance of *f* (focal length of camera lens) from the picture plane.

3. Draw lines *IaA*, *IbB*, *IcC*, etc.

4. In part *b* draw a line to represent the elevation of the water surface of the lake (*WL*).

5. From the known elevation of the camera station above the water level, plot to scale the position of *I′* (part *b*). Through

this point draw a line parallel to the water-surface line. This is the horizon line, intersecting the picture plane at V'.

6. Extend parallel horizontal lines (part a) across to the water-surface line of part b (like a', etc.). From each intersection draw lines to I' (part b).

7. Measure distance $V'd'$ (part b) with a pair of dividers and lay off as Vd'' (part c). Draw line a''–g'' (part c).

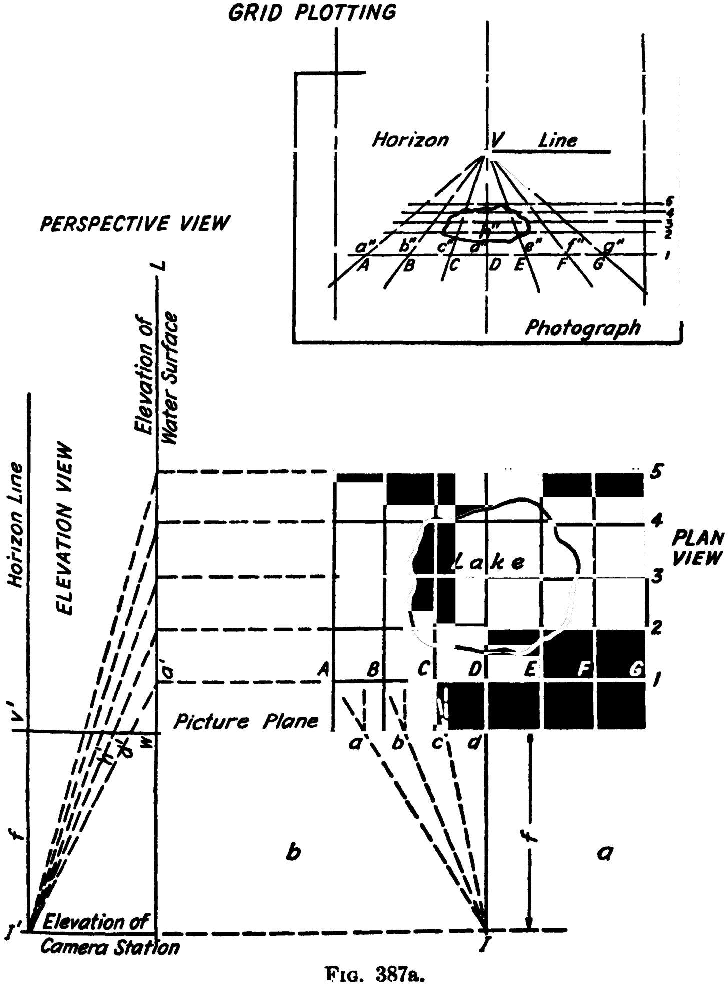


Fig. 387a.

8. Project points *a, b, c, d,* etc. (part *a*), to *a″, b″, c″, d″*, etc. (part *c*).

· 9. Draw lines to vanishing point *V* (part *c*) as *a″V, b″V, c″V*, etc.

10. From part *b* measure *V′h′* and lay off (part *c*) as *Vh″* and draw a line through this point parallel to *A–G*. Proceed in this manner until all horizontal lines are drawn in part *c*.

The grid just described can be laid out on the photograph or on a piece of transparent celluloid or glass. If it is made on a separate sheet, this sheet is used to superimpose on the photograph.

We are now ready to transfer the shore-line points to the plan or map. Transfer, point by point, from the perspective view to the corresponding squares in the plan view. The relative position of points in the perspective view can be estimated quite closely by eye or can be scaled if a more accurate map is desired.

The fundamentals of this scheme are used by the Canadian Government in mapping thousands of square miles of territory from oblique aerial photographs. This method will be discussed later under Example No. 2.

AERIAL PHOTOGRAMMETRY

388. Aerial Photography. Aerial photography has so many uses in all branches of civil engineering that a complete coverage in this chapter is quite impossible. An attempt will be made to cover many of the more common uses of aerial photographs along with some of the practical applications.

The aerial photograph must be of the highest quality and, therefore, the use of a precision camera and low shrink film and paper are the first and most important considerations.

In general, aerial photographs may be classed as (1) verticals; (2) obliques; and (3) composites.

(1) *Verticals* — those photographs taken with the axis of the camera in a vertical position and taken directly beneath the plane.

(2) *Obliques* — those photographs taken at an inclined angle.

(3) *Composites* — a combination of the first two such as those taken with the three-, four-, or five-lens camera. In this case the center picture is a vertical and the others obliques. The obliques are usually rectified into verticals afterward.

389. Fundamental Principles. The fundamental principles as used in terrestrial photographic surveying apply to aerial photographic surveying except that many errors and distortions are introduced into the aerial photograph which are not apparent in the terrestrial photograph. In terrestrial photography the camera is rigidly fixed. Plenty of time is available for exposures, reading angles, etc. Also the ground stations are accurately located by a system of triangulation. These conditions do not exist in aerial work. The camera is in a fast-moving plane where exposures must be made in rapid succession. The altitude of the plane at the instant of exposure must be known. The direction of flight and any tilt and swing are all factors which may cause distortion in the photograph and are all difficult to control.

To cover a large area, the photographs are taken in flight strips so that they are overlapped at least 50 per cent (usually 60 per cent) in the direction of flight (called end lap) and about 30 per cent between adjacent flight strips (called side lap). This overlap assists in matching the photographs and permits the use of a stereoscopic instrument in viewing them. (See Fig. 391.)

390. Four Parts to the Survey. There are really four parts to the making of a map from aerial photographs:

(a) The flying
(b) The photography
(c) The ground control
(d) The construction of the map

The first two are not the duties of the engineer as a rule. The latter two are the duties of the engineer. It is of advantage to the engineer to know how the pictures were taken and a few of the difficulties of the pilot and photographer.

391. Errors and Distortion. The point on the ground directly beneath the camera lens is called the "ground nadir point, sometimes the "plumb point." This is the point that would be indicated by a plumb line if it could be suspended from the lens at the instant of exposure.

The "principal point" of the photograph is as described in terrestrial photography, the point formed by the intersection of the coordinate axes.

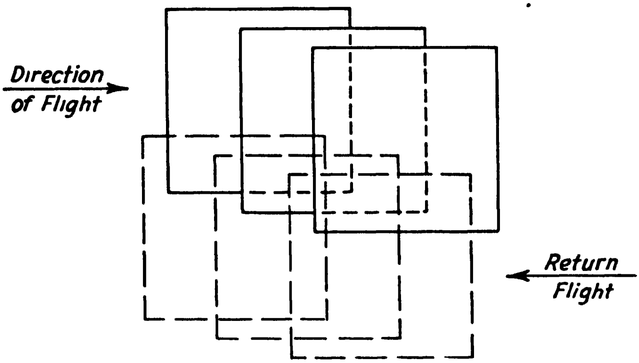


FIG. 391. END LAP AND SIDE LAP.

Aerial cameras are equipped with level bubbles, and some photograph the position of the bubbles at the same time that the exposure is made. Experiments have been made with gyroscopic instruments for leveling the camera but have not met with much success.

If the camera is level when the exposure is made, the nadir point and the principal point will coincide. This condition is difficult to obtain; therefore information on the photograph must be used to determine the amount of tilt and the resulting displacement of these two points.

Swing and crab are introduced owing to the fact that the plane must be headed slightly off the course to compensate for wind and air currents.

The ground speed of the plane and the altitude of the plane at the instant of exposure must be known. The area covered in one exposure must be known and the bearing or azimuth of the courses taken by the plane.

392. Overlap. Overlap means that photographs are taken so that certain portions of the terrain appear in several pictures. The customary overlap in the direction of flight is 60 per cent; the side lap is usually 30 to 50 per cent. Figure 391 illustrates the general scheme of taking these photographs.

REASONS FOR OVERLAP

1. To orient prints to form a continuous flight strip.
2. To permit the use of a stereoscopic instrument. Only the overlapped portion is available for this work (must have at least 50 per cent overlap).
3 The central portion of the print is less distorted than the outer edges. With overlapped photographs the outer portion can be discarded.

In Fig. 391, the photographs are set over to illustrate overlap, but, in actual practice, the edges of a flight strip should be as near to a straight line as possible. It requires very good flying and proper equipment to accomplish a flight mission which gives straight photographic strips and pictures that are free from tilt.

393. Scale of the Photograph. Photographs are perspective views and are formed by the light rays which form a cone as projected from the ground to the lens of the camera. Passing through the lens they form a smaller cone which is intercepted by the negative in the camera. Figure 393 represents the ideal condition, with the ground level and the plate or negative in a horizontal position and parallel to the ground plane.

The *scale of the photograph* is determined by a simple proportion (sometimes called the scale fraction).

$$\text{Scale} = \frac{AB}{ab} = \frac{H}{f}$$

In words it is the ratio of the altitude of the lens to the focal length of the lens. If H is in feet, as is usually the case, and f in inches, the scale is in feet per inch.

The altitude of flight may be anything from 5000 feet to 5 miles. Flights at less than 5000 feet are usually difficult to control because of the uncertain air conditions near the ground.

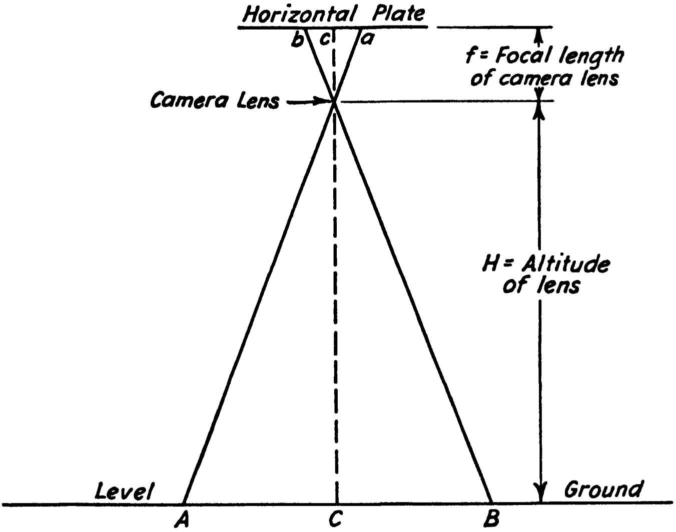


Fig. 393. Determination of Scale.

Area Covered. If the scale of the photograph is known and also the length and width of the exposed portion, the area of the ground covered in one photograph is determined by the equation

Area = (length × scale fraction) × (width × scale fraction)

Example: Assume that the altitude H = 10,000 ft. and the focal length of the lens = 10 in.

$$\text{The scale fraction} = \frac{H}{f} = \frac{10{,}000}{10\text{ in.}} = 1000\text{ ft. per in.}$$

Let the size of the exposed negative be 7 in. × 9 in. The ground area is then found by the following:

$$\text{Area} = (7 \times 1000) \times (9 \times 1000) = 63{,}000{,}000\text{ sq. ft.}$$

Reducing this to acres and then to square miles, we have

$$\frac{63{,}000{,}000}{43{,}560} = 1446\text{ acres (approx.)}$$

$$\frac{1446}{640} = 2.26\text{ sq. mi. (approx.)}$$

This means that each exposure covers an area of approximately 2.26 sq. mi. of territory.

394. Time Interval of Exposure. Now assume that we want to determine the time interval between exposures in making a flight strip (using the information derived in § 393 and assuming a ground speed of 100 miles per hour).

Using an end lap of 60 per cent and the 7 × 9-inch photograph, we should cover, in the direction of flight, a distance of 2800 feet between exposures.

$$7 \times 1000 \times 0.40 = 2800 \text{ ft.}$$

In the above equation

7 in. = the length of the negative

1000 = 1000 ft. per in. the scale of the map

0.40 = 100% − 60%

Since the speed of the plane is 100 miles per hour, this reduces to velocity in feet per second in the following formula:

$$V = \frac{5280}{3600} S = 1.467 S$$

then

$$V = 1.467 \times 100 = 146.7 \text{ ft. per sec.}$$

$$\frac{2800}{146.7} = 19.1 \text{ second (say 19 second.)}$$

This means that exposures should be made every 19 seconds during the flight. In this case and similar problems, use the next lower number to insure at least 60 per cent end lap.

395. Number of Pictures to Cover a Given Area. The total number of photographs required to cover an area will be the total area divided by the area of each photograph. This provides for no overlap. Assume as before that the overlap in the direction of flight (end lap) is 60 per cent and the side lap is 30 per cent.

Then each photograph will in reality add only a small portion of new area. The percentage of new area will be

$$(100\% - 60\%) \times (100\% - 30\%) = 28\%$$

There will then be 100/28 more photographs required than if no overlap were used.

Suppose that an area of 80 square miles is to be covered and the scale is 1000 feet per inch on a 7 × 9-inch print.

$$\begin{aligned}\text{The area in square feet} &= 80 \times 5280 \times 5280\\ &= 2{,}230{,}272{,}000 \text{ sq. ft.}\end{aligned}$$

The number of prints with no overlap then is

$$\frac{2{,}230{,}272{,}000}{7000 \times 9000} = 35.4 \text{ (say 36 photographs)}$$

Then with overlap

$$\frac{100}{28} \times 36 = 128.6 \text{ (129 photographs required)}$$

396. The Stereoscope. Nearly everyone has looked through a stereoscope at one time or another such as the old parlor stereoscope which was common in every home at one time. It consists of two eyepieces with magnifying lenses and a carrier for two photographs. When viewing these photographs, the objects therein stand out in relief; that is, one not only sees vertical and horizontal distances but is able to perceive depth

In order to get stereoscopic vision, the two photographs must be taken from two different viewpoints. In operating the stereoscope, the right-hand view is seen with the right eye and the left-hand view with the left eye. These two views are merged together to form one picture in which length, height, and depth are clearly visible.

397. Stereoscopic Vision. The phenomenon of binocular or stereoscopic vision is exceedingly complex and investigators do not yet agree entirely as to its explanation. The following is an explanation of some of the simple principles which are the only ones in which we are particularly interested.

In binocular vision two things enable a person to judge distances. They are —

1. The relative position and size of the objects
2. The stereoscopic principle of parallax.

In Fig. 397 assume that E and E' represent the position of the two eyes of an observer. The curved arcs below E and E' represent the retina of the two eyes. In viewing a distant object O the retina of the left eye is impressed at o and the right at o' and therefore O is at infinite distance. Now assume that a nearer object is viewed, O_n being a near corner and O_f a far corner.

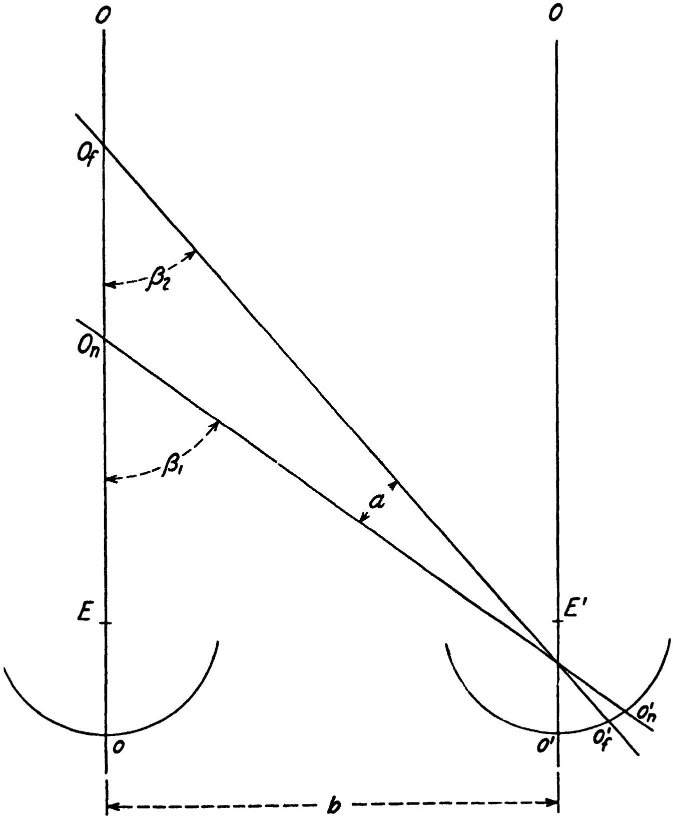


Fig. 397. Stereoscopic Vision.

The retina of the left eye is impressed at o and the right eye at o'_f for the far corner and o'_n for the near corner. The angles β_1 and β_2 are termed the angles of parallax for the two objects and α the differential parallax and is expressed by the following equation:

$$\alpha = \beta_1 - \beta_2.$$

The following physical conditions limit the distance from which stereoscopic vision can be perceived:

1. When the angle β_1 becomes less than about 20 seconds the observer no longer perceives the spatial relation of points. This is the smallest angle which the eye can sense.
2. The normal distance between the human eyes is $2\frac{1}{2}$ inches.

From these limitations the distance *EO* is about 2100 feet. This means that beyond 2100 feet we no longer get stereoscopic vision with the naked eye. It is true that we judge distance beyond this limit but we do so by the relative size and position of objects, light, and shadows.

It is possible to increase the distance of stereoscopic vision in two ways. They are

1. By increasing the base distance *b*
2. By magnifying the field of view

This principle is employed in ordinary binoculars. By means of prisms the objective lenses may be extended so that they are 5 inches or more apart. Also, the magnification of the image is accomplished by means of the lenses. If the base is increased to 5 inches the angle of parallax is increased by an amount equal to the ratio of 5 to 2.5 or 2 times. If the magnification is 4, then the stereoscopic vision is increased $2 \times 4 = 8$ times.

The stereoscope is an indispensable instrument for photogrammetrists and many sizes and forms may be purchased from American manufacturers. A magnifying mirror stereoscope is preferable and the following description of the Fairchild Model F-71 Magnifying Stereoscope will give the reader some of the important features to look for in a high grade instrument. This instrument differs from the usual type of stereoscope in several ways, chiefly in the fact that it uses 4-power binoculars to permit close examination and detailed study of small areas of overlapped pairs of aerial photographs in addition to simple stereoscopic observation.

Figure 397a illustrates this instrument. It is compact, light in weight, sturdy, and accurate. These are essential features of a good stereoscope. The optical system consists of two first-surface evaporated rhodium-coated mirrors free from double images (ghosting); two right angle 45 degree prisms, silvered and lacquered on the hypotenuse surface; and two high quality lenses of approximately $11\frac{1}{4}$-inch focal length. A pair of

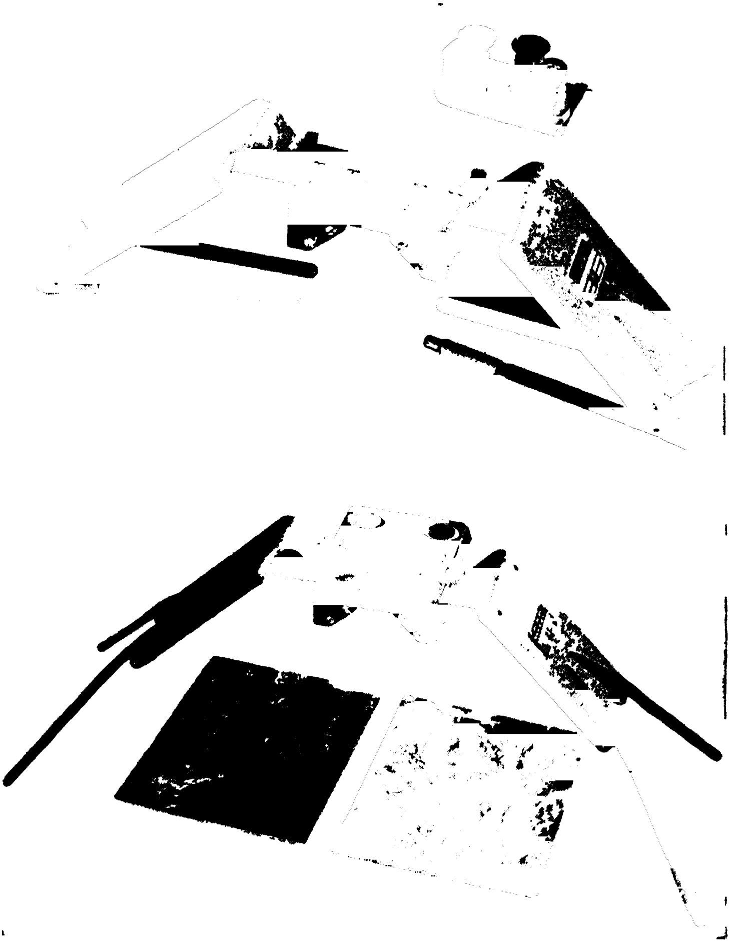


FIG. 397a. THE FAIRCHILD F-71 MAGNIFYING STEREOSCOPE.

4-power binoculars having 20 mm. objective lenses and 5 mm. exit pupils are attached to the top of the frame and may be removed or tilted back out of the way when not in use.

The optics are arranged to accommodate inter-pupillary distances from 52 mm. to 75 mm. which makes it possible to train a large number of persons with normal vision to use the instrument in a very short time. This feature is of great importance to military organizations where various groups such as engineer, artillery, infantry, and intelligence personnel may be called upon to interpret and catalog the information afforded by aerial photographs.

The magnifying feature of a stereoscope is particularly advantageous in the detection and study of camouflage and other details of aerial photographs requiring close study.

398. Stereoscopic Fusion. The following simple demonstration will illustrate the stereoscopic fusion of points. Take a card and place two black dots on it as shown in Fig. 398.

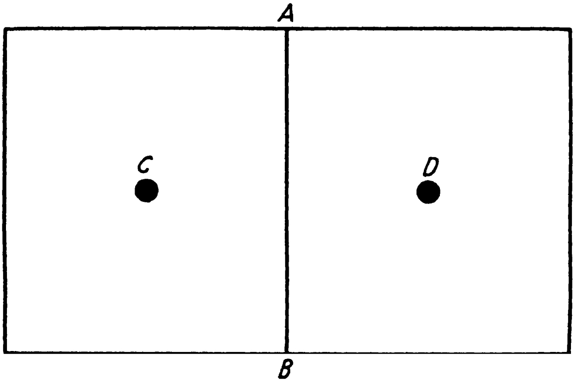


FIG. 398. STEREOSCOPIC FUSION — TWO DOTS.

Next, place a second card perpendicular to the first as at AB. Hold this combination directly in front of the observer's two eyes, so that the right dot is seen with the right eye and the left dot with the left eye. Now imagine that you are looking through Card No. 1. The two dots will slide in toward each other until they are merged into a single dot.

Figure 398a illustrates the geometric condition of this experiment. The two dots have been brought together and appear to the observer to be at the point O in back of the card on which they were drawn. This is *stereoscopic fusion.* The depth or distance to *O* in back of Card No. 1 will vary with the spacing of the dots.

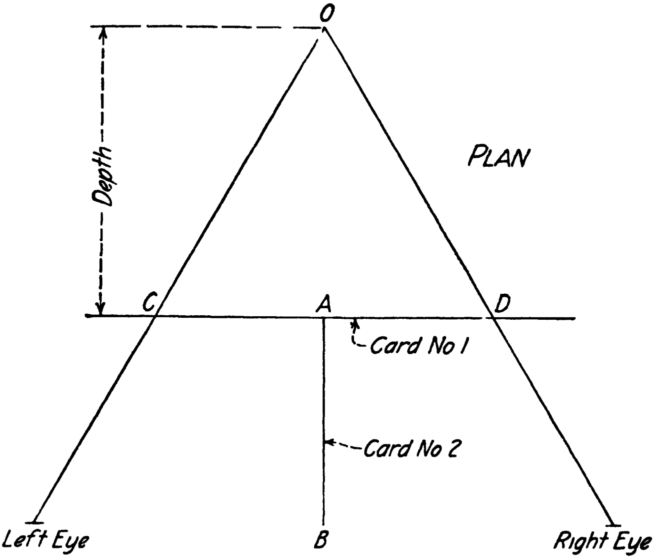


FIG. 398a. GEOMETRY OF FUSION.

Now let us take two pairs of dots as in Fig. 398b. The dots *C* and *D* are closer together than the dots *E* and *F*. As before

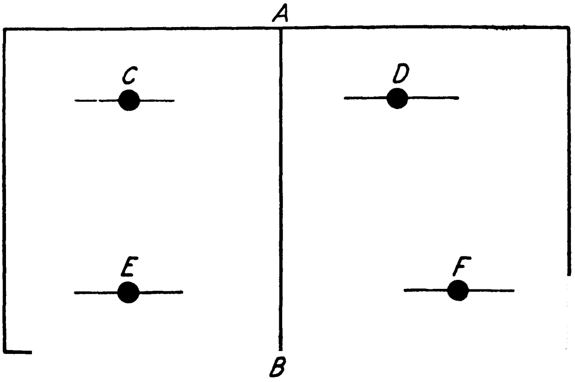


FIG. 398b. TWO PAIRS OF DOTS.

a second card is placed perpendicular to the first and the observer views the dots. The dots *C* and *D* will merge together and the dots *E* and *F* will merge together, but the top dot will appear closer to the observer than the lower dot.

Figure 398c illustrates geometrically the fusion which takes place and indicates why the dots *C* and *D* appear at *O* and the dots *E* and *F* at *O'*. Thus the spacing of the dots is a measure of the depth. This explains why the floating point in the stereocomparator can apparently be moved toward and away from the observer by shifting the right-hand pointer. When we bring the floating point in contact with the top of the hill (as in § 400), we might consider this pointer to be moved from *F* to *D* and in doing so that the floating point has been brought toward the observer from *O'* to *O*. The distance *DF* is the parallactic displacement and is measured on the *p* scale.

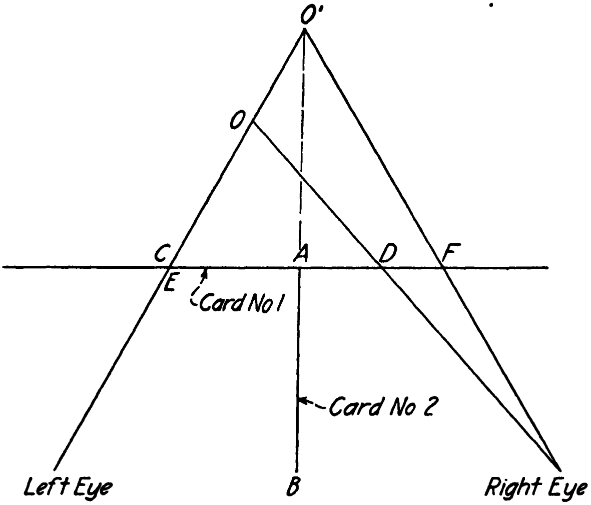


FIG. 398c. GEOMETRY OF FUSION.

399. Fundamental Principles. The following discussion of fundamental principles has been applied to terrestrial photographic surveying, but it is to be remembered that these same principles may be applied to overlapped aerial photographs.

The camera axis is usually horizontal when one talks about terrestrial work and the camera axis is usually vertical when speaking about aerial photographs.

STEREOSCOPIC PARALLAX. *Horizontal Distance.* Assume that two photographs are taken of a range of hills and are represented in Fig. 399 as Photograph No. 1 and Photograph No. 2. The base distance B is known, as is f, the focal length of the camera lens.

To find the distance to the point A, we have by similar triangles

$$\frac{D}{B} = \frac{f}{p} \qquad \text{or} \qquad D = \frac{Bf}{p}.$$

From this equation we can compute D because B and f are known and the value of p can be measured from the photographs. That is,

$$p = a_1c_1 + c_2a_2$$

in which p is called the parallactic displacement or absolute parallax.

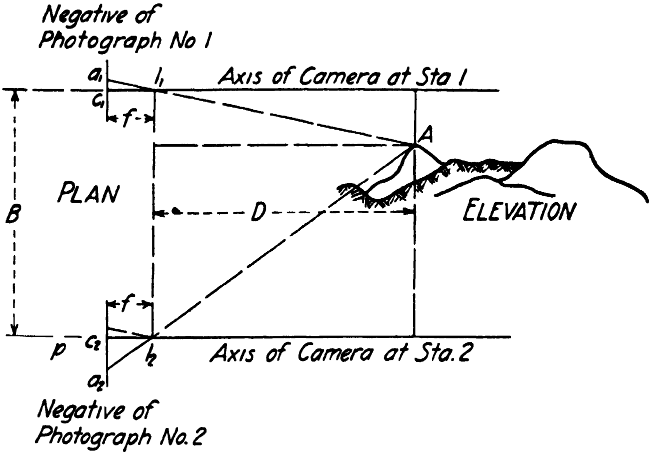


FIG. 399. ABSOLUTE PARALLAX.

Difference of Elevation. In Fig. 399a, take the Photograph No. 2 and use the plate positive instead of the negative. It is

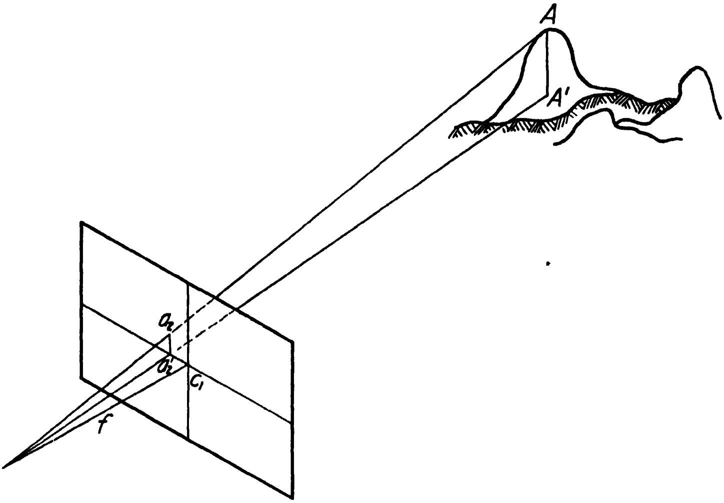


FIG. 399a DIAPOSITIVE FOR FIGURE 398.

possible to measure the x and y coordinates of Point A from the photograph. Figure 399b shows the construction necessary to determine the required information. Referring to that figure,

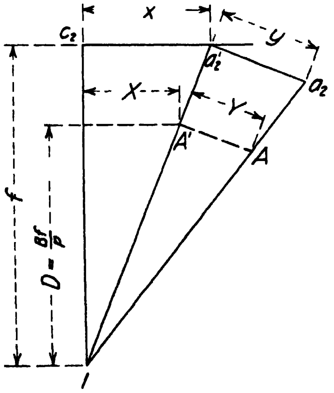


FIG. 399b. STEREOSCOPIC PARALLAX.

1. Draw a line Ic_2 equal in length to f, the focal length of the camera lens (actual length)
2. Measure x on the photograph and lay it off as c_2a_2' and perpendicular to c_2I
3. Draw Ia_2'. This fixes the direction of the ray of light to A'
4. Erect a perpendicular to Ia_2' at a_2' and on this lay off the distance y (measured from the photograph)
5. Draw Ia_2. This will fix the direction of the ray of light to A
6. From the point I lay off $D = \frac{Bf}{p}$ to an appropriate scale and erect a perpendicular intersecting Ia_2' at A'. This locates

the horizontal position of A on the map and gives the value X

7. Erect a perpendicular at A' to Ia_2' intersecting Ia_2 at A. The distance $A'A$ is the difference in elevation between I and the point A and is designated by Y

This process can be repeated for any number of points which are to be plotted.

400. The Stereocomparator. From the preceding section it is seen that the location of a point depends on B (the base line), f (the focal length), p (the parallactic displacement), and x and y.

The measurements x, y, and p can be measured conveniently in an instrument called a *stereocomparator.* Then referring to Fig. 399b the following equations will locate any point desired

$$D = \frac{Bf}{p}$$

$$X = \frac{Bx}{p}$$

$$Y = \frac{By}{p}$$

The stereocomparator as used in practice is a precision instrument, made to make very fine measurements of x, y, and p usually by means of micrometer attachments. The principle, however, can be illustrated by a line diagram such as Fig. 400. A frame holding two overlapped photographs taken from two different viewpoints is shown. These are viewed through a pair of magnifying lenses and are so adjusted that they appear as a stereoscopic pair. On the top of the frame holding the photographs is a movable x scale which can be moved in a horizontal direction. The y scale which is held in a slot in the x scale can be moved in a vertical direction. This y scale is attached to the left-hand floating point. The right-hand floating point is also held in the x scale frame and is moved horizontally when the x scale is moved, but in addition it can be moved independently both horizontally and vertically.

Refer now to Fig. 400 and remember that the observer is looking at the two photographs stereoscopically so that he sees only one picture. Furthermore, he sees only one floating point because the floating points are fused together due to the stereoscopic vision. When the floating points are at the origins of coordinates as shown in Fig. 400 the scales all read 0.

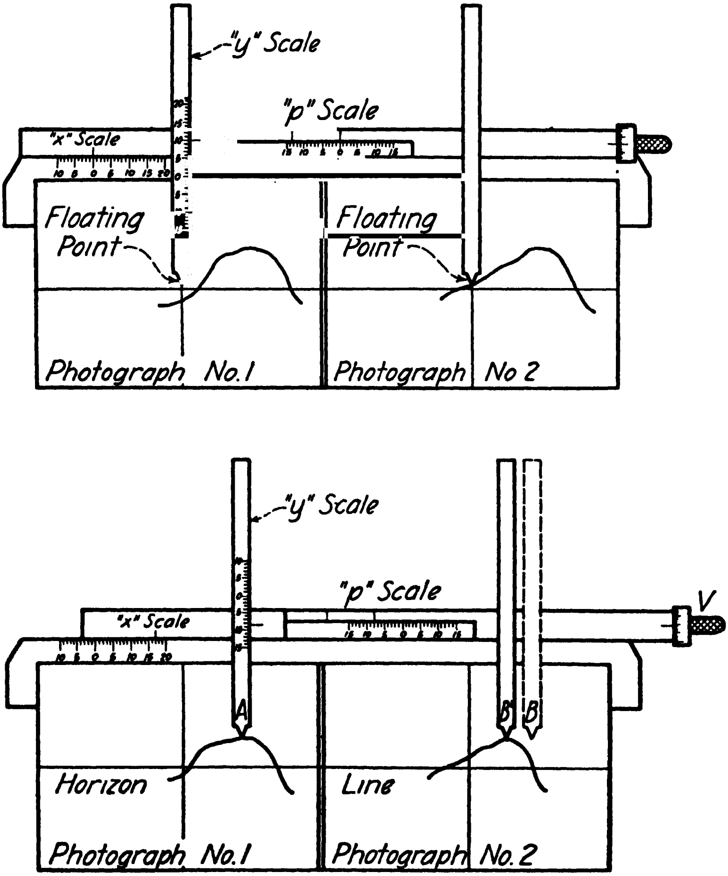


FIG. 400. THE STEREOCOMPARATOR.

Now let us determine the location and elevation of the top of the hill shown. We need to measure x, y, and p to use in the set of equations for D, X, and Y as determined in the first part of this section.

While sighting through the stereoscope, move the frame carrying the scales horizontally to the right until the floating points (only one as the observer sees, but really two) are in line with the top of the hill. The points will travel along the horizon line. The value of x is read from the x scale.

Next move the floating points vertically until the points are in line with the top of the hill. The value of y is read from the y scale. The floating points will be in the positions A and B.

As the observer sees the merged floating point it will appear to be in line with the top of the hill but in back of it. The floating point can be brought forward and in contact with the top of the hill by moving the right-hand floating point toward the left. This is accomplished by turning the vernier V. In this operation, the floating point B is moved to B'. The amount that it is moved is recorded on the p scale. This is called parallactic displacement. Thus we have measured x, y, and p.

The fundamental principles of stereoscopic fusion and displacement are common to nearly all stereoscopic measuring and plotting instruments and the floating marks in some form are always present. The line diagram of the stereocomparator represents a very simple instrument and is used here to illustrate fundamentals.

401. Stereoscopic Principle Applied to Aerial Photographic Surveying.

Two adjacent overlapped photographs from a flight strip may be placed under a stereoscope as illustrated in Fig. 397a.

Since the camera axis was vertical when the photographs were taken, any stereoscopic displacement of images now corresponds to height or difference of elevation. The same fundamental principles of the floating marks may be combined with the stereoscope and any difference in the spacing of the floating marks may be measured. This difference bears a definite relation to the difference in elevation. All stereoscopic plotting instruments have scales (usually micrometer attachments) or graphic attachments for measuring the difference in spacing of the floating marks. A mathematical solution is necessary when the readings are numerical or tables may be prepared to speed up the compilation.

FIG. 402 THE STEREOCOMPARAGRAPH.

402. The Stereocomparagraph. One of the simplest stereoscopic plotting instruments is the *stereocomparagraph.* A description of this instrument will be given, but again it must be remembered that the same fundamental principle is used in other more elaborate and more expensive instruments. The accuracy of stereoscopic plotting instruments is about in proportion to the cost. The prices range from a few hundred dollars to several thousand dollars. The more costly instruments have very accurate measuring devices and complicated optical systems, and only a few companies can afford such equipment.

The *stereocomparagraph,* illustrated in Fig. 402, is a simple stereoscopic plotting instrument. The instrument is attached to a device which allows it to be moved over the photograph while maintaining a parallel position. The instrument consists of a mirror stereoscope and a right and left lens which rest upon the photographs and directly beneath the respective mirrors of the stereoscope. A small black dot is placed at the center of each lens and these dots form the floating mark when fused together stereoscopically. The right lens is adjustable by means of a micrometer while the left lens remains fixed. Thus, the spacing of the two dots may be changed and the principles of § 398 apply. The micrometer is used to record the change in spacing of the dots and this bears a direct relation to the change of elevation in the stereoscopic model viewed by the operator. The theory of parallactic displacement for a stereocomparator may now be applied to determine the difference in elevation of points on the ground. Fig. 402a shows the *Abrams contour finder* with a planimeter attachment.

If the comparagraph or contour finder is to be used for drawing contours directly, the floating mark is made to rest upon a point of known elevation by moving the instrument on its parallel arm and by adjustment of the micrometer. If the instrument is now moved so that the floating mark is kept in contact with the ground, as seen in the stereoscopic model, a contour line will be traced by the pencil in the extended tracing arm.

The tracing arm is illustrated in Figs. 402 and 402a. The next contour level may be plotted by changing the micrometer setting which must be determined for the given contour interval The floating mark is brought in contact with the ground and the contour drawn. The procedure may be followed throughout the entire model. The contours may be superimposed on a photograph or upon a planimetric map previously drawn by one of the methods outlined in this chapter.

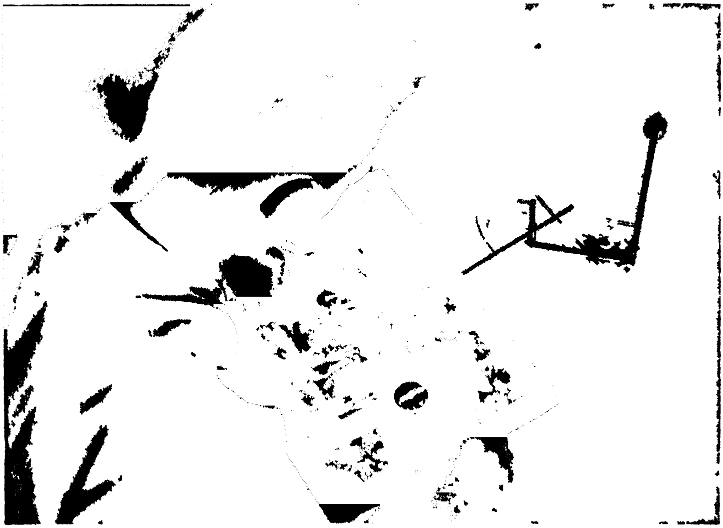


FIG. 402a. THE ABRAMS CONTOUR FINDER.

Stereocomparagraph Theory. The principles of parallactic displacement may be applied to the stereocomparagraph and the following analysis is of importance because it applies to a very important general principle. The theory may be applied to measurements of absolute parallax made with a finely divided scale if no stereocomparagraph or similar instrument is available.

Figure 402b is set up to represent a cross-section through a section of terrain where *B* is the top of a hill and *A* is at a lower elevation. The sea level line represents elevation of 0 feet.

The following letters and symbols are used to represent the elements of the analysis.

The lens of the camera for the first photograph in the series is indicated by L_1 and the camera lens of the second photograph is L_2. The negative is shown at a distance of f (the equivalent focal length) in back of the camera lens and the positive or photograph is shown in front of the lens and the same distance f from L.

The points N_1 and N_2 are the nadir points determined by dropping a perpendicular to the sea level plane from L_1 and L_2 respectively.

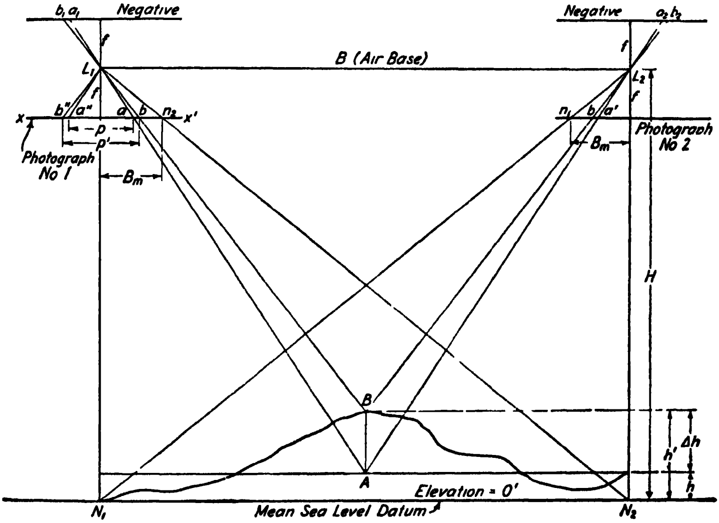


FIG. 402b. STEREOCOMPARAGRAPH THEORY.

Rays are drawn from the ground points through L_1 and L_2.

h = elevation of A above sea level

h' = elevation of B above sea level

Δh = the difference of elevation between A and B

H = the altitude of the camera lens at the time of exposure

n_2 = conjugate center or the positive of N_2 on photograph *1*

n_1 = conjugate center or the positive of N_1 on photograph *2*

a_1 = image of A on the negative No. 1

b_1 = image of B on the negative No. 1

a_2 = image of A on the negative No. 2

b_2 = image of B on the negative No. 2

a = position of A on the photograph No. 1

b = position of B on the photograph No. 1

a' = position of A on the photograph No. 2

b' = position of B on the photograph No. 2

L_1a'' is parallel to L_2A and L_1b'' is parallel to L_2B

p = absolute parallax of A

p' = absolute parallax of B

Δp = difference of parallax

B = Air base, distance between the exposure stations in feet

B_m = Air base projected on the photograph in millimeters if Δd is expressed in millimeters. If Δd is expressed in inches then B_m should be expressed in inches

Derivation of Fundamental Equation.

From Fig. 402b

$$p' = p + \Delta p$$

$$h' = h + \Delta h$$

Referring to Fig. 402b and the similar triangles.

$$\frac{p}{f} = \frac{B}{H - h}$$

or

$$p = \frac{Bf}{H - h}$$

Referring again to Fig. 402b and the similar triangles.

$$\frac{B_m}{B} = \frac{f}{H}$$

or

$$Bf = B_mH$$

Combining these equations

$$p = \frac{B_mH}{H - h} \qquad \text{(For point } A\text{)}$$

and

$$p' = \frac{B_mH}{H - h'} \qquad \text{(For point } B\text{)}$$

or
$$p + \Delta p = \frac{B_m H}{H - h - \Delta h}$$

$$\Delta p = \frac{B_m H}{H - h - \Delta h} - p$$

$$= \frac{B_m H}{H - h - \Delta h} - \frac{B_m H}{H - h}$$

Simplifying
$$\Delta p = \frac{B_m H \Delta h}{(H - h - \Delta h)(H - h)}$$

This equation may now be solved Δh

(415)
$$\Delta h = \frac{(H - h)^2 \Delta p}{B_m H + (H - h)\Delta p}$$

When A is at sea level $h = 0$ and the equation (415) becomes

(415a)
$$\Delta h = \frac{H \Delta p}{B_m + \Delta p}$$

Example: To determine the elevation of B from a known elevation A.

Let Elevation of A = 295 feet above sea level and let it be required that the elevation of B is to be determined by the use of a stereocomparagraph. Both points, A and B, must appear in photograph I and photograph II of a flight strip. The centers or principal points of the two photographs are first carefully marked. Then mark the center of photograph I on photograph II and also mark the center of photograph II on photograph I. The distance between centers on each photograph should be indicated by a straight line. Measure these distances and express the measurement in millimeters. This value is B_m, the air base as plotted on the photograph. The two distances should be practically the same if the centers were transferred properly. The two air bases are now used to "set up" the photographs under the stereocomparagraph. The air bases should be in the same straight line and parallel to the straightedge of the stereocomparagraph.

The photographs should be spaced so that good stereoscopic vision is accomplished. If difficulty is experienced, use the following simple procedure:

1. Measure the distance between the floating marks (the black dots) when the micrometer is about in the middle of the scale.

2. Place the photographs under the stereocomparagraph so that the air bases are in a straight line and with the distance from the center of photograph I to its spotted position on photograph II equal to the distance measured between the floating marks. Great care should be used in setting up the photographs before proceeding with the reading of parallax.

Assume that the photographs have been properly oriented under the stereocomparagraph. First, take a reading on the point of known elevation A. This reading is 4.92 mm. Now move the stereocomparagraph to point B and take a reading. This reading is 4.61 mm. The altitude of the plane above sea level is 5760 feet and the measured air base on the photograph (B_m) is 23.5 mm.

Substituting these values in the following equation gives the difference in elevation

$$\Delta h = \frac{(H - h)^2 \Delta p}{B_m H + (H - h)\Delta p}$$

$$\Delta p = 4.92 - 4.61 = 0.31 \text{ mm.}$$

$$H - h = (5760 - 295) = 5465 \text{ feet}$$

$$B_m = 23.5 \text{ mm.}$$

Then $$\Delta h = \frac{(5465)^2 \times 0.31}{23.5 \times 5760 + 5465 \times 0.31} = 78.7 \text{ feet}$$

The elevation of B is then

$$295 + 78.7 = 373.7 \text{ feet above sea level}$$

In Fig. 402b the Δp was measured in the plane LXX' or the plane formed by the camera station L and the air base or the XX' axis. The question may arise as to whether the same relation of Δp exists if the points in question are on other places on the photograph.

It will be remembered that all displacements due to topographic relief are measured parallel to the X axis, and the stereocomparagraph when moved over the photographs is attached to a universal drafting arm and therefore any measurements made on the micrometer are parallel with the X axis.

Figure 402c is a graphical method of showing that the Δp will be the same for a given difference of elevation whether made along the *X* axis (like *ab*) or along a line parallel to the *X* axis (like *a′c′*).

It is true that the displacement on the photograph due to topographic relief is radial from the center or principal point of the photograph when the camera axis is vertical, but the *X* component of this displacement is the quantity measured.

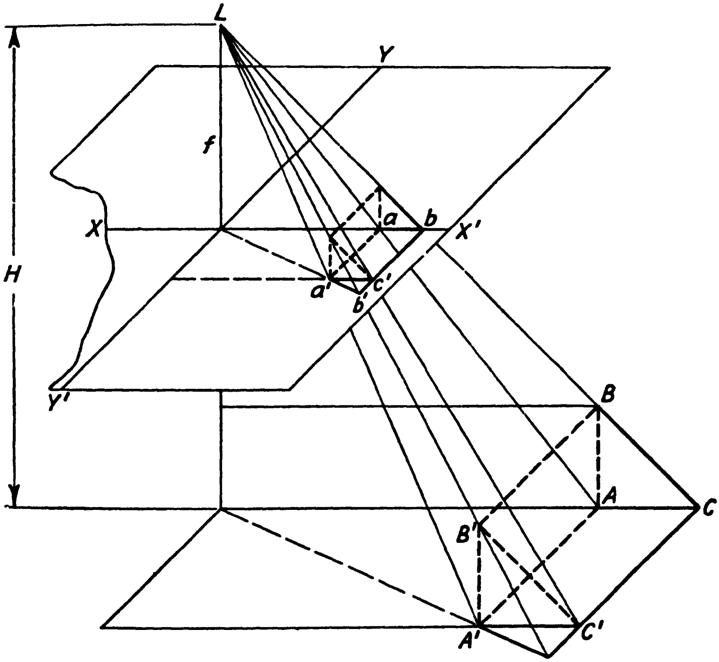


FIG. 402c.

CLASSIFICATION OF MAPS

403. Classification. Maps may be classified in the following order —

1. Uncontrolled aerial mosaic
2. Controlled aerial mosaic
3. Aerial line map
4. Aerial topographic map
5. Aerial-stereotopographic map

404. Mosaics. Strictly speaking, a mosaic is not a map. It is a photograph or a group of photographs matched and assembled to cover the desired area. The assemblage of photographs is usually rephotographed so that the final print is on one sheet of paper.

An *uncontrolled aerial mosaic* is made from contact prints without much reference to ground control. The photographs may be slightly out of scale and may contain small amounts of tilt.

A *controlled aerial mosaic* is composed of an assembled group of photographs, but in this case ground control is used and the photographs are rectified to a common scale with the tilt removed.

405. Maps. An *aerial line map* is made on a separate sheet of paper by transferring data from photographs and is sometimes called a *planimetric map.* It does not show contours.

An *aerial topographic map* is made in the same manner as an aerial line map, but in addition to the planimetry it also shows contours. The contours are put on by field methods such as the plane table.

An *aerial-stereotopographic map* is made from aerial photographs and the contours are put on by stereoscopic means.

CONTROL SURVEYS

406. General. Control may be classified as horizontal ground control, vertical ground control, and picture control. There may be hundreds or even thousands of photographs taken on one project, and these photographs must be properly oriented one with another to make a continuous plot. The purpose of control is to furnish the necessary information to accomplish this matching. Opinions vary as to how much control is necessary in each photograph but it may be said that the more one has the better the orientation. The running of ground control may be costly, particularly in rough country; therefore research is constantly being carried on to eliminate as much ground work as possible. In general, it may be stated that at least one ground control point should appear in each photograph and three are desirable.

407. Horizontal Control. Horizontal control surveys are made by the usual geodetic and plane surveying methods such as triangulation, traverse, and astronomic position (Chapters XX and XXI). The general practice is to select the ground control stations after the photographs are taken. In following this procedure, well defined points which show up well on the photograph can be selected. This is of great importance in the process of orientation.

408. Vertical Control. This type of control is obtained by spirit leveling, trigonometric leveling, and barometric leveling. Control elevations are of extreme importance, particularly when contours are to be shown on the resulting map.

409. Picture Control. This type of control consists of points selected on the photograph which have no corresponding ground control and are of importance when used in conjunction with ground control in the template methods which are discussed later.

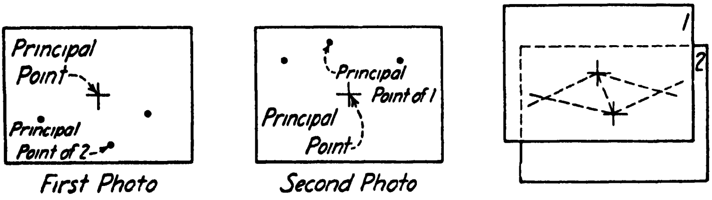
410. Plotting Control. Three common methods are in use for plotting control. These are

1. Radial line method
2. Template method
3. Slotted template method

The same fundamental principles are used in these three methods, the only difference being the mechanical way in which the work is accomplished. The fundamental requirements are that the pictures be taken or rectified so that they are nearly free from tilt and to the same scale. This means that points near the center of each photograph are free from errors caused by tilt or elevation. Also, the error at any point caused by small amounts of tilt will be in a radial direction from the principal point (the intersection of the axes joining the fiducial marks) of the photograph. The true position of any point, therefore, lies on a line drawn from the principal point through the photographed position of the point. The location of points are thus made in a manner similar to that used in plane table plotting. The position of the principal point on the map corresponds to the station occupied by the plane table, and plane table directions taken on any points correspond to the radial lines on the photograph drawn from the principal point to other points on the photograph. The radial line may be drawn on a tracing medium instead of on the photograph.

If a second photograph is taken which shows the same points as photograph number one shows, its principal point would correspond to a second instrument station. This second photograph may be oriented with respect to the tracing in a manner similar to that used in plane table surveying. This requires that the principal point of each photograph must occur in the next adjacent photograph, and that radial lines must be drawn from each principal point through the plotted position of the other on the same photograph. For this purpose, the position of the principal point of photograph No. 2 should be carefully spotted on photograph No. 1 which is best accomplished by placing the photographs under a stereoscope and using point selectors or needle points.

Radial lines are drawn from principal point number one to other carefully selected control points in photograph number one. These same control points are spotted on photograph number two and radials are drawn through them from principal point number two. The intersections of the radial lines locate the points as shown in Fig. 410.



First Photo Second Photo

Fig. 410. Radial Line Method.

It will be seen that in this procedure the scale has not been fixed and that the two principal points may be mapped at any distance apart, thus establishing the scale. If three ground control points occur in these two photographs they may be oriented in the same manner as that used in the three point problem with the plane table. This is to be preferred, as the map may then be constructed to any desired scale. These methods, therefore, are methods of graphic triangulation which may be carried through any desired number of photographs.

411. Preparation of Photographs. The photographs are prepared in the same way for the radial line, template, or slotted template methods. The principal point should be located and marked on each photograph. The principal point of each photograph should also be marked on each adjacent photograph of the flight strip. If this is found difficult, another point very near the principal point, which may be identified in the two adjacent pictures, may be used. This point should be as nearly as possible on the line connecting the principal points of the two adjacent photographs. This point is used in the same manner as the principal point is used in orienting the picture, but the radial lines are drawn from the true principal point.

Six other picture control points are marked on the photograph, three along the right edge and three along the left edge, two being on a line with the principal point of each photograph

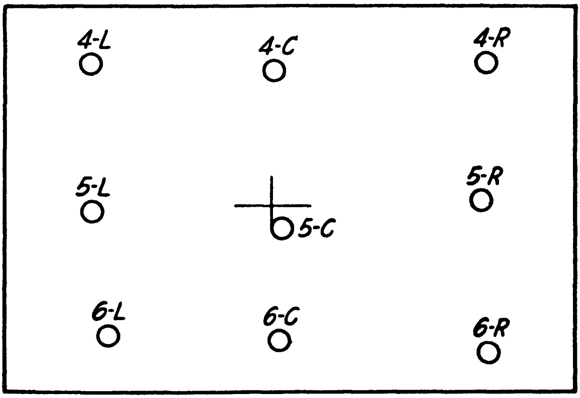


FIG. 411. NINE PICTURE CONTROL POINTS.

as shown in Fig. 411. These points should be marked with a pin prick and circled with a colored pencil. In selecting these points the photographs should be examined under a stereoscope and points near the average elevation of the territory which is being mapped should be chosen. These points should be such that they may be easily identified and great care should be used in marking them.

The side control points should be common to two adjacent flight strips and the adjacent strip should be studied to choose points which are satisfactory for both strips. All ground control points should be marked on all photographs in which they occur in a similar way but a different color should be used so that these points will not be confused with the picture control points.

412. The Fundamentals of Graphical Plotting. The term *radial line* was first applied to a graphical method of building up a triangulation control on one large sheet of transparent celluloid or acetate sheeting. The method as originally used, has been modified and the general practice now is to use separate transparent templates or slotted templates to accomplish the same results. The objection to the original radial line method was the difficulty in making the necessary adjustments for scale and matching to the control. A short description of the radial line method may be in order but most of the following description will be devoted mainly to the template methods. It should be remembered that the photographs are prepared in the same manner and the same fundamental principles used in all of the methods.

Radial Line Method. We will assume that the ground control and picture control points have been selected and marked on each photograph.

Photograph No. 1 of a flight strip may now be placed under a large sheet of transparent celluloid on which the ground control has been plotted to scale. The photographic image of the ground control in photograph No. 1 is made to coincide with the plotted position of the same control point on the transparent map sheet. Next the center of the photograph and radial lines are drawn on the map sheet. The radials are made to pass through each of the control points marked on the photograph.

Photograph No. 2 in the flight series is now placed under the map sheet and No. 1 may be removed. Assuming that the same ground control point appears in photograph No. 2, we may proceed in the same manner as with photograph No. 1. It is evident that all points which appear in both photograph No. 1

and photograph No. 2 will now have a two-line intersection on the map sheet.

Next, photograph No. 3 may be placed under the transparent map sheet and the process repeated. Points which appear in all three photographs will now have a three-line or three-point intersection.

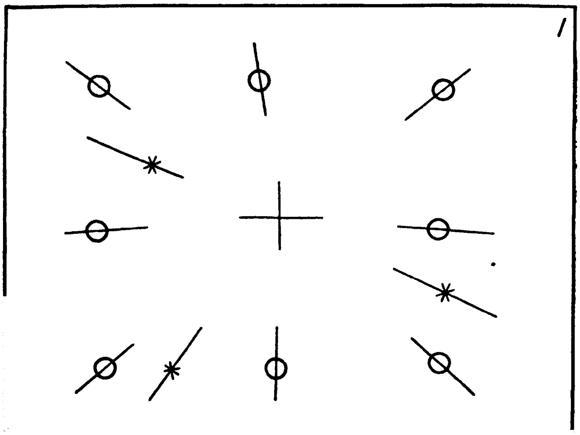
Adjustments may be necessary if proper checks are not made on the ground control points and it is here that the template methods are more convenient.

413. The Transparent Template Method. In this method, transparent templates with radials are made for each photograph instead of drawing the radials on one transparent control sheet. Figure 413 illustrates this procedure with the control points on the photograph showing through the templates and the radials which are drawn on the transparent template. In preparing a template, a pin is stuck into the celluloid directly above the principal point of the photograph and this is used as a guide for the straightedge which is used in drawing the radial lines. Radial lines are now drawn on the template through all the picture control points and ground control points. A template for each of the photographs is prepared and each template will have the appearance of Fig. 413c.

After a transparent template has been prepared for the second photograph in a series, they may be overlapped as shown in Fig. 413a.

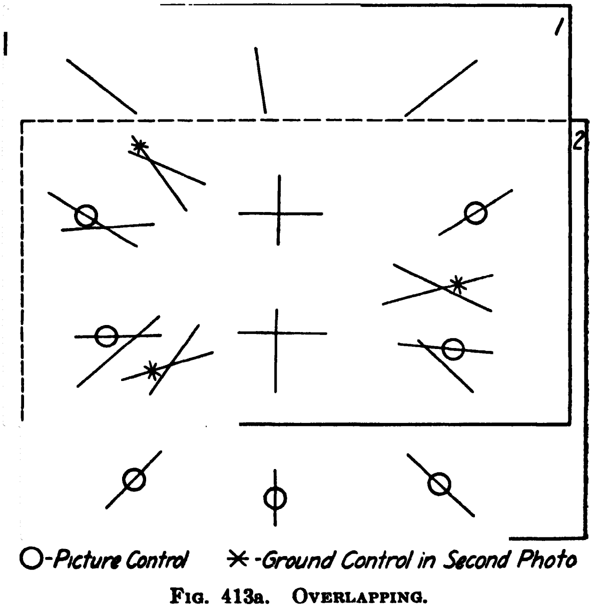
A third template is next prepared for photograph No. 3 and this procedure may be continued throughout the entire flight strip. The first three templates of a flight strip are shown in Fig. 413b. Overlapped transparent templates such as these may be adjusted for scale and the three point intersections transferred to a control sheet. This control sheet forms the base upon which the detailed map is drawn.

The templates are usually numbered to correspond with the number appearing on the photograph. Various transparent materials have been used for templates. One product known as *vinylite* has been successful and is manufactured by the Bakelite Corporation. Another, manufactured by Eastman Kodak is sold under the name of *Matte Acetate.*



○-*Picture Control* ✳-*Ground Control* +-*Principal Point*

FIG. 413. EIGHT POINTS TOWARD COMMON CENTER.



○-*Picture Control* ✳-*Ground Control in Second Photo*

FIG. 413a. OVERLAPPING.

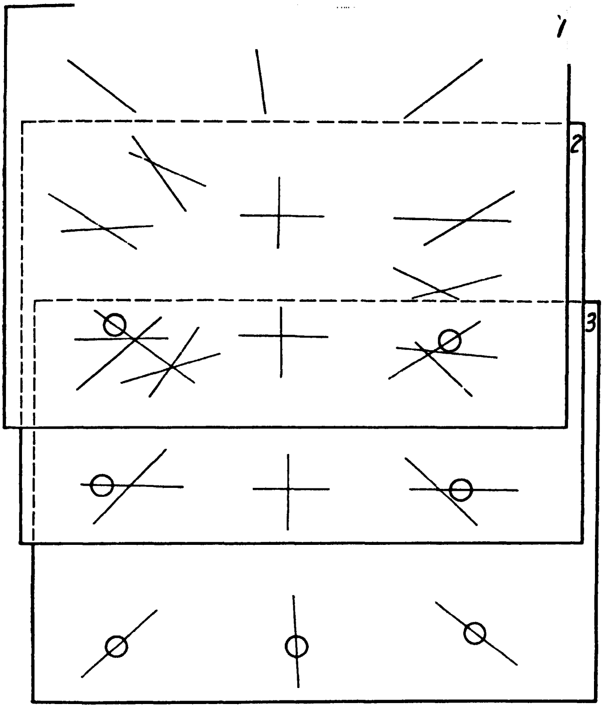


FIG. 413b. THREE OVERLAPPED.

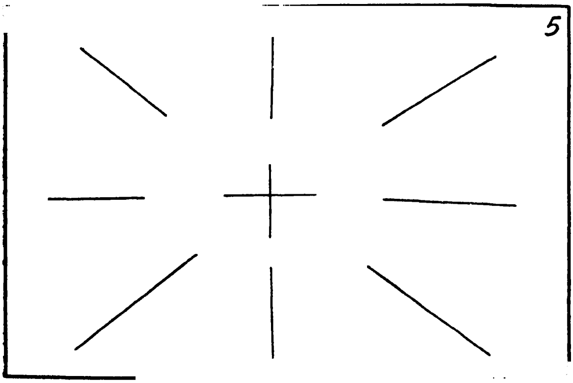


FIG. 413c. TRANSPARENT TEMPLATE.

414. Slotted Template Method. The slotted template is a modification of the template method in which the templates are oriented mechanically. Stiff cardboard or vinylite is used for the templates instead of celluloid. The photographs are first placed on their corresponding template and the principal point, picture, and ground control points are pricked through to the cardboard. These points on the templates should then be circled with colored pencil for identification. Radial slots 3/16 inches wide and 2½ inches long are then cut in the template as shown in Fig. 414. The first template is placed on the drawing table with a "rivet" placed in each slot with the head under the template. The second template is placed on the table overlapping the first so that the slots to control points fit on the corresponding rivets placed in the first template. The rivets have a hole through the center through which a pin may be

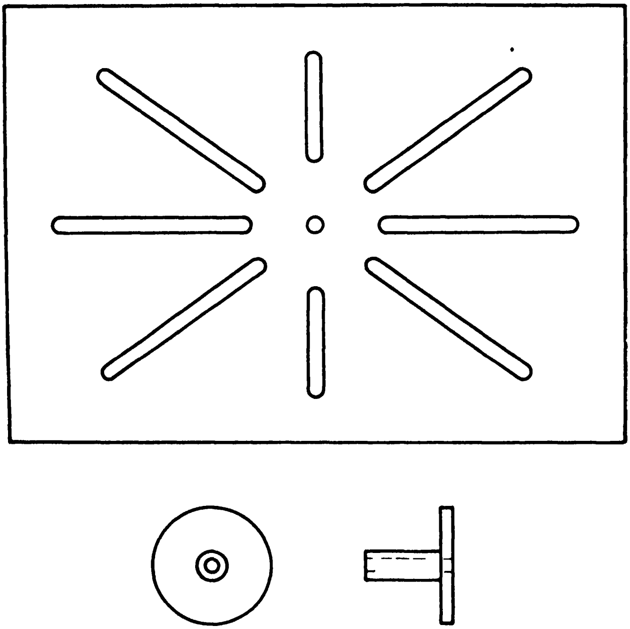
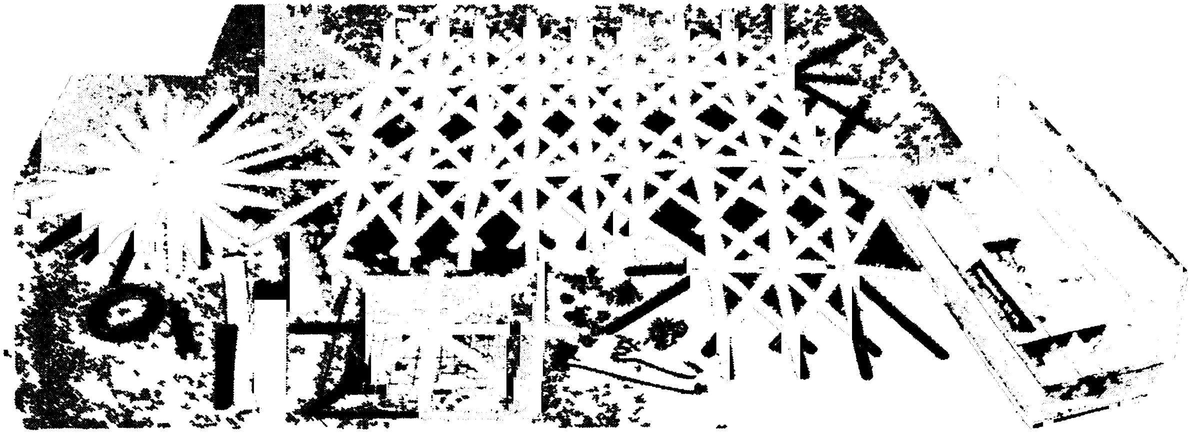


Fig. 414. Slotted Template and Rivet.



Courtesy Abrams Aerial Survey Corp

FIG 414a LAZY-DAISY MECHANICAL TRIANGULATOR

inserted to prick a point which locates the position of the control point on the celluloid mapping sheet.

Any desired scale, within the limit set by the length of the slots, may be used. After two templates have been placed on the table, as described, the distance between the principal points of the two photographs may be adjusted to establish the desired scale. The rivet for each ground control point should fall exactly over its corresponding point as plotted on the celluloid mapping sheet. The templates are thus assembled with the adjoining flight strips which tie on at the sides. The position of each ground control point must check the location of that point on the mapping sheet as the work progresses. If it does not, adjustments must be made by shifting each template slightly until all these points check.

When the slotted template method is used, photographs should be free from tilt. If tilt exists, the templates will buckle and cause difficulty in the assembling process. Slotting machines are now available which cut the slots and eliminate pricking through the photograph.

An adaptation of the slotted template has been made in the use of the "Lazy Daisy Mechanical Triangulator" or, as they are sometimes called, "Mechanical Triangulators." These are made from thin sheet steel with slots and holes which enable the construction of so-called spider templates (see Fig. 414a). A spider template is assembled for each of the photographs in an overlapped flight strip. One of these assemblies is shown in the lower left portion of Fig. 414a. The arms are held in their correct radial position by a center nut tightened down, and then the entire spider is removed from the photograph. These spiders, made from the consecutive photographs in a flight strip, may be assembled in the same manner as slotted templates, transparent templates and the like. An assembly is shown in the right portion of Fig. 414a.

415. Detailing Maps. Control such as described in § 407 is used to orient the photographs in case a controlled mosaic is to be made. The photographs are assembled and then the overlapped portions are cut away so that the entire assemblage may lie flat on a cardboard backing.

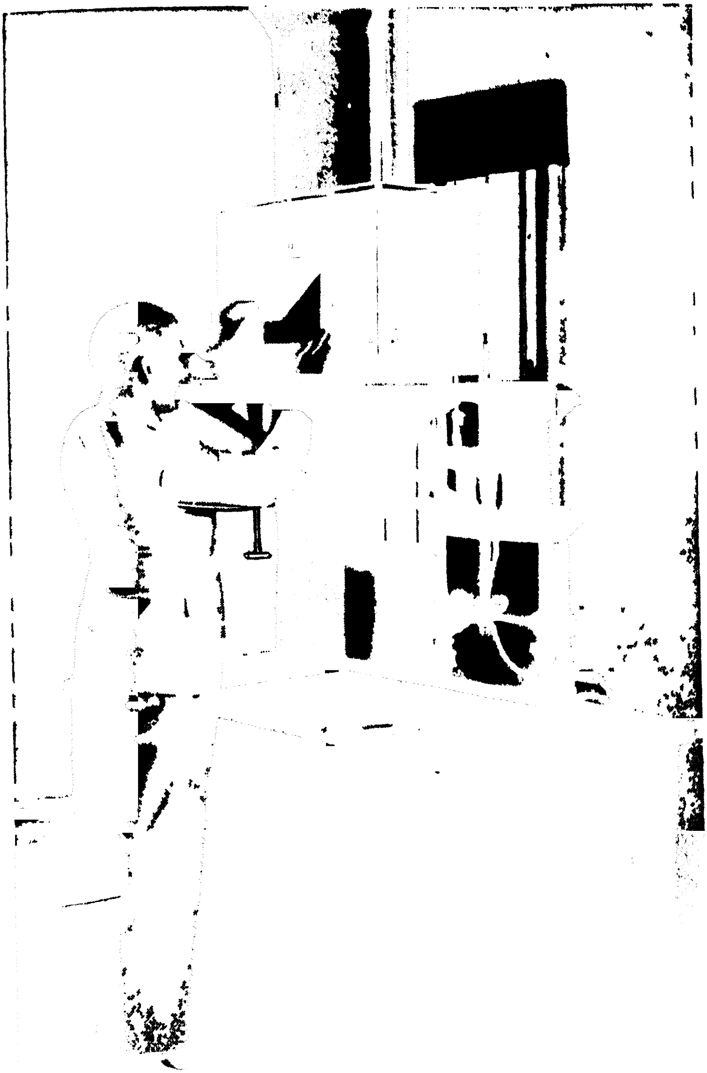


FIG. 415. THE OVERHEAD RATIO PROJECTOR.

Planimetric maps are made by transferring the detail fron the photograph to a map sheet. If direct tracing is to be used, the resulting map will be to the same scale as the photograph Remembering that the control has been plotted on a large sheet of transparent matte sheeting, it next remains to place the photographs under the matte sheeting for tracing. They are placed singly by making the control points on the photograph coincide with the plotted control points on the matte sheeting. The set-up is now ready for tracing. Only the central portion of each photograph is used. This is made possible by the end lap and side lap in the photographs

The *overhead ratio projector.* The instrument illustrated in Fig. 415 is used to project control and detail on to a map sheet to any desired scale. The procedure is as follows:

1. The control sheet, which has been plotted, is placed in the projector and the control points are projected downward to a sheet of mapping paper to the desired scale and carefully marked.

2. The control sheet is now removed from the projector and the first aerial photograph inserted. Since the control points have been marked on the photograph, their images will be projected on to the map sheet. The projector is now adjusted so that the image of each control point coincides with the plotted position of the corresponding control point. When this condition exists, the projection is properly oriented so that detailing may be penciled in by the operator. Succeeding photographs are placed in the projector in like manner and the detailing is completed.

416. Contouring by Direct Method. Contours may be plotted from a stereoscopic pair of photographs by one of several methods; the method used will depend upon the available equipment and the accuracy which is directly related to the scale of the map desired. Standard specifications used by most engineers and mapping agencies require that 90 per cent of the elevations as indicated by the contours shall be correct to one-half of the contour interval. This means that on a map with 20 foot contours, 90 per cent of the elevations shall check within 10 feet. Photogrammetric methods and equipment have been developed to a point where maps of 1″ = 200′ with 2′ contours may be constructed and meet the standard specifications.

Maps of this kind naturally require expert photography and special expensive plotting equipment not usually available in a small organization. It is not the purpose of this chapter to go into the great refinements of these methods but rather to cover the fundamentals.

Field Methods of Contouring. This is a method which any engineer may use who has the average plane table equipment or is familiar with the transit stadia method of making a contoured map. Photograph enlargements or planimetric sheets prepared from photographs may be taken into the field and used as base maps upon which the contours are drawn following the normal stadia procedures used for many years.

Elevation by Altimeters. Elevations for contour plotting may be interpolated from elevations established by altimetry. Terrestrial or aerial photographs may be used to plot the planimetry and the elevation of a number of points then established by the so-called "Single Base" or "Two Base" method.

Fundamentally, these methods involve a moderate use of the engineer's level because a few ground control stations are necessary. However, the bulk of the elevations, which may run into hundreds are determined through the use of altimeters.

Surveying altimeters are available which are primarily an aneroid barometer. The instruments differ from the older types in that they are essentially a precision instrument and therefore, give more dependable readings and are built sturdy enough to be carried about in all kinds of terrain. A thermometer is also included in each instrument. On some models, the scale may be read by estimation to the nearest foot of elevation; on others, with ranges up to 15,000 feet, this accuracy cannot be attained.

Single-Base Method. The single-base method gives good accuracy. One altimeter and a thermometer at the "base" are read at five-minute intervals. A "roving" altimeter, previously compared for reading with the base instrument, is taken to the other point or points to determine the difference in elevation between these locations and the base station. Whenever, the roving altimeter is read, time and temperature are recorded simultaneously.

To obtain the difference in elevations, three computations are required: Interpolation of base-instrument readings to the time of roving readings; subtraction of interpolated base and roving observations; and correction of difference in elevation for temperature variation.

Altimeter manufacturers provide simple table or nomographs for temperature correction. The table or nomograph is entered with the observed difference of elevation and the air temperature, and the total correction is quickly picked off. At 50° F, there is no correction. Above 50 degrees, the correction is added to the observed difference of elevation, and below 50 degrees the correction is subtracted.

When a table or nomograph is not available, the following "rule of thumb" will be found helpful and fairly accurate: The correction for each degree of temperature above or below 50° F is 0.2 ft. for each 100 ft. of observed elevation. For example, if the observed difference of elevation is 343 ft. and air temperature is 57° F, the correction would be $7 \times 3.43 \times 0.2 = 4.8$ ft. and the corrected difference of elevation would be $343 + 4.8 = 348$ ft. (approximately).

It should be noted that this temperature correction is necessary for the air temperature, not for the instruments themselves, which are fully temperature-compensated.

The Two-Base Method. The two-base method, requiring a minimum of three altimeters, is the most accurate procedure. This method utilizes two points of known elevation called the "upper" and "lower" base and assumes that weather conditions are uniform between the two bases.

Before field work is started, the three altimeters are compared simultaneously at one base. Altimeters left at the upper and lower bases are then read every five minutes and the roving instrument is taken to each point where the elevation is desired and read. Temperature is also recorded.

Since the difference in altimeter readings between the lower and upper bases will not represent the true difference in elevation between these points—because of temperature and humidity—the elevation of field points is determined from a simple proportion. This formula is

$$\frac{El.F.P. - El.L.B.}{El.U.B. - El.L.B.} = \frac{R.F.P. - R.L.B.}{R.U.B. - R.L.B.} \tag{1}$$

where

$El.$ = elevation
R = reading
$F.P.$ = field point
$L.B.$ = lower base
$U.B.$ = upper base

The proportion is derived from similar triangles (Fig. 416). Elevations of the upper and lower bases are plotted to scale on a vertical line (U and L) and a horizontal line is drawn through L. An arc with radius equal to the difference in altimeter readings between the upper and lower bases is swung from U to intersect the horizontal line through L at L'. If the difference between altimeter readings at the lower base and field point is then plotted to scale along $L'U$ starting at L', point F' is found. A horizontal line drawn through F' to intersect UL determines point F. Line FL will then be the desired difference in elevation between the lower base and field point. A similar construction determines difference in elevation between the upper base and field point. As a result of the graphical solution, the analytical equation given in equation (1) may be rewritten:

$$\frac{FL}{UL} = \frac{F'L'}{UL'}$$

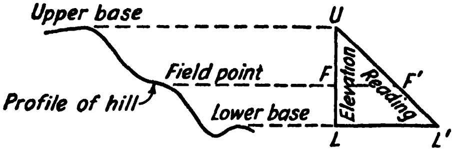


Fig. 416. In the Two-Base Method, the Elevation of the Field Point Is Found from a Proportion Based on Similar Triangles.

Spot Elevations. This method might be compared to plane table or transit stadia methods where spot elevations are determined at high and low points with interpolation between these points for the contour elevations. The difference when using a stereoscopic instrument is that the spot elevations are deter-

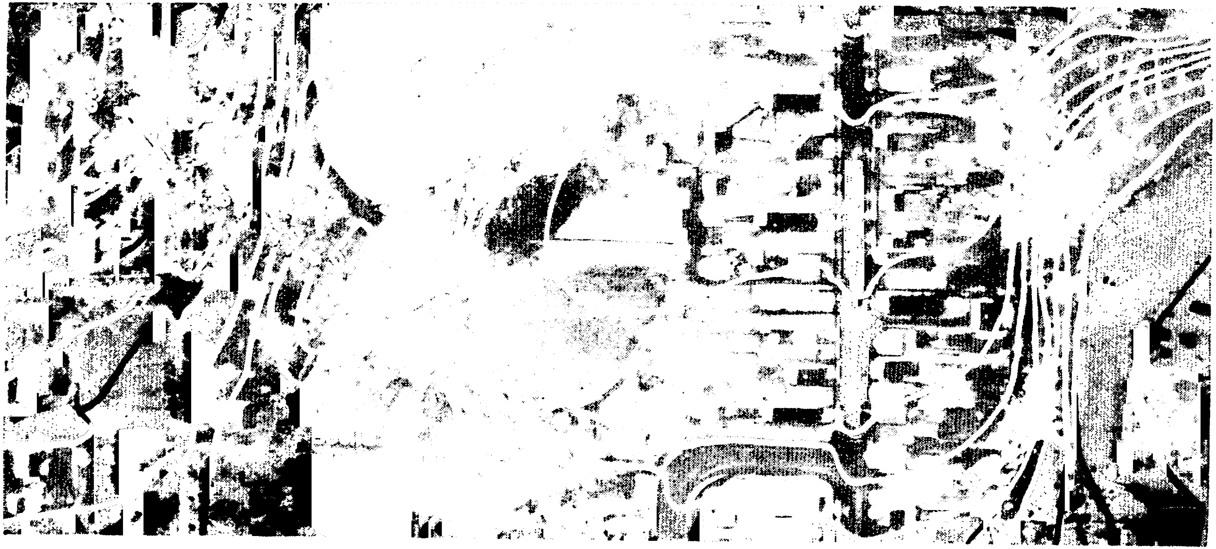


Fig. 416a. Aerial Photographic Enlargement with Contours.

mined in the office by reading the stereoscopic parallax for the high and low points. With the difference in stereoscopic parallax for two points in the photograph it is possible to enter equation (415a) and determine the difference of elevation of the two points.

Direct Contours. Many stereoscopic plotting instruments are available but they all operate on the fundamental principle of stereoscopic parallax which was illustrated by the dots and the stereocomparagraph.

Fundamentally, there is but one spacing of the floating marks for a particular elevation. It is possible to compute the change of stereoscopic parallax for a particular change in elevation.

STEREOSCOPIC PLOTTING INSTRUMENTS

The Aerocartograph, Fig. 416b, is a universal topographic mapping machine. It provides a means for drawing a complete topographic map from a stereoscopic pair of photographs. Furthermore, due to a special optical switch, it permits, by aerial triangulation, the extension of ground control throughout a flight of stereoscopic photographs.

When contours are plotted with this instrument, the floating mark is brought in contact with some point of known elevation on the photograph and the footplate is locked. Then by operating the two hand wheels, the floating mark is kept in contact with the ground. If one or both hand wheels are turned too much, the floating mark will appear to float in space or under the ground. By keeping the floating mark in contact with the ground, the contour of the known elevation is traced on the map.

The Stereoplanigraph, Fig. 416c, is a larger instrument than the aerocartograph and differs somewhat in operation. In the aerocartograph, the photographs are viewed directly through two eyepieces and a series of prisms, whereas in the stereoplanigraph, projections are viewed. Both use the principle of the floating point and the operation is somewhat similar. The x and y motions are manipulated by hand wheels and the z motion by a footplate. Both machines are rather complicated and require several months of training for efficient operation. They are probably the most precise of all the plotting instruments.

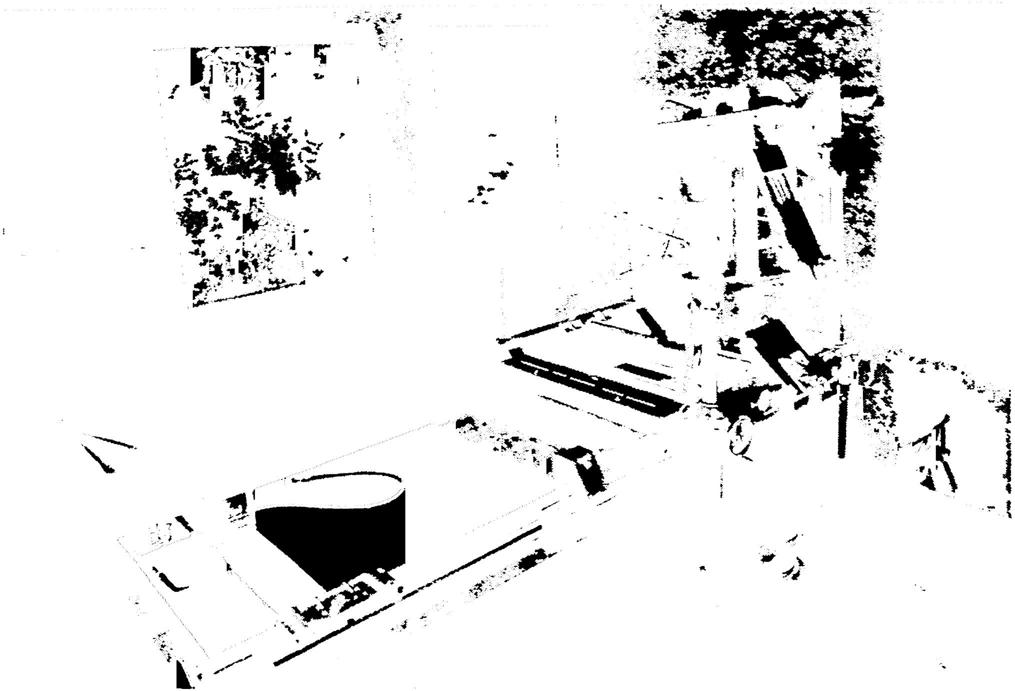


FIG. 416b THE AEROCARTOGRAPH.

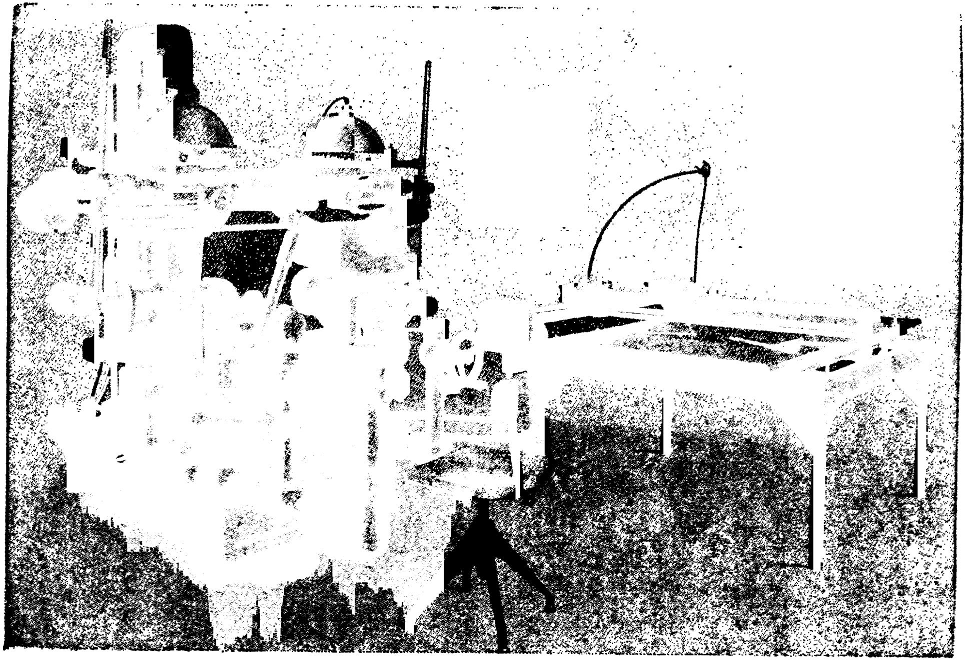


FIG. 416c. THE STEREOPLANIGRAPH.

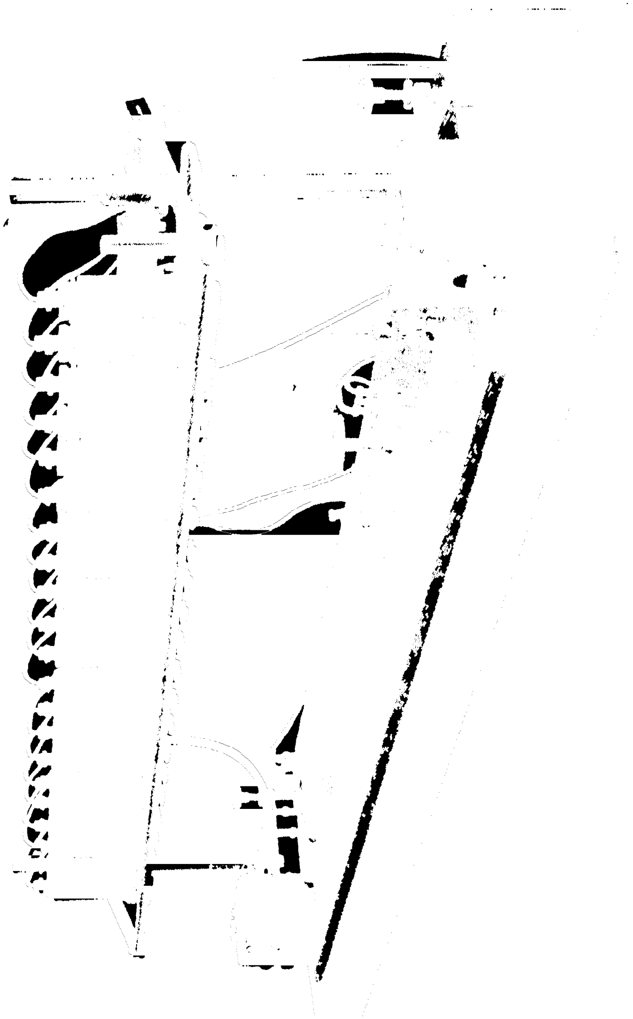


FIG. 416d. THE MULTIPLEX AEROPROJECTOR.

The Multiplex Aeroprojector, Fig. 416d, is a stereoscopic plotting instrument for producing maps by means of the simultaneous spatial projection of several aerial pictures of a series. The multiplex is a double-projection apparatus developed from the original ideas of Scheimpflug and Gasser. A relief model of the ground is generated by reversal of the photographic process and by projection with rays of two complementary colors. Corresponding rays are caused to intersect in space by suitable adjustment of the projectors. This spatial model is stereoscopically viewed through filters of complementary colors in the form of spectacles, and is gone over and measured with the aid of a spatially adjustable measuring mark. A pencil connected to the measuring mark carrier permits the direct generation of a map by the continuous tracing of topography and contour lines.

This instrument is much simpler to operate than the aerocartograph and the stereoplanigraph and is being widely used in the United States.

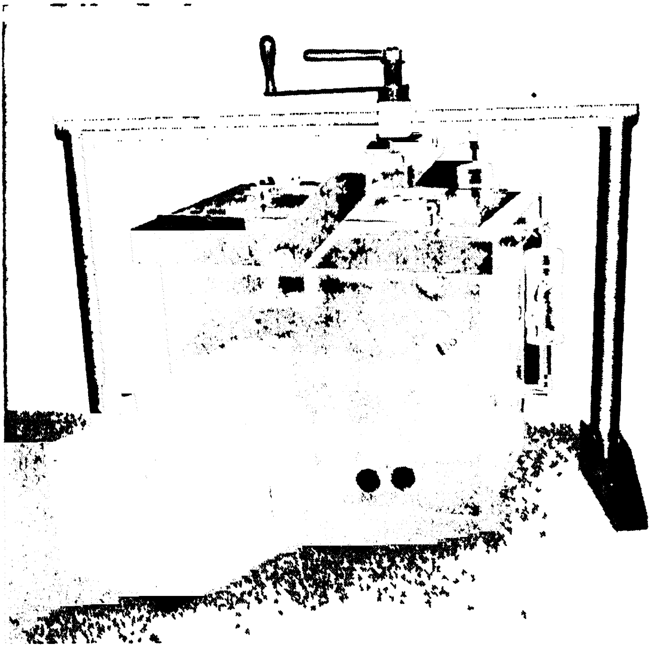
THE BROCK PROCESS

The *Brock Process* is a unique method of photogrammetric map compilation in that the two major functions of rectification and contouring are performed separately. The equipment was designed and built by Brock and Weymouth Co. in the early 1920's, and in 1923 they compiled the first topographic map produced in this country by American plotting equipment.

Photography. The Brock Process uses glass aerial negatives and glass diapositives for rectification and contouring to assure maximum flatness and stability in the emulsion base. Interchangeable cones, each with an integrally mounted lens, are used in the cameras, with a range of focal lengths from four to 12 inches. The aerial camera holds a magazine containing 48 sensitized glass plates 6½″ by 8½″.

Image area of the negative is 6″ x 8″ with the longer side parallel to line of flight. A forward lap of 56 per cent and side lap of 16 per cent provides a net gain of 17.5 square inches per model. Photographs are taken at an altitude up to 1500 times the contour interval to provide the usual accuracy of one-half contour interval.

Field Control. Elevations of points which can be identified on the photographs are obtained in the field by spirit or trigonometric leveling. The vertical control points are distributed through the photographs to provide an elevation near each center and an elevation on each side of the center near the edge of the plate. Each model* thus has six vertical control points as minimum control, with additional points in the center of the model as check points.



Courtesy Aero Service Company

FIG. 416e. BROCK PRECISION AERIAL CAMERA.

Horizontal positions are obtained to provide basic control for radial line template assembly, the distribution and density being governed by the required horizontal accuracy of the map.

Preliminary Steps. The aerial negatives are enlarged two diameters on a precision projector and these glass positives are

* A model is considered to be made up of a pair of overlapped photographs.

set up in stereo-pairs in the Brock Stereometer. The horizontal and vertical control points, and the principal points are identified and marked stereoscopically on the emulsion of the plates.

Courtesy Aero Service Corporation

FIG. 416f. BROCK ENLARGING PROJECTOR.

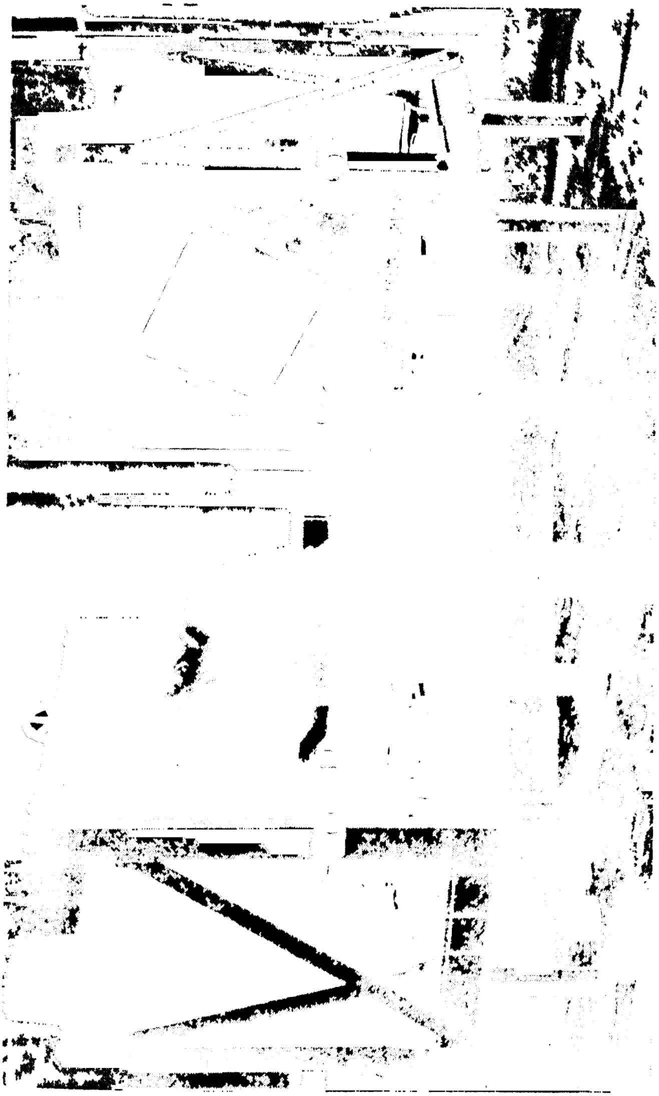
Radial line templates are made from the marked plates, and from the assembly of these templates is obtained the horizontal positions of the photo centers and the vertical control points. Measurement of the radial distances of these points on the plot and on the plates provide data for computing the height of the camera lens for each exposure. Parallax differences for the vertical control of each model are then computed.

Rectification. The glass positives of the stereo-pair are inserted in two Correcting-Projectors which are horizontal rectifying projection cameras with a fixed ratio of 1 : 1 along the lens axis. In the negative plane is a plate holder to carry the glass positive and in the receiving plane is a comparator grid plate capable of X and Y motions which are measured by micrometer gauges. The positive and the grid can be rotated 360° within their own plane and each plane can be swung up to 6° about its own vertical axis. This tilt is controlled through a mechanical linkage to provide for equal and opposite tilt angles of the two planes simultaneously.

The coordinates of the projected images of the control points on each plate are measured by use of the grids which are centered on the principal points and aligned on the conjugate principal points. These measurements give the actual parallaxes of all points and differ from the computed parallaxes because of the tilt in each plate. A determination of the tilt is made by analyzing these differences, and the computed tilt for each plate is set up in the projectors. Grids and plate-holders are rotated to bring the computed axis of tilt into alignment with the mechanical axis of each plane and the two planes are swung for the amount of tilt computed. As the front and back planes are tilted a mechanical linkage provides for displacement of plate and grid along the X axis by an amount which is a function of the angle of tilt, thus bringing the isocenter on to the optical axis of the projector and the center of the grids on to the nadir points. The grids are then aligned to have the X axis pass through the conjugate nadir points.

Verification of the computed tilt is made by reading the coordinates of the control points with such tilt settings. When the correct tilt has been obtained through several such trials the grids are replaced by plate holders carrying sensitized glass plates which are exposed in the receiving planes of each projector. These rectified negatives are in turn projected to obtain rectified positive pairs at an equal scale.

Contouring. The rectified positives are set up on the Brock Stereometer and are ready for contouring. This measuring



Courtesy Aero Service Corporation

FIG. 416g. BROCK RECTIFYING CAMERA

stereoscope has a fixed optical system which provides a magnification of four to six diameters depending on eyepieces used. The plate holders rotate to provide for alignment through the nadir points and the pair of plates can be moved in the X and Y direction under the optics. The separation between plates is adjustable and is measured by a micrometer gauge. This separation controls the datum of the cross-hairs in the eyepieces, which fuse to form a floating mark. Settings are computed for all contours and the operator traces the path of the cross-hair as it touches the ground in the model. The contours are drawn by hand on a vinylite sheet which is attached to the right-hand plate.

Scale Equalization. The contours drawn on the stereoscope retain the perspective properties of the aerial photograph, with each contour at a different scale. The contoured template is set up in a projector and each contour is traced off at a different enlargement factor, thus obtaining an orthographic projection of the template.

The Brock Process has been in continuous operation by Aero Service Corporation since 1938 and has compiled topographic maps with contour intervals of 2, 5, 10, and 20 feet. Maximum efficiency with high standards of accuracy is found on the large scale, small interval map used for special purposes such as highway and railroad location, city planning, mining operations, and reservoir studies.

The Ryker Stereoscopic Plotter. This instrument is used for plotting contours and map detail from stereoscopic pairs of vertical photographs. The instrument uses the floating dot principle and the application of these dots differs from such instruments as the stereocomparagraph in that the dots are not in contact with the photographs. In the stereocomparagraph the dot is made to rise and fall by varying the spacing of the two so-called *half-dots.* In the Ryker Stereoscopic Plotter, the dot is made to rise and fall by moving the two-half-dots vertically, while keeping the parallactic or X separation of the dots constant. For objects of low elevation the dots are lowered to almost touch the photograph while for points of greatest elevation the dots are raised to a higher position.

Since the dots and the photographs are not viewed in the

Courtesy Aero Service Corporation

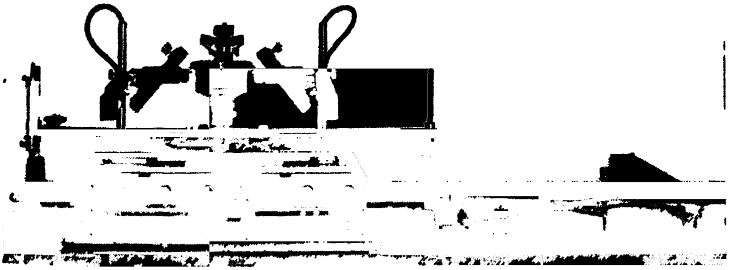
FIG 416h BROCK STEREOMETER

Courtesy Aero Service Corporation

FIG. 416. STEREOMETER PLATE-HOLDER

same plane, there might be some doubt as to the ability of the observer to focus the dots and the photograph at the same time. No difficulty is experienced as the natural depth-of-focus of the eye keeps dots and photographs in sharp focus. Most operators can accommodate a difference up to about 2 inches between dots and photographs, which on this instrument, represents a difference of elevation of 3300 feet in a 1/20,000 scale photograph. Only under extreme conditions is this much separation required. When larger differences of elevation are encountered, the model may be divided into two elevation zones, and thereby enabling the operator to adjust the *X* separation of the half-dots so that they remain as close as possible to the surface of the photograph.

The plotting mechanism which transmits motion of the dots to the pantograph comprises two frames slidable upon one another. The so-called upper frame slides in an *X* direction and the lower frame slides in a *Y* direction. The *X* and *Y* directions mentioned, are the normal designations as used to indicate *X* and *Y* axes in any rectangular plot. The *Z* direction is perpendicular to the table and corresponds to the normal *Z* axis as used in analytics and other spatial representations. A record of the vertical movement of the dots is made on the vertical scale as shown in Fig. 416j.



Courtesy Harrison C. Ryker, Inc

FIG. 416j. MODEL PL-3 WERNSTEDT-MAHAN PLOTTER.

417. Some Applications of Photogrammetry. The photogrammetrist should plan carefully before a project is started. He should consider such items as the degree of accuracy, the map scale, and the contour interval that are appropriate to the region.

The most important of these is the contour interval, because upon it depends the altitude of flight, the scale, and the accuracy that can be attained. Experience indicates that an altitude of 590 times the contour interval is about correct for photographs to be used in the multiplex projector. This will locate elevations so that 90 per cent of the contours will be correct to within one-half the contour interval, a requirement usually specified in standard specifications.

The preliminary planning should take into account such matters as the type of camera to be used, the determination of its optical characteristics and constants, and all details relating to the actual completion of the aerial photograph. The camera should be tested in a laboratory by competent technicians, preferably the Bureau of Standards.

The problem of obtaining undistorted photographs depends upon the photographic base material as well as the camera. The film base should possess low shrink characteristics, and any shrinkage should be uniform throughout. Modern film gives best results when used soon after exposure. The emulsion support tends to distort with age as its solvent evaporates, and as a result a permanent distortion takes place from long storage of the film. The following list prepared for the Massachusetts Department of Public Works, gives a good example of the items which must be considered when best results are desired.

(1) The most suitable scale*
    (a) Taking scale
    (b) Degree of enlargement
(2) The most suitable focal length
(3) The most suitable season
(4) The most desirable film and filter combination
    (a) Black and white
    (b) Color
(5) Specifications for contact prints
(6) Methods of numbering and indexing negatives

* From Aerial Photographs for Highway Location by E. W. Wood, Jr. and E. C. Houdlette.

(7) Most suitable types of mosaics for various situations
(8) Methods of presenting topographical data
(9) Possible applications of oblique photography

As a result of the excellent studies made, the following conclusions were made and printed in the report entitled "Aerial Photographs for Highway Location."

CONCLUSIONS

Experience with different types of pictures and mosaics indicated the following conclusions:

1. The most suitable taking scale for urban work is 400 feet per inch. For rural work a taking scale of 600 feet per inch is sufficiently large.
2. Under normal conditions negatives should not be enlarged more than three diameters; however, enlargements of four or six diameters of critical areas are entirely usable if the specifications have been drawn with this possibility in mind.
3. The aerial cameras should have a focal length of 12 inches or more.
4. The most suitable season for the aerial photography is when the leaves are off the trees and there is no snow on the ground. Every effort should be made to plan the projects sufficiently in advance to permit photography under these conditions.
5. Panchromatic film should be used for the black-and-white photography, and in rural areas the choice of filter should be left to the contractor so that he may use the one that will best meet the haze conditions existing on each flight. In urban areas where high buildings will cast long shadows, the contractor should be limited to the use of yellow filters and these should be permitted only when the resulting photographs contain sufficient shadow detail to satisfy the inspector. Infrared film should not be used.
6. Color photography should be confined to pictures needed for use in public hearings where the added sense of reality inherent in color would justify the added expense.

7. Contact prints should be on double weight paper for durability and dimensional stability, and the surface should be semi-matte (smooth lustre) to permit easy marking and reduce objectionable reflections.
8. Each negative should be marked clearly with the Order Number of the Project followed by the serial number of the particular negative within the project (thus 21721–17 indicates the seventeenth exposure on Order Number 21721). In addition, the date of photography should appear on each negative, while the first and last negatives of each strip should also show the time of exposure, the scale, and the focal length of the camera used.

For the index map, the appropriate Geological Survey sheet should be used as a base, and the centers of each photograph plotted, numbered, and connected by a straight line to indicate the general path of the airplane. In addition the area covered by the first and last picture on each strip should be outlined on the map. Photographic reproductions should then be made at a size that is convenient for filing.

9. Topographic information in regards to the right of way may be presented by reproducing the Geological Survey map beside the mosaic, or the contours may be reproduced on a transparent overlay kept in register with the mosaic. Neither method is entirely satisfactory.
10. Oblique photographs serve a useful purpose in planning interchanges and illustrating proposed takings at public hearings. Such obliques should be taken with a camera having a focal length of 12 inches or more. The flight altitude for 12-inch lenses should be 1,000 feet in rural areas and 500 feet where property values are high, and may be increased proportionally with longer lenses.

VERTICALS VERSUS OBLIQUES

Often times a good oblique photograph will give sufficient information for reconnaissance surveys and preliminary studies. It may take as many as 100 or more vertical photographs to cover the same ground area that is covered in one oblique photograph. It is true that the oblique photograph will have different scales for the near areas than for the more remote parts of the area but this may not be a serious objection for the studies to be made. It may be said that oblique photographs are not good for topographic mapping and in general it may also be said that the oblique photograph is not good for relatively large scale planimetric mapping. The oblique photograph, however, does furnish a convenient, economical basis for small scale reconnaissance and preliminary mapping which is both rapid and accurate where only planimetry is to be shown and great detail is not involved. This is evident in the Canadian problem discussed in this chapter.

The vertical photograph is used where a finished mosaic or topographic map is required with a relatively large scale. It is apparent from the discussions in this chapter that verticals furnish the best medium for stereoscopic plotting by the use of the principles of stereoscopic parallax. Likewise, a good vertical photograph, taken at a reasonably high altitude, may be traced or projected without much cause for error in plotting planimetric detail.

ERRORS, DISPLACEMENT, AND SCALE

The scale fraction $\frac{H}{f}$ will change with the flying height and the focal length of the camera used. For the same focal length, a larger scale will result from a low flying altitude than from a high altitude. Low flying may be difficult because of rough air conditions. In this case, a camera with a longer focal length may be used.

Displacement. Objects are displaced on vertical photographs and the displacement is greater on photographs taken at low altitudes than on those taken at higher altitudes. This may or may not be an advantage. If low-altitude flying was used then a given parallactic displacement will correspond to a smaller difference of elevation than will the same parallactic displacement at a high altitude. At an altitude of 10,000 feet, a difference in parallax of .01 mm. corresponds to a difference of elevation of 20 feet on the ground. If the altitude had been 20,000 feet the same difference of parallax would correspond to a difference in elevation of 40 feet on the ground. The stereo-comparagraph, as an example, reads directly to hundredths of millimeters. Therefore, if all other things were equal it would be better to fly at low altitudes when closer differences of elevation on the ground are required.

Certain limitations of the photograph and plotting equipment may not justify a reduction of the flying height and it is important that the photogrammetrist know these limitations before a change in altitude is specified.

Scale. Large scale photographs are convenient to use when a detailed map is to be produced at a large scale (say 1″ = 200′ or 1″ = 400′). It must be remembered, however that each photograph taken at a low altitude will cover less area per photograph than one taken at a high altitude The following sample computation will illustrate the comparative coverage when using a camera with a focal length of $8\frac{1}{4}''$ at two different altitudes.

Let the size of the photograph be 9″ × 9″.

Coverage at 5,000′ Altitude

$$\text{Scale of Photograph} = \frac{5000}{8.25}$$

$$= 606 \text{ ft inch (approx.)}$$

$$\text{Area covered} = \frac{(9 \times 606) \times (9 \times 606)}{43{,}560}$$

$$= 682.9 \text{ acres (approx.)}$$

Coverage at 10,000′ Altitude

$$\text{Scale} = \frac{10{,}000}{8.25} = 1212 \text{ ft/inch (approx.)}$$

$$\text{Area covered} = \frac{(9 \times 1212) \times (9 \times 1212)}{43{,}560}$$

$$= 2731.5 \text{ acres (approx.)}$$

This demonstrates that if the altitude is doubled, the area covered by one photograph at the higher altitude will be four times the area covered at the lower altitude. The desired scale of the finished maps is therefore, an important economic consideration and the adapted scale will determine the flying height and focal length of the camera to be used with the proper resulting scale for the photograph.

The scale of the photographs may be changed by enlarging the originals and 2½ to 3 times enlargements may be accomplished without loss of detail.

Errors. In all of the preceding general discussion, it was assumed that the camera axis was vertical at the instant the photograph was taken. Tilt of the camera axis with resulting tilt of the focal plane may introduce errors in a photograph. Errors due to this cause are troublesome as the amount of tilt may vary in succeeding photographs in a flight strip. Modern specifications are rather rigid and may require that there shall be no more than one degree of tilt. This small amount of tilt is not serious for most mapping projects. The tilt may be removed from a photograph by reprojection in a rectifying camera and some plotting instruments may be equipped with the necessary adjustments to remove the effect of tilt during the process of plotting. Instruments of this type are usually of the projection type, such as the Multiplex.

Change of scale caused by considerable ground relief may be great enough to cause error in plotting directly from a photograph either by tracing or projection. Some methods and some instruments make a correction for this change of scale and the Brock Process used by Aero Service Corporation is a typical example of this method.

There are also several minor errors which may be caused by (1) lens distortion; (2) incorrect assembly and calibration of the camera; (3) the negative may not have been held flat at the time of exposure; (4) slight errors may be introduced when photographs are taken through glass pressure plates. This is more pronounced at the edges of a photograph since the oblique rays are refracted as they pass through the glass. The errors due to this cause are usually small and may be neglected for normal mapping work. Changes in the dimensional characteristics of the negative and the paper upon which the photograph is printed may also cause errors. The handling and processing of negative and prints may cause serious difficulties when accurate dimensional characteristics are required.

PRACTICAL APPLICATIONS

Ever since World War I, aerial mapping has grown in utility for government and industry. Today the mapping plane can explore big areas broadly, and in detail, anywhere. It flies easily over the muskeg country of the far north to speed the search for new mineral resources; it photographs the bush country of Africa for oil geologists. Here at home accurate aerial topographic maps help highway and city planning—aid industry with plant maps and development information—and deliver needed facts for flood control and forestry surveys.

Once a crude tool, the aerial camera now is a precise mapping instrument. Topographic maps with contour intervals as small as one foot can be made to rigid engineering accuracy through modern photogrammetric methods, and the contour interval may vary from 1 to 2, to 5, to 20, or more, feet.

In addition, aerial mapping affords very worthwhile economies in both time and costs. In New York State, for example, topographic facts were needed for a main highway link 31 miles long. The transit and line method estimate was $74,400 and two years' time. The mapping plane flew the route in a day and a half, producing an accurate topographic map in 90 days' time. Cost was $17,600.

Another highway design and location requirement—140 miles for the Pennsylvania Turnpike location -was completed by photogrammetric methods within five months' time, at a cost of $82,900. According to the Turnpike design engineers, the aerial method was "five times faster—one fifth the cost" of ground surveys.

In the field of city mapping, like economies are effected through photogrammetric methods. According to G. Brooks Earnest, Director of the Cleveland Regional Geodetic Survey and Professor of Engineering Surveying, Case Institute of Technology, the aerial mapping of Cleveland is being accomplished at less than half the cost and one-fiftieth the time of previous plane table methods. Cost per square mile in Cleveland by the aerial method is $3200 per square mile as compared with $7500 per square mile for ground methods; the time schedule for this 450-square mile area is three years, compared with approximately 150 years for plane tabling.

Like everything else, the cost of aerial mapping has risen and is higher in 1949 than before. However, the very favorable relationship between the cost of aerial mapping and ground methods still remains, for there have been like increases in ground costs. Aerial mapping costs vary according to the size of the area and its location in relation to airport facilities, as well as according to the contour interval and other requirements.

In new industrial developments also aerial mapping brings major savings in time and costs. One large mining enterprise in Canada recently was completed years ahead of schedule, thanks to aerial mapping's aid in photo-reconnaissance of their area for geologic study, plus precise aerial mosaics to assist in the preliminary planning of roads, harbor facilities, power and plant sites, and other development work.

APPLICATION No. 1

Flight Map. Flight maps are usually prepared in advance of the aerial survey. They should furnish the basis for cost studies and general planning. The most satisfactory method of planning the photographic flight is to specify definitely where each flight shall be made by indicating its position on the flight map to be used by the pilot. A flight map should give enough information so that the pilot will have little trouble in running his flight strips to the boundaries of the area to be photographed. The highways and streams should be indicated with important structures, mountains, woods, transmission lines, railroads, and all detail which stands out clearly from the air.

A geological map or a photographic mosaic may serve as a base upon which a flight map may be constructed. If a geological map is used it should be enlarged to about 1/2 mile per inch and only the important details transferred to the new map. Too much detail may be confusing to the pilot as he must make quick decisions while in the air.

A simple flight map is represented in Fig. 417 where the area *ABCD* has been designated on a topographic map or a photographic mosaic.

The flight starts at the upper left-hand corner and proceeds toward the right along flight line No. 1. Only the first two exposures #1 and #2 are shown and the time of exposure was figured so that an end lap of 60 per cent was obtained. This means that each new exposure will cover 40 per cent new area in the direction of flight. (See § 394.)

Flight No. 1 continues, and photographs taken until the end of the area is reached. The airplane then turns and swings into flight No. 2. A series of photographs are again taken so that the same 60 per cent end lap is obtained and so that a side

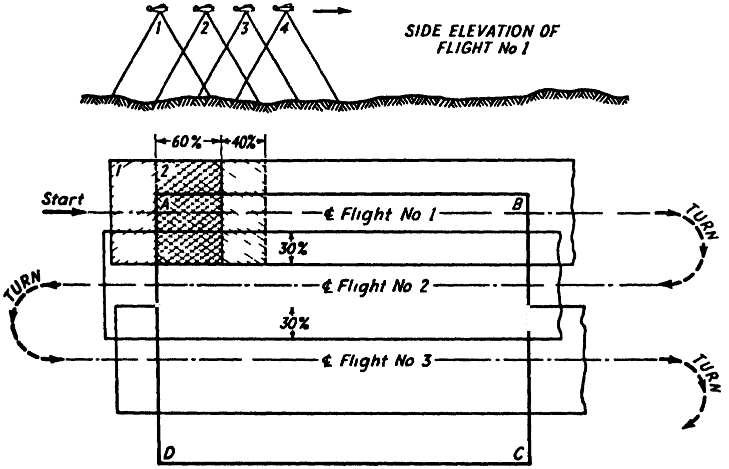


FIG. 417. FLIGHT MAP.

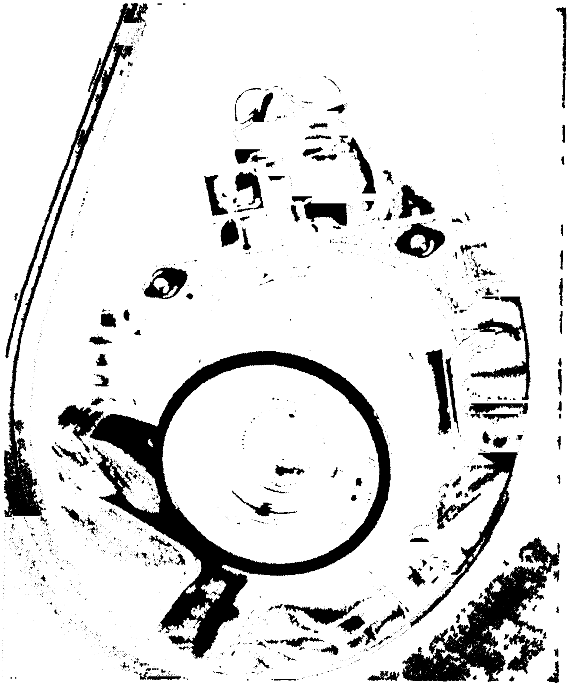
lap of 30 per cent between flight No. 1 and flight No. 2 is obtained. At the end of flight No. 2, the airplane turns and flight No. 3 is made in a similar manner. This process continues until the entire area is covered.

Care must be taken to cover enough area with overlapped photographs so that stereoscopic pairs are available for the area in question.

The flight map furnishes a work sheet for cost studies, flight lines and flying information for the pilot and photographer when they take to the air.

APPLICATION NO. 2

Oblique Aerial Surveying. Extensive use of oblique aerial photographs has been made by the Canadian Government in mapping the great "Lake of Woods Country" and other inaccessible regions. A greater part of the wooded sections of Canada were never charted, and those which have been mapped in the past were very crudely done. Only the water courses were sketched with a few of the trails shown.



Courtesy Fairchild Camera and Instrument Corporation

FIG. 417a. OBLIQUE AERIAL PHOTOGRAPHY.

With the advent of the airplane and air-camera the mapping presents a comparatively easy task. Thousands of square miles of this region have been covered in the past few years.

The oblique photograph presents a rapid method for this type of mapping. The scale of the finished map as furnished by the Canadian Government is four miles to the inch, with no contours. The general character of the country is particularly suited to oblique work because there is no great difference of elevation to cause excessive displacement, and the horizon line is visible in all photographs, an essential element in oblique work.

The advantage of this method over the vertical is that each photograph covers a much larger area, with consequent lower flying cost.

This method does not do away with ground surveying entirely because some ground control is necessary for determining the correct altitude of flight and in orienting the photographs. The known ground distance between two points is required on each photograph.

The information required for oblique work may be classed as follows:

1. The horizon line must appear on the photograph and about 7/8 inch from the top margin.
2. Dip of the camera axis at instant of exposure.
3. The altitude of the plane at the instant of exposure.
4. The focal length of the camera lens.

In general, the method is somewhat the same as that of Fig. 387a under terrestrial photographic surveying, more complicated however because of the uncertain altitude at the time of exposure. Flights are usually made at 5000 feet elevation or as near as this as possible.

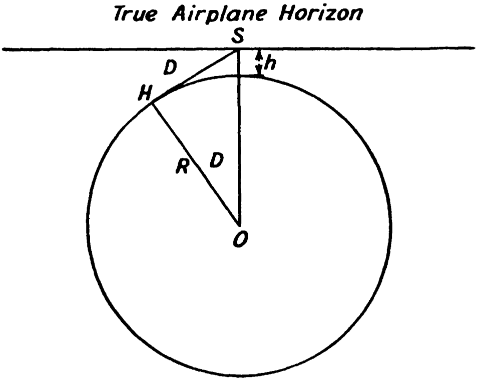


Fig. 417b.

A transparent grid is constructed on glass or celluloid and superimposed on the photograph. The lakes, rivers, islands, etc., are then transferred to drawing paper on which a rectangular grid has been plotted. The map is made by transferring point by point to the respective squares and then connecting these points. In practice these squares are ten chains to a side, and correspond to squares ruled on sheets of tracing paper on a scale, usually of one mile to one inch. The maps which are printed for sale to the public (similar to our geological maps) are on a scale of four miles to the inch.

The following derivation will give an equation for the angle D of Fig. 417b.

$$SO = R + h$$

$$\tan D = \frac{SH}{OH} = \frac{\sqrt{\overline{SO}^2 - \overline{OH}^2}}{OH} = \frac{\sqrt{(R+h)^2 - R^2}}{R}$$

$$= \frac{\sqrt{R^2 + 2Rh + h^2 - R^2}}{R} = \sqrt{\frac{2h}{R} + \frac{h^2}{R^2}}$$

Since $\frac{h^2}{R^2}$ is small, it may be dropped without much effect on the angle D.

The equation then becomes

$$\tan D = \sqrt{\frac{2h}{R}}$$

$$= \sqrt{\frac{h}{10{,}444{,}314}}$$

Also D being a small angle, we may assume

$$\tan D = D \tan 1 \text{ sec.}$$

Therefore
$$D = \frac{\tan D}{\tan 1 \text{ sec.}}$$

Substituting the value of tan D from above, we have

$$D \text{ sec.} = 206{,}265 \sqrt{\frac{h}{10{,}444{,}314}} = 63.82\sqrt{h}$$

Multiply D by 0.9216 to correct for refraction and

$$D \text{ sec.} = 58.82\sqrt{h}$$

D is called the dip of the apparent horizon from the airplane horizon at the altitude h.

Let Fig. 417c be a photograph on which the apparent horizon is as shown and is 0.72 inch below the upper margin by actual measurement. This line is not exactly a straight line, but in this method is considered to be straight and perpendicular to the principal line.

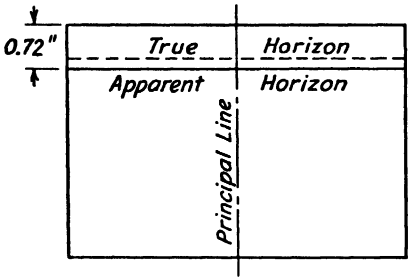


FIG. 417c.

The true horizon will be a line parallel to the apparent horizon and above it as shown. The reason for having these two horizon lines is that the airplane is elevated above the earth's surface. The horizon then appears lower than it would if the observer in the plane were to project his line of sight in a horizontal plane. Remembering then that the apparent horizon is as seen from the plane and the true horizon as seen along a horizontal line from the plane, we have the following computations:

The equation for the dip is:

$$D = 58.82\sqrt{h}$$

Using a flight altitude of 5000 feet, we have

$$D = 58.82\sqrt{5000} = 1^\circ\, 9'\, 19''$$

Depression Angle. To determine the depression of the camera axis below the apparent horizon:

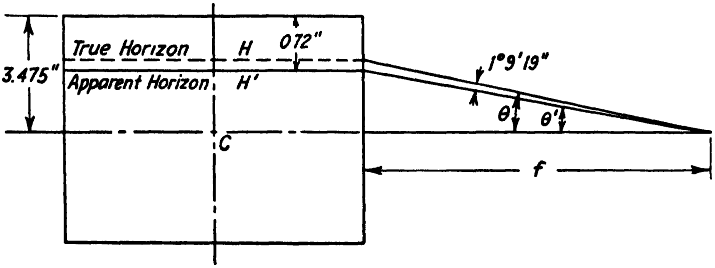


FIG. 417d.

1. Mark the point C (plate center) by drawing the horizon line and principal line from the marks on the margin of the photograph.

2. Measure the distance 3.475 inches on the photograph. Then

$$3.475 \text{ in.} - 0.720 \text{ in.} = 2.755$$

$$\tan \theta' = \frac{CH_1}{\text{focal length}} = \frac{2.755}{8.709} = 17° 33' 15''$$

$$\text{Angle } \theta = 17° 33' 15'' + 1° 9' 19'' = 18° 42' 34''$$

3. The distance from plate center to the true horizon line $= CH = f \tan \theta = 8.709 \tan 18° 42' 34'' = 2.9494$ in.

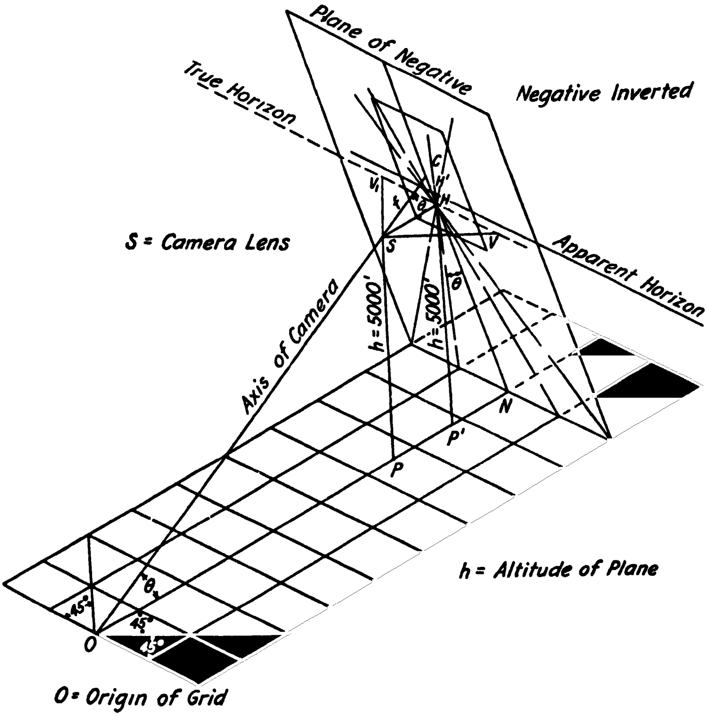


FIG. 417e.

Construction of Plotting Grid

1. The distance from the airplane true horizon to the ground plane measured in the plane of the axis of the camera = *h* (see Fig. 417e).

2. HN (Figs. 417e, 417f) = 5000 sec θ = 5000 × sec 18° 42′ 34″ (in feet).

$$HN = \frac{5000 \times \sec 18^\circ\, 42'\, 34''}{66} \text{ (in chains)}$$

Then reduced to a scale of ten chains = 1 inch, HN (Fig. 417e) becomes HG (Fig. 419g) $= \dfrac{5000 \times \sec 18^\circ\, 42'\, 34''}{66 \times 10} = 7.9984$ in.

3. In Fig. 417e ($HV_1 = HS = HV_2$) because angle $V_1SH = 45^\circ = SV_1H$. In Fig. 417e, from triangle SCH,

$$SH = SC \sec \theta$$

Then

$$HV_1 = HV_2 = f \sec \theta = 8.709 \sec 18^\circ\, 42'\, 34'' = 9.1949 \text{ in.}$$

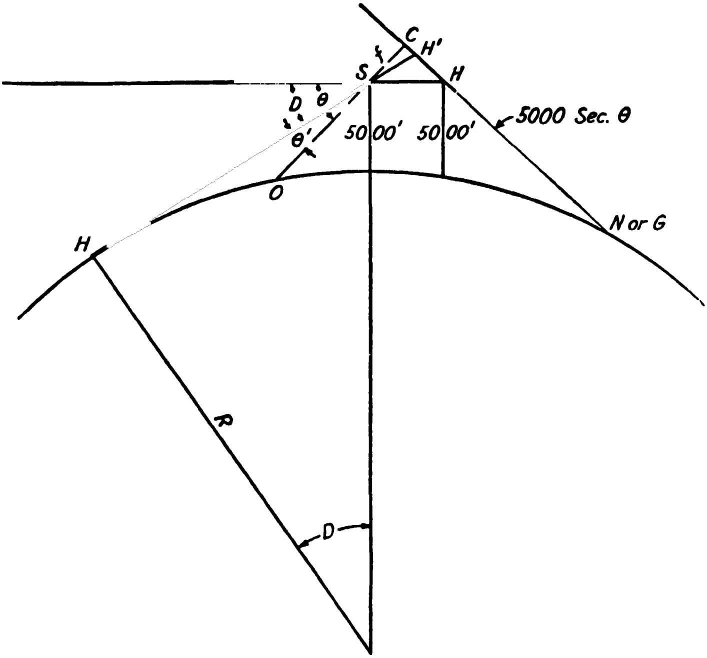


FIG. 417f.

We now have the necessary information to construct a trial grid, trial grid because the altitude of 5000 feet may not have been correct. As stated before, it is difficult to maintain a uniform altitude, and it is the purpose of the trial grid to determine the exact altitude from which any particular photograph was taken. When the correct altitude is determined, a final grid must be made for plotting the map.

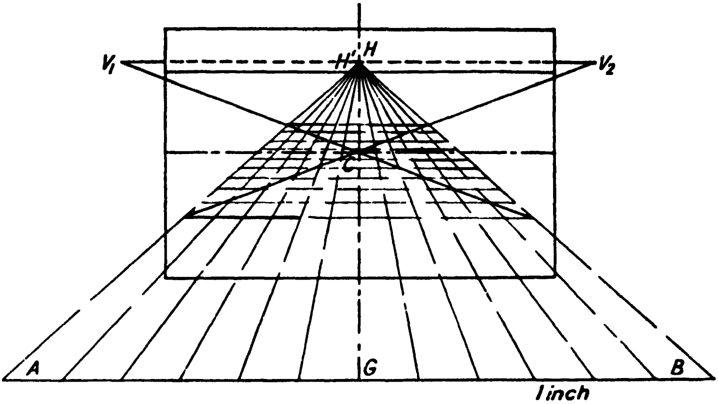


FIG. 117g.

Trial Grid. While the grid is eventually drawn on glass or transparent celluloid, the construction can be made on paper and then traced to the transparent material.

Steps in Making the Grid.

1. Draw on the photograph the coordinate axes which give the point C.

2. Measure (on the photograph) the distance from top marginal line to the center = 3.475 inches and the distance to the apparent horizon = 0.72 inch.

3 On a large piece of drawing paper lay out a rectangle to represent the size of the photograph.

4. With the above distance plot on the drawing paper the coordinate axes and the apparent horizon.

5. With the computed distance $CH = 2.9494$, locate H on the principal line and draw a line parallel to the x axis. This is the true horizon line.

6. On the line CH produced downward measure the distance $HG = 7.9984$ inches.

7. Through G draw an indefinite line A–B perpendicular to the principal line and parallel to the x axis.

8. On this line lay off equal distances of 1 inch and from these points draw lines converging to point H (since all lines perpendicular to the picture plane meet on the true horizon and along the principal line). The perspective of these lines has thus been drawn.

9. On the true horizon line lay off distances HV_1 and HV_2 = 9.1949. The points V_1 and V_2 are the vanishing points for the diagonals of the squares on the grid (since they make 45 degrees with the vertical).

10. Draw lines from C to V_1 and V_2. Where these diagonals cross lines AH, GH, BH, etc., draw parallel lines at right angles to HG. The perspective of lines parallel with the picture plane have thus been drawn.

11. If necessary, draw an auxiliary ground line one-third the distance HG from H and lay off spaces 1/3 inch instead of 1 inch as on the real ground line.

12. The trial grid is now ready for tracing to the glass or celluloid surface. It is next superimposed on the photograph, making the apparent horizon line coincide with the apparent horizon on the photograph and the center of the grid coincide with the center of the photograph.

Two control points are selected whose ground locations are known and therefore the distance between them known. The distance between these two points is determined by the grid and, if this does not agree with the ground distance, the wrong grid altitude has been used.

The correct grid altitude is then found from the proportion:

$$\frac{\text{Correct grid elevation}}{\text{Grid elevation used}} = \frac{\text{correct ground distance}}{\text{measured grid distance}}$$

In the problem under discussion the correct ground distance between two known points is 8332.5 feet, and this distance determined by the grid is 8830.8. Substituting in the proportion, we have

$$\frac{X}{5000} = \frac{8332.5 \text{ ft.}}{8830.8}$$

$$X = 4717.5 \text{ ft. (true altitude)}$$

The following computations then will give the required information for plotting the final grid:

$$D = 58.82\sqrt{4718} = 1^\circ\,7'\,20''$$

$$\theta' = 17^\circ\,33'\,15''$$

$$D = \underline{\;1^\circ\;\;7'\,20''}$$

$$\theta = 18^\circ\,40'\,35''$$

$$CH = f \tan\theta = 8.709 \tan 18^\circ\,40'\,35'' = 2.944 \text{ in.}$$

$$HG = \frac{4718 \sec 18^\circ\,40'\,35''}{660} = 7.546 \text{ in.}$$

$$HV_1 = HV_2 = f \sec\theta = 8.709 \sec 18^\circ\,40'\,35''$$
$$= 9.193 \text{ in.}$$

With this information plot a final grid in the same manner as the trial grid. This grid is used in plotting the map.

In practice, where a large number of photographs are to be used, grids made with various altitudes and marginal distances are on file and the proper one selected for use on each photograph.

The grids are usually drawn to a large scale to increase the accuracy of plotting and then reduced to the required size by photographic methods.

As in Fig. 387a, the plotting is done by transferring from the perspective grid on the photograph to the drawing paper by plotting in the respective squares on a rectangular grid. The drawing paper has plotted on it a rectangular system of squares to correspond with the perspective grid.

APPLICATION No. 3.*

The project is a Super-Highway, Detroit to Willow Run, a distance of 25 miles.

1st Step. Gather together all available information, which in this case was the following:

State highway maps
Regional highway maps
County roads maps

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* Courtesy· Talbert Abrams.

Township road maps
City maps
Aerial photographs, scale 1 : 20,000, 5 years old
Aerial photographs, scale 1 : 10,000, 10 years old

2nd Step. A study was made in the office of all maps and aerial photos and it was decided that a reconnaissance flight was necessary. Therefore all project engineers, with notebooks, pictures, and maps, were flown over and around the area until they not only had a good idea of the layout, but had taken notes on the principal controlling factors.

3rd Step. A conference was held and the several possible locations were sketched, thus narrowing the location down to a minimum for the aerial survey.

4th Step. A set of air photos was made at a scale of 1″ = 800′ of the reduced area and studied stereoscopically, to note any unusual obstructions that would interfere with a road location

5th Step. The tentative road location was plotted on the office pictures which were enlarged to a scale of approximately 1″ = 200′ without ever having had a man on the ground in the field.

6th Step. The land buyers were then given a set of pictures, scale 1″ = 400′, without the road location but with just the outline of the pieces of property which they were to buy.

7th Step. Center line of highway having now been tentatively decided and the future location narrowed down to the bare minimum necessary, it was decided to have still larger pictures made of the restricted areas.

8th Step. Stereoscopic aerial photos, scale 1″ = 200′ with enlargements at the scale of 1″ = 40′, were made on which to plot all grade separations, crossovers, under-passes and drainage structures and with the land layout in visual form showing the location of all trees, shrubs, surface stones, buildings, existing drainage, and structures. Practically all questions could be answered in the office and a final alignment and road layout was designed onto the photos. As property lines were being surveyed and monumented in the field, the ground control was secured to make the enlargements as accurate as the ground control could be plotted.

9th Step. Many hand-drawn line maps were made from the aerial photos as follows:

- Maps were traced directly from the enlargements.
- Maps were plotted with the sketchmasters.
- Maps were drawn off a varying scale with the Pantagraph.
- Maps showing contours were made with the Contour Finder.
- Road maps were made showing first, second, and third class roads, trails, paths and bridges, farm lanes, railroads, and conditions of roads and materials used.
- Maps showing legal property descriptions and buildings and fences were made by the legal department.
- Comparative tax values were plotted on the photos.

Please keep in mind that this route map, made by the aerial survey, was in a suburban area adjacent to the fourth largest city in the United States. The area was relatively flat and had been mapped and re-mapped many times for almost every imaginable purpose, but still when a new project such as the Willow Run Super-Highway was conceived, the very latest information was necessary and could best be gotten by the aerial method.

APPLICATION NO. 4*

The opposite of the above-described problem is a road to be built through poorly wooded, rugged north country. The project is in the planning stage, the route is to connect existing trunk lines along a better and shorter route, but money is not available for the shorter route if it costs too much.

Timber cruisers have a good general idea of the country and recommend the old logging routes be improved. No good existing map is available. The area is 100 miles long by 25 miles wide.

1st Step. The area is photographed at a scale of 1 : 40,000 and the stereoscopic pictures studied to get a better idea of the country. A photo index gave us the first reasonably accurate map of the area showing rivers, lakes, trails, hills, swamps, and mountains.

---

* Courtesy: Talbert Abrams.

2nd Step. Monumented ground control established around the perimeter of the area by the U.S.G.S. was recovered and plotted on a new base, scale 1 : 20,000, and new astronomical positions were established for additional control at about one point for every 250 square miles or one fix as near the center as possible of every 15 minute quadrangle.

3rd Step. A Lazy Daisy Mechanical Triangulator lay down was built up and a new map projected and with this control a mosaic was made at the scale of 1 : 20,000, thus giving us the first map of the area with a relatively high degree of accuracy.

4th Step. The appearance of the mosaic and the shadows gave a general appearance of relief; however, this could not be depended upon for locating a highway route so the Contour Finders were used without any vertical ground control. By using just the height of the airplane and measured distance between print centers and the Photogrammetric Computer, we gained a factor suitable for a preliminary check of the relief in the area.

We laid out several prospective center lines for the new route and then with the Contour Finder, checked the varying elevations along the routes and for a distance of about 2000 feet both sides of the route. With this information we were able to estimate the cost of cuts and fills, the number of culverts and bridges required, and also the scenic value of comparable routes. Finally, having established the relative cost, utility to the best parts of the area, and the scenic values, a choice of many routes was narrowed down to several possibilities fairly close to each other.

5th Step. Now with a good air map at hand that could be used as a flight map for low-altitude picture, we went back and flew a single flight strip at a scale of 1 : 4800 along the center line of the area best adapted to the route. This flight strip was 3600 feet wide and covered the best possibilities. The flight at low altitude was made in the early spring before the leaves came out on the trees.

6th Step. The air photos, scale 1 : 4800, were studied stereoscopically, and with the Contour Finder to check spot elevations a final line was picked along the easiest grades avoiding swamps and difficult areas.

Air photo enlargements, scale 1 : 1200, were made for use as plane table sheets.

The proposed center line was marked on these sheets and the survey crew went into the field to accurately measure the horizontal and vertical distances and stake out the route. With a protractor in the drafting room, compass bearings were planned and plotted on the photographic plane table sheets, distances were scaled off to points of intersection where the route turned to a new tangent.

In the heavily wooded areas, transits were set up to the predetermined angles and lines were cut through the woods for several miles at a time, the only check being the information available on the aerial photographs. The 110-mile line was staked in the field without any off-line surveying, points of beginning and ending were the same as originally planned.

7th Step. With the vertical control established along the center line, the Contour Finder completed the necessary contouring of both sides of the center line for a distance of 200 feet where necessary, and in some places a watershed line was extended around a basin, the drainage of which had to be accommodated by bridge or culvert. In areas like this, no other contouring was necessary.

HIGHWAY STUDIES

The engineers of the Public Roads Administration have analyzed and developed excellent material which should be of great value to all engineers interested in the use of Photogrammetry. The material is too voluminous to attempt a complete description in this chapter but some of the material from their studies will be given. Engineers will find that the Public Roads Administration is very cooperative, if additional information is desired.

"In explaining how photogrammetry is used in highway engineering, it may be well to consider the stages in development of a highway. These stages can be defined as: selection of terminal points, location and design, construction, maintenance and betterment, condition and inventory surveys and the improvement and reconstruction. Then we can start the cycle all over again when highways become obsolete."*

"Once the terminal points are designated, highway location and design follows, to be accomplished in four successive stages, namely:

1. Reconnaissance of area for selection of feasible alternate routes.
2. Reconnaissance of alternate routes comparing one with another to select the best route for survey.
3. Preliminary location survey and design to determine the place for the highway on the selected route.
4. Location surveys and project (contract) plans; the actual staking of highway alignment, grade lines, cross sections and structures on the ground in readiness for construction."

EXPLANATION OF FIG. 417H (CHART)†

At the bottom of the chart, as an abscissa, three stages of the highway location are indicated: reconnaissance of area, reconnaissance of alternate routes, and preliminary location surveys and designs. In the ordinate direction of this chart, designated as steps 00 to 99, steps are shown from the initial stage of aircraft pilot, cameraman, and aerial camera through photography, ground control surveys, and photogrammetry to the finished product of picture or map that will be used by the highway engineer. From left to right, it represents a progressive increase in the amount of detail and accuracy in information and factual data that will be obtained by photogrammetric methods for highway engineers' use.

Starting with columns 1 and 2 on the left are oblique photographs and trimetrogon photographs. Obliques are used to illustrate a highway problem and proposed solution in perspective. Trimetrogon photographs can be used for mapping to small scale providing a general pattern of the topography and an idea of where the mountains, the rivers and the central habitations are in an undeveloped or heretofore unmapped country.

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* Selected parts from a paper presented before the American Society of Photogrammetry by William T Pryor and printed in "Photogrammetric Engineering," Volume XIV, No 1, Page 17

† Acknowledgement is made for the assistance given by W. T. Pryor in writing this part of the text.

Following that is another choice —the preparation and use of uncontrolled mosaics, and radial plot controlled planimetric maps. The planimetric maps may be made by projection methods using such instruments as sketchmasters, reflecting projectors, the radial line plotter or the multiscope.

Then, when the highway engineer needs to know more about topography by seeing its character and shape and the use of land in three dimensions, he should use paired photographs and stereoscopes.

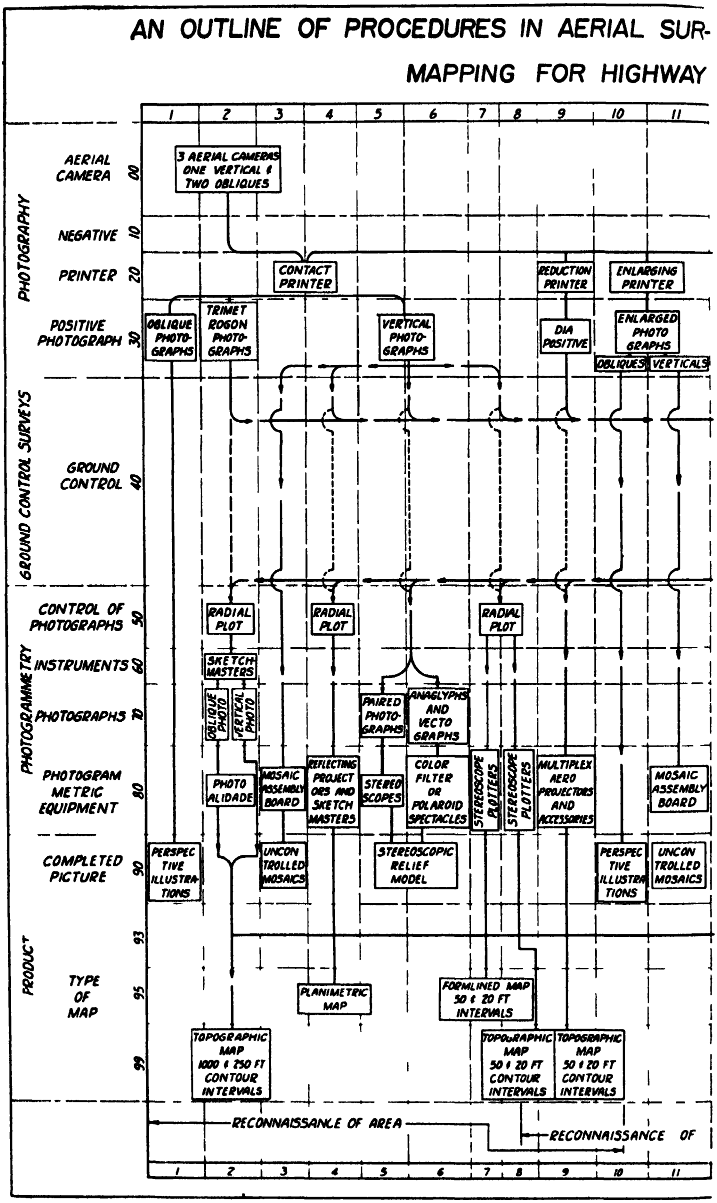
If the task is to illustrate in three dimensions the proposed solution of particular highway engineering problems to officials, the public and administrators, analglyphs and vectrographs may be used. These will require use of the color filter or polaroid spectacles.

When the engineering job requires small-scale large-contour-interval mapping, use stereoscopic plotters such as the stereocomparagraph, the contour finder, the Wernstedt-Mahan stereoscopic plotter, the K:E.K. stereoscopic plotter and other instruments that might be available. The first two instruments named provide form-line maps, column 7; the latter two make a topographic map, column 8. When more accuracy is required, use multiplex equipment, column 9.

Lacking any of the equipment named thus far, the highway engineer may use uncontrolled mosaics, column 11, as map substitutes to obtain a pattern and idea of the shape of the topography and some rough dimensions of it for his use in the reconnaissance stages. Controlled mosaics, column 13, are preferred for the same purpose. Use of mosaics would be supported by stereoscopic study of successively paired photographs, column 15.

An alternative choice for small-scale large-contour-interval mapping required in reconnaissance stages is precise photogrammetric equipment, provided it is available and circumstances make it economical to use such equipment.

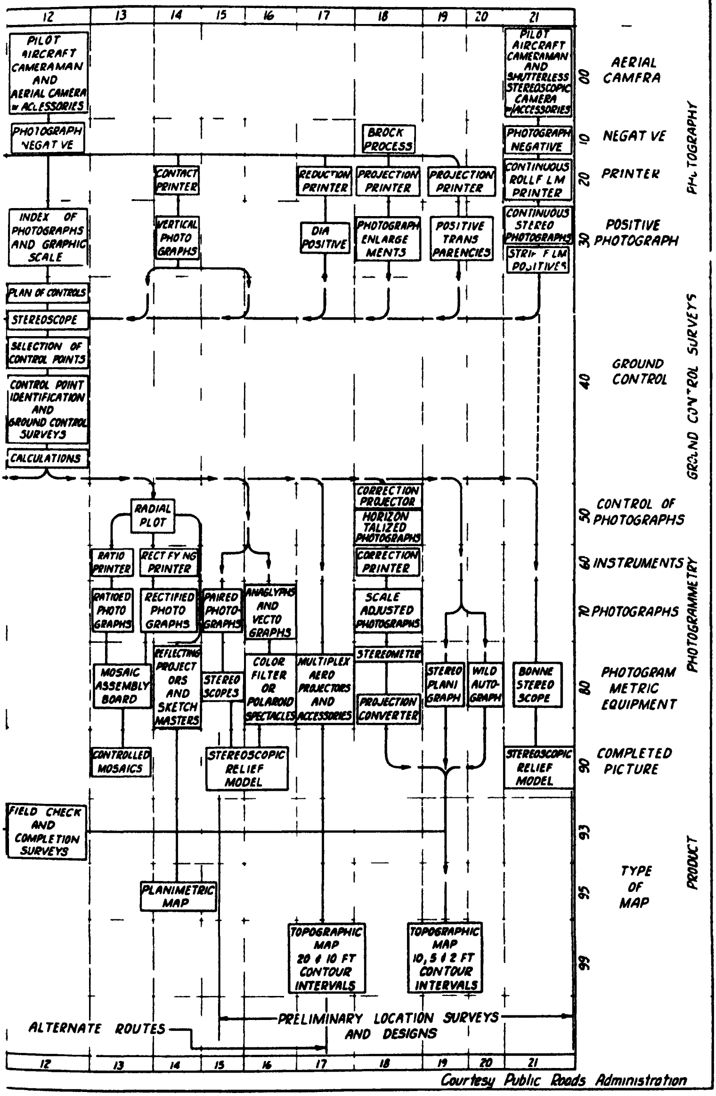
In areas of low relief and concentrated use of land, controlled mosaics and planimetric maps of large scale may be used for



AN OUTLINE OF PROCEDURES IN AERIAL SUR-
MAPPING FOR HIGHWAY

FIG

VEYING AND PHOTOGRAMMETRIC METHODS OF ENGINEERING PURPOSES



Courtesy Public Roads Administration

417h.

route studies. Their preparation would be accomplished as indicated in columns 13 and 14.

For the preliminary survey, large-scale small-contour-interval mapping may be accomplished by use of any one of five types of precise photogrammetric equipment. Four of these are shown in columns 17 to 20, namely: Multiplex, Brock Process, Stereoplanigraph and Wild Autograph. The fifth is the Kelsh Stereoscopic Plotter.

Primarily Figure 417h is a reference guide generally describing how a highway engineer would select his photogrammetric equipment according to the product of completed picture or map required. The entire chart shows the progressive procedure and use of equipment from the aerial camera to the picture or map.

A shutterless aerial camera is listed in column 21. This camera may be used to take very large-scale continuous strip photographs of the located highway. Such photographs viewed under the stereoscope provide the means of obtaining a stereoscopic relief model to scales of from 200 to 20 feet to one inch showing everything on the earth's surface along the highway location before the highway is built. Similar photographs taken after the highway has been constructed will show the changes caused by construction. If desired, the highway engineer can have a third continuous-strip photograph taken to show certain kinds of failures after the highway has been used. Where three separate record-photographs have been taken at designated intervals of time they will provide valuable factual information showing all changes that occurred from the beginning. Study of the changes will reveal where it has been most successful This suggested practice has great promise.

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CHAPTER XXV

MAP PROJECTIONS AND STATE COORDINATES

418. General Statements. A map of an area, in the usual meaning of the word, is a scale representation upon a flat surface of everything contained in the area. All north and south lines on it are parallel and at right angles to all east and west lines. Any point can be located by rectangular coordinates and the shapes and areas of all portions are correctly shown. Such conditions are true only when the area mapped is so small that the effect of the curvature of the earth's surface is less than can be plotted to the chosen scale.

Careful measurements have established that the shape of the earth is that of an oblate spheroid. The Clarke spheroid, computed in 1866, is the standard for all work in the United States. It has an equatorial axis of 41,852,124 feet and a polar axis of 41,710,242 feet. This difference in the lengths of the axes, approximately 26 miles, can be visualized by assuming the equatorial diameter to be 25 feet, which would make the polar diameter about 24 feet 11 inches.

The surface of a sphere cannot be developed into a plane as can the surface of a cone or cylinder. Therefore, it is impossible to map or represent on a flat surface any large portion of the earth's surface without distortion of some kind. This can be illustrated by attempting to flatten half a hollow rubber ball upon a plane. The edges will stretch or tear before the center will touch the plane and distortion of various amounts and in different directions will result.

419. Projections. Since our map must be on a plane, many different schemes of representing the surface of a sphere upon a developable surface have been devised. These map projections consist of different arrangements of meridians and parallels of latitude which constitute the framework for the plotting. None of them is free from distortion but the distortion can be made to appear in different ways by varying the choice and the position of the developable surface upon which the projection is made. The subject of map projections is a broad one, some parts of which require complex mathematical treatment. Only a few general considerations can be given here with some emphasis on those projections most used by surveyors.

In certain projections, areas near the equator are shown correctly but with great distortion near the poles. In others, east-west areas project well and north-south areas poorly, or vice versa. Some maintain relative shapes but with variable scales over the map while others will hold areas equal but greatly distort their shapes.

The projection that should be employed depends entirely upon the purpose for which the map is to be used and how much of the earth's surface is to be shown. For example, the projection that would show best the shapes and relative sizes of the United States would not necessarily be the best for a navigation chart of the Atlantic Ocean.

420. Classification of Projections. Specific classification of the various projections cannot be made because of overlapping properties. However, for the sake of convenience certain class names are in common use as follows:

Equal Area. This type of projection maintains a constant ratio of areas, i.e., one square inch taken at any location on the map represents a certain constant portion of the earth's surface. This does not mean, however, that the shape of the area contained in the square inch is as it really exists on the earth because the scale in a north-south direction is different than that in an east-west direction. Such a projection is valuable for political and statistical work.

Conformal. A conformal projection is one in which each small area shows in its proper shape. This means that at any point the scale along the parallel and the meridian is the same. Obviously, a large area cannot be shown in its correct shape since the distortion must be taken into account by changing the scale from point to point. Actually, the general shape of a large area can be obtained better by projections that are not conformal. The conformal type is of advantage for delineating a section of very irregular coast line.

Azimuthal or Zenithal. This type shows the azimuth or direction from the central point of the map to any other point as it is on the earth's surface. For example, if Chicago were the central point, the map would give true directions to all other cities to which airplanes would fly. However, the map would not give these results at any city other than the one at the central point.

421. The Mercator Projection. One of the common forms of projection is that by which points on the earth's surface are represented upon a cylinder touching along the terrestrial equator. The cylinder is then rolled out into a plane by cutting it along any one of the equally spaced meridians. It is obvious that the equatorial belt will be well represented but that serious distortions will be shown in the polar regions. In order to reduce this effect, the Mercator Projection, Fig. 421, increases the spacing of the parallels at the same rate the parallel representation of the meridians increases the longitude. For example, at 60° latitude the scale along the meridian is doubled because the representation of longitude is just twice the amount it is at the equator. Thus, Alaska shows more than four times

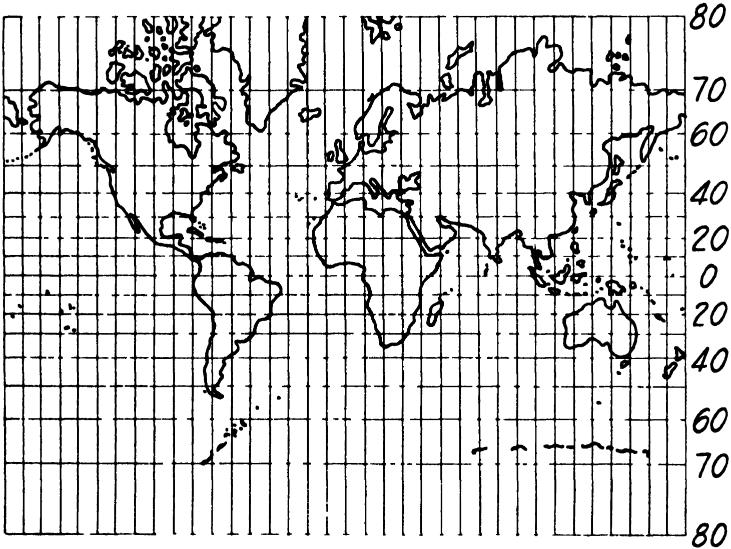

FIG. 421. MERCATOR PROJECTION.

its actual size and although South America is nine times as large as Greenland it appears to be smaller.

The Mercator is commonly used among the large number of projections because the entire surface of the globe can be shown on it. Also, it is the only projection upon which a line making a constant angle with all meridians, called a *rhumb line*, appears as a straight line. This makes it of value in the construction of nautical charts.

422. The Lambert Conformal Conic Projection. This is one of the most important of the many projections constructed upon the general principle of representing points on the earth's surface upon a cone or series of cones tangent to parallels of latitude or cutting through pairs of them. The developed cone, cut along any meridian and rolled out on a plane, will show the meridians as straight lines meeting at a point outside the map and the parallels as concentric circles. This form of projection represents accurately an east-west strip of the earth's surface for

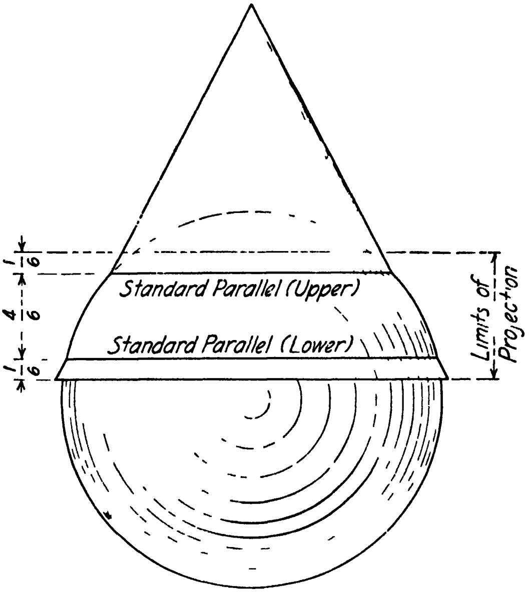

FIG. 422. LAMBERT CONIC PROJECTION.

a short distance on each side of the parallel touching the cone. Obviously, the distortion increases north or south of this line.

The Lambert Conformal Conic projection widens the belt of latitude within which the scale distortions are not serious by taking the cone through two standard parallels, Fig. 422, instead of tangent to one parallel mid-way between them. This arrangement makes the scale between the standard parallels somewhat too small and beyond them somewhat too large.

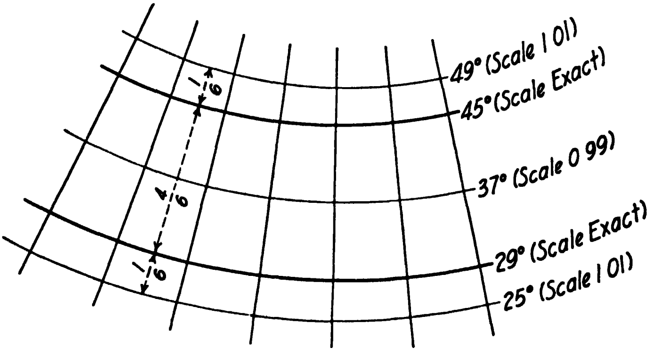

FIG. 422a. SCALE DISTORTION.

Figure 422a shows the distortion of scale of a map covering the United States.

If the distance between the standard parallels is decreased, the effective belt is narrowed and the distortion of scale becomes less. This becomes of great importance in the establishment of rectangular coordinates over a state having a predominant east-west dimension.

423. Other Projections. Among the many other forms of projection the following might be mentioned as most important or most used: Gnomonic, Albers Equal Area, Polyconic (used as the basis for the U. S. Military Grid System and extensively by the United States Coast and Geodetic Survey), Bonne, and Stereographic. Each of these possesses some feature tending toward a more nearly perfect representation of shape, area or direction. They present very interesting problems both from the standpoint of mapping and from that of the mathematics involved in their construction.

424. State Plane Coordinates. The United States Coast and Geodetic Survey has established more than 40,000 miles of well-distributed control surveys. This control net, Fig. 161, is being extended steadily and soon it will be available for use for any engineering work involving measurements on the surface of the earth. However, this control system has been computed geodetically; that is, the results are given as latitude, longitude, geodetic azimuth,* and distance. The average engineer is not able to handle geodetic computations and even if qualified he would find it easier to set up local plane coordinates for a particular survey. Such practice results in a complete lack of coordination between survey areas that might border on each other.

The need for some simple and practical scheme that would make possible the use of the geodetic coordinates without geodetic computation has arisen in recent years. Therefore, the United States Coast and Geodetic Survey, as explained in § 458, has computed plane coordinates with a state or a large part of a state as the area unit. This is a map projection problem for an area the size and shape of which are fixed. In addition, the plane coordinates must be calculated more accurately than would be necessary merely for the plotting of the map since they are to be used as a basis for computation of various kinds of surveying operations.

Since even a large part of the area of a state cannot be projected upon a plane without distortion, it is evident that variations in scale cannot be avoided. However, these variations can be made to appear in definite directions and correction tables may be prepared for them by adopting a form of projection in which meridians and parallels are at right angles to each other. These considerations lead to the selection of one of the conformal projections. The projection chosen for states with greatest extent in an east-west direction is the Lambert Conformal Conic with two standard parallels. For states extending principally north and south the transverse Mercator with two standard meridians, which is essentially the Mercator projection turned 90°, has been chosen. Both of these projections are conformal and each is especially suitable for the states or parts of states in which it is used. An idea of the amount of the scale factor or change of scale from the original projection to that of the plane coordinates may be obtained from the computations for any state. Connecticut, Fig. 424, for example, which is computed on the Lambert projection with two standard parallels, has a maximum variation of 1 part in 59,000 mid-way between the parallels. Indiana, Fig. 424a, is divided into east and west parts each of which is computed on the transverse Mercator with two standard meridians, 64.6 miles apart, giving a maximum variation of 1 part in 30,000 mid-way between them. The amount of this variation is a function of distance between the chosen standard parallels or meridians along which the scale is held exact.

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* Geodetic azimuth is defined as the horizontal angle to the right from true south to the line or direction in question when the horizon is perpendicular to a normal to the spheroid.

A study of the advantages to be derived from the use of state coordinates will cause progressive engineers to urge their early establishment in every state. All state highway work would be definitely coordinated and would carry the control system to every corner of the state. City surveys would tie up directly with the surrounding country and a control system for the construction of good county maps, so sadly lacking at present, would be available. In addition, property corners, once given their x and y coordinates, can never be lost; the errors in the original land surveys would cause no more difficulty; and legal deed descriptions could be written with greater ease and accuracy than ever before. See also §§ 455 and 458.

425. Plane Coordinate Tables. The computations necessary to reduce geodetic coordinates, distances, and azimuths to plane coordinates, distances, and grid azimuths * are long and somewhat difficult. The variable factors are principally differences in latitude, longitude, and the spheroidal shape of the earth. Fortunately, most of the calculations can be made for any one state and the results can be given in tables. These reduce the necessary calculations to a minimum that is neither long nor difficult after the computer becomes familiar with the tables.

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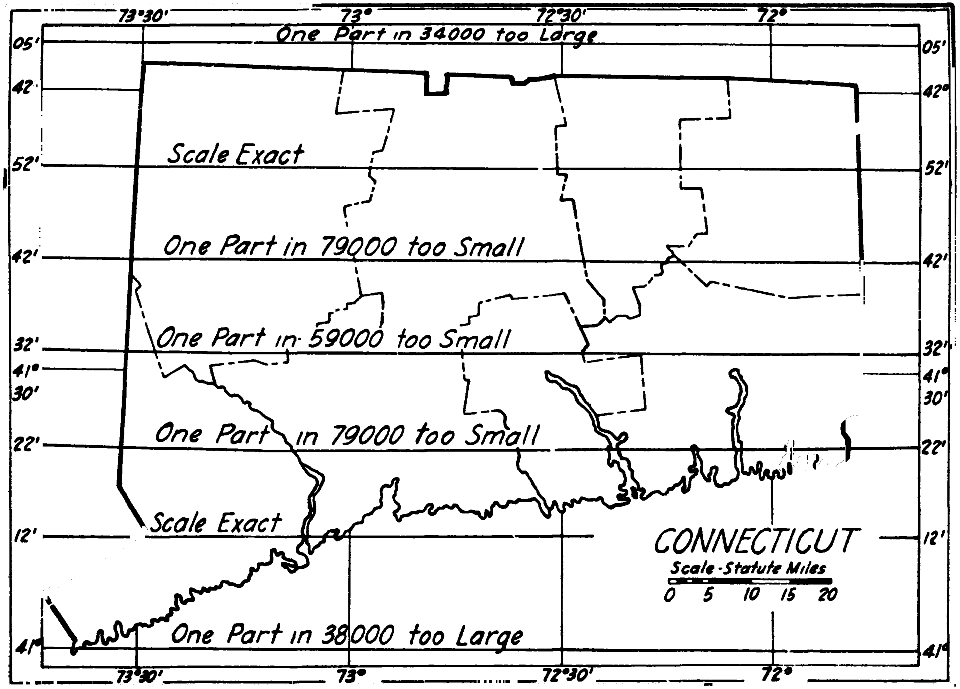
* Grid azimuth is defined as the angle that any given line makes with a north-and-south grid line. It differs from the true or astronomic azimuth (except at the central or initial meridian) by the amount of the convergence of the meridians.



73°30'
73°
72°30'
72°
One Part in 34000 too Large
05'
42°
Scale Exact
52'
One Part in 79000 too Small
42'
One Part in 59000 too Small
32'
41°
30'
One Part in 79000 too Small
22'
Scale Exact
12'
CONNECTICUT
Scale-Statute Miles
0 5 10 15 20
One Part in 38000 too Large
41°

FIG. 424. STATE COORDINATES — CONNECTICUT.

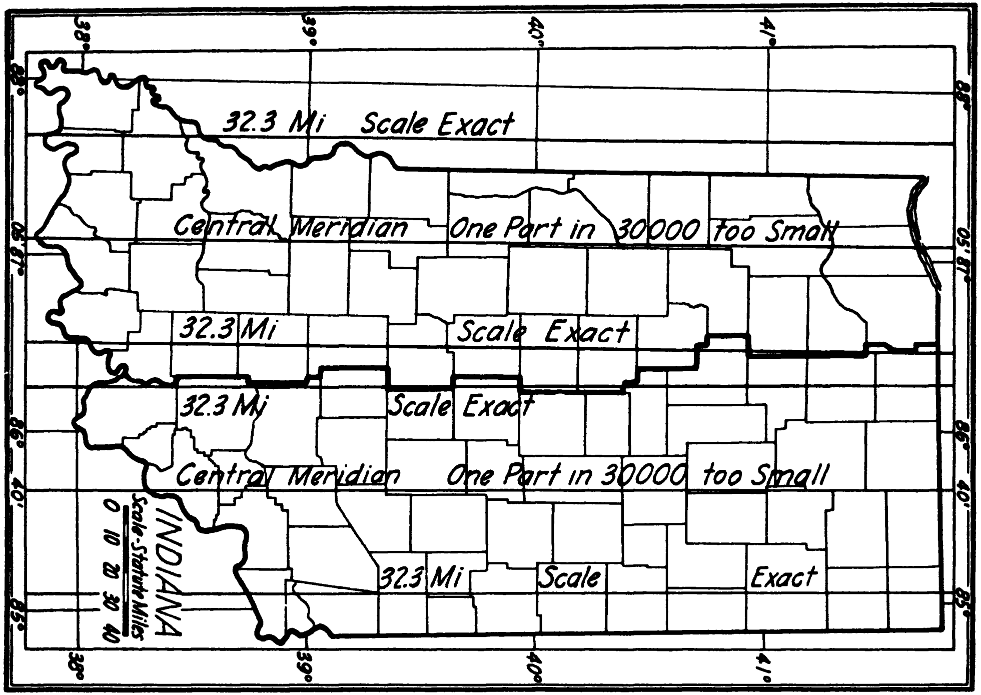


FIG. 424a. STATE COORDINATES — INDIANA.

The necessary tables for any state together with small maps showing such data as may be needed for those desiring to make use of the projections can be obtained from the United States Coast and Geodetic Survey.

For those who desire complete information on the subject, special reference is made to the Manual of Plane Coordinate Computation, Manual of Traverse Computation on the Lambert Grid, and Manual of Traverse Computation on the Transverse Mercator Grid -- Special Publications numbered 193, 194, and 195 respectively of the United States Coast and Geodetic Survey.

426. Military Grid System. A system of plane coordinates has been established over the United States for military purposes. It differs from state plane coordinate systems in that the entire country is covered and the coordinates are stated in yards instead of feet. The basic projection used is the polyconic projection developed for the country by the United States Coast and Geodetic Survey.

For purposes of superimposing the grid system on maps the United States has been divided into seven zones designated by letters, A to G. Each zone, Fig. 426, representing 9° of longitude overlap into adjacent zones E and W, is 30′. The rectangular grid is fitted over the polyconic projection of each zone by making the north-south grid line agree with the central meridian of the zone. The origin of each zone is the intersection of its central meridian with the 40° 30′ N parallel of latitude. The assigned coordinates of this point are $X = 1{,}000{,}000$ yards, $Y = 2{,}000{,}000$ yards. Figure 426a illustrates the superposition of 100,000-yard grids upon zone C.

Because of the convergence of meridians true north and grid north do not agree except along the central meridian. This variation exceeds 3° at the edge of the grid zone. There is also a discrepancy in distance measurement due to the magnification of scale in a north-south direction incident to the polyconic projection. However this scale error, approximately two-tenths of one per cent at zone edges, is of no military consequence since it probably is exceeded by distortions that occur in paper map sheets due to climatic conditions.

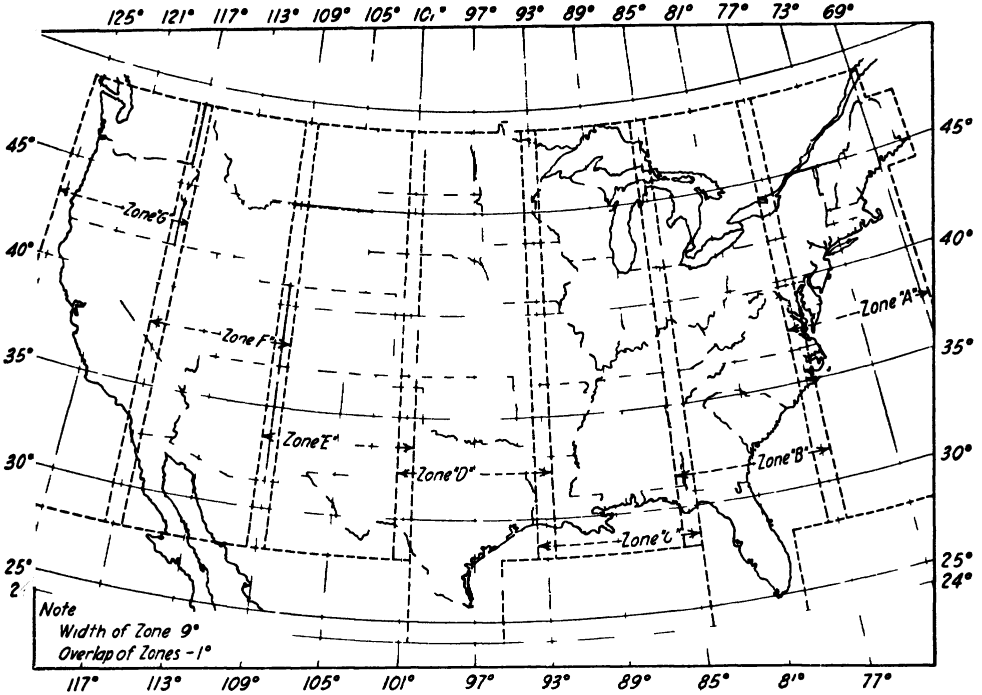


Fig 426 U S Army Grids

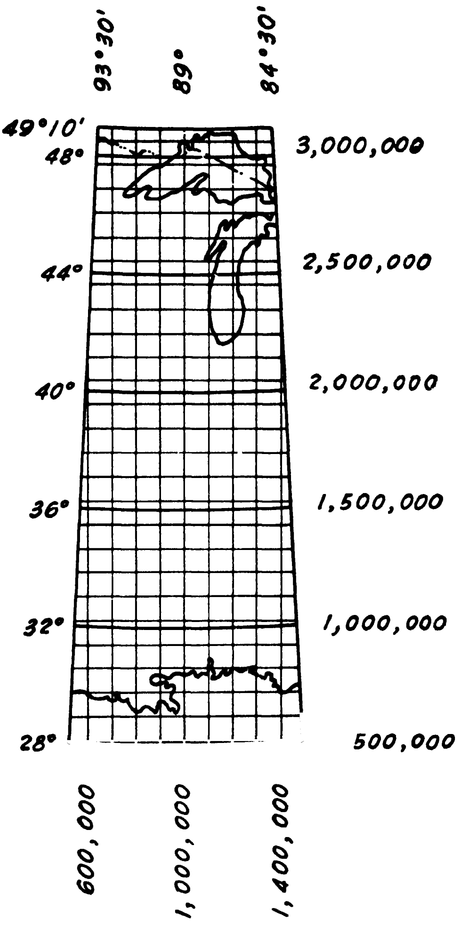


FIG. 126a. ONE GRID ZONE (C).

REFERENCES

1. SHELDON, R. C., *Practical Use of Horizontal Geodetic Control, and Discussion*. Transactions of the American Society of Civil Engineers, Vol. 103, p. 791, 1938.
2. STEERS, J. S., *An Introduction to the Study of Map Projections*, 1929.
3. UNITED STATES COAST AND GEODETIC SURVEY, *Special Publications 5, 47, 49, 52, 53, 57, 59, 60, 68, 112, 130, 153, 193, 194, 195, 235*. Government Printing Office, Washington, D. C.
   State Plane Coordinates, Geodetic Letter 1, Volume 4, January, 1937.

CHAPTER XXVI

HYDROGRAPHIC SURVEYING AND FLOW MEASUREMENTS

By Professor W. I. Freel
Purdue University

HYDROGRAPHIC SURVEYING

427. Definition. Hydrographic surveying usually involves the mapping of topography along the shore line of a body of water as well as the conformation of the earth's surface beneath the water. In addition, a hydrographic survey may require the determination of velocities, directions of flow, and fluctuations of level in the body of water.

428. Purposes of Hydrographic Surveys. Hydrographic surveys may be made for one or more of the following purposes:

1. Mapping bottom and shoreline of a body of water for navigation purposes
2. Estimating yardage on dredging projects
3. Determining volume of impounded water
4. Measuring fluctuation of tides
5. Determining direction and velocity of currents
6. Determining drainage areas.

429. Equipment. The equipment employed in hydrographic surveying includes all of that used in regular topographic mapping with the addition of boats, tackle, and the special instruments peculiar to this field which are described later.

430. Hydrographic Charts. A map or chart for navigation purposes is little different from any good topographic map except that tide fluctuations, navigation markers, water depths, and character of bottom material are indicated. Fig. 430 is an example of a hydrographic map or chart.

431. Control. The control of the hydrographic map or chart should be of the same order of accuracy as that specified for

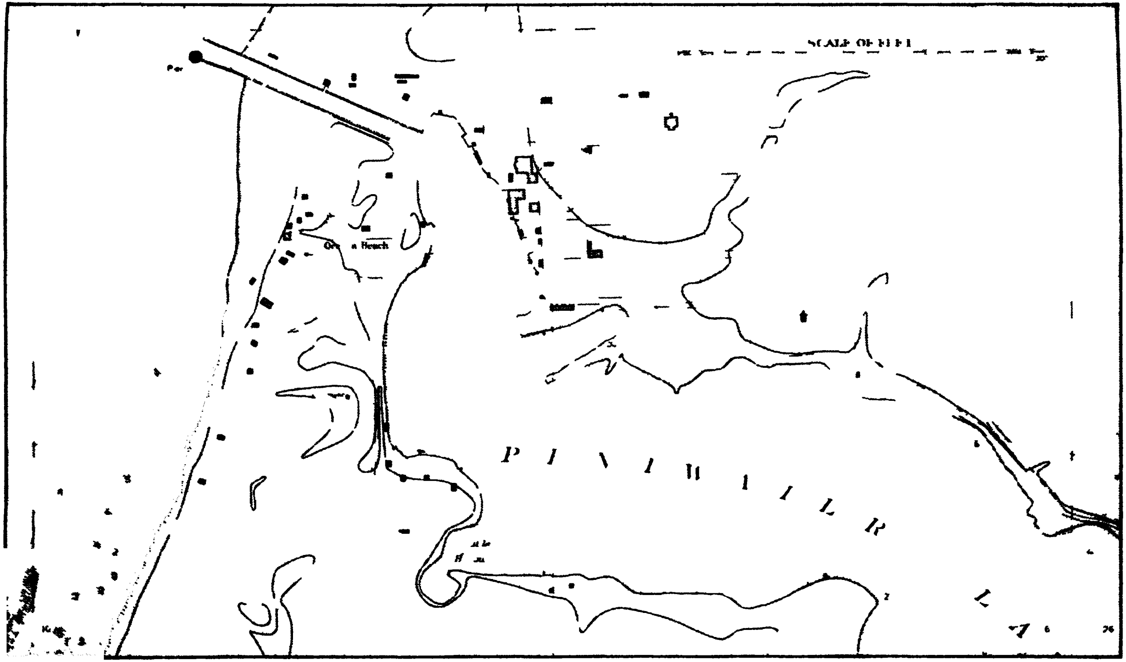


FIG 430 PENTWATER, MICHIGAN HARBOR CHART

Courtesy of Great Lakes Survey

general topographic mapping (see § 275) or a higher order and should include a careful determination of the datum by methods suggested in Special Publication No. 135 of the United States Coast and Geodetic Survey, *Tidal Datum Planes.* When points on the surface of the water are being used as part of the control system, they are usually marked by short staffs attached to anchored buoys. Control points consisting of special shore stations, visible from the water over which the boat is employed in making measurements, are constructed of timber and painted for high visibility. Wherever possible, as in plane table surveying (Chapter XXIII), steeples, stacks, tall buildings, flag poles, and salient landmarks are used as part of the control system.

The scope, completeness and accuracy required in work of this character are exemplified in the splendid charts produced by the U. S. Coast and Geodetic Survey and the U. S. Lake Survey Office of the War Department Corps of Engineers.

432. Filling in Detail. Filling in the detail of the subaqueous surfaces requires the accurate determination of depth by sounding and the accurate location of the point at which the sounding was made.

The principles most used in making observations for locating points are the same as those employed in plane table work, namely, intersection and resection, as explained in Chapter XXIII. When intersection is used, the position of a sounding is determined by plotting rays, the directions of which have been established by transit measurements from the ends of a base line established on shore. Where the resection principle is used, the angles to shore stations are measured by sextant from a boat. Approximate methods for locating points are sometimes used, such as uniform rowing along range lines marked on shore.

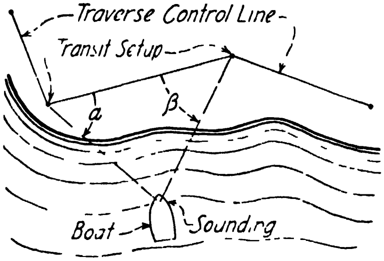


Fig. 132. Locating Soundings by Intersection.

When depths are shallow, a stadia rod, equipped with a shoe which prevents the rod from penetrating soft bottom, is employed for taking soundings. The azimuth, rod intercept, and vertical angle establish the position and elevation of the foot of the rod according to the principles of stadia as outlined in Chapter IX.

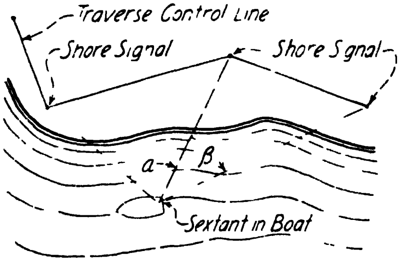


FIG. 132a LOCATING SOUNDINGS BY RESECTION.

In moderately deep water, soundings are taken by hand with regular sounding weights suspended from a calibrated line of hemp, cotton, or chain. Where the character of bottom material is required, the sounding weight is provided with a cupped recess at its lower end in which a sample of the bottom material wedges when the weight touches the bottom. Where the bottom material is granular, the cupped recess may be filled with wax or tallow to which sand grains will adhere. In deeper water, the line suspending a heavier weight may be manipulated by an engine-driven or motor-driven winch.

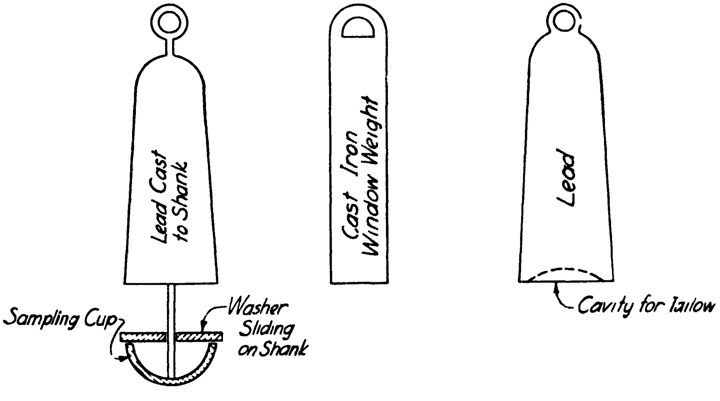
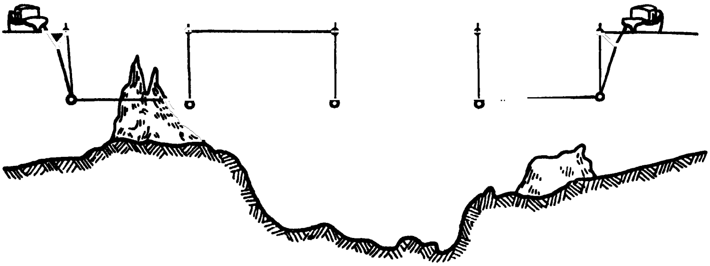


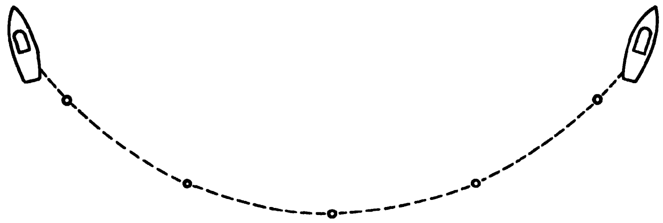
FIG. 132b. SOUNDING WEIGHTS.

The location of all physical features and markers of interest to navigation, such as buoys, lights, bars, channels, and submerged obstructions, are carefully marked and identified on hydrographic maps.

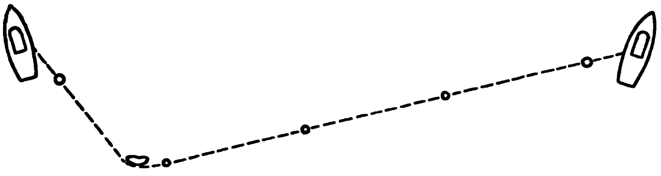
A long taut wire is sometimes stretched between two power boats and dragged through the water to discover pinnacle rocks or other hazards to navigation. The wire is supported at intervals by floats to prevent excessive sag. Although this practice does not yield complete contours of the bottom, it locates high points and makes possible the quick determination of safe depths over large areas. By changing the lengths of the vertical supports of the drag wire and successively dragging



Drags Set at Different Depths



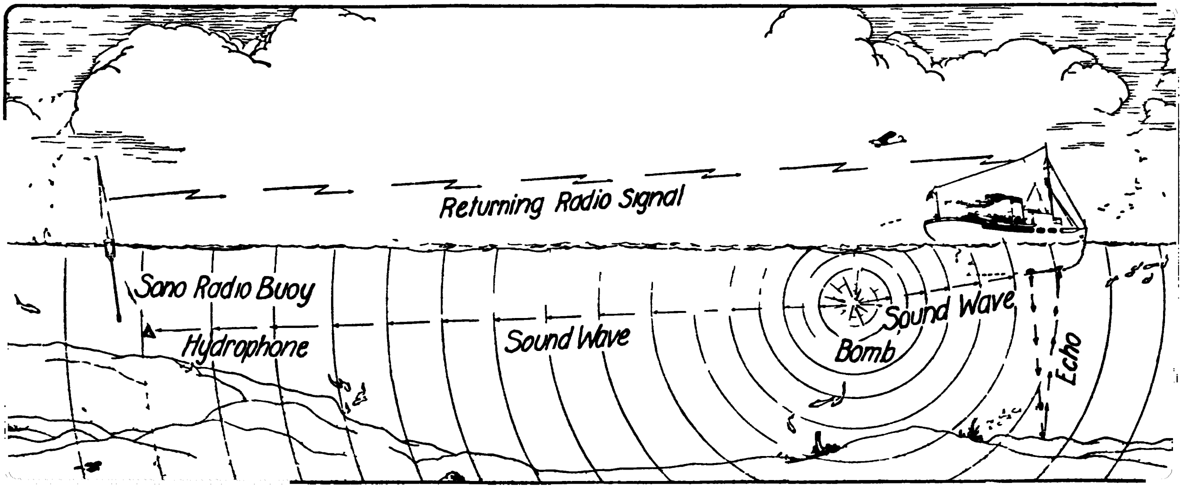
Plan of Drag under Way



Plan of Drag Aground

Courtesy of United States Coast and Geodetic Survey

FIG. 432c. WIRE DRAG.



Courtesy of United States Coast and Geodetic Survey

FIG. 432d. FATHOMETER SOUNDINGS AND RADIO ACOUSTIC RANGING.

the area being mapped, it is possible to establish the locations and depths of obstructions in considerable detail. See Fig. 432c.

432a. Radar, Shoran, and Loran.* Soundings may be taken with a Fathometer (Fig. 432d). This instrument determines depth by measuring the time interval required for a sound wave to travel from ship to ocean bottom and back again. The principal advantage in the use of this instrument is the rapidity with which observations may be made. This makes possible frequent depth measurements from a ship in motion.

433. Yardage on Dredging Projects. Yardage on dredging projects may be estimated by soundings of the bottom taken before and after the dredging operation, or by the principles of mensuration after the excavated material is loaded on barges. Another common method of yardage determination is to measure the draft of a scow before and after loading, and thus to estimate by dimensions the volume of water displaced during the loading process. The specific weight of the water in which the barge floats and the unit weight of the excavated material are essential data in these computations.

434. The Volume of Impounded Water. The capacity of reservoirs or lakes is usually expressed in acre-feet. An acre-foot is the volume of water one foot deep and one acre in extent, or 43,560 cubic feet. In estimating capacity, soundings may be made and plotted to scale. The volume of water may then be computed by the application of the principles outlined in § 208, and as described in Chapter XII of Rubey's *Route Surveys*.

435. The Fluctuation of Tides. Information concerning the fluctuation of tides is of importance to navigators and is essential information on a shore line map. In measurements of tide fluctuations, the determination of a correct datum is essential. Mean sea level is usually taken as datum. Since tides are subject to cyclic variation in amplitude, long observation is usually necessary to establish precisely the elevation of mean sea level at a station. Accurate observations of water levels may be

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* Recent developments of electronic methods of surveying are covered by a symposium on 'Radar Shows Promise in Mapping,' *Civil Engineering*, July 1946, Vol 16 No 7, pp 294-9

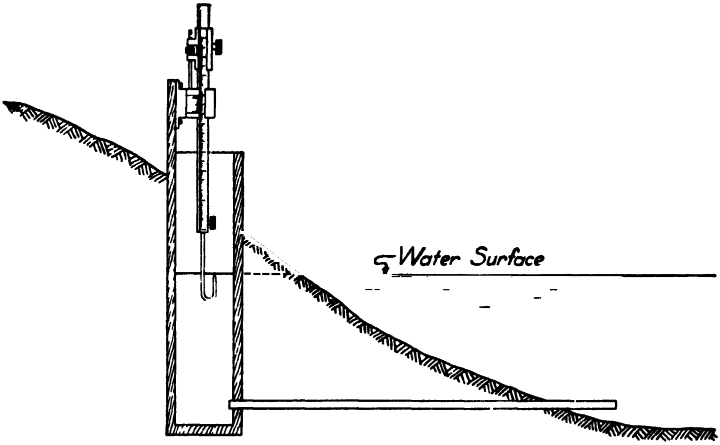


FIG. 435. HOOK GAGE IN STILLING BOX.

obtained by reading a hook gage suspended in a stilling box as in Fig. 435. Continuous records of water level fluctuations may be obtained by use of a water level recorder (Fig. 435b).

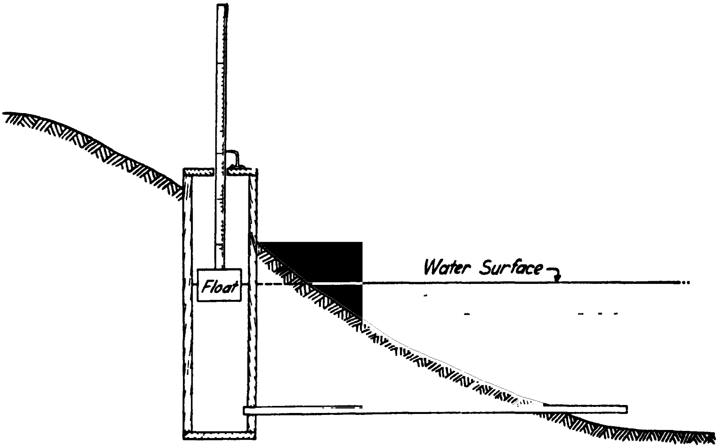


FIG. 435a. STAFF FLOATING IN STILLING BOX.

For work of ordinary precision observations for the determination of a datum should extend over one lunar month. For precise surveys the period of observation is extended.

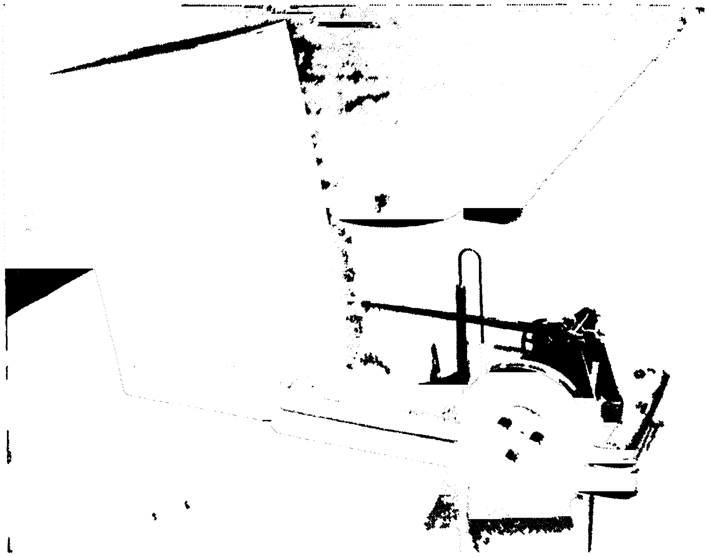


FIG 435b WATER LEVEL RECORDER

436. Measurements of Tidal Currents. The direction and velocity of currents may be measured by the use of floats of various types, some of which are illustrated in Fig 436 The application of each type is suggested by its construction

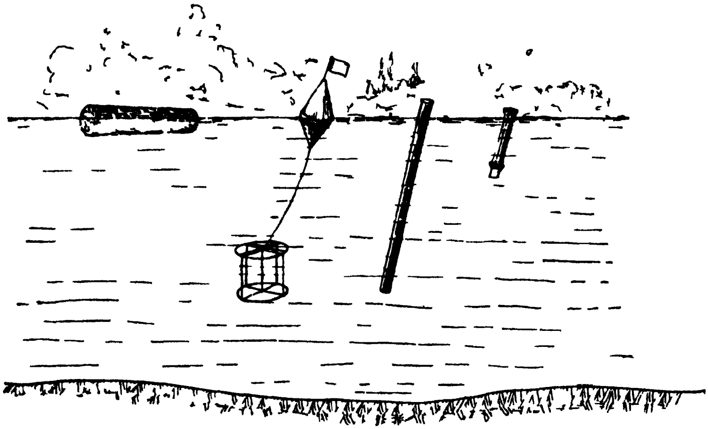


FIG 436 TYPES OF FLOATS

Special floats devised to suit the needs of the problem may be better than any of those suggested here. The principal precaution to be observed in connection with the use of a float is to make sure that the float has accelerated to the velocity of the current before the initial observation of its position is made. High velocities may be measured by the use of the current meter which is described later.

437. Drainage Basin Areas. Complete maps of drainage areas may be constructed by the principles of topography or photogrammetry. When only the size of the drainage area is required, a traverse is run which follows the high ground and sufficient side shots are made for plotting the location of the ridge line. In many cases, however, there are reasons for requiring complete contouring of the physical features within the drainage area. The slope of the ground, the character of the soil and cover, the routes traveled by drainage, and the cross-section of streams within a basin are essential data where a particular problem is being completely solved.

A few examples of engineering problems in which such information is required are listed below:

1. Determination of size of bridge or culvert opening
2. Design of flood retention works including retention reservoir, dam and spillway
3. Design ot levees for flood protection
4. Rational design of sewerage systems
5. Design of storage reservoirs for water supply and water power projects
6. Design of artificial waterways for navigation.

FLOW MEASUREMENTS

438. Discharge Measurements. Discharge of streams, canals, ditches, flumes, and other waterways usually is expressed in cubic feet per second, commonly abbreviated sec.-ft. In discharge measurements the information sought is the number of cubic feet per second flowing past a station. This may be required for the minimum flow, the maximum flow, or for the variations in flow extending over a long and uninterrupted period of time. The nature of the problem to be solved governs

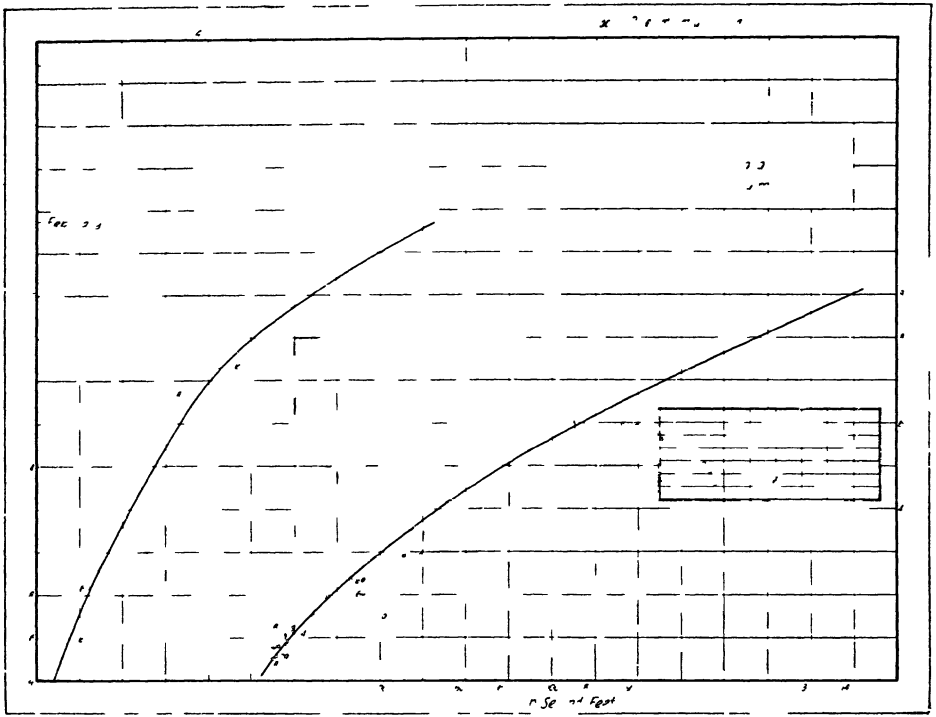


FIG. 438. STATION RATING CURVE

the information desired and determines the methods to be used.

No effort has been made in these pages to outline completely the entire field of stream discharge measurement or to cover completely the application of stream discharge measurement to engineering practice. The professional technique of stream discharge measurements is covered in the references.

Most methods of determining discharge comprise finding the average velocity of the flowing water and multiplying this by its cross-sectional area. The result is more accurate when the cross-section of the whole moving body of water is divided into several smaller sections, the velocity of each section determined separately and multiplied by its area, and the sum of these products used as the total discharge. A single discharge figure is seldom useful. It becomes effective as a part of the analysis of an engineering problem only after it is combined with several other discharge measurements made at the same station at various other stages. These are used to construct a curve known as a *Rating Curve* for the station under consideration. A rating curve is illustrated in Fig. 438. More complicated rating curves may be constructed to show the effect upon discharge of a rising or falling stage. No attempt is here made to illustrate such a curve or to explain its construction. Examples of such rating curves and instructions for their construction may be found in Water-supply Paper No. 888, United States Geological Survey.

439. The Price Current Meter. The method of velocity measurement which here will be termed standard is the one commonly used by the United States Geological Survey. It employs as its principal instrument the small Price current meter shown in Fig. 439. The essentials of this meter are a bucket wheel rotated by the passage of the water and a battery circuit through meter and headphones which permits the revolutions of the bucket wheel to be counted. The meter is used by pointing the bucket wheel directly against the current whose velocity is to be measured, and counting the revolutions for a stated period of time. With the speed of the bucket wheel as an argument, the velocity at the point of its suspension

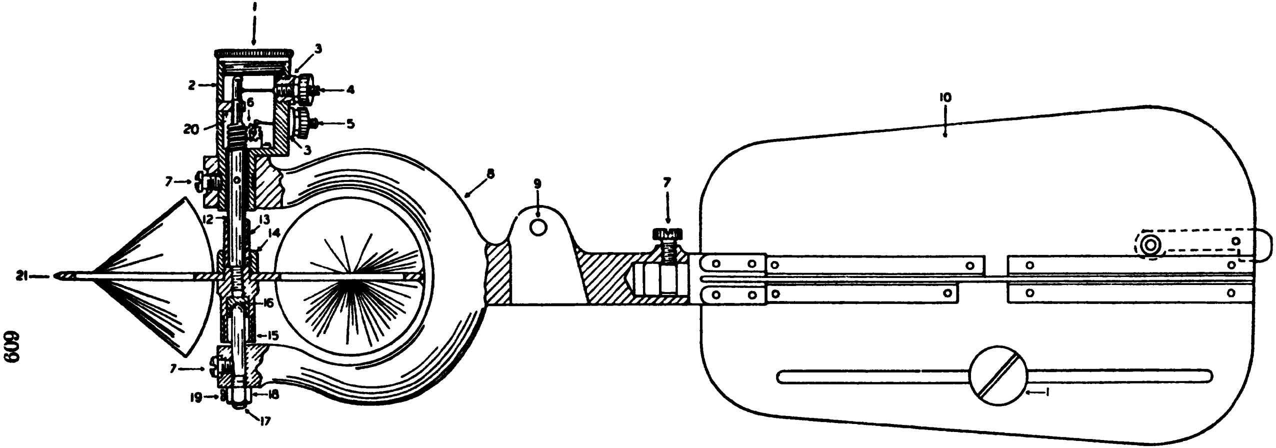
TABLE 439.

DEPARTMENT OF INTERIOR—GEOLOGICAL SURVEY
Water Resources Branch
RATING TABLE FOR TYPE *A*
CURRENT METER NO. *1851*
SUSPENSION *ROD* DATED *OCT 23, 1939*

INDEX *508—504*
EQUATIONS $V = 2.320\,R + 0.025 (2.34) 2.280\,R + 0.065$
Limits of Actual Rating *0.2* to *8.2* feet per sec. at
Bureau of Standards, Wash., D. C. Condition of Meter *NEW*

NO. *1851*
ROD
OCT. 23, 1939

Time in Seconds	Velocity in Feet per Second									Time in Seconds	Time in Seconds	Velocity in Feet per Second									Time in Seconds
	Revolutions											Revolutions									
	3	5	7	10	15	20	25	30	40			50	60	80	100	150	200	250	300	350	
40	.199	.315	.431	.605	.895	1.18	1.48	1.76	2.34	40	40	2.92	3.48	4.62	5.76	8.62	11.46	14.32	17.16	20.02	40
41	.195	.308	.421	.591	.874	1.16	1.44	1.72	2.29	41	41	2.85	3.40	4.51	5.63	8.41	11.19	13.97	16.75	19.53	41
42	.191	.301	.412	.577	.854	1.13	1.41	1.68	2.23	42	42	2.78	3.32	4.41	5.49	8.21	10.92	13.64	16.35	19.06	42
43	.187	.295	.403	.565	.834	1.10	1.37	1.64	2.18	43	43	2.72	3.25	4.31	5.37	8.02	10.67	13.32	15.97	18.62	43
44	.183	.289	.394	.552	.816	1.08	1.34	1.61	2.13	44	44	2.66	3.17	4.21	5.25	7.84	10.43	13.02	15.61	18.20	44
45	.180	.283	.386	.541	.798	1.06	1.31	1.57	2.09	45	45	2.60	3.10	4.12	5.13	7.66	10.20	12.73	15.27	17.80	45
46	.176	.277	.378	.529	.782	1.03	1.29	1.54	2.04	46	46	2.54	3.04	4.03	5.02	7.50	9.98	12.46	14.93	17.41	46
47	.173	.272	.371	.519	.765	1.01	1.26	1.51	2.00	47	47	2.49	2.98	3.95	4.92	7.34	9.77	12.19	14.62	17.04	47
48	.170	.267	.363	.508	.750	.992	1.23	1.48	1.96	48	48	2.44	2.92	3.86	4.82	7.19	9.56	11.94	14.32	16.69	48
49	.167	.262	.356	.498	.735	.972	1.21	1.45	1.92	49	49	2.39	2.86	3.79	4.72	7.04	9.37	11.70	14.02	16.35	49
50	.164	.257	.350	.489	.721	.953	1.18	1.42	1.88	50	50	2.34	2.80	3.71	4.62	6.90	9.18	11.46	13.74	16.02	50
51	.161	.252	.343	.480	.707	.935	1.16	1.39	1.84	51	51	2.30	2.75	3.64	4.54	6.77	9.01	11.24	13.48	15.71	51
52	.159	.248	.337	.471	.691	.917	1.14	1.36	1.81	52	52	2.26	2.70	3.57	4.45	6.64	8.83	11.03	13.22	15.41	52
53	.156	.244	.331	.463	.682	.900	1.12	1.34	1.78	53	53	2.21	2.65	3.51	4.37	6.52	8.67	10.82	12.97	15.12	53
54	.154	.240	.326	.455	.669	.884	1.10	1.31	1.74	54	54	2.17	2.60	3.44	4.29	6.40	8.51	10.62	12.73	14.84	54
55	.152	.236	.320	.447	.658	.869	1.08	1.29	1.71	55	55	2.13	2.55	3.38	4.21	6.28	8.36	10.43	12.50	14.57	55
56	.149	.232	.315	.439	.646	.854	1.06	1.27	1.68	56	56	2.10	2.51	3.32	4.14	6.17	8.21	10.24	12.28	14.32	56
57	.147	.229	.310	.432	.636	.839	1.04	1.25	1.65	57	57	2.06	2.46	3.26	4.06	6.06	8.06	10.06	12.06	14.06	57
58	.145	.225	.305	.425	.625	.825	1.02	1.22	1.62	58	58	2.02	2.42	3.21	4.00	5.96	7.93	9.89	11.86	13.82	58
59	.143	.222	.300	.418	.615	.811	1.01	1.20	1.60	59	59	1.99	2.38	3.16	3.93	5.86	7.79	9.73	11.66	13.59	59
60	.141	.218	.296	.412	.605	.798	.992	1.18	1.57	60	60	1.96	2.34	3.10	3.86	5.76	7.66	9.56	11.46	13.36	60
61	.139	.215	.291	.405	.595	.786	.972	1.17	1.55	61	61	1.93	2.31	3.06	3.80	5.67	7.54	9.41	11.28	13.15	61
62	.137	.212	.287	.399	.586	.773	.960	1.15	1.52	62	62	1.90	2.27	3.01	3.74	5.58	7.42	9.26	11.10	12.94	62
63	.135	.209	.283	.393	.577	.762	.946	1.13	1.50	63	63	1.87	2.23	2.96	3.68	5.49	7.30	9.11	10.92	12.73	63
64	.134	.206	.279	.388	.569	.750	.931	1.11	1.48	64	64	1.84	2.20	2.92	3.63	5.41	7.19	8.97	10.75	12.53	64
65	.132	.203	.275	.382	.560	.739	.917	1.10	1.45	65	65	1.81	2.17	2.87	3.57	5.33	7.08	8.83	10.59	12.34	65
66	.130	.201	.271	.377	.552	.728	.904	1.08	1.43	66	66	1.78	2.13	2.83	3.52	5.25	6.97	8.70	10.43	12.16	66
67	.129	.198	.267	.371	.544	.718	.891	1.06	1.41	67	67	1.76	2.10	2.79	3.47	5.17	6.87	8.57	10.27	11.98	67
68	.127	.196	.264	.366	.537	.707	.878	1.05	1.39	68	68	1.73	2.07	2.75	3.42	5.09	6.77	8.45	10.12	11.80	68
69	.126	.193	.260	.361	.529	.697	.866	1.03	1.37	69	69	1.71	2.04	2.71	3.37	5.02	6.67	8.33	9.98	11.65	69
70	.124	.191	.257	.356	.522	.688	.854	1.02	1.35	70	70	1.68	2.01	2.67	3.32	4.95	6.58	8.21	9.84	11.46	70



Courtesy of the United States Geological Survey

EXPLANATION

1 Cap for contact chamber
2 Contact chamber
3 Insulating bushing for contact binding post
4 Single-contact binding post (upper)
5 Penta-contact binding post (lower)
6 Penta gear
7 Set screws
8 Yoke
9 Hole for hanger screw
10 Tailpiece
11 Balance weight
12 Shaft
13 Bucket-wheel hub
14 Bucket-wheel hub nut
15 Raising nut
16 Pivot bearing
17 Pivot
18 Pivot adjusting nut
19 Keeper screw for pivot adjusting nut
20 Bearing lug
21 Bucket wheel

FIG. 439. STANDARD PRICE METER ASSEMBLY.

in the stream can be determined from a calibration for the meter used. Meter calibrations are usually made by The National Bureau of Standards at Washington, D C, but may be made

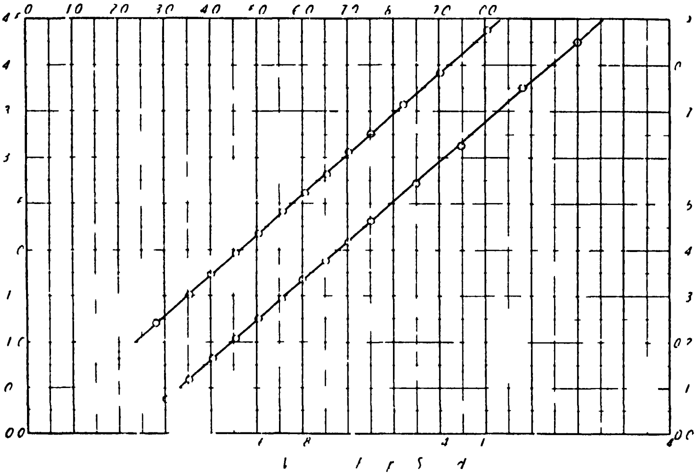
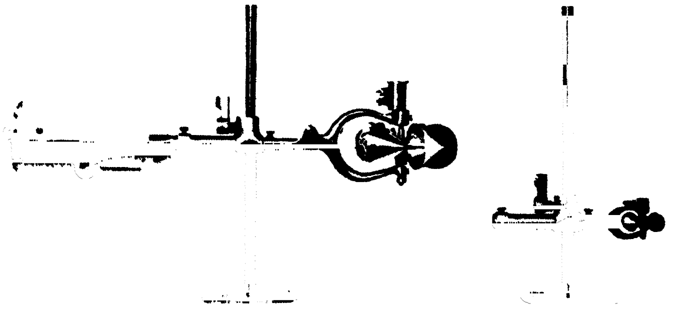


Fig. 439a Meter Rating Curve

in the field by determining the relation of the speed of the bucket wheel to the velocity of the meter as the meter is propelled through still water

Paper No 1778, Vol 95, Transactions of the American Society of Civil Engineers, entitled "Effect of Turbulence on



Courtesy of United States Geological Survey

Fig. 439b Comparative Sizes of Standard Price and Pigmy Meters

the Registration of Current Meters," adequately covers the technicalities of the use of current meters and is accompanied by an appendix listing published papers covering all types of current meters used up to 1927. A recent innovation in current meters is illustrated in Fig. 439b.

440. Variations in Velocity. At any one station, the variation in velocity of the various parts of the cross-section of the stream is very marked and consequently must be taken into account in making measurements. The velocity usually increases from the bank toward the center of the stream and varies from the surface to the bed. The variation from the bank towards the stream center is difficult of prediction and is determined by dividing the cross-section into a number of subdivisions. The variation from surface to bed, however, is understood, and the rather consistent velocity variation in this direction is depended upon in making measurements. Fig. 440 shows the common distribution of velocity from surface to bed. The consistent shape of this curve makes possible the practice of taking the velocity at one or two depths in each section of a station and computing the average velocity for that section. An examination of Fig. 440 will reveal that the average velocity occurs at 0.6 the depth; that the average of the velocities at the 0.2 depth and 0.8 depth is the average velocity; and that the average velocity is approximately 0.9 the surface velocity. This gives rise to the practice of using 0.9 of the surface velocity or averaging the velocities at the 0.2 and 0.8 depths, or using the velocity at the 0.6 depth as the average for the section being con-

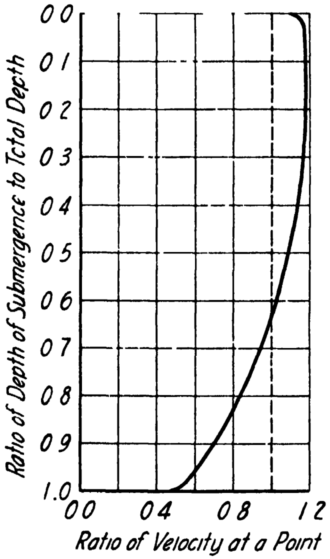


FIG. 440. VELOCITY DISTRIBUTION.

sidered. The velocity obtained by averaging the 0.2 and 0.8 depth readings is the most reliable of the three methods mentioned, but because of the impracticability of always using this method, the velocity at the 0.6 depth is sometimes used alone. Nine-tenths of the surface velocity is sometimes used as the average velocity especially at flood stages where meter determination is difficult. Velocity measurements at this point are most affected by wind, waves and currents produced by channel roughness and are therefore likely to be unreliable. On deep streams the vertical velocity curve may be determined somewhat more precisely by additional readings at other depths.

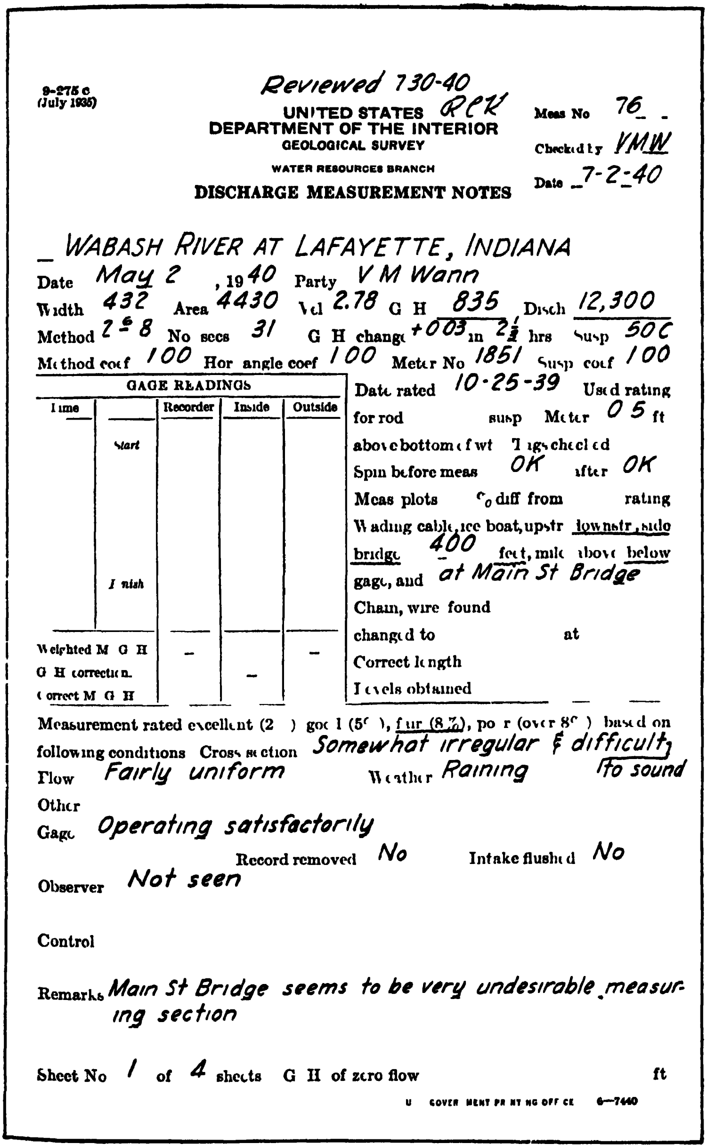
441. Use of the Price Current Meter. The application of the small Price meter to making a stream discharge measurement comprises the following:

1. Choosing a suitable station
2. Dividing the cross-section of the station into sections
3. Determining the area of each section and its average velocity
4. Summing the products of area and velocity to arrive at the stream discharge.

The process may be better understood by an examination of the sample notes of Fig. 441.

A station should be chosen where the channel is fairly straight, of uniform cross-section for a reasonable distance above and below the station, and where the bed material is relatively stable. Straightness of channel contributes to freedom from eddies or cross currents, and stable bed material avoids changes in cross-section resulting from scouring. Care should be taken to establish the station where changes in the flow of the stream will be quickly reflected in the change of the stage reading, thus insuring reasonable sensitiveness of the system, or, in other words, assuring that a change in discharge is accompanied by a change in stage.

This condition is most readily obtainable where some natural control exists just below the station such as a ledge across the stream or a steep rapids. Artificial controls are sometimes constructed for the express purpose of facilitating discharge



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(July 1935)

Reviewed 730-40
RCK

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES BRANCH
DISCHARGE MEASUREMENT NOTES

Meas No 76
Checked by VMW
Date 7-2-40

WABASH RIVER AT LAFAYETTE, INDIANA

Date May 2, 1940 Party V M Wann
Width 432 Area 4430 Vel 2.78 G H 835 Disch 12,300
Method 2-8 No secs 31 G H change +003 in 2½ hrs Susp 50C
Method coef 100 Hor angle coef 100 Meter No 1851 Susp coef 100

GAGE READINGS				
Time		Recorder	Inside	Outside
	Start			
	Finish			
Weighted M G H		—		—
G H correction			—	
Correct M G H				

Date rated 10-25-39 Used rating for rod susp Meter 0 5 ft above bottom of wt Tags checked
Spin before meas OK after OK
Meas plots % diff from rating
Wading cable, ice boat, upstr [✓ downstr, side bridge] 400 feet, mile above [✓ below] gage, and at Main St Bridge
Chain, wire found
changed to at
Correct length
Levels obtained

Measurement rated excellent (2%), good (5%), [✓ fair (8%)], poor (over 8%) based on following conditions Cross section Somewhat irregular & difficult to sound
Flow Fairly uniform Weather Raining
Other
Gage Operating satisfactorily
Record removed No Intake flushed No
Observer Not seen

Control

Remarks Main St Bridge seems to be very undesirable measuring section

Sheet No 1 of 4 sheets G H of zero flow ft

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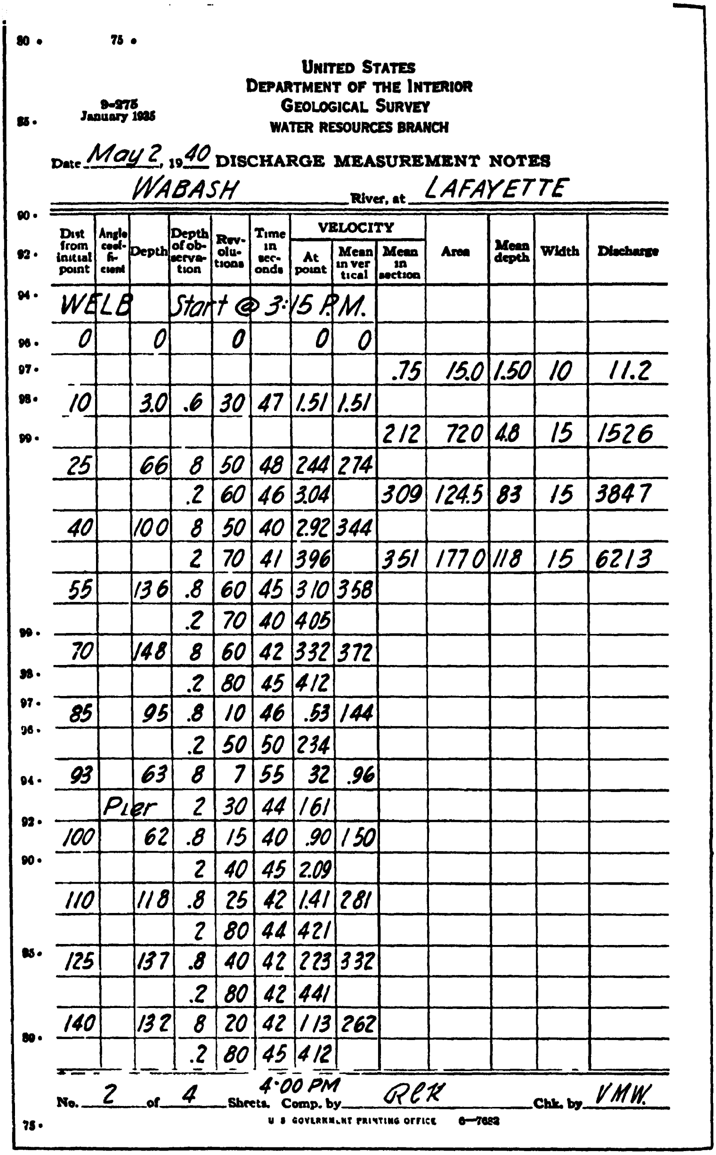
FIG 441 FIELD NOTES



9-275
January 1935

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES BRANCH

Date May 2, 1940 DISCHARGE MEASUREMENT NOTES

WABASH River, at LAFAYETTE

Dist from initial point	Angle coefficient	Depth	Depth of observation	Revolutions	Time in seconds	VELOCITY			Area	Mean depth	Width	Discharge
						At point	Mean in vertical	Mean in section				
WELB			Start @ 3:15 P.M.									
0		0		0		0	0					
								.75	15.0	1.50	10	11.2
10		3.0	.6	30	47	1.51	1.51					
								212	720	4.8	15	1526
25		66	8	50	48	244	274					
			.2	60	46	3.04		309	124.5	83	15	3847
40		100	8	50	40	2.92	344					
			2	70	41	396		351	1770	118	15	6213
55		136	.8	60	45	310	358					
			.2	70	40	405						
70		148	8	60	42	332	372					
			.2	80	45	412						
85		95	.8	10	46	.53	144					
			.2	50	50	234						
93		63	8	7	55	32	.96					
	Pier		2	30	44	161						
100		62	.8	15	40	.90	150					
			2	40	45	2.09						
110		118	.8	25	42	1.41	281					
			2	80	44	421						
125		137	.8	40	42	223	332					
			.2	80	42	441						
140		132	8	20	42	113	262					
			.2	80	45	412						

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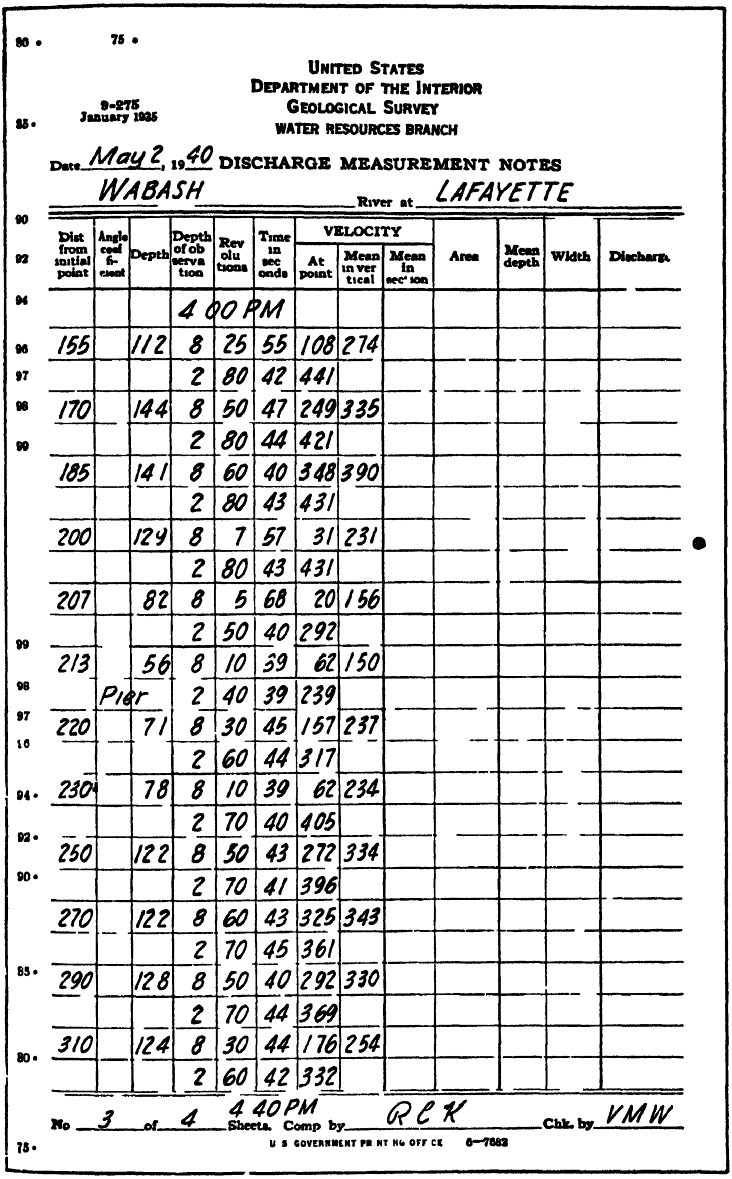
FIG. 441 (*continued*).



9-275
January 1935

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES BRANCH

Date May 2, 1940 DISCHARGE MEASUREMENT NOTES

WABASH River at LAFAYETTE

Dist from initial point	Angle coefficient	Depth	Depth of observation	Revolutions	Time in seconds	Velocity: At point	Velocity: Mean in vertical	Velocity: Mean in section	Area	Mean depth	Width	Discharge
			4 00 PM									
155		112	8	25	55	108	274					
			2	80	42	441						
170		144	8	50	47	249	335					
			2	80	44	421						
185		141	8	60	40	348	390					
			2	80	43	431						
200		129	8	7	57	31	231					
			2	80	43	431						
207		82	8	5	68	20	156					
			2	50	40	292						
213		56	8	10	39	62	150					
	Pier		2	40	39	239						
220		71	8	30	45	157	237					
			2	60	44	317						
230		78	8	10	39	62	234					
			2	70	40	405						
250		122	8	50	43	272	334					
			2	70	41	396						
270		122	8	60	43	325	343					
			2	70	45	361						
290		128	8	50	40	292	330					
			2	70	44	369						
310		124	8	30	44	176	254					
			2	60	42	332						

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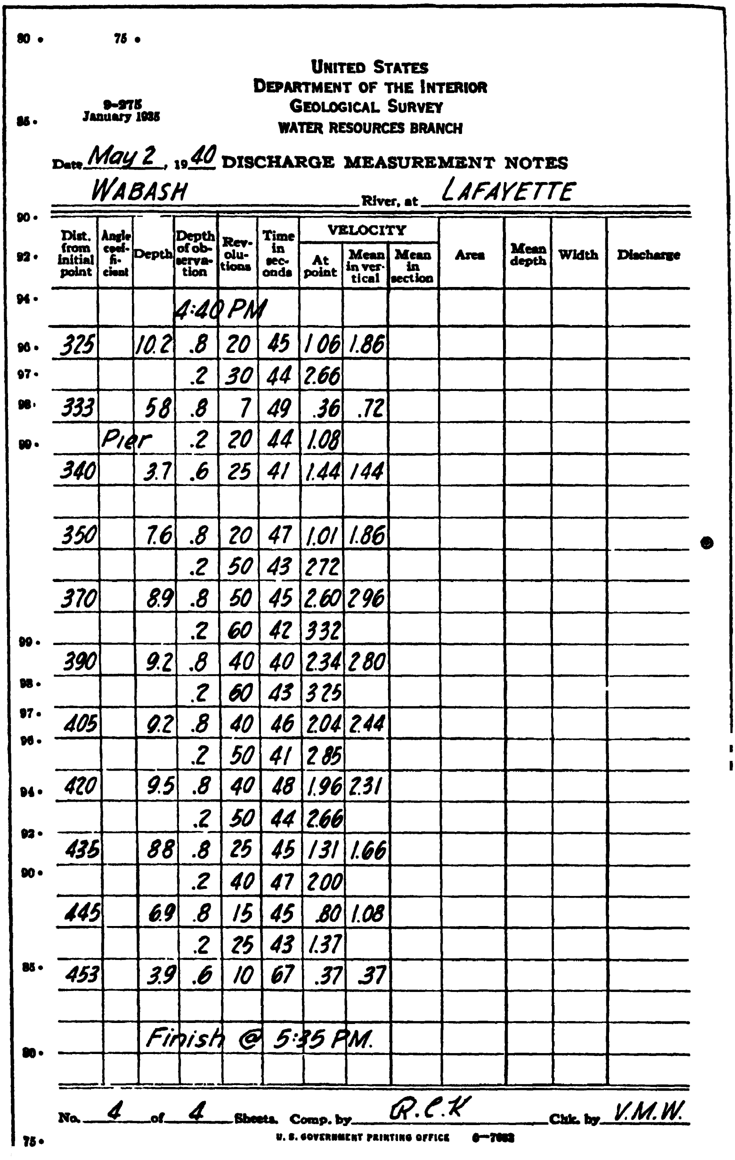
FIG. 441 (*continued*)



9-275
January 1935

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES BRANCH

Date May 2, 1940 DISCHARGE MEASUREMENT NOTES

WABASH River, at LAFAYETTE

Dist. from initial point	Angle coefficient	Depth	Depth of observation	Revolutions	Time in seconds	VELOCITY At point	VELOCITY Mean in vertical	VELOCITY Mean in section	Area	Mean depth	Width	Discharge
			4:40 PM									
325		10.2	.8	20	45	1 06	1.86					
			.2	30	44	2.66						
333		58	.8	7	49	.36	.72					
	Pier		.2	20	44	1.08						
340		3.7	.6	25	41	1.44	144					
350		7.6	.8	20	47	1.01	1.86					
			.2	50	43	272						
370		8.9	.8	50	45	2.60	296					
			.2	60	42	332						
390		9.2	.8	40	40	2.34	280					
			.2	60	43	325						
405		9.2	.8	40	46	2.04	2.44					
			.2	50	41	285						
420		9.5	.8	40	48	1.96	2.31					
			.2	50	44	2.66						
435		88	.8	25	45	131	1.66					
			.2	40	47	200						
445		6.9	.8	15	45	.80	1.08					
			.2	25	43	1.37						
453		3.9	.6	10	67	.37	37					
		Finish @ 5:35 PM.										

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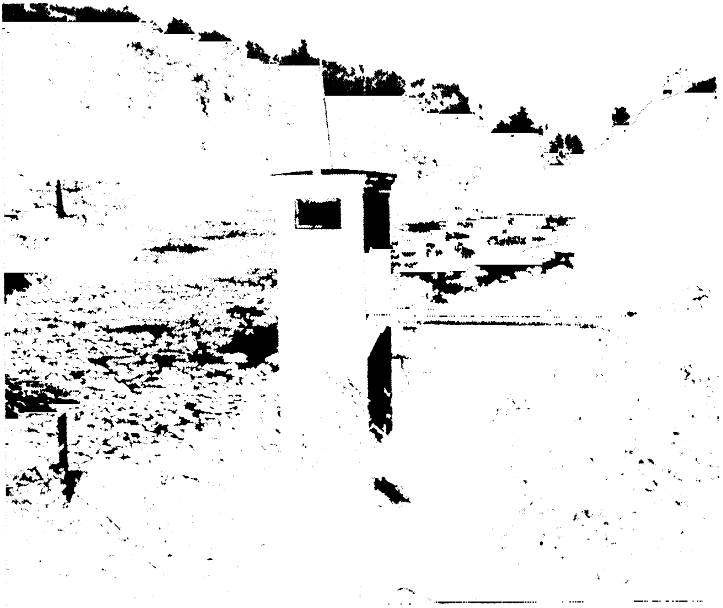
FIG. 441 (*concluded*).

measurements. Hydraulic structures having some other purpose often form a suitable control for a gaging station. The control serves to avoid conditions under which back water might increase the stage without affecting the stream discharge appreciably. A gaging station should not be located where it will be affected by back water from a downstream tributary subject to quick floods.



Courtesy of United States Geological Survey

FIG 441a. NATURAL CONTROL.

A station should be sectioned at right angles to the current and a suitable number of sections established as in Fig. 441c. Too few sections will result in an approximate value — too many in unnecessary labor or time consumption. It is desirable to keep the time consumed in making a single stream discharge measurement to a minimum in order to obviate the difficulty which arises when the stage materially changes during a set of readings. The width of the sections is usually uniform, but conditions at the station should govern and it may be found advisable to have the sections of unequal width. Ordinarily, flow in the largest section should comprise less than ten per cent of the discharge at the station.

As often as possible, two velocity readings should be made at each section, one at the 0.2 depth and one at the 0.8 depth, and the average used as the section velocity. Where this is not feasible, the velocity at the 0.6 depth should be read and used as the average velocity. In extreme cases the meter should be submerged for a surface velocity reading and 0.9 of this value considered average velocity. Variation from the above procedure will yield good results providing the operator uses discretion in the application of the velocity curve of Fig. 440.

Courtesy of United States Geological Survey

FIG. 441b. ARTIFICIAL CONTROL.

When a single stream discharge is obtained, it constitutes a valid point for plotting on the rating curve, stream discharge being the abscissa and stage reading the ordinate. A series of stream discharge measurements taken at various stages of the stream is necessary for the construction of the complete rating curve. The simple form has been shown in Fig. 438. The more complicated forms of rating curves are justified by the influence upon discharge of rising stage, falling stage, backwater, or other complications, but the discussion of them is omitted here. For simplicity, the relation of stage to discharge is sometimes set up in tabular form.

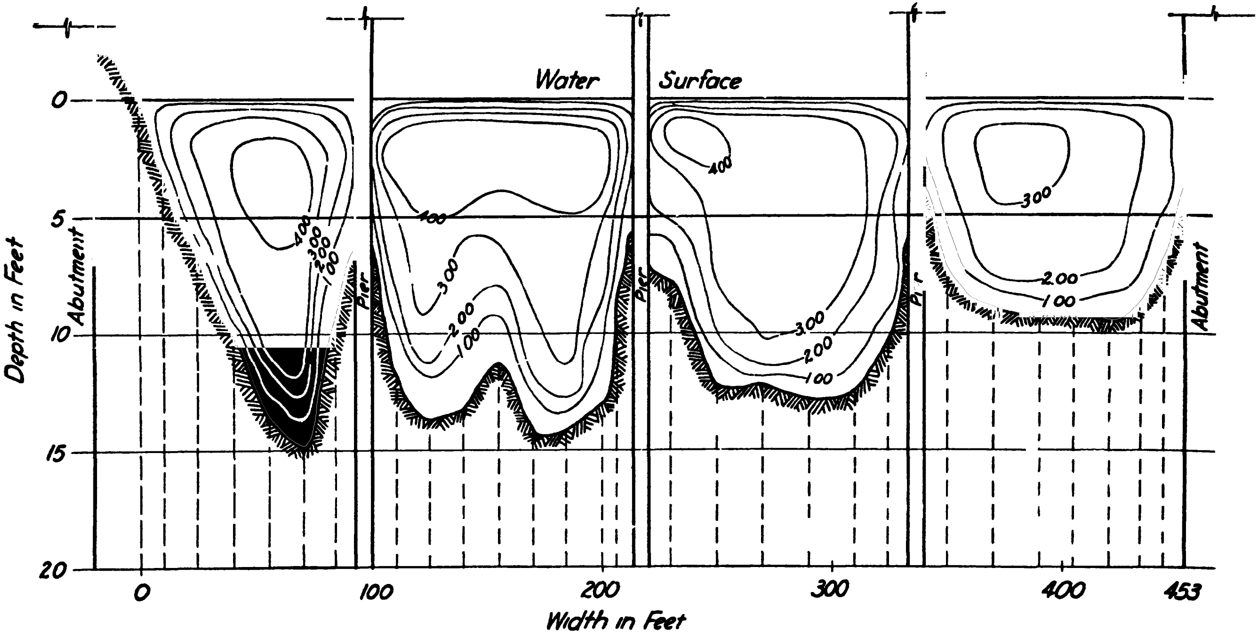


FIG 441c WABASH RIVER AT LAFAYETTE INDIANA

442. Stage Readings. Stage readings are usually expressed in feet and decimals. They may be read from a permanently mounted scale painted on a board where the level of the water may be observed hourly or daily, or a continuous record of stage may be obtained by the use of a water level recorder.

443. Effect of Changing Stage on Stream Discharge. When flow is uniform the stage indicates the stream discharge at a station. When the stage is changing the relation between stage and discharge is shown only by the more complicated form of rating curve mentioned in § 438. As the stage rapidly increases, the actual stream discharge is greater than the simple rating curve indicates and as the stage rapidly decreases, the discharge is less than the curve indicates.

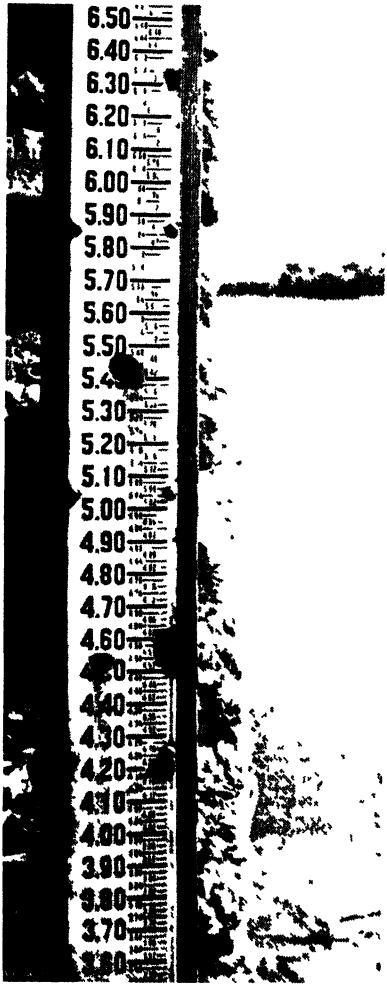
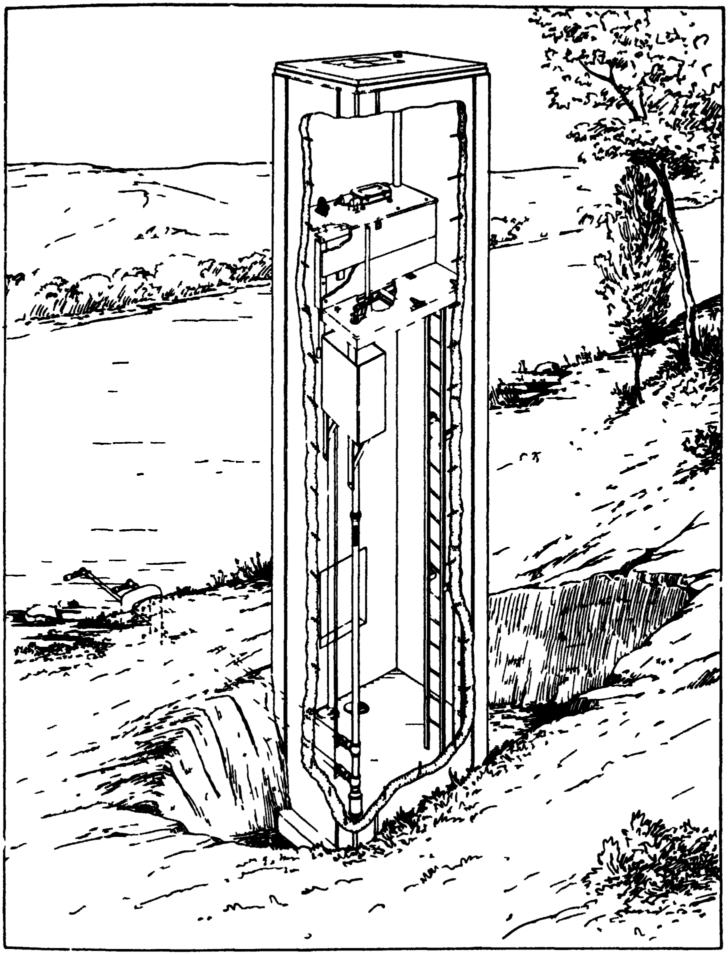


FIG. 442 RIVER GAGE.

444. Manipulation of Current Meter. The handling of the current meter in the field contributes in a great measure to the success of its application. It may be suspended from a bridge, cableway, or boat or through holes in ice; or it may be attached to a rod as the operator wades the stream. The essential precautions to be observed are the following:

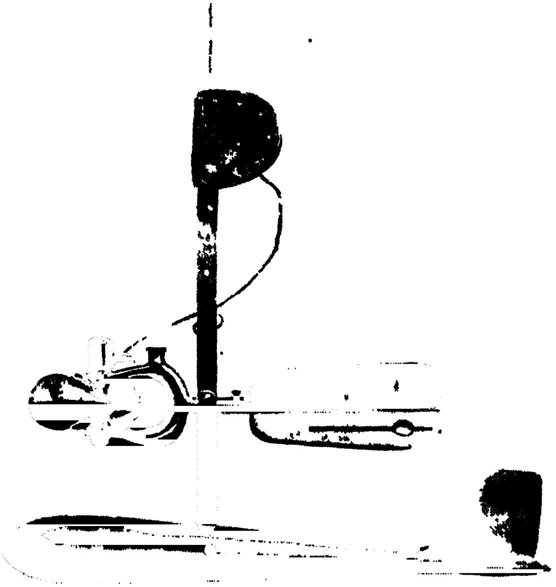


Courtesy of United States Geological Survey

FIG. 442a. TYPICAL RIVER MEASUREMENT STATION

1. The depth of submergence should be definitely known
2. The meter should be level and directed against the current
3. The effect of obstructions should be strictly avoided
4. When suspended by a cable, the meter should be properly weighted to make sure that the measurements are being made at the proper depth. This precaution should be carefully

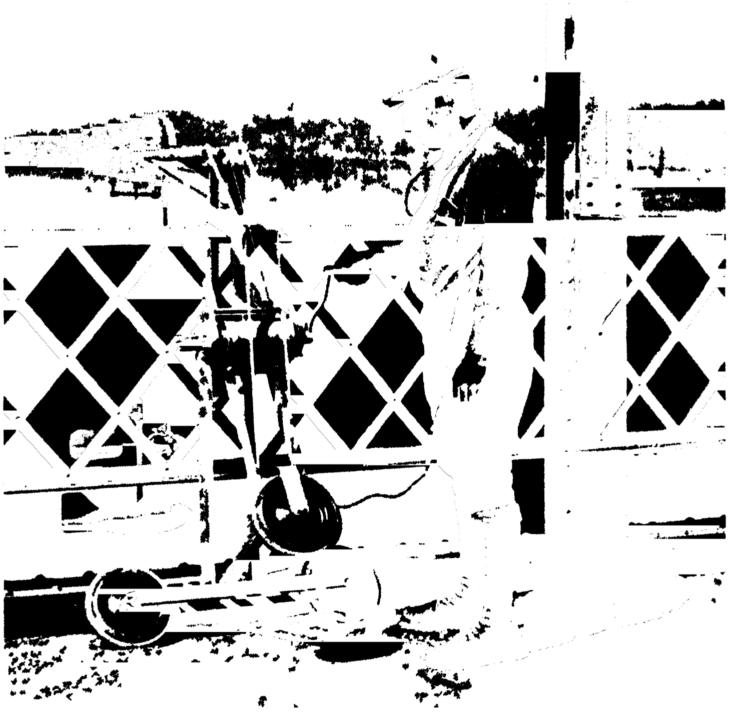
observed where the water is deep and the current swift. Where the weights essential to the accomplishment of these measurements become excessive, special tackle is used as in Figs. 444a and 444b.



Courtesy of United States Geological Survey

FIG. 444. PRICE CURRENT METER, TYPE A, WITH 15 POUND STREAMLINED WEIGHT.

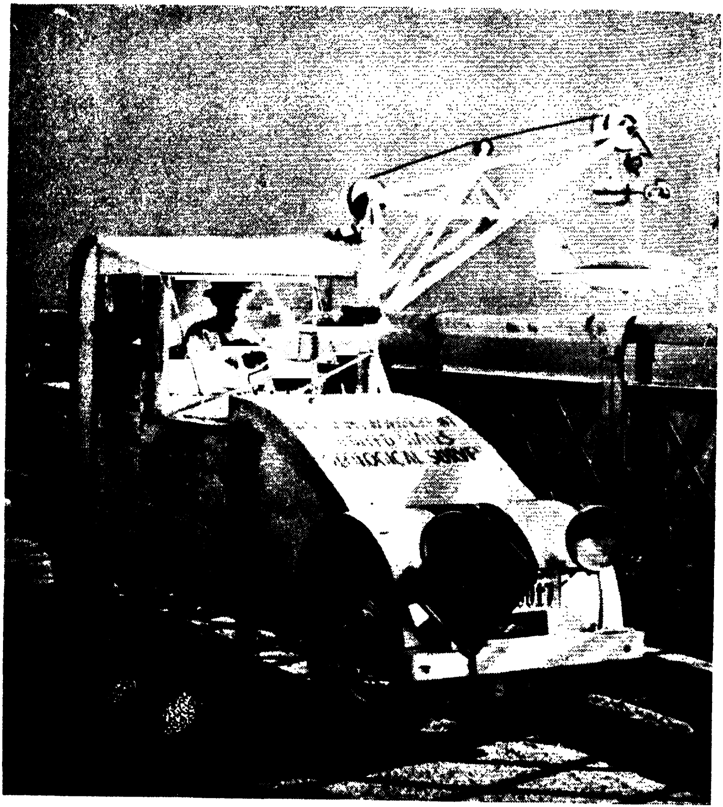
445. Floats for Velocity Determinations. Floats like those suggested in § 436 may be used to obtain the average velocity between two stations. The average velocity thus determined may be used in conjunction with the area to obtain the discharge. In this method of measurement a course is established and a sufficient number of trials made to establish the velocity in the various sections of the stream. The floats should be released far enough above the point at which the initial observation is made to insure that the float attains the speed of the



Courtesy of United States Geological Survey

FIG. 444. TYPE *A* BOOM AND REEL

current before its timing begins. In computing the velocity indicated by the time required for the float to traverse the course, allowance should be made for the depth-velocity relationship, bearing in mind the variation in velocity previously illustrated in Fig. 440. Float measurements seldom yield as reliable or accurate results as do current meter measurements, therefore extreme caution should be exercised in their use especially in the presence of wind. Float measurements are apt to be found more useful in artificial channels of uniform section than in natural channels of variable section.



Courtesy of United States Geological Survey

FIG. 444b. POWER-DRIVEN EQUIPMENT.

446. Use of Open Channel Formulas for Determining Average Velocity. When a reach of stream has a uniform cross section and a gentle slope, the average velocity is occasionally obtained by the application of an empirical formula. A common formula is that of Chezy,

$$v = C\sqrt{rs}$$

in which v is the average velocity in the channel in feet per second; r is the hydraulic radius in feet, obtained by dividing the cross section of the flowing stream by the wetted perimeter of the same; s is the slope of the water surface; and C is an empirical constant. A widely used formula for the computation of C is that of Robert Manning,

$$C = \frac{1.486}{n} r^{\frac{1}{6}}$$

A few values of n, the roughness coefficient, for use with Manning's formula for C, are shown in table 446.

TABLE 446.* VALUES OF n FOR FAIRLY STRAIGHT CHANNELS

Planed timber, glazed or enameled surfaces, smooth clean cement, in perfect order	0.010
Unplaned timber, newly and well laid brickwork, and moderately clean iron pipe	0.012
Smooth stonework, iron, ordinary brickwork and concrete and vitrified clay pipe	0.013
Smooth fine gravel or tuberculated iron	0.020
Corrugated metal	0.021
Earth in ordinary condition	0.025
Earth in rather poor condition with stones or weeds	0.030
Earth in poor condition about one-third full of vegetation	0.035
Channels in exceptionally bad condition, two-thirds full of vegetation, up to	0.050

In the application of the Chezy formula for determining average velocity, the area and hydraulic radius may be obtained by methods already discussed, n for use in computing C is chosen with the surrounding conditions as an argument, but s must be measured by methods not yet discussed. This is usually done by suspending hook gages in stilling boxes at the end points of the reach of the stream under consideration and referring the hook gage readings to a precise level circuit connecting the two hook gages. In some cases, four hook gages instead of two are employed, measurements being taken on both sides of the stream and the measurements averaged. Differences in interpretation of the various definitions of n result in a variation in the discharge value even though the other channel properties are well known. This leaves much to be desired in such measurements.

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* Based on the use of the Manning C with the Chezy formula as diagrammed by Mr. Fred C. Scobey, Bureau of Agricultural Engineering, U. S. Department of Agriculture.

447. Determination of Cross-Section at a Station. Accurate determination of the dimensions of the cross-section at a station is necessary in obtaining good results. An examination of the sample of notes, Fig. 441, reveals that the soundings for depth proceed hand in hand with the velocity measurements where the small Price current meter is used. Where other methods of velocity measurement are employed, a separate determination of area of cross-section is carefully made.

448. Use of Weirs and Flumes for Discharge Measurements. Conditions of channel occasionally lend themselves to the use of a weir as a means of measuring rate of flow. A weir is a notch in an artificial or natural barrier to the flow of a stream. It may be of various forms, but is usually rectangular or triangular. The relation of depth of flow over a weir to the discharge is a definite one for any particular case, but the improper application of the principles of weir flow to discharge measurements may lead to gross error. The best procedure to follow in applying the weir to stream discharge measurements is to determine the correct formula for use by a careful examination of reliable papers on this subject, making sure that the conditions surrounding the derivation of the empirical constants in the formula were the same as those now existing at the site of the intended measurements. Since it is likely that difficulty with floating debris and sediment will be encountered in natural streams, suitable measures should be taken to cope with such interference. The customary way of handling this difficulty is to station an attendant at the weir to keep it in perfect operating condition.

Where it is necessary to avoid accumulation of sediment or debris, the modified Venturi flume, commonly known as the Parshall flume, is increasingly used for slightly less precise measurements.

Water-Supply and Irrigation Paper No. 200 of the United States Geological Survey has been much used as a basis for discharge measurements by means of weirs. Paper No. 1711, Vol. 93, Transactions of the American Society of Civil Engineers, Precise Weir Measurements, by Schoder and Turner, extensively discusses the finer points of such measurements.

PROBLEM

From the notes shown in Fig. 441 plot the elevation view of the station at which the notes were taken and compute the discharge of the Wabash River for the complete set of notes.

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CHAPTER XXVII

LEGAL PRINCIPLES OF BOUNDARY SURVEYING

449. General. A complete treatment of the law of boundaries would occupy volumes, especially if court decisions were quoted, for the law of boundaries is "extremely complicated and characterized by endless exceptions." The most satisfactory references on this subject are given in the list at the end of this chapter.

This brief chapter must be confined largely to generalized statements true in most of the states and territories of the United States, but subject to variation in these fifty-odd jurisdictions. The boundary surveyor must familiarize himself with the law in the jurisdiction in which he practices.

The private surveyor * or engineer possesses no formal legal status other than that of expert witness, but actually and informally he must advise his client and conduct his surveys in accordance with accepted legal principles. In many cases, the surveyor must cooperate with an attorney both in the preparation of a case and in the negotiations or court proceedings connected therewith.

The actual surveying, that is, the making of measurements and calculations, might frequently be classified as sub-professional work. But when the surveyor so engaged is called upon to exercise legal knowledge and judgment in connection with surveys, the work takes on a professional character and in many cases becomes highly professional. As repeatedly mentioned in this chapter the surveyor has no formal judicial function, but from the moment he is approached by a client regarding a boundary survey his advice and action must be based on sound legal knowledge and professional judgment.

The interests of the client, the surveyor, and the community are best served by securing the facts, by studying the case, by definite solution from a survey, by settling the controversy through agreement, or by arbitration. Finally, if these fail, the

---

*** The word "surveyor" is commonly used in boundary surveying although the function is frequently performed by an engineer.**

dispute must be decided by court action which may entail a maximum of expense and hard feeling, frequently far beyond the value of the disputed land.

450. Review. The treatment in this chapter is predicated upon, and supplementary to, that given for *Land Surveys* in Chapter XII. Some of the legal principles there stated will be briefly summarized here, but it would be better to re-read the chapter on Land Surveys. Chapter XIV of Rubey's *Route Surveys* covers *Rights of Way* for engineering projects.

The engineer is interested in boundary surveying as such; as a part of construction projects and the preparation therefor of descriptions of land boundaries for deeds, mortgages, conveyances of rights of way, and other documents of title; and as an industrial executive who finds it necessary to conduct real estate transactions.

Information regarding surveys is secured from abstract and title insurance companies, from court decisions, from the official records of counties or cities, from survey records and monuments of original surveys made by the federal government or by private surveyors, from descriptions of the property in deeds or other legal documents, and from testimony of informed persons.

In order to secure legal precedence over subsequent claimants of interest in the land, legal documents such as deeds and mortgages are recorded by the owner with the county or city registrar or recorder of deeds or other designated officials, and are there available for public reference. Private abstract companies are prepared to give a complete abstract of all recorded documents affecting a given piece of property, in such form that the validity of the title may be passed upon by an attorney versed in land transfers. In some localities, in New England for example, one attorney commonly functions as abstractor, title examiner, and accumulator of abstract records. Title guarantee companies combine the functions of the abstractor and the attorney, and in addition insure the title of the property. Such insurance assumes no liability regarding the location of the property lines on the ground as determined by surveys, and as a consequence careful dealers in real estate require a survey as well as title insurance from the owner.

Every effort should be made to secure an agreement between interested property owners without recourse to the courts, and friendly relations should be fostered. Agreement between owners failing, only one agency can decide authoritatively the position of a boundary line, namely, a court of proper jurisdiction. *Special land courts* are in operation in California, Massachusetts, Minnesota, Ohio, Hawaii, and the Philippines. Some twelve other states have enacted statutes similar to those in the jurisdictions mentioned which are not yet functioning effectively. Claimants refer questions regarding land surveys and titles to these special land courts where a moderate fee is charged. Detailed surveys conforming to definite specifications are used by these courts whose decisions are seldom reversed.

It is held by the courts that in making resurveys the surveyor must attempt to re-establish the corners of the property in their *original locations on the ground.* It is not the function of the surveyor, or of any one else, to change the positions of corners by correcting original surveys.

One exception may be noted. In most states, exclusive, actual, continuous, visible, hostile, and notorious possession of land under claim of right or color of title, or the position of a land boundary, which has existed for a statutory period of ten or twenty years, gives title for the land to the adverse possessor under the legal principle of *adverse possession.* A somewhat similar doctrine of *acquiescence* is in effect in Iowa. These principles seldom apply against governmentally owned property. Where competent legal advice indicates that these principles do control a boundary, no time is to be spent in relocating the original corners except as they may be necessary in locating other boundaries not determinable by adverse possession. Even when the period of possession is less than the statutory one, it is frequently undesirable to disturb long established and generally accepted property lines.

Boundary re-surveys are something of a specialty which will best be left to those qualified for such work in a particular locality and other engineers should avoid all but the simplest land surveys. Perhaps a dozen reasons may be advanced for making this statement (see page 201), as, for example, that the engineer is legally liable for damages as explained later.

The proper descriptions, maps, and field notes must be secured; then the existing or easily identified corners found on the ground, using local witnesses. Only after a thorough search, including digging for corners not evident on the surface, and all possible testimony from local people, should the attempt be made to reestablish the corner by surveys. Following the surveys, a new search for the corner is sometimes successful since the probable location is then more definitely established.

A land description is to be interpreted as forming a closed survey. Natural and ascertained objects on the ground are normally given the greatest weight, then artificial monuments, then a statement of adjoining property (which in a sense is an independent designation of a boundary). As between distance and bearing of lines, that should be given the greater weight which was probably the more accurately measured. Errors in bearings or distances are usually proportioned over the lines intervening between known monuments. Area is given the least weight in determining land boundaries unless area is the essence of the deed or unless the error is large. In one case an error of eight per cent in area was given special cognizance.

451. Additional Legal Principles of Boundaries. Few engineers will attain a mastery of this extensive and complex phase of law although they may acquire knowledge of the principles generally applicable in the jurisdiction in which they are practicing. As a matter of fact, this is a specialized field of law in which thoroughly qualified attorneys are rare. However, "Ignorance of the law excuses no man," and this applies with special emphasis to the engineer engaged in boundary surveys.

The fundamental fact of boundary location is that a boundary is properly located only when it is in the position which a competent court of proper jurisdiction would approve.

The courts follow such general principles as ". . . to place ourselves as nearly as possible in the seats which were occupied by the parties at the time the instrument was executed," and "the cardinal rule for the interpretation of deeds and other written instruments is the expressed intention of the parties, gathered from all parts of the instrument, giving each word its due force, and read in the light of existing conditions and circumstances.

It is the intention effectively expressed, not merely surmised. This rule controls all others."

The surveyor's duty is to meet these general requirements in a specific case.

Where the true boundary is in dispute, and unknown, it may be located according to parol (verbal or informally written), or formally written, agreement by the interested adjoining property owners. It is desirable, although not always essential, that a formal written copy of the carefully drawn agreement, preferably under legal advice, be filed with the proper recording official. It is helpful if action is taken on the agreement, such as building on, or to, the line or otherwise occupying the property. In cases where the true boundary is later discovered, the owners only rarely may claim and occupy land to that true boundary. The general purpose of the courts is to approve and encourage such agreements regarding unknown and disputed boundaries. The competent surveyor is usually the best arbitrator in such cases, if an arbitrator is needed.

Under the principle of *Adverse Possession* or *Acquiescence*, property may under certain circumstances be acquired by long occupancy as stated on page 199, although corners so established have no weight in determining the location of other property corners.

Re-surveys usually are made to prevent or to settle controversies between property owners. As a matter of policy, the surveyor should attempt to secure agreement between the interested parties, either with or without the aid of such surveys as he has made. In this effort it may become advisable to refer one or both of the parties to the dispute to a qualified lawyer. Should such attempt fail and should the case go into court, then the surveyor enters simply as an expert witness to present evidence as to where, in his opinion, the corners should be relocated; and the final position of the relocated corners is determined by the court. It is thus evident that the surveyor, even though he be an employee of federal, state, county or city governments, has no authority to re-locate a corner himself. It has been stated that the surveyor in these cases is an expert witness rather than a judge.

Were space available, an entire chapter might well be devoted to the ethics, strategy, and techniques of the surveyor in his capacity as an expert witness.

Riparian property rights (those concerning land adjoining water courses) assume many forms, have been the subjects of extensive litigation, and require special study. They are largely statutory, and consequently vary in different jurisdictions. An extensive treatment may be consulted in Clark's *On Surveying and Boundaries*.

452. Subdivisions. In the matter of subdividing land into city lots or into small acreage tracts (see also Chapter XXVIII, *City Planning Surveys*), a well defined map of the entire tract and of the individual tracts and roads must be prepared on which the permanent monuments are carefully shown. All information necessary for a surveyor subsequently to retrace the lands must be shown accurately. Certificates must be made upon the plat; first, by the surveyor stating the work which he has done; second, by the owner stating the nature of his title to the property and dedicating the streets and alleys to the public; third, by some legal public officer such as a city engineer, county surveyor, or city plan commission approving the layout, particularly with regard to roads and alleys and other public or jointly owned property; and fourth, by a notary before whom the execution of the first and second certificates may be acknowledged. There are numerous other requirements under special circumstances and in certain localities.

453. Maps, Plats, and Field Notes. *Eleven Corpus Juris Secundum, Boundaries*, Sec. 113, page 709 covers the admissibility of plats and maps as evidence, as follows:

"As a general rule, maps or plats of surveys are admissible in evidence. These include maps or plats made by the county surveyor or other official surveyors; plats of unofficial surveys when made by a qualified surveyor; maps from the land office, although made subsequent to the beginning of the action; and maps made by order of the state land board. Likewise, plats and maps attached to and made a part of deeds or grants or referred to therein, either expressly or by clear implication, maps which have been recognized as correct by a former owner of the land, and ancient maps and plats when duly authenticated, are admissible to prove a disputed boundary. A map or plat referred to in the testimony is admissible as explanatory of it.

"In all cases the map or plat must be verified or authenticated, and in the case of a surveyor's plat proof offered that the surveyor making the same actually traced the lines on the ground as depicted."

Loose, miscellaneous memoranda have frequently been excluded in testimony. Field notes kept in regular bound field notebooks are most acceptable. While, so far as is known, no decision regarding loose leaf notes is available, it is probable that properly authenticated loose leaf notebooks would be admissible.

The field notes of a survey are usually kept by the surveyor as his personal property, although plats and reports based on the survey are given to the client.

454. Land Descriptions. Perhaps the most important of the boundary surveyor's duties is to furnish an adequate description of the land from which other surveyors may surely trace the boundaries and by which the land may be identified by land conveyancers for the purpose of tracing the title. Such descriptions are given on pages 204 and 219 of this volume, and on pages 208 and 209 of Rubey's *Route Surveys*. For a city lot, or for the similar case of a small acreage tract which is part of a formally filed plat, the description is a simple one, somewhat as follows:

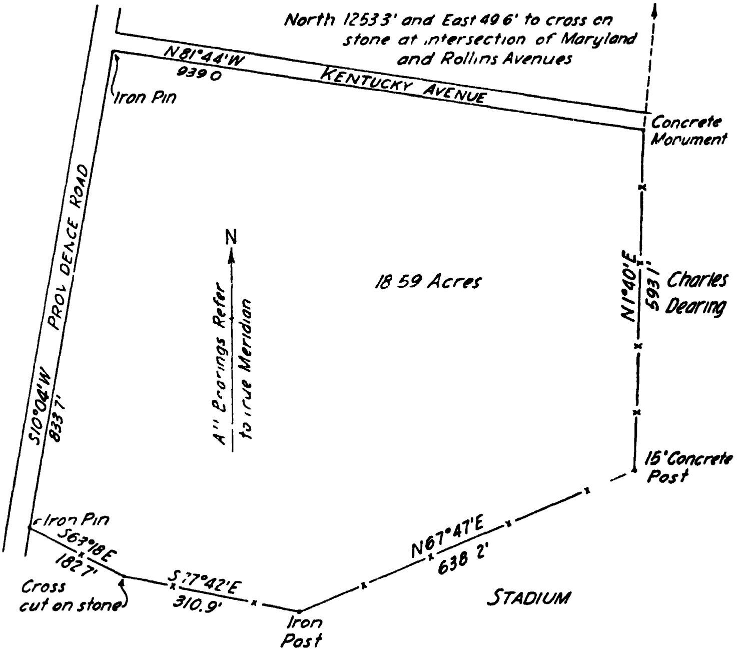
Lot 24, Block 60 of Smith's Third Addition to the City of St. Louis, (City Block 6274), as shown on the plat thereof recorded in Plan Book . . . on page . . . in (state place of filing).

In nearly all cases it is most desirable to refer in the description to "the accompanying plat (or map) which is made a part hereof." The plat should be complete, as shown in Fig. 226a, or in Figs. 454 and 455, and should be certified by the surveyor and recorded with the instrument in which the description occurs.

In such description as that given above for Lot 24, or for regular subdivisions of the United States Public Land Surveys (§ 184), it is usually satisfactory to refer to an already filed plat.

It is desirable to incorporate state systems of plane coordinates (§§ 191 and 458) in the description and plat.

PROPOSED ADDITION TO GOLF COURSE,
SOUTH OF COLUMBIA, MISSOURI



I, John Doe, hereby certify that I surveyed the property shown hereon on March 30, 1941, and that the plat is a true representation thereof

John Doe
Civil Engineer

Subscribed and sworn to before me this 2nd day of April, 1941

Richard Roe

NOTARY PUBLIC

FIG. 454. PLAT TO ACCOMPANY DEED.*

Professor Kissam† summarizes the requirements for a good land description as follows:

"The only permanent records that remain after a land surveyor's work is done are the descriptions that are recorded in the deeds. The marks that he sets are lost or destroyed; the field notes are filed in some attic; the laborious computations and careful measurements obtained with such effort are forgotten with time. The most important written evidence bearing on the location of property lines is the recorded descriptions. On them depends the ownership of the greater portion of the wealth of the country. They provide the most useful means of tracing titles, and their importance cannot be overestimated.

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* See Survey Data in Figs 253 and 253a, and on page 131 of Table X.

† *Proposed Standards for Land Descriptions* by Philip Kissam, Civil Engineering, November, 1910 pp. 696–697.

"As the responsibility for drafting descriptions is usually in its hands, the engineering profession might well develop standards for descriptions that could be used generally. Under present conditions title companies, real estate brokers, lawyers, and title examiners, all have their particular requirements which must be met and it becomes a difficult task to draft a description that will satisfy everyone and in addition be adequate for the land surveyor.

"The description has two functions and unless each is properly fulfilled the description is inadequate. It is natural that in reviewing a description the lawyer looks to the identification of the property. Proper identification and precise location are so much alike that frequently both seem complete when only one is fulfilled. On the other hand surveyors often inadvertently fail to provide identification. Hence the need of these tests:

"1. Does the standard description completely identify the property so that the title can be surely traced?

"2. Does it include all the necessary data to mark the boundaries on the ground?

"Six rules, which are here proposed for standard descriptions, ensure the fulfillment of these requirements and can be applied immediately with very little change in our present form of description.

"*Rule 1.* Each description in a recorded deed should bear the name of the person responsible for the description.

"*Rule 2.* All recorded descriptions should include the date when the person whose name appears upon it was assured of its accuracy.

"*Rule 3.* The source of the survey data upon which the description is based should be definitely stated.

"*Rule 4.* The identity of the property must be clearly stated. For example, 'Being the same tract (or part of the same tract) conveyed to A by B by warranty deed dated June 3, 1934, and recorded in Book 100, page 100, July 1, 1934.'

"*Rule 5.* The description should include survey ties to at least two durable monuments.

"*Rule 6.* All dimensions of property lines and all bearings shall be stated, and no 'more or less' distances shall be used. All dimensions shall be based on a closed survey which has been checked by latitudes and departures.

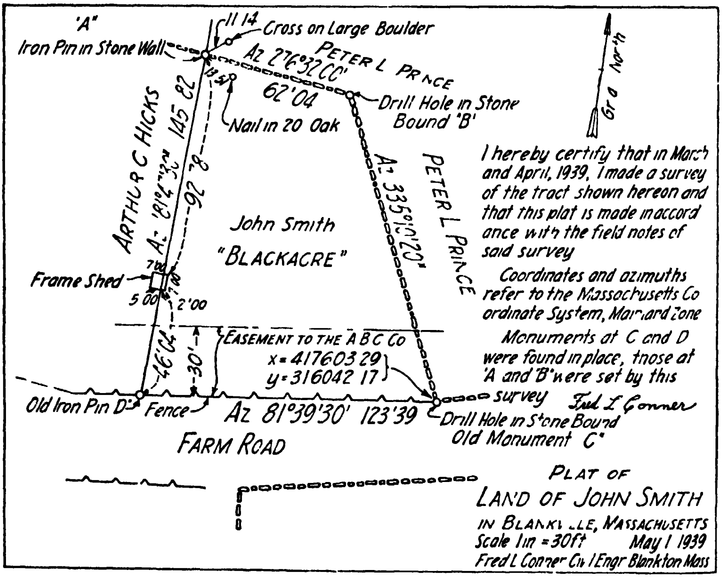
"These six rules can be carried out with little change in present methods of writing descriptions. In fact nearly all of them have been customary from time to time. They could be adopted at once without disrupting present practice."

455. Complete Description by Means of a Plat Only. Professor Kissam then continues by proposing that a plat only, without worded description, will meet all requirements for a good description:

"Consider for a moment what would be the ideal form for a description to take. Above everything else it should be expressed in such a way that it could be understood as easily by the layman as by the land surveyor. And yet how many persons can understand a land description as it is written today? No one, not even a land surveyor, attempts to do so. Every time a land description is used, a sketch is made of it. Why shouldn't a sketch be made at first, once and for all? I believe that a tracing on cloth should be substituted for the metes-and-bounds description. The original tracing should be attached to the deed and a lithograph copy on tracing cloth should be inserted in the book of record. This method has been used from time to time, with the modification that a metes-and-bounds description has also been included. In this case the metes-and-bounds must be compared with the tracing, an unnecessary waste of time. The question may be asked, 'Will such an arrangement be safe?' In Massachusetts it is now exactly the procedure for registered property.

"All the six rules mentioned would be included in the drawing. When the drawing includes references to the state coordinate system, no question can possibly exist as to the location of the property."

In such a procedure as recommended by Professor Kissam and as under consideration by others, such as the Joint Committee of the American Bar Association and the American Society of Civil Engineers, the only necessary description in a deed may be accomplished by referring to the land as "... *situated in the Town of ..., County of ..., and State of ..., shown upon a plat drawn by John Doe, Civil Engineer, dated ..., and filed (state here place of filing), to which plat* (as Fig. 455) *reference is hereby made for particular description.*" For rights of way, see Proceedings, Am. Soc. C. E., Nov. 1941.


'A"
Iron Pin in Stone Wall
11 14
Cross on Large Boulder
Az 276°32'00'
PETER L PRINCE
62'04
Drill Hole in Stone Bound 'B'
Nail in 20 Oak
13.51
Grid North
ARTHUR C HICKS
Az 181°4'32"
145 82
92 8
John Smith
"BLACKACRE"
Az 335°11'20"
PETER L PRINCE
Frame Shed
7 00
7 00
5 00
2'00
EASEMENT TO THE A B C Co
x = 417603 29
y = 316042 17
46'04
30'
Old Iron Pin D
Fence
Az 81°39'30' 123'39
Drill Hole in Stone Bound
Old Monument C"
FARM ROAD
I hereby certify that in March and April, 1939, I made a survey of the tract shown hereon and that this plat is made in accordance with the field notes of said survey
Coordinates and azimuths refer to the Massachusetts Coordinate System, Mainland Zone
Monuments at C and D were found in place, those at 'A and 'B" were set by this survey
Fred L Conner
PLAT OF
LAND OF JOHN SMITH
IN BLANKVILLE, MASSACHUSETTS
Scale 1in = 30ft May 1 1939
Fred L Conner Civ l Engr Blankton Mass

Courtesy of American Society of Civil Engineers

FIG. 455 COMPLETE DESCRIPTION BY PLAT ONLY.

456. General Remarks. The owner's attention should be called to the desirability of allotting sufficient funds to permit properly monumenting the survey on the ground, so that future boundary disputes may be minimized or eliminated.

After all is said, let it be understood that as a practical matter perhaps most boundary questions are actually settled by surveyors. A competent surveyor with a good legal understanding is really the best qualified person, and he is generally so recognized, even though his "judicial function" is exercised by common consent rather than by legal right.

457. The Surveyor's Legal Rights and Responsibilities in Boundary Surveys. In most of the states, those persons making land surveys must be licensed by a state examining board. These licensed surveyors will be found to have something more than a casual knowledge.

There is a general legal ruling that a professional man must exercise that degree of care which a skilled man of his profession, using ordinary prudence, would exercise under like circumstances; and this applies to those making boundary surveys. "For failure to exercise that degree of care he would be liable in damages to the extent of the injuries, to those who employed him."

The surveyor will so adapt his surveying procedure and methods as to give the required precision for the particular land in question, ranging from very precise work of 1/20,000 or better for valuable property to work of low precision such as 1 5000 for less valuable property. Owners are frequently unwilling to pay enough to secure good work and the engineer should avoid such assignments since their poor quality will remain to plague him long after their low cost is forgotten.

The surveyor may testify as an expert in most matters connected with his work such as stating his opinion as to whether a certain pile of stone or a tree indicates a monument, the age of the monument, the meaning of certain lines on a map, mistakes on the part of a conveyancer in copying field notes and other cases where "the court applied the ordinary rule of allowing parol evidence to explain a latent ambiguity and the surveyor to testify as an expert, applying his professional skill and knowledge in giving his answer."

In the matter of trespass while engaged in surveying, no trespass occurs if the surveyor is carrying out in a reasonable manner an undertaking directed and authorized by the legislature, and "the entry is reasonably necessary, . . . temporary, . . . and accompanied by no unnecessary damage." The right of eminent domain usually carries with it the right to make surveys on private property. Actual damage must be paid for, and excessive or unnecessary damage may be enjoined.

In the absence of such an official mission, the surveyor is a trespasser even though no damage is done. In some states statutes have been enacted recently giving the surveyor the right to make surveys on private property. Usually tact and diplomacy will secure the owner's consent.

The law pertaining to libelous things written or slanderous remarks made of the surveyor in his professional capacity is practically the same as in other professions. "It is not necessary to descend to vulgar abuses or to make specific charges of crime in order to expose one to hatred, contempt, or ridicule, or to injure him in his business or occupation." If the language used concerning the surveyor is untruthful and tends to injure him in his reputation as a surveyor, it is actionable libel or slander.

458. State Systems of Plane Coordinates. These systems have been worked out for the various states of the Union by the United States Coast and Geodetic Survey and adopted by statute in several states. The main purpose of such state systems is to provide many permanent and inter-connected monuments at frequent intervals throughout the state to which property surveys and other surveys may be tied by direction and distance in such a manner that they may forever be traced regardless of whether the corners of the property itself or other survey points become lost or obliterated. The following quotation * expresses rather definitely the purpose of such state systems.

"The Committee finds further that, in order to provide proper reference marks for fixing position and direction, a system of survey monuments should be established, connected by properly controlled precise surveys, and that the positions of these monuments, so determined, should be expressed in terms of a coordinate system, preferably by plane rectangular coordinates. Descriptions of the monuments, together with their coordinates, must be recorded by some public authority and the record made available for public use.

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* Land Surveys and Titles, First Progress Report of the Joint Committee of the Real Property Division, American Bar Association, and the Surveying and Mapping Division, American Society of Civil Engineers, Proceedings of the American Society of Civil Engineers, November 1938, page 1879.

"The United States Coast and Geodetic Survey has developed a plan for the establishment of such inter-related monuments and systems of plane coordinates for defining positions. This plan has already been put in operation in several states and has won the immediate approval of practicing land surveyors and title examiners. Many thousands of monuments have been set and their coordinates have been determined. The plan is known as the 'State System of Plane Coordinates.' It has been thoroughly studied by the members of the Committee, and it is believed to offer an adequate remedy for the deficiencies described. The utility of the plan extends to so many fields other than that of land surveys and descriptions that the cost to the taxpayer for its adoption will be returned manyfold. It provides a basis for checking and controlling instrumental and aerial surveys for maps and plans required for all engineering development. The time and cost of establishing special control for each project are thus obviated. Furthermore, all existing data can be compiled, and boundary lines can be correlated without field determination. . . .

"Since an existing net of triangulation stations* extends over the entire United States, it is necessary, due to the curvature of the earth, to express the positions of the stations in terms of a spherical coordinate system; that is, by precise latitude and longitude. As it is unnecessary to consider the curvature of the earth in the reduction of most survey data, the mathematics of this procedure is unfamiliar to all but a few specialists. It is clear then, in order to make them generally usable, that some form of plane rectangular coordinates must also be used to express the position of these stations. The United States Coast and Geodetic Survey has developed a plane coordinate system for every State, according to which the positions of the triangulation stations may be expressed. In the larger States several zones have been introduced, each zone having a separate origin. The zones are made as large as possible without introducing appreciable differences, caused by earth curvature, between measurements on the ground and distances as expressed by the coordinates. The variations thus introduced are so slight that they cannot be discovered by ordinary survey methods.

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* The net of the United States Coast and Geodetic Survey, described in §§ 161 and 424.

"It is planned that monuments will be established in convenient locations along highways and streets, and precise surveys will be made connecting these monuments with triangulation stations. The coordinates of the monuments can be determined by using the plane coordinates of the triangulation stations. In many States this work is well under way, and many municipalities are extending the monument system throughout their streets. By this plan the position of every monument in the system can be determined with relation to that of every other monument, so that it is inconceivable that the position of the system should ever be lost.

"To utilize the system a land surveyor must make a proper survey connection with two monuments. He can then either state the plane coordinates of the property corners in his description, or give the bearings and distances of two or more property corners from the monuments. With these data in the description there can be no question as to the precise position of the property. Its location is described accurately and permanently. Moreover, it is possible to determine, by examination of descriptions of this kind, whether all the parcels lotted in a certain tract will, in fact, fit into the tract.

"When once the title examiner has become familiar with such a system he can determine, without a field survey . . ., whether the requisite land exists for the title he is examining. Surveys are thus coordinated, and, in fact, made part of one great survey which is all-inclusive.

"There are many applications of the system to engineering and mapping. It has proved of extreme value for tax maps and has been used considerably for that purpose.

"It is desirable that each State legislature enact enabling laws defining the system used in the State and naming it so that any coordinates used can be referred to by name. A model act has been prepared and an example of it is attached hereto as an Appendix. Certain obvious changes are necessary, of course, to adapt the act to other States.

"It is also desirable that a bureau of surveying and mapping be established in each State to administer the coordinate system. Such a bureau should be the depository of control-survey data, should disseminate them to the public, and should maintain the monuments. The bureau should be equipped to aid counties, municipalities, and other political subdivisions, to extend the monumentation along the roads and streets. By proper supervision of the work and by careful checking of the results, the bureau would be able to include these surveys in the State systems.

"The Committee recommends, therefore, that each State legislature enact an enabling law as described herein and designate by law the establishment of a bureau of surveys either independent of existing State Departments or as a division of an existing department. The necessary appropriations should be made at the same time to start the bureau, to be increased as the work of the bureau results in savings in survey costs and litigation."

Figure 455 shows a plat referenced to state coordinates and Figs. 421 and 424a show typical state coordinate axes.

The State systems of plane coordinates are established, as described in § 421, in such a manner that no discrepancy greater than one part in 10,000 exists between the distance measured on the ground between any two points and the corresponding distance computed from the coordinates of the two points. In most localities the discrepancy is very much less than 1/10,000.

Special Publication Number 235, 1945, *State Coordinate Systems*, of the United States Coast and Geodetic Survey is a manual in which such systems are described and procedures are explained and illustrated for the use of surveyors.

459. Retracing the United States Public Land Surveys. In connection with the retracing of the original surveys, which is covered in Chapter XII, it is important for the surveyor to know the instructions in effect with reference to the original survey of the particular area with which his present survey is concerned. Often these were contained in letters, were not published, and are not easily obtained. They have been changed from time to time. The current instructions are contained in the "Manual of Instructions for the Survey of Public Lands of the United States," 1947 (Reference 4). Many of the older instructions are available in Reference 21.

Clark, Patton, Skelton, and Wait (References 6, 17, 20, and 23) cover the laws of boundaries and surveying rather fully. Clark's book is the latest and perhaps the best single reference.

Attention is called to the pamphlet *Restoration of Lost and Obliterated Corners and Subdivision of Sections* (Reference 22). This is a compendium of rules applicable to, and within the area of, the United States Rectangular Surveys. Long accepted general rules are there summarized as follows:

"*First.* That the boundaries of the public lands, when approved and accepted, are unchangeable.

"*Second.* That the original township, section, and quarter-section corners must stand as the true corners which they were intended to represent, whether in the place shown by the field notes or not.

"*Third.* That quarter-quarter-section corners not established in the original survey shall be placed on the line connecting the section and quarter-section corners, and midway between them, except on the last half mile of section lines closing on the north and west boundaries of the township, or on the lines between fractional or irregular sections.

"*Fourth.* That the center lines of a section are to be straight, running from the quarter-section corner on one boundary to the corresponding corner on the opposite boundary.

"*Fifth.* That in a fractional section where no opposite corresponding quarter-section corner has been or can be established, the center line must be run from the proper quarter-section corner as nearly in a cardinal direction to the meander line, reservation, or other boundary of such fractional section, as due parallelism with the section boundaries will permit.

"*Sixth.* That lost or obliterated corners are to be restored to their original locations whenever it is possible to do so."

The book by Hodgman, and Johnson and Smith's Appendix A (References 8 and 13) are old, but no better brief treatments for the retracing of public land surveys have been published; and they are well worth reading.

460. Historical Incident. These matters were under discussion in 1859 when a convention of surveyors of Illinois called upon Abraham Lincoln, for the reason that he was a practical land surveyor as well as a good lawyer, and secured from him the following written opinion—interesting historically as well as because of its sound position on a then debated phase of boundary laws.

"The 11th section of the Act of Congress, Approved Feb. 11, 1805, prescribing rules for the subdivision of sections of land within the United States system of surveys, standing unrepealed, in my opinion, is binding on the respective purchasers of different parts of the same section, and furnishes the true rule for surveyors in establishing lines between them. That law, being in force at the time that each became a purchaser, becomes a condition of the purchase. And by that law, I think the true rule for dividing into quarters, any interior section, or section which is not fractional, is to run straight lines through the section from the opposite quarter section corners, fixing the point where such straight lines cross, or intersect each other, as the middle, or center of the section.

"Nearly, perhaps quite, all the original surveys are to some extent erroneous, and in some of the sections, quite so. In each of the latter, it is obvious that a more equitable mode of division than the above might be adopted; but as error is infinitely various, perhaps no better single rule can be prescribed.

"At all events, I think the above has been prescribed by a competent authority.

"Springfield, Jany. 6, 1859.

A. Lincoln"

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20. Stewart, *Public Land Surveys*, Collegiate Press, Ames, Iowa, 1935. This is concerned mainly with the history, instructions (both original and subsequent), and methods as particularly applicable to Iowa and adjoining territory.
21. U. S. Land Office pamphlet, *Restoration of Lost and Obliterated Corners and Subdivision of Sections*, 1939, Superintendent of Documents, Washington, D. C.

CHAPTER XXVIII

CITY PLANNING SURVEYS

461. Introduction. The tendency of government at all levels toward the solution of community problems by applying the principles of planning has become increasingly apparent in recent years. The fact that many of these problems, especially those of the smaller communities, are primarily of a physical nature and that, therefore, the engineer has a definite professional responsibility in their solution emphasizes the need for an understanding of certain phases of planning technique.

The more important fundamental problems of city planning deal with transportation, streets, recreational areas, public buildings and structures, public utilities, and land use. That these physical problems are related directly to the social and economic well-being of every individual in a community is an easily demonstrated fact. For example, because of the great increase in the use of the automobile, the traffic congestion problem in every modern city has become acute. Streets and highways are more dangerous than battlefields as evidenced by the fact that more than 32,000 persons were killed and more than 1,100,000 were injured in traffic accidents in 1948.

The automobile is also credited with the present-day tendency toward decentralization of business which has resulted in the so-called "decay" of central business districts and a consequent loss of tax revenue by the city. In the older residence districts, decreased property values, resulting from migrations to outlying suburban districts and lack of proper zoning regulations, have caused "blighted" districts. Such districts are a definite economic burden upon the city and cause expenditures for city services such as fire, police, and health protection far in excess of the tax income from these same areas. For example, it is said that in St. Louis city services in such districts are two and one-half times as much as the taxes received therefrom, and cause an annual deficit of five million dollars. In Indianapolis also, the results of a survey conducted by the Chamber of Com-

merce indicated that two districts of the city inhabited by bu
10 per cent of the population required an expenditure of 26 pe
cent of the amount devoted to the whole city for social services

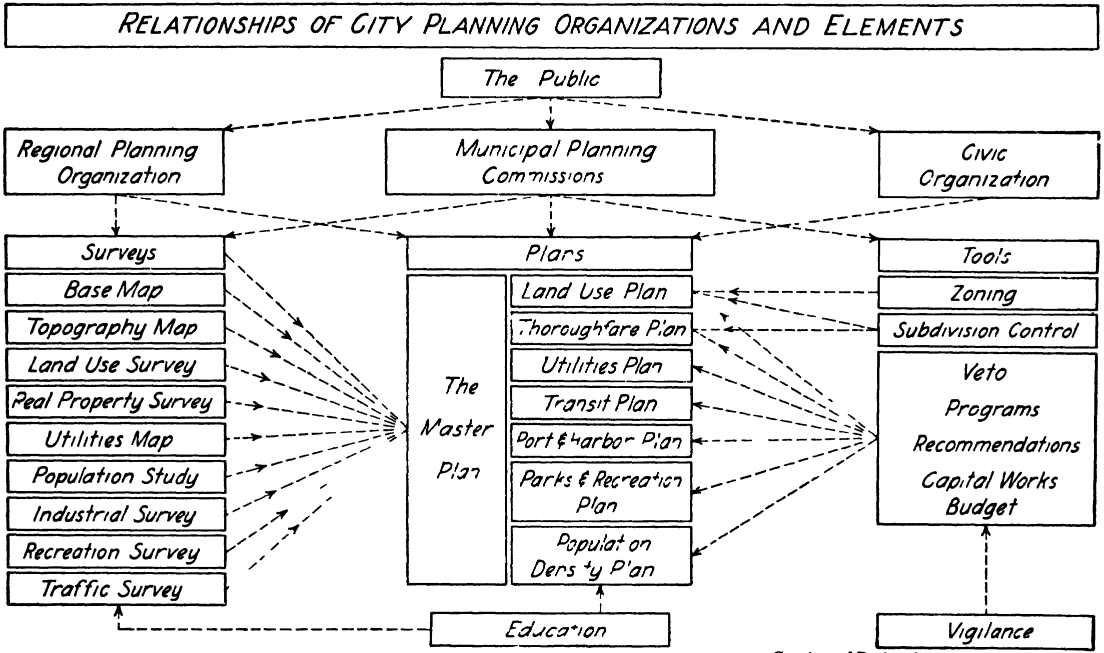
The tendency toward a shorter work-week which results in more leisure time for workmen is another economic and social problem. This forces recognition of the need for more recreational areas in or near the city.

The engineer must play his part in this movement. In most states, the city engineer is, by law, a member of his local plan commission and as such he should utilize this opportunity to discharge his responsibility as a citizen and as an engineer.

It must be remembered that the Plan Commission serves as an advisory body to the administrative officers. On request or on their own initiative the commissioners study conditions, formulate policies, draw plans, and prepare programs. The citizens of the community through their elected and appointed officers either accept or reject the plans, adopt the policies, and adhere to the programs.

462. Planning Agencies. The first American city plan commission was organized at Hartford, Connecticut, in 1907. For the following fifteen years the movement was not very rapid but since 1920 many cities have adopted the principle until at the present time more than one thousand city plan commissions have been formed. In addition to these, five hundred counties, forty-seven states, and many regions such as New England and the Pacific Northwest are also engaged in planning activities, placing special emphasis on those problems which are of county-wide, state-wide, or regional importance and at the same time are attempting to correlate the plans of the smaller governmental units within their boundaries.

From 1933 to 1943 a National Resources Planning Board served as the Federal Planning agency. Although organized primarily to study planning problems affecting the nation as a whole, the Board served also as a coordinator of the work of the State Planning Commissions. Since 1943 these State Boards have continued their studies of those problems which are of state-wide importance and have served as advisors to local planning boards in counties, cities and towns.



Courtesy of Regional Association of Cleveland

FIG. 462. AN ANALYSIS OF THE PLANNING FUNCTION.

In general, planning studies made for the smaller units such as cities and towns emphasize the physical conditions surrounding the problem while those for regions, states, and nation consider also the social and economic aspects.

463. The Civic Survey. One of the first steps taken in a community planning or replanning study is a complete inventory of the physical conditions which exist in that community. Because of its daily contacts with the physical city the engineering department serves as the supervising agent for this inventory. The head of that department is the planning engineer and as such he must prepare maps and collect data which will enable the other members of the commission to understand the facts concerning the civic structure. This is particularly true in small cities. In larger communities, the planning function may be delegated either to a planning staff employed by the planning commission or to planning engineers assigned to the city engineering department.

Many of the surveys are applicable only to planning studies while others involve the general principles and practices of plane and topographic surveying, the results of which are of value to all departments of the city government in addition to their special use as base maps for planning.

A survey of physical conditions for a city planning study should result in the following:

A map of the city, reliably accurate in its alignment of streets, alleys, transportation lines, etc. and also in its arrangement of property lines.

A land-use map upon which the present use of each lot in the city is indicated by color or conventional sign.

A traffic flow map, which shows the volume of traffic flowing through the streets of the city.

Detail maps of particularly important places in the city such as hazardous, complicated street intersections or railway crossings at grade.

464. City Maps. Most city maps, especially those of the smaller and average-size cities, are definitely not to be relied upon for accuracy. Many of these have been developed from an accumulation of miscellaneous surveys and records of varying degrees of accuracy, and have been made by different individuals. Some of these individuals were well qualified while others had little appreciation of appropriate surveying and mapping technique.

As a consequence, mistakes in mapping and errors in recording made by one city engineer and accepted without checking by others have resulted in the existing unsatisfactory and unreliable maps. The only remedy for this condition is an accurate resurvey followed by competent calculations and mapping. Such a solution involves additional items in the budget request of the city engineering department and it may be difficult to convince the members of a city council budget committee that the expenditure is justified.

An excellent argument in support of this additional activity on the part of the city engineer is presented in "Technical Procedure for City Surveys," a manual published by the Surveying and Mapping Division of the American Society of Civil Engineers in 1934. The following is a quotation from that manual:

"The City Survey is an inventory of the physical facts relating to the land and its occupation. Its use is essential in operating existing facilities and in planning extensions thereto, as conditions of growth and progress may require. It is usually initiated either as a necessary basis for some particular improvement project, such as large extensions to sewerage and water systems, or as the first step in a broad planning program. Since it is of permanent value it is properly a capital expense, frequently financed with bond funds set up for the creation of permanent improvements. Whether financed by this method or by appropriations from general revenue, the expense is not great when considered in relation to the benefits derived. The cost of the entire City Survey (except that of the Property Survey, which necessarily varies widely) usually ranges from 0.1 per cent to 0.2 per cent of the aggregate cost of providing the area with water, sewers, and passable roads. This comparative estimate takes no account of the value of the land itself or of the many other public and private improvements and structures that will be built upon it.

"In any consideration of costs it should be emphasized that the City Survey is only a part of the city's engineering expense, and that even total engineering costs are never, in themselves, significant. Planning upon paper is infinitely preferable to building on a basis of information so scanty that the structure will not satisfy the need for which it was designed. Such building is, in fact, experimental, for the structure that does not fulfill its purpose will ultimately be replaced by one that does. What counts is the total building cost and the gain or loss in public welfare that results from the success or failure of the project. Money saved in engineering, or planning, is frequently lost, many times over, in ill-advised construction.

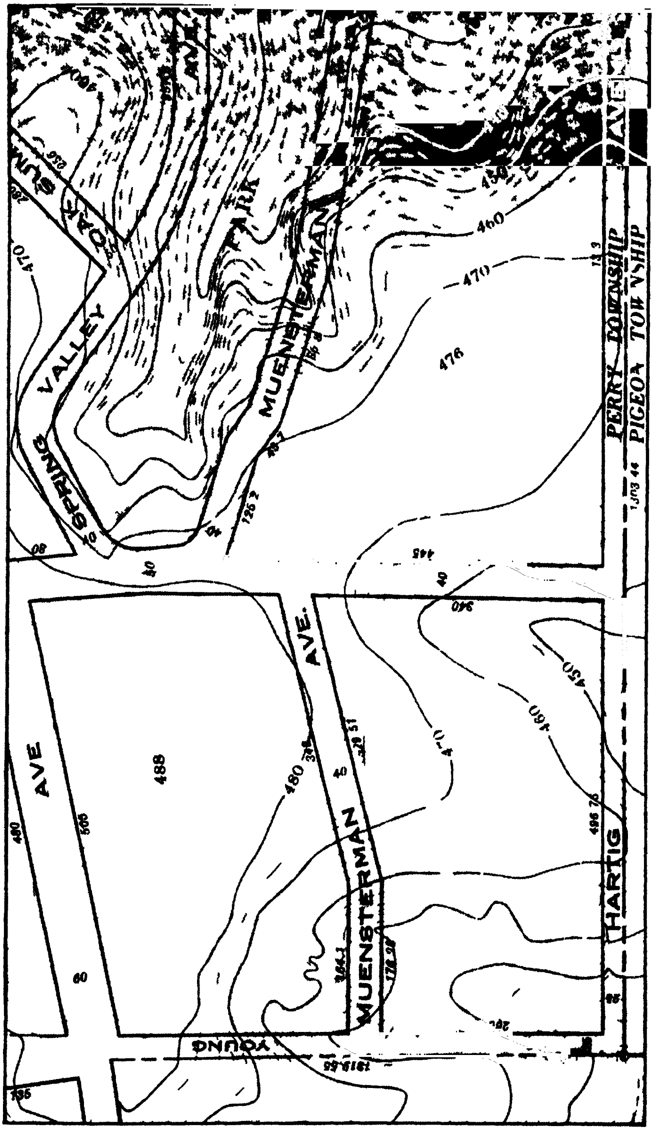
"While the value of a broad City Survey is obvious to the engineer, lay commissioners and disbursing authorities frequently are not sufficiently informed as to its benefits. For this reason it is good policy, in some instances, to start the survey in a small way. With a relatively small appropriation some definite, visible accomplishment can be made, preferably in an area where information is vitally and immediately needed. After this, the common sense worth of the investment becomes speedily apparent to all concerned."

465. The Survey. As in all surveys for maps which delineate many physical features, the detailed city survey is best scheduled as follows:

Horizontal control by triangulation and traverse,
Vertical control by leveling, and
Topographic details by aerial photography, by plane table, or by transit-stadia.

The manual issued by the American Society of Civil Engineers recommends that control for the survey be of first order accuracy; that the map scale be 200 feet to the inch; that plane table traverse closures shall not exceed 1 in 500; that plane table level closures shall not exceed one-fourth the contour interval; that map elevations obtained by stadia shall be correct to within one-half the contour interval; and that the error in the plotted position of any point shall not exceed 0.01 inch.

In the 92 cities with populations of more than 100,000 (see Table 465), property values, availability of qualified personnel, and availability of funds for the purchase of the necessary



Courtesy of Evansville City Plan Commission

FIG 464 PART OF EVANSVILLE INDIANA, TOPOGRAPHIC MAP SCALE 1' = 200'

precision equipment justify the accuracy suggested in the manual.

TABLE 465 NUMBER OF CITIES IN THE UNITED STATES BY SIZE GROUPS

Population Group	Number
2,500 to 10,000	2387
10,000 to 25,000	665
25,000 to 100,000	320
100,000 to 250,000	55
250,000 to 500,000	23
500,000 to 1,000,000	9
1,000,000 and over	5
Total	3464

In the other 3372 smaller cities, however, reliance must be placed upon ordinary field equipment and upon engineers with little or no training in precise surveying. Under these conditions, traverse control of second or even third order accuracy can be employed to advantage if the necessary precautionary measures are taken in the field. Second order accuracy is discussed in Chapter XX. For third order accuracy:

All equipment must be in good condition. Transits and levels must be in good adjustment, and particular emphasis must be placed upon making the vertical axis truly vertical at every set-up of the transit.

Direct horizontal measurements must be made with a standardized tape and plumb-bobs and these measurements must be corrected for the average temperature.

Values of all angles must be determined by double-sighting with the least count of the vernier as the maximum allowable variation between one-half the doubled angle and a single value of the same angle.

Latitude and departure computations of closed traverses must yield errors of closure of not more than 1 in 5000.

Vertical control differential levels must be run in circuits and close with errors not greater than ± 0.05 ft. $\sqrt{M}$, where M is the length of the circuit in miles. Careful work in the field will yield ± 0.04 ft. $\sqrt{M}$ or even ± 0.035 ft. $\sqrt{M}$ but such results are obtainable only if the level is in excellent adjustment, the rod is plumb, solid turning points are used, the bubble is in the center when the rod reading is taken, and the backsight and foresight distances are balanced between successive bench marks.

466. Topographic Details. After the control system has been established, calculated and plotted, the map details may be filled in by any method as outlined in Chapter XXIII. The plane table or stadia method, or a combination of both, seems to be preferred by most engineers.

Map accuracy recommendations listed in § 465 should be complied with even if only ordinary field equipment is available. Field maps plotted on mounted paper so as to eliminate distortion, careful plotting of points, and accurate determinations of stadia constants will help materially in meeting these requirements. Every opportunity should be taken to check important distances, angles, directions, and elevations.

467. Planning Surveys and Maps. Map and survey data vary somewhat with the type of planning study. As mentioned in § 463, an essential requirement for the general preliminary study of a community is the best map procurable. It should be drawn to a scale of from 200 to 1000 feet to the inch and with a contour interval of 2, 5, or 10 feet, depending upon the roughness of the area, the scale of the map, and the need for information on relative elevations. Accompanied by an aerial map[1] to the same scale, this topographic map is the foundation upon which the *master plan* of the community is constructed. That plan is drawn only after a careful analysis of the present and probable future physical conditions. Economic and social facts and tendencies must also be considered, especially when the attempt is made to predict future trends. The master plan is a comprehensive, coordinated long-range plan for the future development of the community. It shows, in a general way, the future community; the pattern of streets, and the relative importance of different streets; the approximate subdivision of vacant land into blocks; an approximate zoning scheme; and the general location of park, playground, and school sites. Such a plan serves as a guide to the commission for the adoption from time to time of individual planning projects, all of which must fit into the general or master plan.

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[1] See § 221

Detailed project plans for street and sewer construction, traffic studies, subdivisions, zoning studies, and the like are best made to scales of from 20 to 200 feet to the inch. Such plans involve much more detail than those made for the preliminary planning study. They are more easily understood by the commissioners if aerial photographs to the same scale are made available.

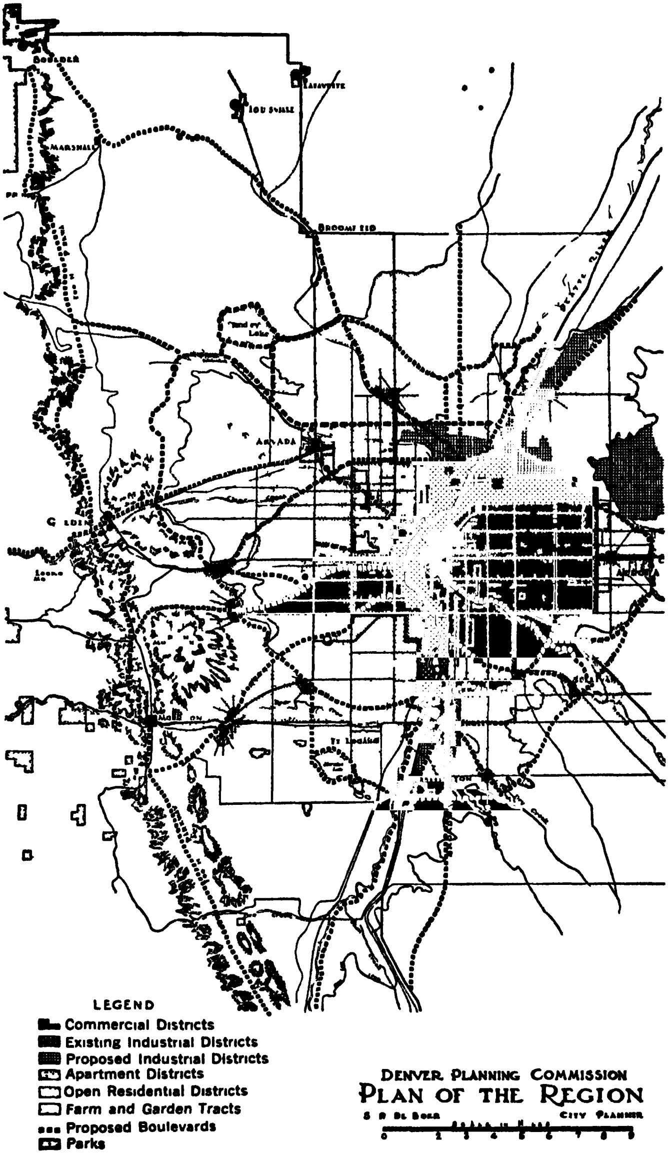
468. Traffic Surveys. Of the many perplexing problems of the modern city which challenge the ability of the engineer, relief of traffic congestion and elimination of traffic hazards are of major importance. The operation of 30 million automobiles over American streets and highways, many of which are incapacitated by width, grade, and alignment for their proper traffic function, creates not only a local planning problem but also one of state, regional, and nation-wide importance.

The analysis of this problem must be based upon special surveys and investigations. Such surveys should include a census of the traffic flowing through the streets, accompanied by information as to the character of that traffic and, in certain cases, its origin and destination. Population distribution data with particular reference to automobile ownership are also needed.

In addition to the study of the moving vehicle, data must be secured concerning the parked vehicle; the present widths, grades, alignments, and surfaces of streets; and the regulatory methods in effect at the time of the survey. A record of accident facts is also needed in order to determine the more hazardous locations in the city.

The average week-day conditions and average weather should prevail during the collection of these traffic facts. Traffic conditions are subject to constant change and resurveys are needed at least annually.

For the planning engineer, traffic police, and planning commissioners traffic surveys make possible the selection of major and secondary traffic thoroughfares, the determination of necessary street widenings, the location of short, new, connecting links



Courtesy of Denver Planning Commission

FIG 467 REGIONAL MASTER PLAN

in the present street system, the selection of by-pass routes for through traffic, and the recognition and elimination of hazards and obstructions to free-flowing traffic.

469. The Traffic Census. A preliminary study of traffic flow in any community reveals by observation those streets which are used intensively. Having determined these, census takers supplied with some form of tally sheet are stationed at each of these important street intersections in the down-town district and at each traffic inlet to the central city. Thus on the day selected as representing average conditions, a cordon count of traffic in and out can be made and during the same time the traffic movements can be surveyed by those stationed at the important down-town street intersections.

Location........................ Date........................

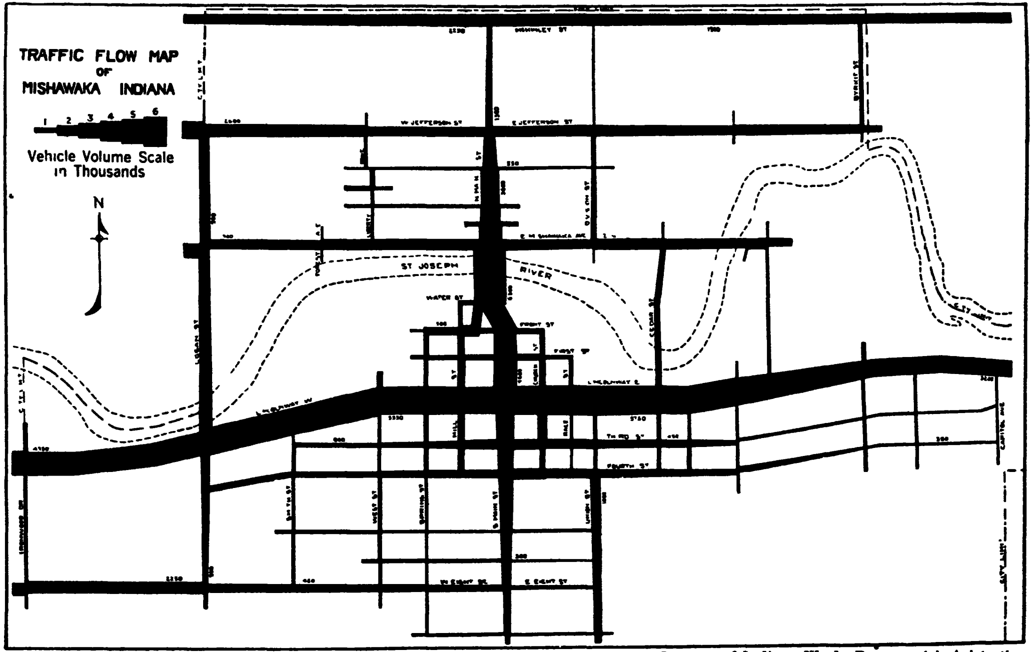
Observer........................

	Direction	6 00 A M to 6 15 A M	6·15 A M to 6 30 A M	Total for 12 hrs
Passenger Automobiles				
Trucks				
Taxis				
Busses				
Street Cars				
Horse-drawn				
Total				

NOTE This form provides for quarter-hour counts The form may be modified to meet other requirements such as half-hour counts or intersection studies

FIG 469 TRAFFIC CENSUS TALLY SHEET

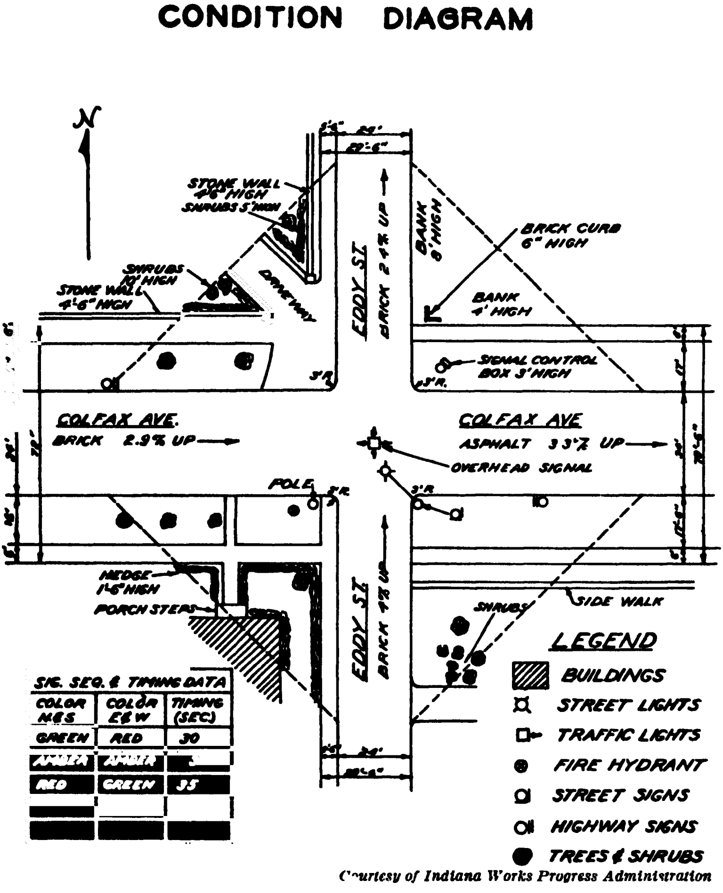
The tally sheet should provide space for the name of the census taker and his location. Then, in tabulated form, a tally space for each 15-minute or 30-minute interval of time between



Courtesy of Indiana Work• Progress Administration

Fig. 469a. Traffic Flow Map.

6 A.M. and 6 P.M. must be provided. The character of the traffic and its direction may be recorded by providing separate spaces for trucks, passenger cars, taxicabs, street cars, busses, and possibly horse drawn vehicles. The latter item is important only if such traffic is of sufficient volume to warrant its inclusion in the survey. Figure 469 suggests a convenient form of traffic tally sheet. Figure 469a is a typical traffic-flow map resulting from a 12-hour census of moving traffic, the band widths being plotted to a definite traffic density scale.



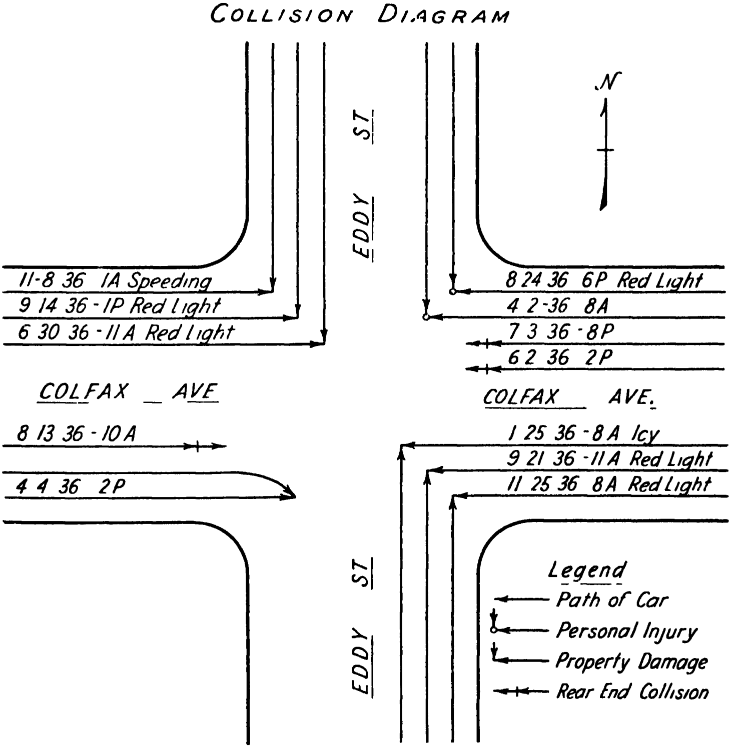
SIG. SEQ. & TIMING DATA

COLOR N&S	COLOR E&W	TIMING (SEC)
GREEN	RED	30
AMBER	AMBER	[illegible]
RED	GREEN	35

Courtesy of Indiana Works Progress Administration

FIG. 470. INTERSECTION STUDY.

470. Intersection Studies. At important street intersections, both volume and movements of traffic are such that several observers (usually four) are needed. If each observer is supplied with a tally sheet such as Fig. 469, modified by providing three instead of two tally spaces for each type of vehicle, he can record the right-turn, left-turn, and straight ahead movements for each traffic unit. Figures 470, 470a, and 470b are typical results of an intersection study.

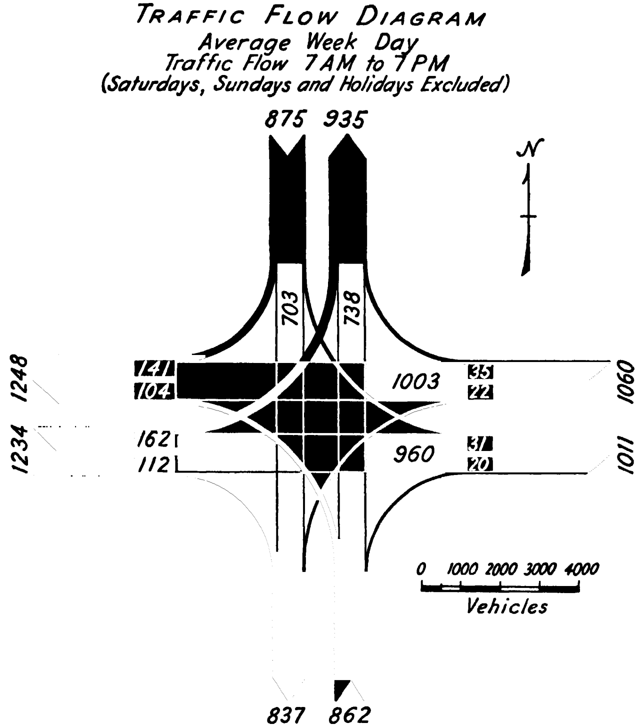


Courtesy of Indiana Works Progress Administration

FIG. 470a. INTERSECTION STUDY.

471. Zoning Surveys. A necessary preliminary step in the development of a zoning plan and ordinance for a city is a survey of the use and intensity of use of every lot.

The survey party, equipped with a 50-foot metallic tape, colored pencils, and a city map (preferably to a scale of 200 feet to the inch) inspects each separate lot and indicates the present use on the map in appropriate colors. The type of structure is shown by conventional symbol* or abbreviation, and in certain necessary key places, the set-back distances from the front property line to the structure and from the side lines to the structure are measured and recorded on the map.



Courtesy of Indiana Works Progress Administration

FIG. 470b. INTERSECTION STUDY.

When colors* symbolize use, many zoning plans and surveys have adopted the following:

Black to indicate industrial use
Green to indicate parks, playgrounds, wooded areas
Red to indicate business use

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* See Reference Number 12, "Mapping for Planning," 1948.

Yellow to indicate residential use
White to indicate open land.

The darker shades of each color represent more intense use as, for example, black for heavy industry and light gray for light industry.

Symbols indicating type of use are being standardized.* Simplicity is essential in the adoption of such symbols and in many cases rubber stamps can be used to advantage. The following suggested symbols might serve as the basis for use survey field parties:

□	single-family dwelling
$\boxed{2B}$	two-story, brick, single-family dwelling
$\boxed{2 \mid F}$	two-story, frame, two-family dwelling
○	business or commercial building
$\textcircled{\boxed{2B}}$	two-story, brick, business use, occupied also as a single family dwelling
△	industrial building

Simple combination of letters, numerals, and forms of symbols such as these are easily made and easily interpreted. A more elaborate and detailed set of symbols is given in a bulletin published by the National Resources Committee in June 1938.

Data from the field maps are transferred to the office map which is then used to establish the boundaries of the various use, height, and area districts for zoning purposes. This zoning map becomes a part of the ordinance which designates the appropriate use and intensity of use of each lot in the city. Maximum heights of structures are also fixed by districts. The public benefit of this exercise of police power is so apparent that zoning of cities has become the rule rather than the exception. Many counties are also zoned for different kinds of agricultural land uses or are in the process of being so zoned.

472. Land Subdivision. The subdivision of vacant land into sites for buildings by establishing street and lot lines is one of the most important responsibilities of the planning engineer. The old idea of sweating the land by squeezing as many lots as possible out of acreage property is fortunately no longer recognized as good practice. Subdivision standards established by lending agencies, platting guides adopted by planning boards, and the realization of both engineer and realtor of their social and economic responsibilities to the future welfare of their community have all contributed to a decided change in theory and practice of land subdivision.

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* See Reference Number 12, "Mapping for Planning," 1948.

Courtesy of Federal Housing Administration

FIG 472. A WELL-PLANNED SUBDIVISION.

473. Subdivision Standards. Passage of the National Housing Act which resulted in the establishment of the Federal Housing Administration has produced a large volume of private dwelling house construction during the last few years. In order to safeguard mortgage loans made on this improved real estate, F.H.A. has found it necessary to establish certain minimum requirements for subdivisions "submitted as suitable sites for homes financed under the Federal Housing Administration's Insured Mortgage Program." * These following eight basic requirements should be applied to all plans for subdivisions even if completely developed by private capital because adherence to these principles will result in economically and socially sound additions to the civic structure:

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* *Subdivision Standards, Circular No. 5*, Federal Housing Administration, August 15, 1938

"1. There shall be convincing evidence of a healthy and continuing demand for homes of the type contemplated in the location proposed and at the prices asked. This requirement is of first importance and is based on the general experience that untimely subdivision development does not succeed. It results in haphazard city growth, blighted areas, and serious financial loss to home buyers, mortgage lenders, and land developers.

"2. The subdivision site shall be plainly suitable for the type of residential development contemplated. It shall have no serious hazards such as smoke, fog, noxious odors, nuisance industries, possibility of subsidence, or the probability of floods. It shall not be unattractive to the average, reasonable prospective resident on account of the existence of any hazard or any other objectionable feature.

"3. The subdivision shall be accessible by highway or by public transportation at reasonable cost and with reasonable expenditure of time to places of employment, schools, and shopping centers.

"4. Public and private utilities and street improvements, appropriate and necessary to the neighborhood shall be provided. The extent of the utilities and the method of construction and installation shall conform to all applicable laws, ordinances, and regulations.

"a. There must be definite assurance of an adequate supply of pure water at reasonable rates.

"b. There shall be means of disposing of domestic sewage in a sanitary and unobjectionable manner which fully meets the approval of the local and state health authorities.

"c. Streets must be graded and surfaced in such manner as to provide safe and convenient access to all dwellings at all times.

"5. The subdivision shall comply with the provisions of existing zoning and subdivision regulations and shall be protected by appropriate and recorded restrictive covenants.

"a. Carefully compiled zoning regulations are effective, if properly enforced, because they not only control the subject subdivision but also the surrounding area.

"b. Subdivision regulations present an effective means of regulating matters not covered by zoning regulations.

"c. Well-drawn and properly recorded restrictive covenants aid in establishing the character of the neighborhood and in establishing and maintaining value levels through control of the use of land, the type and location of the structures, occupancy, and many other factors."

These deed restrictions should include definite statements as to the specific uses of all areas shown on the subdivision plans; the minimum cost of each proposed building; regulations concerning the placement of buildings on the lots; prohibition of resubdivision of lots into smaller parcels; prohibition of more than one dwelling house on each lot; establishment of a property owners' committee to pass on the architectural design of each proposed building; prohibition of undesirable uses such as stables, pig pens, temporary dwellings, and outside toilets; and provision for legal enforcement of deed restrictions by property owners.

"6. Whenever the subdivision, or any part of it, falls within the jurisdiction of a city, county or regional plan, or of subdivision or platting regulations, or state laws, the design and development shall comply with such plans, regulations, and laws.

"7. The subdivision must be suitable for the site and appropriate for the use intended.

"8. The debt for which the subdivision is security, whether mortgages, taxes, or assessments, shall be reasonable in amount. Where there are delinquent taxes or other encumbrances a program shall be set up which will provide funds for regular debt payments in amounts sufficient to amortize the debt in a reasonable period of time."

Success in modern subdivision planning for both realtor and engineer depends upon adherence to these basic minimum requirements. Standards higher than the minimum are very desirable and definitely possible in many communities. Better standards result in a higher level of community improvement at no appreciable loss to the realtor. In fact, better subdivision practices might conceivably result in quicker sales and in the end greater profit to the realtor.

474. Platting Guides. Rules for the guidance of planning engineers and realtors are adopted by planning boards in order that plans submitted to them for approval may fit into the master plan and also in order that uniform practices in the making of subdivision surveys and maps may be established.

The Land Subdivision Manual published in 1939 by the American Society of Civil Engineers suggests "rules and regulations for land subdivision" for adoption by planning boards. Many details of these rules are not applicable to a city of average size but the general principles are sound and must be included in any platting guide.

It is suggested that the following outline be used in writing specifications for the guidance of subdividers:

1. General requirements
    A. Topographic information, if needed
    B. Adherence to Master Plan
        a. Widths and arrangements of streets
        b. Adequate easements for utilities
        c. Recreation areas, school sites, etc. where needed
        d. Zoning
        e. Public utility extensions
    C. Lots
        a. Side lines, perpendicular or radial
        b. Minimum width and depth
2. Surveys
    A. Permanent monuments, kind and location
    B. Ties to existing monuments
    C. Legal descriptions
    D. Datum plane information
    E. Allowable errors of closure, traverse and levels
3. Preliminary map
    A. On tracing cloth or paper, with prints
    B. Scale, 100 feet to the inch, or less
    C. Data on map
        a. Street and lot lines, scaled dimensions
        b. Title under which subdivision is to be recorded
        c. Name of property owner
        d. Scale, date, and surveyor's name
        e. North point
        f. Names of adjacent property owners
        g. Proposed street names
        h. Topographic information
        i. Utility information, proposed easements, present sewers, water lines

4. Final Map (record map)
   - A. On tracing cloth, india ink, with prints
   - B. All details including distances, angles, and bearings, as in 3, above after field measurements have been made
   - C. Location and description of monuments and easements
   - D. Curve data (intersection angle, degree, P. C. station, P. T. station)
   - E. Registered surveyor's certificate with signature notarized
   - F. Owner's certificate of dedication of traffic ways to public use, with signature notarized
   - G. Certificate of approval by plan commission and of any other official bodies
   - H. Private restrictions, which may be lettered on the tracing or recorded separately. These must have approval of the plan commission.

475. Subdivision Design. In delineating the layout of streets and lots in a new subdivision, the engineer must be ever conscious of the fact that he is arranging a part of the future street pattern of the community. If his design is proper for that area, minimum costs of street, sewer, water, and other public improvements result. In addition, the traffic which will flow through the streets will do so with least hazard and difficulty. The engineer's obligations to his client and to the purchasers of home sites are also fulfilled in that his client realizes a reasonable profit from his business venture while the home owners find themselves situated in a desirable neighborhood and blessed with an investment of permanent value.

In order to achieve distinction as a designer of land subdivisions, the engineer must adhere to the standards as outlined in § 473 and comply with platting rules and regulations such as are suggested in § 474. In addition, an appreciation of and conformity to other principles of design details will result in better planned neighborhoods. Some of the more important of these principles are:

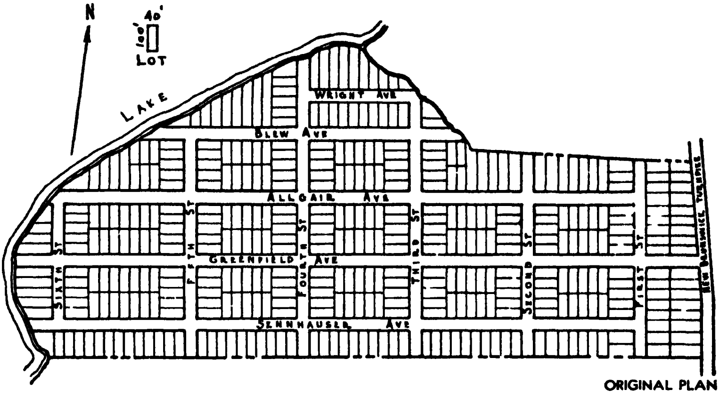
1. Alignment of streets should fit the topography

2. Dead-end streets are to be avoided. When topographic conditions make their use necessary, such streets should end in a turn-around, the radius of which should be not less than 30 feet

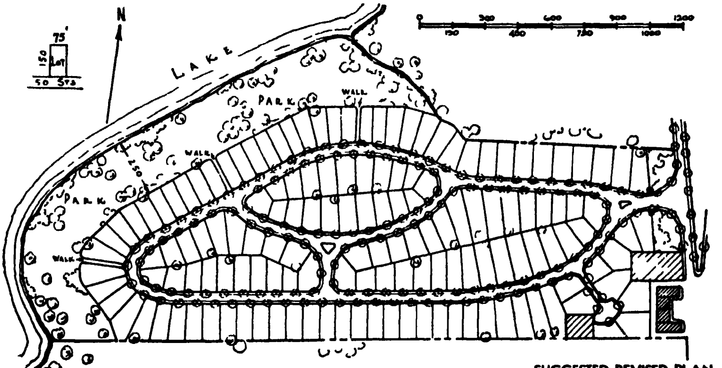
3. Compound and reversed curves are to be avoided in street alignment. Tangents should connect curves of the same or contrary flexure

4. Long blocks, up to 1200 feet, are acceptable if the length is in the direction of heavy traffic flow

5. Minor streets should intersect major streets at right angles, or nearly so



ORIGINAL PLAN



SUGGESTED REVISED PLAN

Courtesy of Federal Housing Administration

FIG. 475. GOOD AND POOR DESIGN.

6. Side lines of lots should intersect street lines at about 90° or along the radii of curves

7. Acute angles at lot corners should be avoided

8. Long lots are generally wasteful. For average conditions, lot depths of 120 or 130 feet are best. Depths greater than 150 feet are undesirable except for estate property developments

9. Lot widths should be such as to admit of from 10 to 2 feet minimum distance between buildings. 50, 60, or 75 feet are adequate widths of lots designed for the average home. Corner lots should be at least 10 feet wider than interior lots

10. Lots should be planned to face desirable views such as parks, school grounds, and the like

11. Tree growth on the site should be preserved if at all possible, but not at the expense of the proper development of the neighborhood as a whole, or if the preservation introduces traffic hazards

12. If not established by the platting guide or master pla, widths of streets should be determined by the traffic volume each is expected to carry. Eighty feet is generally accepted as the minimum width for a major traffic thoroughfare and sixty feet as the width for streets of secondary importance. The minimum width of local or minor residential streets should be fifty feet.

PROBLEMS

1. Prepare a traffic census form, modified as suggested in § 470. This should be prepared for a 12-hour count. Using this form, make a traffic survey at a street intersection in your community. From the results of the census, plot the traffic-flow map at that intersection. See Fig. 470b.

2. Prepare a set of symbols to be used in making a zoning survey for your community using the suggested symbols given in § 471 as the basis.

3. Equipped with a city map (or portion thereof) and the symbols prepared in problem 2, make a use survey of a section of your city. Study the results of the survey carefully and determine if each lot is being used properly. Suggest proper use zoning.

4. Using Fig. 464, design subdivision plans for the blocks as shown, reserving the park area for public use. This is a suburban area.

(a) All lots to be occupied by single family dwellings costing about $5000 each. Zoning regulations require a minimum of 5000 square feet of lot per family.

(b) All lots to be occupied by single family dwellings costing about $20,000 each. Zoning regulations require a minimum of 10,000 square feet of lot per family.

(c) Vacate Muensterman Ave. and the 40-foot north-south street and redesign (a) and (b).

(d) Assume that the zoning ordinance permits apartment buildings along the 60-foot avenue at the northwest and also permits business uses at the corner of Young Street and the 60-foot avenue. Redesign (a), (b), and (c).

5. Prepare a platting guide for your community based on the suggested outline given in § 474.

REFERENCES

1. American Society of Civil Engineers, *Manual of Engineering Practice No. 10. Technical Procedure for City Surveys.* New York, 1934.

   This manual gives details of the procedure for first order accuracy surveys of cities. Suggested forms of notes and sample calculations are included.

2. American Society of Civil Engineers, *Manual of Engineering Practice No. 16. Land Subdivision.* New York, 1939.

   This manual was prepared by a committee of the City Planning Division of the Society. In addition to the problems of subdivision design, the economic and social aspects are also considered. The discussions of the problems of redesigning existing subdivisions and of regulating subdivision practice are important contributions to the literature on this subject.

3. Branch, Melville C., Jr., *Aerial Photography in Urban Planning and Research.* Harvard City Planning Studies, Vol. XIV, Harvard University Press, 1948.

4. Federal Housing Administration, *Technical Bulletin No. 7. Planning Profitable Neighborhoods.* Washington, D. C., 1938.

   An excellent, nicely illustrated bulletin emphasizing the principles or proper land subdivision. It shows examples of good and poor design.

5. Federal Housing Administration, *Circular No. 5. Subdivision Standards.* Washington, D. C. Revised 1938.

   This booklet states the objectives of establishing standards, sets up minimum requirements and suggests methods of correcting defects in existing plans.

6. Lautner, Harold W., *Subdivision Regulations.* Public Administration Service, Chicago, 1941.

   This is a comprehensive analysis of platting regulations, both present and past. The strong and weak points of the different kinds of regulations and their effect upon community planning are discussed. Legal problems, procedures, standards and design requirements are considered.

7. Lewis, H. M., *City Planning, Why and How.* Longmans, Green and Company, New York, 1939.

   A popular treatment of the subject. The text is written "so that the reader may think of city planning not as a science to be dealt with only by trained technicians, but as something which is intimately tied in with his daily routine and his personal welfare and comfort."

8. Lewis, Harold M., *Planning the Modern City,* Vol. I and Vol. II, John Wiley and Sons, New York, 1949.

   Mr. Lewis is a consulting engineer and city planner. As such, his analysis of city planning problems is in language that the engineer can appreciate.

9. **Menhinick, Howard K.**, *Local Planning Administration.* The International City Managers' Association, Chicago, 1948.

This is the Segoe book of 1941, revised, condensed and added to by Mr. Menhinick, now serving as Director of Regional Studies, T.V.A.

10. **National Resources Committee**, *Urban Planning and Land Policies, Volume II of the Supplementary Report of the Urbanism Committee.* United States Government Printing Office, Washington, 1939.

This report supplements *"Our Cities – Their Role in the National Economy."* Case studies were made of American communities which have been developed by various agencies, industrial governmental, real estate, and philanthropic. Urban living conditions and land policies are discussed also.

11. **Walker, Robert A.**, *The Planning Function in Urban Government.* The University of Chicago Press, Chicago, 1941.

This volume is the result of a year's study of city planning agencies in thirty-seven cities. The author analyzes the planning function as it has developed in these cities. Weaknesses are pointed out and remedies for improvement are suggested.

12. **Wilkens, E. B.**, *Mapping for Planning.* Public Administration Service, Chicago, 1948.

A guide for the classification and mapping of Land Uses. Suggested standard colors, signs and symbols.

CHAPTER XXIX

UNDERGROUND SURVEYS

476. Introduction. Underground surveys are made for mining, tunneling, and other sub-surface work. They are quite similar to surface surveys but are more difficult to execute because of unfavorable working conditions underground such as poor light, smoky atmosphere, and cramped and wet surroundings; because the "inclusive checks" of closed traverses and closed level circuits seldom are possible; because instrumental and procedural requirements are more exacting; and because interference with routine operations must be minimized.

Underground surveying is a difficult specialty. If much of it is to be done, the references at the end of the chapter should be consulted. It is hoped that the treatment in this chapter will suffice for occasional cases. Special or extensive surveys can be made more economically and perhaps more precisely by following specialized procedures such as are given in the references.

It is necessary for the underground surveyor to appreciate that accuracy is essential, since an inaccurate survey is worse than useless and may involve loss of life as well as legal and financial difficulties.

477. Definitions. A few of the terms that are more commonly encountered and not elsewhere defined are

Adit. An approximately horizontal underground passageway for drainage or ventilation. The word tunnel is more frequently used at present.

Back (or *roof*). The top of a passageway.

Bore-hole. A hole, sometimes of great depth, drilled for exploration or to facilitate operations.

Collar. Timbers or concrete around the top of a shaft.

Connection. A passageway driven from one part of underground workings to another.

Dip. The angle at which beds or strata are inclined from the horizontal. The dip is always measured at right angles to the strike.

Heading. A preliminary passageway driven for exploration or as an aid to future work.

Levels. Horizontal passageways at different elevations.

Outcrop. The portion of the vein or stratum exposed at the surface of the ground.

Raise. An opening like a shaft made in the back of a level to reach a level above

Spad. The small nail or hook in the roof which marks a survey station and from which a plumb-bob may be hung.

Stope. An excavation from which ore has been extracted either above or below a level.

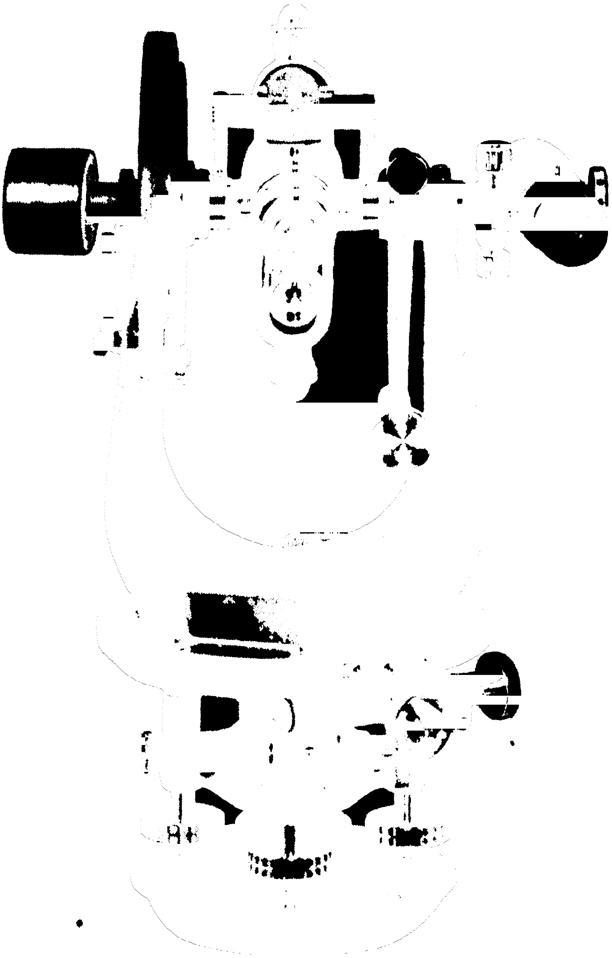
Strike. A horizontal line on a vein or stratum, or the bearing of such a line.

Tunnel. A relatively horizontal passageway open at both ends.

478. Differences from Surface Surveying. While in general the methods of underground and surface surveying are similar, certain differences will be pointed out in this chapter and their usefulness will be evident to interested readers. It is frequently possible to use surface methods underground, but the work is facilitated by special equipment and procedure.

Most surface surveying is run as closed traverses and closed level circuits which give assurance of the accuracy of the work. Such closures are rarely possible underground and resort to other more laborious methods of assuring accuracy is necessary. Usually this consists of repeated and independent measurements and calculations. Computation of the coordinate positions of points is essential in most underground surveying. Elevations are sometimes obtained by slope or vertical taping rather than with the engineers' level.

479. Transits and Levels. Since this work involves short sights, the transit and level (preferably of the dumpy type) should be of low magnifying power and wide field of vision. They should be as nearly moisture-proof and dust-proof as possible. A split-leg tripod with legs which are adjustable in length so as to facilitate instrumental set-up in awkward locations must be provided. In steep shafts and other particularly difficult set-up situations, special procedures or apparatus may



Courtesy of C. L. Berger and Sons

FIG. 479. MINING TRANSIT WITH STRIDING LEVEL AND SIDE TELESCOPE.

be necessary. In Fig. 496, the transit may be used with plates inclined if the third leg is weighted or otherwise attached to hold it down. A bracket which may be screwed or bolted to a timber provides a tripod head on the opposite end.

Courtesy of Federal Works Agency

FIG. 479a UNDERGROUND SET-UP.

Because the transit is frequently set up *under* the plumb-bob, as in Fig. 479a, it is necessary that the position of the vertical axis be plainly marked on the top of the telescope. The transit should be quite free of dust and moisture after use and tissue paper or soft cloth should be provided for cleaning during use. Due to varying temperature conditions, the eyepiece and the vernier may fog up and the instrument must then be allowed to reach the surrounding temperature before it can be used. The lenses must not be touched in such a way as to scratch them, to dull their polish, or to leave oil on them. Platinum crosswires are preferable to spider-hairs because they are not subject to distortion from moisture.

480. Special Requirements in Transit Adjustments. Since underground surveys involve short sights and consequently a large number of instrumental set-ups, and since precision is usually necessary, the transit must be kept in the best possible adjustment and a survey procedure must be used which will eliminate most of the residual errors of adjustment. No unusual attention is needed for the engineers' level.

Sensitive plate bubbles are necessary on the transit in order to make the vertical axis truly vertical, since unless this is accomplished no instrumental procedure or manipulation will eliminate certain residual errors of adjustment. Should the plate bubbles be broken or insensitive, the vertical axis may be made vertical with the telescope bubble by leveling the transit approximately, centering the telescope bubble, reversing the transit 180° in azimuth, and bringing the telescope bubble halfway back to the center with the footscrews. The procedure is repeated over the other set of footscrews. The telescope bubble may then be centered with the tangent screw, and it should remain centered throughout a complete revolution around the vertical axis. This is the more precise method since the telescope bubble is more sensitive than the plate bubbles, and the added precision is particularly necessary where no striding level, Fig. 479, is provided.

The vertical axis may be made precisely vertical with a striding level mounted on the horizontal axis, using a procedure similar to that just described for the telescope bubble.

While it is desirable that the plate bubbles be kept in adjustment, it is to be noted that this is not essential. If out of adjustment, they will not be in the center when the vertical axis is vertical; rather they will remain exactly the same distance off center throughout a complete revolution of the telescope around the vertical axis.

It is desirable that the horizontal axis of the transit be truly horizontal and at right angles to the line of sight so that the instrument will generate a vertical plane when the telescope is revolved on the horizontal axis.

Because both close and distant sights are numerous in underground surveys, the objective slide, or the internal focusing slide, should be in good adjustment.

481. Using the Transit. It is customary to measure horizontal angles as *angles to the right,* such angles being turned clockwise from the back sight and having values anywhere from 0° to 360°. These angles are doubled, once with the telescope normal (bubble underneath) and once with the telescope inverted (bubble on top). Half of the doubled angle should not differ from the first value by more than the least count of the vernier, which is usually either one minute or one-half minute.

In the rare case of eccentricity in the centers or verniers of the transit, both verniers must be read to eliminate it. This is a desirable precaution on all important angles.

Where vertical angles are important in underground work, they should be read once with the telescope normal and once with the telescope inverted, the mean value being used. Usually they are read at the same time that the horizontal angles are being doubled. *The vertical axis must be vertical.* If the two values are not identical, then the same differences in angles should exist for all pairs of vertical angles since the difference measures the lack of adjustment of the instrument.

This instrumental procedure of measuring both horizontal and vertical angles with the telescope normal (bubble underneath) and again with the telescope inverted (bubble on top) also minimizes opportunity for undetected movement of the transit and of mistakes in reading or manipulations, since by comparison of the first and second values of these angles it is possible to detect such movements or mistakes. It may at times be advisable to double the horizontal angle and to measure the vertical angle twice with the telescope normal, and to repeat the entire process with telescope inverted.

As an alternative method of measuring vertical angles when traversing, either with the regular telescope or with the side telescope, and again assuming that the vertical axis is kept vertical, it is only necessary to measure a single vertical angle forward from each station and another angle backward from the next station, with the telescope normal (or inverted) in both measurements. The mean of the two computed differences in elevation gives a very close approximation to the true value, regardless of adjustment errors, as in Fig. 481.

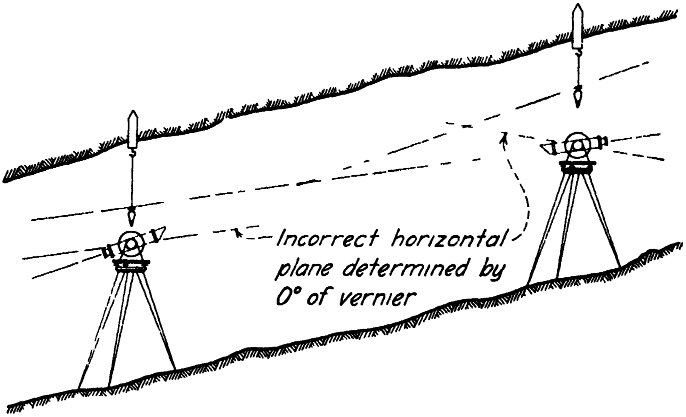


Fig. 481. Difference in Elevation with Transit.

In all transit work, it must be reiterated that the vertical axis must be made truly vertical, otherwise no doubling, reversing, or other instrumental procedure will eliminate all errors of adjustment. Certain of these errors are eliminated when a striding level bubble (Fig. 479) is centered on top of the horizontal axis in both normal and reversed positions.

In order that the transitman need not shift his position about the transit and to avoid disturbing the transit, it is best to read the horizontal angle first, and then to rotate the transit horizontally with the lower motion and read the vertical angle. This is followed by the second set of readings. With mining transits where the vertical circle can be read at the eyepiece end of the telescope (edge graduations) this procedure is unnecessary.

482. Using the Side Telescope. There are several ways in which the auxiliary top or side telescope, Fig. 479, may be used

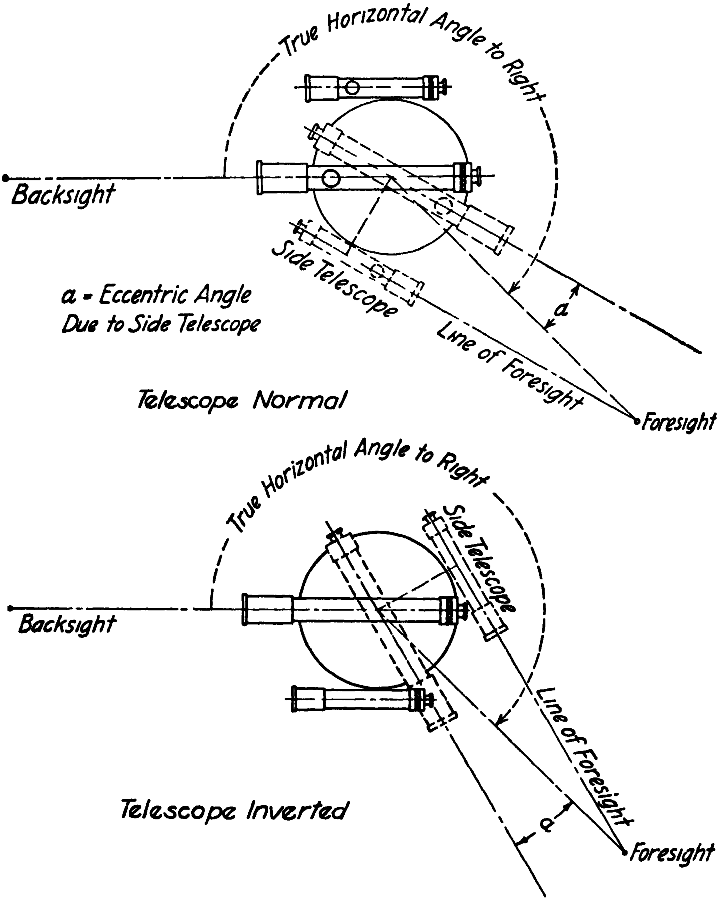


FIG. 482. OUT-OF-ADJUSTMENT USE OF THE SIDE TELESCOPE IN MEASURING A HORIZONTAL ANGLE.

and these are described in the references at the end of the chapter. Only the most common and generally useful procedure is given here, namely, the "out of adjustment use" with the side telescope. If used as shown on Fig. 482, and provided that the vertical axis of the transit has been made truly vertical, then half of the doubled horizontal angle and the mean of the two vertical angles will be true values regardless of imperfect adjustment, and regardless of the eccentric position of the side telescope.*

483. Other Equipment. A 5-foot to 8-foot steel pocket tape is quite useful for measuring the many short distances, offsets, heights of instruments, and heights of stations.

For setting most traverse stations and for fairly precise work, a steel tape 200 feet long and $\frac{3}{8}$ inch wide, graduated throughout to hundredths of a foot, is desirable. It should include a tape reel with a folding handle since the tape is only unreeled as used and is carried on the reel rather than dragged as is customary in surface work. It is usually necessary only to unreel a part of the tape length. The tape should be dried, cleaned, and oiled before it is put away.

Six-ounce, eight-ounce, and ten-ounce transit plumb-bobs are used, and more are needed than for surface work.

The transit reading glass should be mounted in a non-inflammable (metal) frame to prevent its ignition from open-flame lamps.

The small Brunton compass (§ 42) is used for rough surveys of unmapped mines and for other surveys where an error of several degrees in direction is permissable. It is frequently necessary to take forward and backward readings from a point and then to compute the included angle, so as to eliminate local attraction of the needle.

484. Illuminating Instruments and Points. Open-flame carbide lamps or flashlights may be used according to preference for illuminating the cross-wires, the points sighted, and the verniers.

It is frequently necessary to shine a light down the barrel of the telescope from a position in front and to one side of the objective lens so that the cross-wires may be seen. Mining transits have a special reflector shade to place over the objective for facilitating this procedure. Lacking such a shade, a piece of white paper may be wrapped around the objective or held in place inside of the sunshade so as to form a continuation of the telescope. The light is then reflected into the telescope from the inside of this white paper cylinder. Rarely, transits are equipped with a battery and with a light bulb inside the telescope.

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* Pairs of vertical angles should show the same angular discrepancy. Pairs of horizontal angles will show angular discrepancies varying with the distance from the transit (Fig. 482), and a table may be prepared showing this variation.

For sighting at a point, a plumb-bob backed by white paper may be illuminated by shining a light onto it from the front but care must be taken that the shadow of the plumb-bob string is not mistaken for the string. A plummet lamp may be suspended by a cord from the roof station and sights may be taken on the wick. Frequently a light is held behind the plumb-bob with a piece of transparent paper or tracing cloth between the light and the plumb-bob.

Level rods are similar to those used in surface work but shorter. The light may be directed onto the face of the rod from the front, or a horizontal slit may be cut in the target behind which a light may be held.

485. Traversing. Underground traverses are usually run with doubled *angles to the right* (clockwise from the back sight) and with slope taping as in Fig. 485. The tape may, of course, be held level where this is feasible. In slope taping, the rear end of the tape is usually held at one end of the horizontal axis of the transit and the front end of the tape is either held directly on the forward station, at a fixed distance above a floor station, or at some point definitely marked on the plumb-bob string which is suspended from a station in the roof. The top of the plumb-bob itself is usually satisfactory for such a sight. The slope distances multiplied by the sine and cosine of the vertical angle give the vertical and horizontal distances respectively. Some engineers prefer to make these computations with traverse tables or with computing machines similar to the manner in which latitudes and departures are computed. With careful work, a precision of the order of 1/10,000 is possible by this method without using spring balances or temperature corrections except in unusual cases.

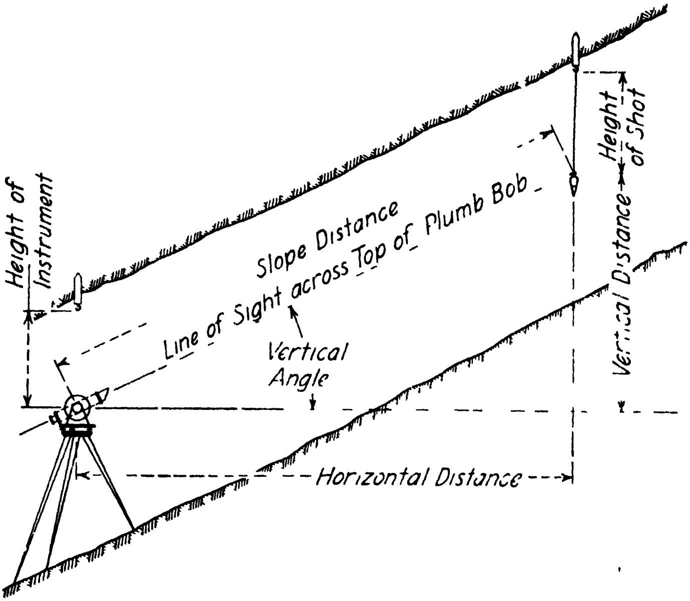

FIG. 185 UNDERGROUND TRAVERSING.

486. Curves. The alignment for a curve is determined by the deflection angle method as in Fig. 486, using chords as long as are possible or convenient. A series of spads is set in the roof from which the foreman suspends plumb-bobs, and then sights are taken to successive points around the curve as the heading is extended. The chords are usually much shorter than in surface work and on sharp curves may be only a few feet in length. For less important curves, the chord offset method as described in Rubey's *Route Surveys*, page 73, might be used.

487. Marking Stations. Stations usually consist of a wooden peg about 1 inch in diameter driven into a hole in the roof, into which a small hook or spad is driven and from which a plumb-bob may be suspended. Flattened and notched horseshoe nails make good spads, or specially made ones may be purchased. Copper spads are used if mine water is acid. Stations seldom can be maintained in the floor because of traffic. The roof stations are also useful to the foreman who can suspend plumb-bobs

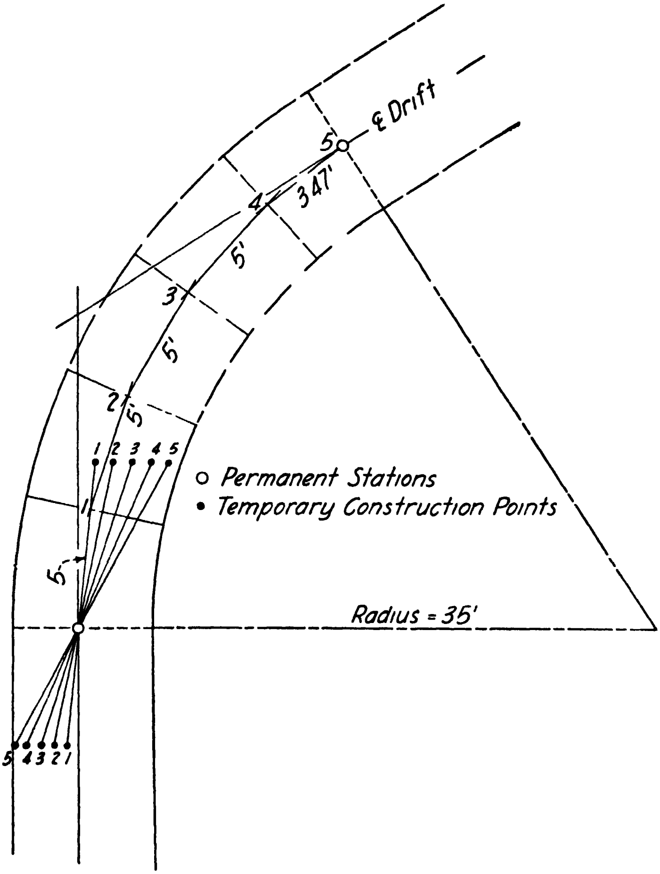

FIG. 486. ALIGNING A CURVE.

from them at the proper elevations so that he can mark the alignment and elevation needed at the face of the heading by sighting along the top of the plumb-bobs, as in Fig. 487.

Stations should be marked with copper or brass tags held in place by the spads. The station number is stamped into the tag before it is placed in position. White painted crosses or arrows are helpful but are seldom permanent.

Along the lines of tunnels or other underground workings such as those for railways, highways, aqueducts, and so on, the regular 100-foot surface taping is continued underground and, emerging at the far end, is continued on the surface. A typical station might be 212 + 43.84.

In mines with many passageways, the system of numbering stations is apt to become involved and confusing unless carefully planned from the beginning. It is customary and desirable to number the stations according to the level of the working. For example, the first station from the shaft on the second level below the collar would be 201, and the fifth station of the fourth level would be 405.

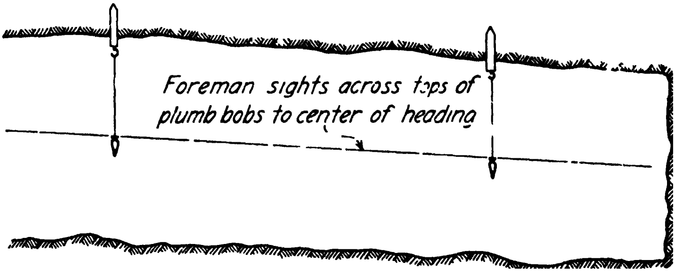
Foreman sights across tops of plumb bobs to center of heading

FIG. 487. CARRYING STATION ALIGNMENT INTO HEADINGS.

It is sometimes advisable to have one series of numbers for all stations on a certain level north of the shaft and to place N before the station numbers, as N201, N405; and a similarly prefixed S before stations south of the shaft. Separate systems of numbering are usually best for crosscuts and subordinate workings.

Under certain circumstances, stations are numbered as they are set, beginning with 100, regardless of the location. This necessitates reference to a map and the card index of § 489 before the station can be identified. This simple method has its disadvantages but is used extensively in large mines.

488. Notes and Calculations. Because of their complexity, it is especially desirable to have permanent printed forms on which to record survey data. Although many engineers prefer bound field notebooks, loose leaf field note sheets are frequently used in wet mines and only exposed a minimum time on a clipboard. They should be numbered serially to avoid loss or misplacement. Many engineers prefer to use the ruling of level *notebooks for transit notes because of the advantageous columnar ruling.*

Level notes are similar to those ordinarily used on the surface except that where the rod is held inverted on a roof station, an algebraic minus sign precedes the backsights and an algebraic plus sign precedes the foresights. Roof elevations are taken on the bottom of the wooden plug, not on the spad, since the spad is often knocked out.

Suggested forms for field transit notes and for office records follow.

489. Field and Office Forms. Much difference of opinion exists as to the exact forms to use, but those of Fig. 489 come as near to an acceptable standard as is possible.

490. Maps. Accurate maps of underground workings are more essential than surface maps since many things which are evident on the surface are entirely concealed underground. Because of great variety, it is impracticable to show examples in this limited treatment.

Due to varying strikes and dips of veins and because workings overlap one another in the map or plan view, it is necessary to use care and judgment in showing those plans and sections which will convey the maximum of clear information. In mining it is usual to prepare many plans, such, for example, as

1. Surface maps

a. Showing property and claim boundaries on a scale such as 1 inch = 400 feet

b. Showing improvements such as buildings, shafts, tramways, pipe lines, roads, railroads, and so on, on a scale such as 1 inch = 40 feet

c. Showing topography with scales such as 1 inch = 100 feet, or 1 inch = 40 feet for more important territories. Contour intervals are ordinarily 5 feet

d. Showing surface geology, and similar to c

e. A general progress map for the general affairs on a small scale such as 1 inch = 400 feet or smaller

2. Underground maps

a. Showing workings on a scale such as 1 inch = 40 feet, and perhaps another one on a smaller scale of 1 inch = 100 feet

SUGGESTED FORM FOR TRANSIT NOTES

Engineer..........

Card No........ Location.................. Helper...........

DATE	B.S.	INST.	F.S.	HOR. ANGLE	SLOPE DISTANCE	VER ANGLE	H.I.	H.S.	STA. TO FLOOR	REMARKS
	1	2	3	131° 26′ 262° 53′ Doubled Angle 131° 26′ 30″	95.41	+ 1° 31′	− 2.13	+ 2.79	7.8	

NOTE. Last 3 columns (H.I., H.S., and Sta. to Floor) may be omitted if elevations are obtained from a separate level survey.

SUGGESTED FORM FOR LEDGER (THE OFFICE RECORD)

Survey of..........

Field Book No................Page.....

Computation Book No...... Page.....

Entered by......................

B.S.	INST.	F.S.	HOR. ANGLE	BEARING	DISTANCE	PLUS LAT.	MINUS LAT.	PLUS DEP.	MINUS DEP	COORDINATES		ELEV.	STA.
										North	East		

SUGGESTED FORM FOR SYSTEMATIZING STATION NUMBERS (THE CARD INDEX)

STATION NUMBER	LOCATION	COORDINATES		ELEV.	LEDGER REFERENCE	
		North	East		Book No.	Page No.
176	4th level east	728.19	976.42	1247.29	1	20
177	5th level west	816.24	413.29	1143.17	1	20

FIG. 489.

b. Showing underground geology, on a scale as in 2a
c. Showing assay sampling, on a scale as in 2a
d. Showing ventilation, to scale as in 2a
e. Stope maps showing workings on a somewhat larger scale, perhaps 1 inch = 20 feet.

It is frequently necessary to show different levels in different colors or to show them on different maps. Dates are often marked on the various workings. It is convenient to superimpose certain plans, and such plans are drawn on tracing cloth so that one or two may be seen through the plan which is on top.

Mine maps are plotted by coordinates with the coordinate lines an inch or a few inches apart so that the distance between them scales some even figure such as 100 feet, 500 feet, or 1000 feet. They should be lightly inked in red or orange, perhaps with some emphasis on the fifth or tenth lines.

491. Sections. Sections are usually drawn to the same scale as a map although the vertical scale may be exaggerated if necessary.

Longitudinal sections may be any vertical section along the general direction of the workings; along a cardinal direction of N, E, S, or W; or it may be a section lying in the plane of the vein, when it is known as a "plan on the vein."

Transverse sections are usually plotted at right angles to the longitudinal sections.

In plotting the sections, a datum level is chosen and drawn on transparent paper, usually below the lowest workings, and this line or some parallel line is placed along the line of the section on the map. Light vertical parallel lines are drawn through each object to be represented and the object is shown on the vertical line at its proper elevation. Surface elevations may be read from contours, and other elevations may be obtained from the surveys. All workings, geological formations, coordinate lines, and other objects of interest are shown.

492. Mine Models. A great variety of models has been prepared showing underground workings and they are available in museums and mine offices. While not strictly a matter of surveying, they are prepared from survey data and usually by the mine survey corps.

493. Stope and Tunnel Surveys. Stope surveys are needed to determine outlines of workings, compute tonnage, and facilitate operations. They may be made from a transit set-up which has been established by traversing, or with the Brunton compass. The horizontal angles and slope distances are measured to points defining the stope. A scale of 1 inch = 20 feet is most frequently used in plotting.

If timbering of regular spacing is used, the stopes may be plotted simply by counting the sets, with little instrumental work.

Where a constant tunnel cross-section is needed, such as an accurately aligned tunnel for railways, highways, and aqueducts, the alignment is run underground with 100-foot stations in the same manner as on the surface and cross-sections of the tunnel are taken every 100 feet, or less, to assure accurate alignment, to control the tunnel lining, and to measure the amount of excavated material. Frequently two cross-sections for excavation are specified as described in Rubey's *Route Surveys*, page 176. One is the inner cross-section beyond which no rock shall extend into the tunnel, and the other the outer cross-section beyond which no payment will be made to the contractor for work done.

In such work, the cross-sections at right angles to the center-line may be determined by vertical transit angles and distances taped from the horizontal axis of the transit, by large specially devised frame protractors and tape, or by full-sized templates of proper dimensions which are set up at each cross-section to check the conformity of the excavation.

494. Tunnel Portals. Special surveys are necessary at tunnel portals, especially where ornamental treatment is desired or where hydraulic transition is needed from an open canal section to the tunnel section. These are conveniently made after the tunnel is driven with the transit set-up on a center-line station just outside the portal. Sights are taken on the rock or earth surface around the portals so as to plot a contour map, having a scale of between 1 inch = 10 feet and 1 inch = 40 feet, on which the designer can fit his construction to the contours. Each sight consists of a horizontal angle, a vertical angle, and a steel taped distance from the horizontal axis of the transit. Stadia measurements are not desirable because of the steeply inclined sights.

TRANSFERRING A MERIDIAN UNDERGROUND

Because of limited working space and because of the expert skill necessary, this is one of the most exacting requirements of underground work. In small workings or where precise location is unimportant, no great difficulty arises, but frequently extensive and important workings must be carried thousands of feet and even miles from the shaft. The methods used for tunnels, inclined shafts, and vertical shafts, follow.

495. Tunnels. Tunnels are surveyed by methods similar to those used on the surface but with the greater precision of repeated angles and very careful taping, since errors are more serious. The 30-second transit is usually adequate although 10-second transits are used on important work. Because tunnels are usually close to a horizontal position, slope taping is often unnecessary. Where considerable precision is needed, the surface surveys are frequently accomplished by accurate triangulation rather than by traverse, and the underground traversing is executed and checked with painstaking care. The tunnel references at the end of the chapter should be consulted regarding these precedures.

In extreme cases, tunnels have been driven from both ends for distances as great as thirteen miles with an error of only a few inches in alignment when the headings met.

Tunnels are usually carried horizontally underground at both portals so that the following shaft procedures are frequently unnecessary in tunneling, except that vertical shafts may be sunk at intermediate points of long tunnels.

496. Inclined Openings. When the inclination of the workings is less than about 60° (or not steeper than the particular transit will permit), ordinary traversing methods are used, with special care regarding manipulations and adjustments. For sights inclined steeply upward, a prismatic eyepiece must be attached to the telescope.

Where no auxiliary telescope is available, the horizontal plates may sometimes be tipped as shown in Fig. 496 by weighting down the tripod leg. The horizontal axis must be horizontal, and the transit double sighted.

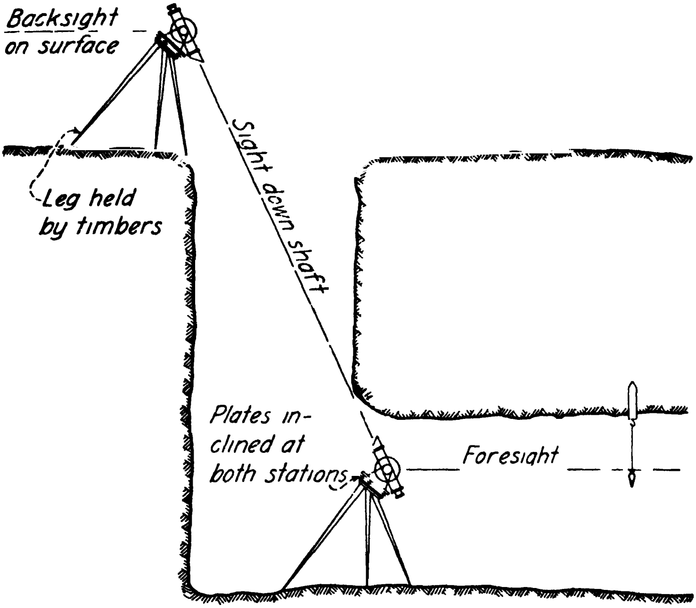

FIG. 496. USE OF TRANSIT ON STEEP SLOPES WHEN NO SIDE TELESCOPE OR WIRES FOR PLUMBING ARE AVAILABLE.

For steeper slopes, it is desirable to use the side telescope, using doubled horizontal angles and twice-measured vertical angles as described in § 482. Half of the doubled horizontal angle, or the mean of the twice-measured vertical angles, then gives the true value even though the eccentric side telescope is used in sighting, as was evident from Fig. 482. All errors of adjustment, except the verticality of the vertical axis, are eliminated by this procedure.

Other less generally satisfactory methods for the side or top telescope are described in the references. Figure 496a shows a method of carrying alignments through steeply inclined slopes by means of a string or wire and plumb-bobs. The precision of this method is not as good as that in which the alignment is determined with a transit.

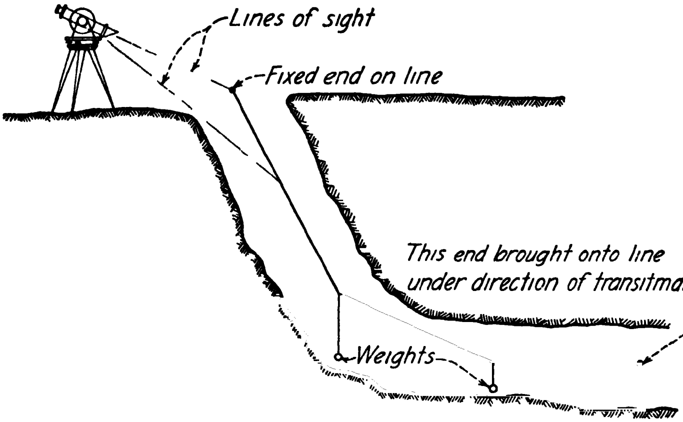

FIG. 496a. ALIGNMENT BY STRING OR WIRE.

497. Plumbing Vertical Shafts. Fine piano wire, or electrician's banding wire, with 10-pound to 40-pound plumb-bobs are suspended from the top of the shaft. Reels must be provided upon which the wire may be wound. A No. 24 piano wire will safely support a 50-pound bob. Lighter bobs may be used for less exacting work, as, for example, No. 3 music wire with a 25-pound bob. Care should be taken not to snap the wire when it is under tension since it is somewhat dangerous. Falling water and air currents with velocities over 100 feet per minute affect the plumb wires and should be eliminated so far as possible, and vertical plumbings of over a few hundred feet should be avoided. The plumb-bobs are submerged in water or oil to minimize swinging, and the container may be provided with a top to avoid disturbance by falling water. Electric conduits in shaft walls sometimes produce magnetic effects which may be eliminated by use of brass or bronze wire.

Since frequently the plumb-bob will not come to rest immediately, and since the wire acts with an oscillating motion rather than as a simple pendulum, it may be necessary to establish the position by several trials. The mean position of the wire is then taken on scales set at right angles behind the wire.

Special devices are sometimes used for centering the wires at the surface.

498. One-Shaft Methods. In this very generally used method, the transit is moved sidewise by trial ("wiggled in" or "jiggled in") in line with two wires as shown in Fig. 498. This is known as *coplaning*. In order to check the accuracy of plumbing, the distance between the two wires, both at the top and at the bottom of the shaft, is accurately measured to thousandths of a foot and alignment results are not considered satisfactory unless the measurements check within a couple of hundredths of a foot of one another, or closer where considerable precision is necessary.

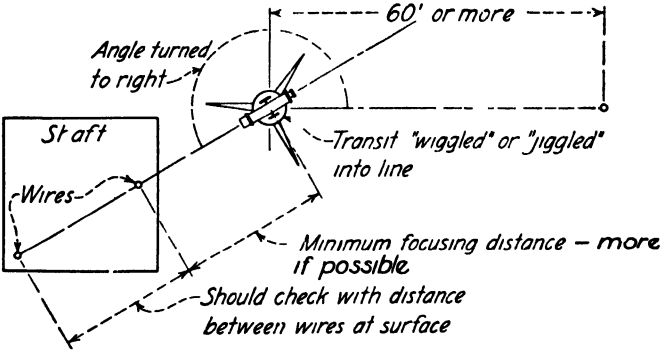
60' or more
Angle turned to right
Shaft
Wires
Transit "wiggled" or "jiggled" into line
Minimum focusing distance – more if possible
Should check with distance between wires at surface

FIG. 498. COPLANING.

The transit is coplaned with the telescope normal and the angle from the wires to the foresight repeated as needed. This procedure is duplicated with the telescope reversed, without changing the position of the transit. The transit must be in good adjustment. Staley (Reference 17) describes a procedure * which eliminates most adjustment errors, but it is somewhat troublesome and frequently unnecessary.

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* The transit is coplaned with the telescope normal, and the horizontal angle is turned to the foresight with as many repetitions as necessary. The position under the transit of the bottom of the plumb-bob point is carefully marked on a smooth lead block some 4" × 4" × 2" laid on the floor, or otherwise. The transit is again coplaned with the telescope inverted (bubble up), the angle is again turned and repeated as often as necessary, and the position of the instrument plumb-bob point is marked on the lead block. The mean horizontal angle is the correct one, and the backsight from the next set-up is taken on the mean of the marks on the lead block.

In order to secure maximum precision of alignment, it is of course desirable to secure the maximum possible distance between the wires and adequate length of foresight and backsight from the transit, through exercise of ingenuity in placing the wires and selecting the transit set-ups.

It is convenient to place a paper on the near wire to distinguish it. The far wire is made visible by moving the near wire, by inserting links in the near wire, or by setting up the transit just far enough from the near wire to focus on it — then the near wire will disappear when focusing on the far wire.

While coplaning is the common one-shaft method, several others are used, though rarely, under different conditions of clearance and available space. Of these the Weisbach method is perhaps preferred. It is described by Staley (Reference 17) and others.

499. Two-Shaft Method. Where precision is required, the best results are obtained by hanging a single wire in each of two shafts which are connected by traverses on the surface and underground, as in Fig. 499. Both traverses, surface and underground, should be checked by independent traverses using different stations.* In first calculating the underground traverse, the first bearing is assumed. The mean calculated length and bearing of the closing lines between wires of both surface and underground traverses are computed. The horizontal distances between the two wires as calculated by the surface traverse and the first underground traverse should, of course, equal one another within the required precision of the work. The assumed bearings of the first underground traverse are then corrected to correspond with the true bearing between the wires. A second true underground traverse and coordinates are computed after underground bearings have been corrected.

It is entirely satisfactory if the wires at the surface are connected by direct measurement or by triangulation rather than traverse.

The underground traverses of Fig. 499a will illustrate the method that was explained above. Referring to Fig. 499, A and B are wires in two vertical shafts. From surface survey, the true bearing from A to B is found to be N 85° 10′ E and the distance is 342.79 feet.

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* This is desirable as insurance against errors. It is *essential* where the closing error of either traverse approaches a right angle relation to the closing line, since the equality of calculated lengths of the closing lines above and below ground would not disclose fairly large errors of bearings.

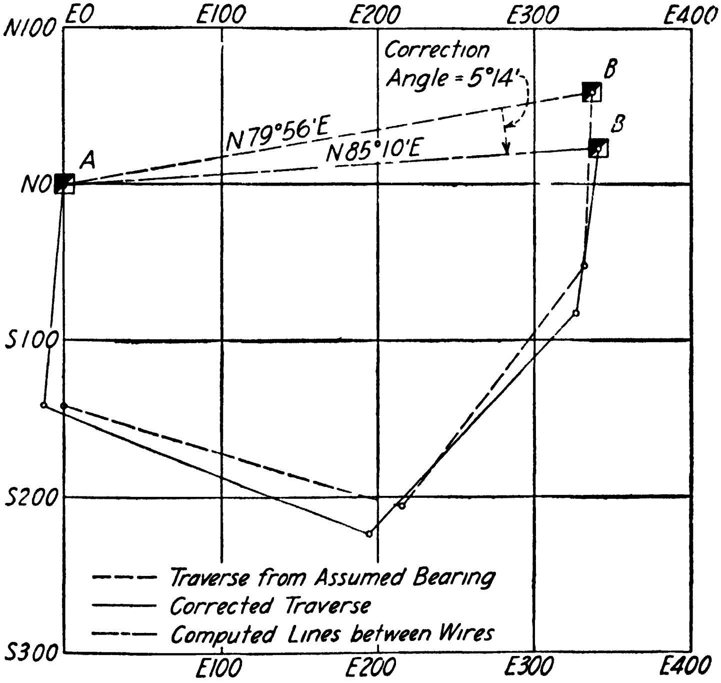


FIG. 499. UNDERGROUND TRAVERSES IN THE TWO-SHAFT METHOD.

OTHER UNDERGROUND SURVEYS

500. Transferring Elevations Down a Shaft. Elevations are taped down a vertical shaft from a point established on the collar and may be taped directly to the lower end if the vertical distance is less than the tape length.

For longer vertical distances, the taping may be conducted along one of the guides for the cage or skip. The positions of the ends of the tape are marked with nails. In precise work, corrections must be made for temperature and for variation from the vertical. In applying the tension correction formula,

FIRST UNDERGROUND TRAVERSE

Bearing from A to Station 1 is assumed as South

B.S.	Inst	F.S.	Hor. Ang.	Bearing	Distance	Lat.	Dep.	Coordinates	
								North	East
	A	1		Assumed South	143.41	− 143.41	0.00	S 143.41	0.00
A	1	2	105° 57′	S 74° 03′ E	224.99	− 61.83	+ 216.33	S 205.24	E 216.33
1	2	3	111° 14′	N 37° 11′ E	192.94	+ 153.72	+ 116.61	S 51.52	E 332.94
2	3	B	145° 10′	N 2° 21′ E	111.53	+ 111.44	+ 4.57	N 59.92	E 337.51
	B	A		S 79° 56′ W	342.79	− 59.92	− 337.51	0.00	0.00
						0.00	0.00		

SECOND UNDERGROUND TRAVERSE, CORRECTED

Correction Angle = 85° 10′ − 79° 56′ = 5° 14′ clockwise

B.S.	Inst	F.S.	Hor. Ang.	Bearing	Distance	Lat.	Dep.	Coordinates	
								North	East
	A	1		S 5° 14′ W	143.41	− 142.81	− 13.08	S 142.81	W 13.08
A	1	2	105° 57′	S 68° 49′ E	224.99	− 81.30	+ 209.79	S 224.11	E 196.71
1	2	3	111° 14′	N 42° 25′ E	192.94	+ 142.44	+ 130.14	S 81.67	E 326.85
2	3	B	145° 10′	N 7° 35′ E	111.53	+ 110.55	+ 14.72	N 28.88	E 341.57
3	B	A		S 85° 10′ W	342.79	− 28.88	− 341.57	0.00	0.00
						0.00	0.00		

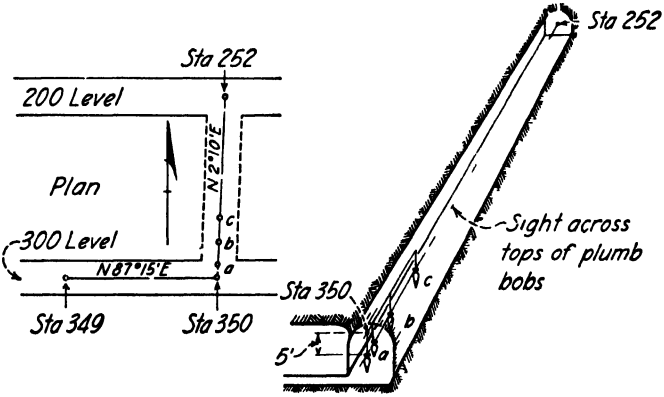
FIG. 499a

one-half of the weight of the tape is to be added to the tension applied at the lower end.

It is frequently possible and convenient to set permanent intermediate bench marks at the various mine levels. Elevations are carried to and away from the shafts by ordinary methods.

501. Bore-hole Surveying. This is a specialized type of surveying requiring special equipment which is used to determine the direction taken by a bore-hole as it departs from the vertical. Unless the organization drills a good many holes exceeding several hundred feet in depth, it will pay to hire the equipment and perhaps the personnel for making and surveying the bore-holes. If extensive work of this nature is contemplated it will be necessary for the engineer in charge to familiarize himself with the references at the end of this chapter and such other literature and experience as he may be able to secure.

502. Underground Connections. The method of setting stations to control the direction of an underground connection is illustrated in Fig. 502 and explained in the following example.



Perspective through Raise

FIG. 502. ALIGNING A RAISE.

EXAMPLE. A raise is to be driven from Station 350 on the 300 level to Station 252 on the 200 level. The center-line of raise is to extend from a point 5 feet below Station 350 to Station 252.

Coordinates of Station 350 are: North 791.20 East 841.15; and its elevation is 1272.11

Coordinates of Station 252 are: North 845.35 East 843.20; and its elevation is 1357.41

The bearing of the line from Station 349 to Station 350 is N 87°15′ E

To find the bearing of the center-line of the raise (five feet below Station 350 to Station 252)

Difference in North coordinates = 54.15

Difference in East coordinates = 2.05

Cot bearing $= \frac{54.15}{2.05} = 26.4146$

Bearing of raise N 2° 10′ E

Horizontal angle to be turned at 350, backsighting 349, is 94° 55′

Horizontal distance from 350 to 252 is $\sqrt{2.05^2 + 54.15^2} = 54.18$

To find the slope of the raise

Elevation of center-line under Station 350 is 1272.11 − 5.00 = 1267.11

Elevation of Station 252 = 1357.41

Difference in elevation = 90.30 Horizontal distance = 54.18

Tan slope $= \frac{90.30}{54.18} = 1.66667$ Slope = 59° 02′

In order to locate the raise, the transit is set up at Station 350, backsighted at Station 349, and turned through a horizontal angle of 94° 55′ to the right.

Locate temporary point *a* several feet from transit. Obtain the elevation of this point and compute the length of plumb line to intersect the center-line of the raise.

By sighting across the plumb line 5 feet below Station 350 and the one suspended from point *a*, the raise can be started. It is probable that Station 350 and point *a* will be blown out by blasting. In that case, Station 350 must be relocated. As soon as the raise has advanced a short distance, points *b* and *c* should be located and the lengths of plumb lines to be hung from these stations determined. Other points may be put in further up the raise if necessary.

503. Geophysical Exploration. The engineer is familiar with the usual drilling, boring, sounding, and testing procedures for sub-surface formations and these are well covered in books on foundations, in Legget's *Geology and Engineering* (Reference 14), and elsewhere. He is not so familiar with the subject covered in this section.

Current literature of this rapidly developing technique is available in the *Engineering Index* and the *Industrial Arts Index* under the head of Geophysics or Geophysical Exploration.

This literature covers mining, oil, ground water, geological, military, and other exploratory uses; determination on the surface of the ground of the location of underground construction such as pipe lines; aids to the civil engineer in construction work such as classification of excavation materials, geological exploration for dams; and the like.

The best summary available of such exploratory methods for the civil engineer is here extracted from an article by C. G. Stipe and Sherwin F. Kelly in *Civil Engineering*, American Society of Civil Engineers, April 1937, pages 264–268 inclusive. The complete article is sufficiently detailed to enable the engineer to decide whether he should secure this specialized service.

"Nearly every civil engineer is frequently confronted with the task of determining the location and nature of subsoil formations. He usually finds it necessary to drill, although this involves considerable expense, and may even require more time than is available. If the desired information can be obtained by geophysical methods, both time and money will be saved. Where local geological conditions favor geophysical methods, the drilling may be confined to checking the crucial geophysical determinations, and to obtaining samples; conversely, the geophysical methods may be used to complete the picture between widely spaced drill holes.

"Although geophysical methods for predicting the position and nature of subsurface formations are not entirely new in America, their application to the service of civil engineers is rather recent — a matter of less than ten years. In the interim development has gone on apace so that now, although it is still young, this field of exploration is well established and used in a variety of construction work These methods relate to the location of rock surfaces for dam, tunnel, harbor, and highway construction; the determination of water-bearing strata, including something of the potability of the water; the prediction of soundness in rock formations; and even the location of sand and gravel deposits. Test pits or borings have checked the accuracy of the findings in many instances. These methods are convenient, owing to the lightness and mobility of the apparatus; also they are rapid and hence economical because they are carried out on the surface. These advantages must commend them to the consideration of civil engineers.

"Differentiations of geological structure by geophysical means can be achieved only where the formations being studied show distinctive differences in certain physical properties. The choice of geophysical method will therefore depend on which properties shown by the formations present sufficiently great variations, as in magnetic permeability, density, electrical resistivity, and elasticity. In certain methods, the geophysicist depends on observations of fields of force natural to the earth itself, and therefore beyond his control to modify. These are the magnetic and gravitational methods. In the former, he uses modified compasses for measuring distortions in the earth's magnetic field — distortions caused by the varying magnetic permeabilities of underlying rock formations. In the gravitational methods, extremely sensitive torsion balances indicate variations in gravitational pull, the result of differing densities of the rocks in the earth's crust.

"The second main category of methods provides the geophysicist with fields of force which he can control at will, and thus suit the scale of operations to the problem in hand. These include seismic and electrical methods. These techniques, especially the electrical ones, have thus far proved the most useful in civil engineering work.

"From the foregoing it is abundantly evident that the geophysicist can render important aid to the civil engineer in many types of construction work. At the present time, the role of geophysics has been principally in the field of dam construction, highway building, and water supply. There are probably other fields, however, to which geophysical methods could be adapted, either by using present techniques, or by developing new procedures applicable to hitherto untried problems.

"It is hoped that civil engineers will consider the possibilities of utilizing the geophysicist and his methods in new problems where it is essential to study the location and character of subsoil formations."

504. Location of Underground Utilities. A pamphlet of this title issued by the American Society of Civil Engineers (Reference 1) covers this subject which is so important in cities. The most recent and comprehensive work of this nature is that of the Cleveland, Ohio, Regional Survey, Underground Division, and publications (Reference 5) of their methods and results may be expected from time to time. It should be indexed in the *Engineering Index*, and perhaps the *Industrial Arts Index*.

505. Mineral Land Surveys. Since this is a specialty, largely under governmental direction, in which only a very limited number of engineers participate and because of its highly specialized requirements in detail, no attempt will be made to treat this subject. Reference to the *Manual of the Survey of the Public Lands of the United States*, and to Underhill's *Mineral Land Surveying* may be sufficient.

PROBLEMS

1. Work problem 9 on page 311.

2. (a) A vein has a strike of S 45° E and a dip of 15° to the Southwest. A tunnel lying in the vein is to be driven on a + 3% grade. What is its bearing? Hint. Work out the trigonometrical relations on a scale drawing as in descriptive geometry. *Ans.* S 51° 26′ E

(b) The same as (a) except for a + 2% grade.
(c) The same as (a) except for a + 4% grade.
(d) The same as (a) except for a − 5% grade.
(e) The same as (a) except for a − 6% grade.
(f) The same as (a) except for a − 7% grade.

3. (a) A coal vein having a strike of S 45° E and a dip of 15° to the Southwest outcrops for several miles along a side hill, but is only exposed at points a mile or more apart. A traverse is to be run as a property boundary along the outcrop and along where the vein produced would outcrop through the overburden. Fill out an abbreviated table giving the vertical angles corresponding to the five given bearings, along which the traverse must be run to follow the vein between outcrop points. Hint. Work out the trigonometrical relations on a scale drawing as in descriptive geometry. The answer to 3(a) is in italics.

Bearing	Vertical Angle
North	*+ 10° 44′*
N 45° E	*+ 15°*
East	*+ 10° 44′*
S 45° E (strike)	*0°*
South	*− 10° 44′*

(b) The same as (a) except for a strike of N 13° E and dip of 10° S.
(c) The same as (a) except for a strike of N 18° W and dip of 20° S.
(d) The same as (a) except for a strike of N 79° W and dip of 25° S.
(e) The same as (a) except for a strike of S 86° W and dip of 30° S.
(f) The same as (a) except for a strike of S 5° W and dip of 35° S.

4. From one of the following sets of transit field data taken with a side telescope, compute the true horizontal and vertical angles as in § 482.

HORIZONTAL ANGLE READINGS		VERTICAL ANGLE READINGS	
First (bubble down)	Second (bubble up)	First (bubble down)	Second (bubble up)
(a) 165° 13′	166° 43′	− 16° 41′	− 16° 49′
(b) 175° 29′	176° 42′	− 13° 17′	− 13° 25′
(c) 179° 14′	180° 24′	− 0° 29′	− 0° 37′
(d) 183° 47′	185° 16′	+ 1° 47′	+ 1° 39′
(e) 186° 54′	187° 03′	+ 2° 18′	+ 2° 10′
(f) 188° 34′	189° 33′	+ 15° 54′	+ 15° 46′

Ans. (a) 165° 58′ and − 16° 45′

5. (a) Three elevations on a vein at *A*, *B*, and *C* are located as follows:

	COORDINATES		ELEVATIONS
	N	E	
A	0′	0′	700′
B	1000′	2000′	500′
C	600′	3000′	1000′

Find the strike and dip of the vein graphically. Precise values are not required in this problem but may be computed laboriously by trigonometry from the drawing. HINT. Use a descriptive geometry drawing to scale, with top, front, and profile (showing the vein as a line) views.

Ans. Strike N 71° E, dip 37°

(b) The same as (a) except that the elevations are 800′, 400′, and 900′.
(c) The same as (a) except that the elevations are 900′, 300′, and 800′.
(d) The same as (a) except that the elevations are 1000′, 200′, and 700′.
(e) The same as (a) except that the elevations are 1100′, 100′, and 600′.
(f) The same as (a) except that the elevations are 1200′, 0′, and 500′.

6. (a) Using the data and the descriptive geometry drawing of problem 5, scale the coordinates and elevation of a point *E* on the vein from which to connect by the shortest shaft with a point *D* on the surface whose coordinates are N 1200′, E 1300′; and whose elevation is 1500′. Precise values are not required in this problem but may be computed laboriously by trigonometry from the drawing.

Ans. N 600′, E 1500′, Elevation 650′

(b) The same as (a) except D has an elevation of 1600′.
(c) The same as (a) except D has an elevation of 1700′.
(d) The same as (a) except D has an elevation of 1800′.
(e) The same as (a) except D has an elevation of 1900′.
(f) The same as (a) except D has an elevation of 2000′.

NOT AS PRACTICAL PROBLEMS BUT TO INDICATE THE NECESSITY OF CAREFUL MANIPULATION AND ADJUSTMENT OF THE TRANSIT, ANSWER QUESTIONS 7, 8, AND 9.

7. (a) If the vertical axis is not vertical by 5 minutes in the plane in which a vertical angle is being measured, what is the angular error remaining after taking the mean of the vertical angles measured with the telescope normal and reversed?
(b) The same as (a) except not vertical by 4 minutes.
(c) The same as (a) except not vertical by 3 minutes.
(d) The same as (a) except not vertical by 2 minutes.
(e) The same as (a) except not vertical by 1 minute.
(f) The same as (a) except not vertical by 6 minutes.

8. (a) If the horizontal axis is not horizontal by 5 minutes, what error (in decimal of a foot) exists in dropping a point 200 feet vertically with the transit, through a vertical angle from +60° to 0°, without reversing the transit? *Ans.* 0.29 foot
(b) The same as (a) except not horizontal by 4 minutes.
(c) The same as (a) except not horizontal by 3 minutes.
(d) The same as (a) except not horizontal by 2 minutes.
(e) The same as (a) except not horizontal by 1 minute.
(f) The same as (a) except not horizontal by 6 minutes.

9. (a) If the line of sight makes an angle of 89° 55′ with the horizontal axis, what error (in decimal of a foot) exists in dropping a point 400 feet (slope distance) from the transit through a vertical angle from +60° to 0°, without reversing the transit? *Ans.* 0.29 foot
(b) The same as (a) except an angle of 89° 56′.
(c) The same as (a) except an angle of 89° 57′.
(d) The same as (a) except an angle of 89° 58′.
(e) The same as (a) except an angle of 89° 59′.
(f) The same as (a) except an angle of 89° 54′.

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CHAPTER XXX

CONSTRUCTION SURVEYS

506. General. Construction surveys and supervision cost several per cent of the total construction budget, and are proportionately important. Such surveys follow those needed for location and design, and immediately precede or accompany actual construction on the ground. Rather complete maps and occasionally aerial photographs are available at this stage of most projects, showing contours where necessary — to smaller scales for general planning purposes and for acquiring real estate, and to larger scales for the limited areas where actual construction is undertaken. Scales of 1 inch = 40 feet, 1 inch = 200 feet, and 1 inch = 400 feet are convenient for maps, since in those cases both engineers' and architects' scales may be used.

Many maps, profiles, and sectional views are prepared covering various features of construction. It is noteworthy that most projects extending over a considerable area are laid out on a coordinate system. Ackerman and Stubbs (References 1 and 8) show many photographs, plans, and sections which indicate the surveying activities of the construction engineer.

507. Other Engineering Associated with Construction Surveys. Those engineers who set construction stakes also usually inspect and supervise construction, and Chapters XII, XIII, XIV, and XV in Rubey's *Route Surveys*, Macmillan, 1940, are devoted to these associated subjects under the headings of cross-sectioning, setting slope stakes, precision in measurements, computing end areas, methods of computing volumes, excavation tables and diagrams, embankment openings, curvature correction, borrow pits, computation of grading over an area — or borrow-pit method, shrinkage and swell, settlement, classification of excavation, grading equipment and methods, haul, overhaul, station to station method, economic limit of haul, mass diagram, influence of rights of way and easements, rights of way, legal rights of surveyors, ordering materials, staking appurtenant structures, change orders, inspection, contracts, monthly estimates, progress reports, final estimate, summary of requirements of supervision of construction, operation after construction, management and administration, allowable bearing power of soils, bearing power of piles, strength of building materials, concrete data, waterway size for culverts or bridges, flow of water in pipes and open channels, flow of water over weirs, conversion factors, weights of materials, and cost of various constructions.

The extent of construction supervision may cover maintaining as many resident engineers and inspectors on the work as may be necessary to secure proper results; giving all necessary lines and grades; inspecting and testing material and workmanship; checking all shop drawings and lists of materials furnished by the contractor; preparing additional details as circumstances arising during the progress of the work may require; preparing monthly and final estimates of work done as a basis of partial and final payments to the contractor; maintaining contact between the home office and the resident engineers through weekly progress reports, mail and telephone or telegraph; visits to the work by representatives of the home office at average intervals of not less than a month (once per week, once per 60 days, or any other period that may seem adequate under the circumstances); preparing a set of "record" plans showing the work as actually completed; preparing a final report containing a history of the work and general instructions for its maintenance and operation; and monumenting the survey for permanent reference.

Legal questions regarding title to property, compliance with ordinances, building permits, various types of insurance, and the like are frequently encountered during the construction period.

While borings and subsurface geological exploration (§ 503, and pages 51 and 175 of Rubey's *Route Surveys*) are usually conducted prior to construction, they occasionally continue along with the construction surveys.

Estimates of quantities of construction are usually made beforehand, periodically (perhaps monthly), and upon completion of the work. These frequently are based on surveys —

estimated roughly for progress estimates, and accurately for final estimates. Progress charts are prepared, unit costs are estimated, and payments to the contractor are made from these surveys. Figs. 507, 507a, and 507b illustrate a typical monthly estimate, a final estimate, and a progress report, respectively. While these vary greatly in form on different projects, particular examples are nevertheless shown, more to call attention to their general style and necessity rather than as samples to be followed.

Field notes, aerial and ground photographs, construction survey maps, and a diary are valuable records when questions or disputes arise later. Field record drawings, usually called *accessions* or *record plans*, form a part of the permanent record of construction as it is finally designed and built. This matter is frequently handled by the field party and is in addition to the preparation and filing of construction survey maps.

508. Necessary Staking. At the beginning of construction, the project usually has been located and staked on the ground, but these stakes were not set primarily for construction purposes and many of them will be missing. The essential stakes must be found or re-set.

Construction preliminary to the main project such as roads, railroad connections, docks, construction buildings, construction plants, storage yards, and the like is undertaken next. Since this is usually temporary and of secondary importance, it is sometimes sufficient to provide a competent and experienced construction foreman with a map from which he may lay out such work. In other cases, it may be advisable for the engineer to stake these construction facilities even though he uses rapid and approximate methods, such as staking a secondary road by setting flagged lath several hundred feet apart on a grade contour with a clinometer clamped on the proper grade angle, or such as setting approximate corner stakes to show the location of temporary buildings and allowing the foreman to stake them more accurately. It is increasingly necessary for the engineer accurately to stake preliminary construction, because of the increasing use of complex equipment — often especially designed for a particular job.

Contract Voucher

THE A, B AND C COMPANY

Contract No 42

TO John Doe Construction Co Dr

Monthly Estimate No 1

920 E Adams, Chicago, Illinois

To October, 25, 1949

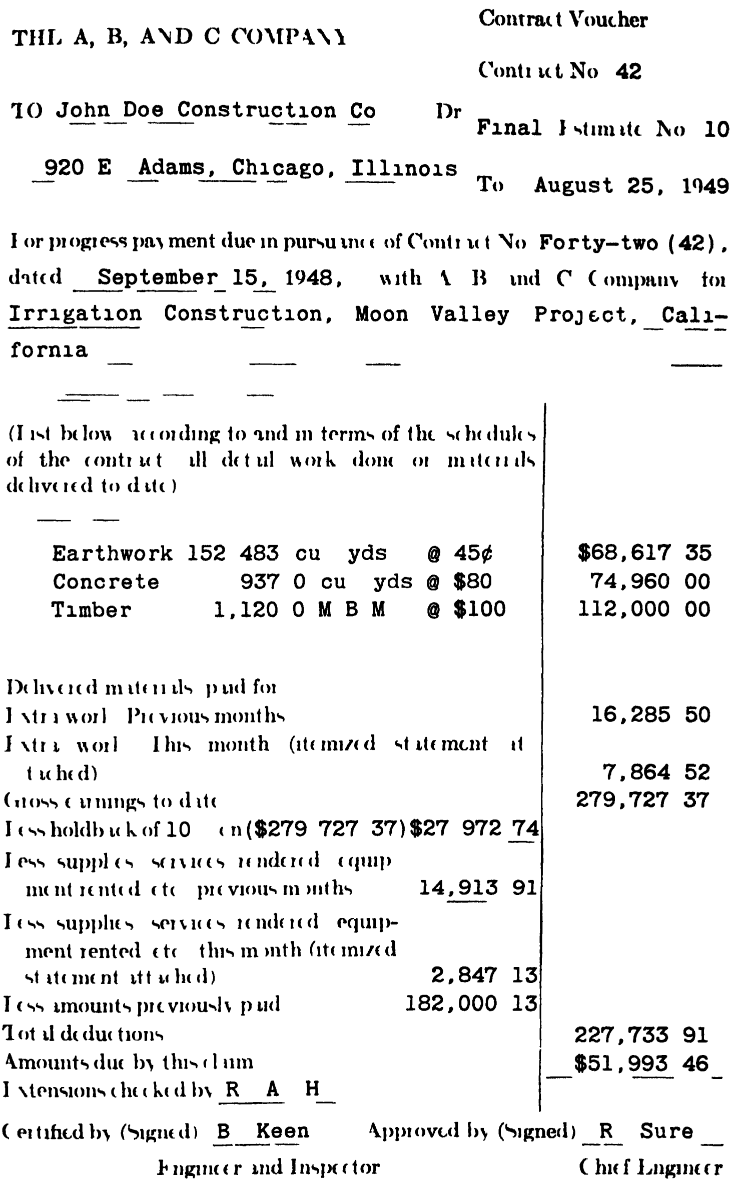
For progress payment due in pursuance of Contract No Forty-two (42) dated September 15 1948, with A B and C Company for Irrigation Construction Moon Valley Project, California

(List below according to and in terms of the schedules of the contract all detail work done or materials delivered to date)		
Earthwork 23,287 cu yds @ 45¢		$10,479 15
Concrete 146 cu yds @ $80		11,680 00
Timber 235 M B M @ $100		23,500 00
Delivered materials paid for		2,846 13
Extra work Previous months		
Extra work This month (itemized statement attached)		5,280 96
Gross earnings to date		53,786 24
Less holdback of 10% on ($53,786 24)	$5,378 62	
Less supplies services rendered equipment rented, etc, previous months		
Less supplies services rendered, equipment rented, etc this month (itemized statement attached)	1,826 31	
Less amounts previously paid		
Total deductions	7,204 93	7,204 93
Amounts due by this claim		$46,581 31
Extensions checked by R A H		

Certified by (Signed) B Keen Approved by (Signed) R Sure

Engineer and Inspector Chief Engineer

FIG 507 MONTHLY ESTIMATE



THE A, B, AND C COMPANY

Contract Voucher

Contract No 42

TO John Doe Construction Co Dr

Final Estimate No 10

920 E Adams, Chicago, Illinois

To August 25, 1949

For progress payment due in pursuance of Contract No Forty-two (42), dated September 15, 1948, with A B and C Company for Irrigation Construction, Moon Valley Project, California

(List below according to and in terms of the schedules of the contract all detail work done or materials delivered to date)		
Earthwork 152 483 cu yds @ 45¢		$68,617 35
Concrete 937 0 cu yds @ $80		74,960 00
Timber 1,120 0 M B M @ $100		112,000 00
Delivered materials paid for		
Extra work Previous months		16,285 50
Extra work This month (itemized statement attached)		7,864 52
Gross earnings to date		279,727 37
Less holdback of 10% on ($279 727 37)	$27 972 74	
Less supplies services rendered equipment rented etc previous months	14,913 91	
Less supplies services rendered equipment rented etc this month (itemized statement attached)	2,847 13	
Less amounts previously paid	182,000 13	
Total deductions		227,733 91
Amounts due by this claim		$51,993 46
Extensions checked by R A H		

Certified by (Signed) B Keen
Engineer and Inspector

Approved by (Signed) R Sure
Chief Engineer

FIG 507a FINAL ESTIMATE

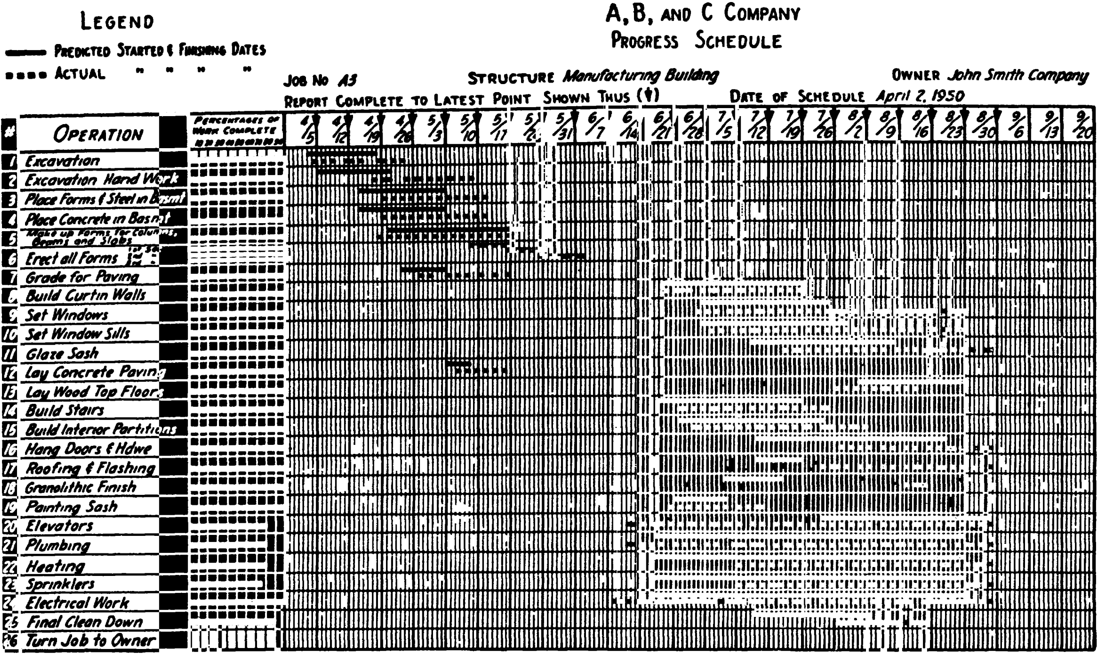


FIG. 507b. PROGRESS REPORT.

For the more important permanent construction, it is essential for the engineer to set a sufficient number of accurate stakes giving line and grade. Too many stakes may be confusing and wasteful of the engineer's time; too few may permit errors of more or less importance to creep into the work. The number of stakes to be set and the plan of staking should be discussed with the construction superintendent or foreman so as to make them of maximum value to him.

In general, the engineer should set all stakes which cannot be properly or accurately set by the superintendent or foreman, and in addition he should set those which his survey party can place more economically than can the foreman with his less well-trained men. The amount of time which the engineer or foreman can spare for such work must also be considered.

Field notes are kept in so many different forms that it would be impractical or misleading to attempt to show them. Most of the larger organizations have standard forms which must be followed closely so that any member of the organization can easily and correctly interpret the notes.

509. Checking All Previous Surveys. The first step in construction surveys is to check all lines, dimensions, elevations, and property boundaries that have been established previously, frequently by other engineers. This may sometimes be done as construction proceeds, but it should not be delayed long enough to permit construction to progress erroneously. It is usually possible to find and identify some of the stakes previously set on the project. Special care must be given to checking bench marks and to establishing additional ones convenient to the construction. All important points must be referenced and monumented. Discrepancies and misunderstandings are often discovered at this time.

510. Avoidance of Disturbance. Stakes must be set in such locations and with sufficient permanence so as to last until construction is completed. It may be desirable to leave the project permanently monumented so as to serve for maintenance, operation, and future expansion.

Of course the stakes should be subject to a minimum of disturbance by excavation and other construction operations, and they *must* be referenced to other nearby stakes or monuments which will not be disturbed. Care must be exercised that stakes will not be found to be behind or under piles of construction materials or otherwise inaccessible as construction progresses.

The main survey lines *must* be maintained in their original position without fail, since if once lost it is difficult to reestablish them so as perfectly to fit the partly completed construction.

It is usually necessary to set stakes on offset lines to avoid the disturbance just mentioned, or perhaps because the actual line is inaccessible as might be the case when the construction is in a stream but close to shore.

While wooden hubs with guard stakes suffice for temporary use, there is an increasing trend toward permanent pipe or concrete monuments sufficiently long to prevent heaving from frozen ground. Drill holes, crows' feet, or crosses on rock or on permanent masonry structures serve excellently for permanent reference points.

511. Elevations. Elevations are taken from bench marks near the job, often set especially for the construction surveys. If at all possible, they should be based on the United States datum, or at least on the generally recognized datum of the community, *not* on a special job datum. They must be free from disturbance, and should occasionally be checked against one another as insurance against settlement or disturbance. It is good practice to set them in pairs so that backsights may be taken on two bench marks when the level is set up at the construction site. Grades may be set on

The top edge of batter boards (see page 278)

Tops of stakes driven to grade, or sawed off to grade — sometimes in a hole below the ground

Some even foot or half foot below or above the top of stake — so marked on the stakes

Nails, notches, or keel marks on the sides of stakes and

Nails inside of concrete forms.

The nearest tenth of a foot is sufficiently precise for rough earthwork, although finish stakes are frequently set to hundredths. The nearest hundredth of a foot suffices for most structures. Inches and eighths of inches are rarely used in engineering leveling, but vertical dimensions are frequently marked on plans and given to workers in these units.

SURVEYS FOR VARIOUS TYPES OF CONSTRUCTION

Experience has demonstrated those methods of construction staking that are most satisfactory for the usual forms of construction. The more important of these will be mentioned, and modifications will suggest themselves which may be adapted to other types of construction.

512. Route Surveys. Methods of construction surveys for railways, highways, transmission lines, pipe lines, canals, and so on are described on pages 161 to 241 of Rubey's *Route Surveys*, Macmillan, 1938. Other data that are useful in the field during the construction period are also given in this reference. Most highway departments and many other organizations specify methods in their manuals.

513. Tunnels. See Chapter XXIX. Underground Surveys, the surface triangulation for tunnel construction on page 191, and reference 2.

514. Sewers and Pipe Lines. The center lines of sewers or pipe lines are established as in other route surveys. Small hubs, spikes, or painted marks are offset a fixed distance from the center line—only so far as construction requires—and are driven flush with the ground or pavement. A guard stake is set alongside the hub on the far side of which the station and the offset is marked, while on the near side the cut is marked to the nearest .01 foot. Stakes are usually set fifty feet apart. Sewers are laid accurately to grade—pipe lines less accurately and with fewer stakes and batter boards.

If excavation is by hand, or after machine excavation is completed, it is customary to establish batter boards on the center line over the sewer trench (Fig. 514) with a chalk line or wire stretched between them on the center line and at some even number of feet above the inside of the bottom (the invert) of the sewer. A measuring stick is provided for the excavators and pipe layers. Batter boards are seldom necessary for pipe lines since they are not often laid to exact grade.

Quantities of earth and rock excavation, and lengths of pip laid, are finally computed from the surveys; and plans shoul be altered to show the adjacent underground utilities that were encountered during the work, as mentioned in § 504.

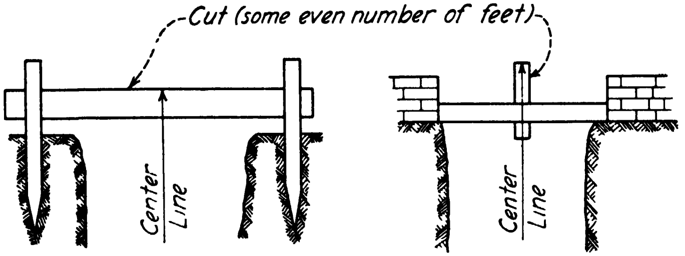


FIG. 514. BATTER BOARDS OVER A SEWER TRENCH.

515. Streets. Preliminary locations and profiles are made as in route surveys, upon the basis of which the location and grad of the street are established so as to give the maximum service as a street, and at the same time to be of greatest benefit to the adjoining property. Where the street is to be curbed, curb stakes are offset some two feet back from the face of the curb and driven to grade. A tack in the top of the stake shows the exact alignment. Special stakes are set for the curbs at street and alley intersections, for curb openings into private driveways, for storm sewer inlets, and other special curb construction.

Sidewalks may be placed by the construction foreman with reference to the curb stakes or to the already constructed curb. Should separate stakes be needed for sidewalks, they are usually offset one foot toward the property from the sidewalk, tacked, and driven to grade in a manner similar to those set for curbs.

For the street proper, it is sometimes necessary to set small stakes or pegs between the two curbs as a guide for grading operations, although more frequently the foreman can use a templet set between tops of the curbs, or he can stretch a chalk line from one curb to the other and conduct the grading accordingly. Rough excavation is staked as local conditions require. Often the engineer sets finish excavation pegs some 50 feet apart along the center line of the street and also sometimes half way between the center line and the curbs. These pegs are driven with their tops at the finished sub-grade. Later, another set of such pegs may be driven into the finished sub-grade with their tops at the elevation of the finished pavement, although frequently the pavement is finished with a templet set on temporary or permanent curbs or from a chalk line stretched between curbs.

An important and intricate feature of street design occurs at the intersections of streets where warped surfaces must be provided for drainage and for adjusting the differences in elevations arising from the differing grades of the intersecting streets. The reader is referred to the old but unexcelled article, *Street Grades at Intersections*, by A. S. Tuttle in Engineering Record, October 16, 1909, page 429, for details of the design of street intersections.

516. Anchor Bolts. Anchorage into concrete foundations is usually provided by steel bolts or angles set in the concrete with the bottoms hooked or otherwise anchored, and with the tops extending above the concrete. Since the superstructure, usually steel, allows for but little error in the position of the bolts or angles, perhaps not more than 1/8 or 1/4 inch, it is necessary to set them with great care.

In the usual practice, a wooden or steel template is so made that the bolts or angles may hang through vertical holes in the template into the space which is to be filled with concrete. The template is set in position with a transit and level, or in less important cases with a mechanic's square and level.

In order that the upper end of the anchor may be sprung for perhaps a half-inch in any horizontal direction, it is sometimes permissible to place a small thin pipe of perhaps two-inch diameter around the upper portion of the anchor whose lower portion is fully imbedded and anchored in the concrete. After the superstructure has been placed, the holes are filled with cement grout; and at the same time grout is poured between the concrete foundation and the superstructure.

Such procedures are used in bridge piers, column footings, transmission-line tower foundations, machinery foundations, and the like.

517. Bridge Sites. The surveys for the sites of smaller bridges and culverts is covered in Rubey's *Route Surveys*, pages

216 and 217; but more elaborate surveys are required for the larger structures. A site map is prepared on a scale of perhaps 1 inch = 100 feet with a contour interval of two feet or five feet depending upon the steepness of the slopes. Most large organizations such as highway departments or railways furnish a printed form upon which pertinent information regarding the proposed bridge is entered. Photographs are usually helpful. Information is assembled, on the ground so far as possible, regarding the characteristics of the stream and site (or of intersecting roadways and the traffic in the case of grade separation structures), and a recommendation is made of the type of structure needed.

The *Bridge Survey Report* of the Missouri State Highway Department consists of a five-page printed form. Extracts from the instructions regarding this form are here quoted.

"The purpose of a bridge survey is to establish three important points: the general dimensions of the structure (length, height, skew, and arrangement of spans); the type, size, and depth of foundation; and the cost of construction. These three points are very intimately related to the required waterway. A restricted waterway means serious scour, and footings must extend deep or be very substantially founded. There is no exact method of determining the proper waterway. Judgment must be based upon the following approximations: Comparison with existing bridges, computation by formula of the discharge of the natural channel, and estimate of the run-off from the drainage area. . . .

"In addition to this report, the survey drawings shall show the following:

"1. An accurate profile of the crossing along the proposed center line of the road, extended on each side of the channel to the limits of extreme high water. That particular portion of the crossing to be occupied by the bridge shall be shown at equal horizontal and vertical scales not smaller than 1 inch = 20 feet, preferably 1 inch = 10 feet. The entire flood valley is not required on this scale.

"2. Extreme and ordinary high water levels; extreme low water level.

"3. A profile along the stream bed extending at least 1,000 feet each way from the bridge site. This should be along the thread of the channel and should show all holes in the stream bed, and all shoals and rapids. This is particularly important for small areas where culverts will be used. For larger areas having perennial streams, it will be sufficient to show the profile of the water surface.

"4. Contour map of the site extending at least 1,000 feet up and down stream from the crossing and covering the whole width of valley submerged by high water. If necessary to properly show bends in the channel, extend the meander lines beyond the 1,000-foot limit. Show also the present road and the proposed center line. Show enough of the valley in any case to determine the direction of flood currents.

"5. Location and elevation of at least two bench-marks. North point. Direction of flow.

"6. Complete details of existing substructures which it may be proposed to use in place. If present structure is to be extended, show complete details of superstructure and substructure at points of attachment.

"7. A profile across the valley between limits of extreme high water. Profile to be at right angles to stream and valley, near bridge site, and preferably where valley is constricted, so that little or no overflow is present. If center-line profile meets these requirements this will be sufficient,—so state on survey. If valley section is very far removed from center-line, care should be used to ascertain the correct high water mark at that point. If any doubt exists as to high water marks reported to the survey party, give location and elevation of all such marks and in every case give dates of occurrence. Too much care cannot be used in determining these data."

The map, profiles, and information are then turned over to the bridge designer who lays out the structure on the map and profile, after which it is returned to the field engineer for staking prior to construction. The setting of the construction stakes is described on pages 278 and 279 and on page 217 of Rubey's *Route Surveys*.

Piers for bridges are often located far off-shore where staking by direct methods is impossible. In such cases it is necessary to establish one or more base lines on shore and to triangulate the pier location from these base lines. Reference marks are established by triangulation on the piling or on the cofferdams. Where the piers are at a considerable distance off-shore or where the bridge spans are long, these surveys must be executed with a precision in distance of from 1 50,000 upward. It would be misleading to show a figure for such triangulation since they vary greatly to fit the adjacent shore conditions. A recent and intricate triangulation for the Tacoma Narrows Bridge is described in *Civil Engineering* March 1941, pages 143–146.

It is frequently necessary to establish permanent reference points and bench marks which may be used in subsequent surveys and for determining possible settlement or other movement.

518. Buildings. Site maps for buildings such as Fig. 518 are prepared for the architect on a large scale such as 1 inch = 40 feet or larger, with elevations and sometimes with contours thereon. It is, of course, necessary that the lot corners be carefully and permanently established by a competent local land surveyor.

After the architect has located the proposed building on the map, it is staked by means of batter boards and reference stakes as described on pages 278 and 279 of this volume, and on page 216 of Rubey's *Route Surveys.*

For larger buildings, it is necessary that stakes be set inside of the building following rough excavation, so as to locate foundations for columns, machinery, or other heavy construction. Elevations are established and marked on these stakes by use of the engineers' level. For large footings, it may be necessary to set batter boards inside of the main building. Lines may be carried inside the building through doors or windows, and stakes outside of the building should be located with this in mind.

Where forms for concrete are used, it is customary to drive nails around the inside of these forms at required elevations such as the top of the concrete.

In large and important structures, it is necessary continuously to check major portions of the building by use of the transit, tape, and engineers' level.

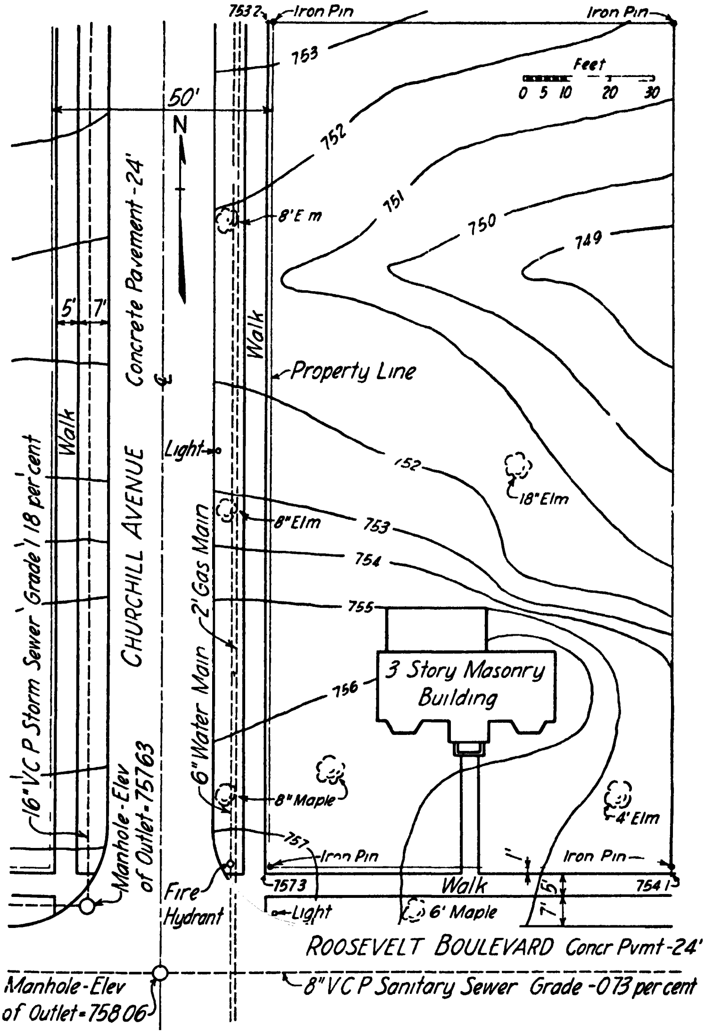


FIG. 518. SITE MAP.

Vertical lines are maintained through the construction period by means of plumb bobs or by use of the engineers' transit. The verticality of the vertical axis of the transit should be obtained with the telescope bubble, where precision is necessary.

519. Groups of Buildings and Industrial Plants. Three horizontal control systems for locating buildings and associated improvements are commonly used for real estate developments, industrial sites, and landscaping, namely —

1. Street lines, either property lines or center-lines
2. Railroad center-lines
3. Coordinated rectangles, sometimes called checkerboard.

In the latter case, it is best to set hubs, with elevations thereon, in rectangles with sides of 100 or 200 feet over the entire site, and to drive these stakes flush with the ground and as permanently as necessary. The stakes may be used as defining rectangular coordinates by which landscaping and the various structures such as roads, railroads, utilities, docks, wharves, and buildings may be designed on a contour map, and staked on the ground. See §§ 209 and 351 of this volume, and pages 188 to 192 of Rubey's *Route Surveys.*

The site map, drawn to a scale of perhaps 1 inch = 40 feet and with a one-foot or two-foot contour interval for relatively level ground, is then prepared showing all improvements and other features of interest together with the coordinate lines corresponding to the staked rectangles, if that method was used.

After the project has been designed and established on the map, it is transferred therefrom and staked on the ground. It is desirable to leave adequate permanent monuments and bench marks for subsequent uses.

520. Reservoirs and Dams. Frequently, United States Geological Survey Quadrangle maps covering the site of the reservoir are available upon which preliminary studies and estimates of quantities and costs may be made, and upon which alternative sites for the dam may be located tentatively. Aerial photographic maps are useful in the early stages of planning, for locating the shore line of a reservoir, and sometimes for acquiring property.

Maps with scales of the order of 1 inch = 40 feet are prepared for each of the alternative dam sites upon which topography, borings, and other exploratory work are shown.

At the same time or later, a topographic map of the reservoir site is prepared on a scale of perhaps 1 inch = 400 feet or more that shows the high and low water levels of the reservoir on which the required property may be selected tentatively.

Later, when the dam site and high water elevation have been determined definitely, it is customary to run levels and a traverse somewhat above this elevation around the reservoir and to tie this traverse to all available adjacent property corners as well as to any control system covering the reservoir site. The necessary property is acquired from this survey. Bench marks are established around the reservoir above high water and particularly at the site of the dam. The area to be cleared is designated by paint marks on trees and otherwise, and the storage capacity of the reservoir may be computed from the map of this survey.

For the dam itself, it is desirable to locate and monument the center-line and the centers of curvature of the dam in such a way as to assure their permanency during and after construction. Where possible, these are set well above the top of the dam where they are less subject to disturbance or submergence. A large scale cross-section along the center-line of the dam is prepared upon which the borings, geologic formations, and elevation of the dam are shown. The ground line cross-section is extended well above the top of the dam in order to facilitate the design of cableways or other construction equipment.

521. Quantity Surveys and Bidding. This important phase of construction engineering should be mentioned since it is definitely associated with construction surveys.

The preparation of a detailed estimate of quantities, the quantity survey, is necessary for nearly all construction projects. One is prepared by the owner's engineers, and another by each contractor bidding on the work. In English practice, though rarely in this country, specialized firms of engineers not otherwise associated with the work make the quantity survey, and their quantities are used by all contractors who bid.

At intervals during construction, and after the work is completed, the quantities usually are computed again as a basis for payment to the contractor, as in Figs. 507 and 507a.

It is probable that the best line of advancement for the young construction engineer is that associated with bidding, which in turn is based on quantity surveys. Bidding involves the preparation of proposals for new work, selection of equipment, selection and computation of costs, sub-letting of certain parts of the work, and many other construction activities, in the course of which the engineer develops construction judgment and makes many desirable contacts with clients and with officials of his organization. It is therefore desirable that every opportunity be utilized to secure experience and responsibility along these lines.

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LOGARITHMIC AND TRIGONOMETRIC TABLES

TABLE I

COMMON LOGARITHMS OF NUMBERS

FROM

1 TO 10 000

TO

FIVE DECIMAL PLACES

1 — 100

N	Log	N	Log	N	Log	N	Log	N	Log
0	———	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
N	**Log**	**N**	**Log**	**N**	**Log**	**N**	**Log**	**N**	**Log**

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	44	**43**	**42**
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	**34**	**33**
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	**31**	**30**
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25	24
1	2.5	2.4
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

	23	22
1	2.3	2.2
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

	21
1	2.1
2	4.2
3	6.3
4	8.4
5	10.5
6	12.6
7	14.7
8	16.8
9	18.9

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

log 2
=.30102 99957

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20	19
1	2.0	1.9
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	18	**17**
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

M
$=\log_{10} e$
$=\log_{10} 2.718\cdots$
$=.43429\ 44819$

	16	**15**
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
3	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	14
1	1.4
2	2.8
3	4.2
4	5 6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

log 3
=.47712 12547
log π
=.49714 98727

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	612	654	666	678	691	703	715	727	739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	415	426	438	449	461	473	484	496	507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	546	557	569	580	591	602	614	625	636	647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	660	671	682	693	704	715	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	13	**12**
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	**10**
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885	895	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	11	**10**
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

log *M*
=log [log *e*]
=9.63778 431
−10

	9
1	0.9
2	1.8
3	2.7
4	3 6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Prop. Pts.

450 — Logarithms of Numbers — 500

N.	0	1	2	3	4	5	6	7	8	9
450	65 321	331	341	350	360	369	379	389	398	408
451	418	427	437	447	456	466	475	485	495	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725	734	744	753	763	772	782	792
455	801	811	820	830	839	849	858	868	877	887
456	896	906	916	925	935	944	954	963	973	982
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66 087	096	106	115	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	276	285	295	304	314	323	332	342	351	361
461	370	380	389	398	408	417	427	436	445	455
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	745	755	764	773	783	792	801	811	820	829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	*006	*015
468	67 025	034	043	052	062	071	080	089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	669	679	688	697	706	715	724	733	742	752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	461	469	478	487	496	504	513	522	531	539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	log 5
506	415	424	432	441	449	458	467	475	484	492	=.69897 00043
507	501	509	518	526	535	544	552	561	569	578	
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	
564	128	136	143	151	159	166	174	182	189	197	
565	205	213	220	228	236	243	251	259	266	274	
566	282	289	297	305	312	320	328	335	343	351	
567	358	366	374	381	389	397	404	412	420	427	
568	435	442	450	458	465	473	481	488	496	504	
569	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	
585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
588	938	945	953	960	967	975	982	989	997	*004	
589	77 012	019	026	034	041	048	056	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	
608	390	398	405	412	419	426	433	440	447	455	
609	462	469	476	483	490	497	504	512	519	526	
610	533	540	547	554	561	569	576	583	590	597	
611	604	611	618	625	633	640	647	654	661	668	
612	675	682	689	696	704	711	718	725	732	739	
613	746	753	760	767	774	781	789	796	803	810	
614	817	824	831	838	845	852	859	866	873	880	
615	888	895	902	909	916	923	930	937	944	951	
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	
623	449	456	463	470	477	484	491	498	505	511	
624	518	525	532	539	546	553	560	567	574	581	
625	588	595	602	609	616	623	630	637	644	650	
626	657	664	671	678	685	692	699	706	713	720	
627	727	734	741	748	754	761	768	775	782	789	
628	796	803	810	817	824	831	837	844	851	858	
629	865	872	879	886	893	900	906	913	920	927	
630	934	941	948	955	962	969	975	982	989	996	
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	
638	482	489	496	502	509	516	523	530	536	543	
639	550	557	564	570	577	584	591	598	604	611	
640	618	625	632	638	645	652	659	665	672	679	
641	686	693	699	706	713	720	726	733	740	747	
642	754	760	767	774	781	787	794	801	808	814	
643	821	828	835	841	848	855	862	868	875	882	
644	889	895	902	909	916	922	929	936	943	949	
645	956	963	969	976	983	990	996	*003	*010	*017	
646	81 023	030	037	043	050	057	064	070	077	084	
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	
662	086	092	099	105	112	119	125	132	138	145	
663	151	158	164	171	178	184	191	197	204	210	
664	217	223	230	236	243	249	256	263	269	276	
665	282	289	295	302	308	315	321	328	334	341	
666	347	354	360	367	373	380	387	393	400	406	
667	413	419	426	432	439	445	452	458	465	471	
668	478	484	491	497	504	510	517	523	530	536	
669	543	549	556	562	569	575	582	588	595	601	
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	
685	569	575	582	588	594	601	607	613	620	626	
686	632	639	645	651	658	664	670	677	683	689	
687	696	702	708	715	721	727	734	740	746	753	
688	759	765	771	778	784	790	797	803	809	816	
689	822	828	835	841	847	853	860	866	872	879	
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	log 7
705	819	825	831	837	844	850	856	862	868	874	=.84509 80400
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	
708	85 003	009	016	022	028	034	040	046	052	058	
709	065	071	077	083	089	095	101	107	114	120	
710	126	132	138	144	150	156	163	169	175	181	
711	187	193	199	205	211	217	224	230	236	242	
712	248	254	260	266	272	278	285	291	297	303	
713	309	315	321	327	333	339	345	352	358	364	
714	370	376	382	388	394	400	406	412	418	425	
715	431	437	443	449	455	461	467	473	479	485	
716	491	497	503	509	516	522	528	534	540	546	
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	
723	914	920	926	932	938	944	950	956	962	968	
724	974	980	986	992	998	*004	*010	*016	*022	*028	
725	86 034	040	046	052	058	064	070	076	082	088	
726	094	100	106	112	118	124	130	136	141	147	
727	153	159	165	171	177	183	189	195	201	207	
728	213	219	225	231	237	243	219	255	261	267	
729	273	279	285	291	297	303	308	314	320	326	
730	332	338	344	350	356	362	368	374	380	386	
731	392	398	404	410	415	421	427	433	439	445	
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	
737	747	753	759	764	770	776	782	788	794	800	
738	806	812	817	823	829	835	841	847	853	859	
739	864	870	876	882	888	894	900	906	911	917	
740	923	929	935	941	947	953	958	964	970	976	
741	982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87 040	046	052	058	064	070	075	081	087	093	
743	099	105	111	116	122	128	134	140	146	151	
744	157	163	169	175	181	186	192	198	204	210	
745	216	221	227	233	239	245	251	256	262	268	
746	274	280	286	291	297	303	309	315	320	326	
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
749	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	
765	366	372	377	383	389	395	400	406	412	417	
766	423	429	434	440	446	451	457	463	468	474	
767	480	485	491	497	502	508	513	519	525	530	
768	536	542	547	553	559	564	570	576	581	587	
769	593	598	604	610	615	621	627	632	638	643	
770	649	655	660	666	672	677	683	689	694	700	
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	
783	376	382	387	393	398	404	409	415	421	426	
784	432	437	443	448	454	459	465	470	476	481	
785	487	492	498	504	509	515	520	526	531	537	
786	542	548	553	559	564	570	575	581	586	592	
787	597	603	609	614	620	625	631	636	642	647	
788	653	658	664	669	675	680	686	691	697	702	
789	708	713	719	724	730	735	741	746	752	757	
790	763	768	774	779	785	790	796	801	807	812	
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
809	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
811	902	907	913	918	924	929	934	940	945	950	
812	956	961	966	972	977	982	988	993	998	*004	
813	91 009	014	020	025	030	036	041	046	052	057	
814	062	068	073	078	084	089	094	100	105	110	
815	116	121	126	132	137	142	148	153	158	164	
816	169	174	180	185	190	196	201	206	212	217	
817	222	228	233	238	243	249	254	259	265	270	
818	275	281	286	291	297	302	307	312	318	323	
819	328	334	339	344	350	355	360	365	371	376	
820	381	387	392	397	403	408	413	418	424	429	
821	434	440	445	450	455	461	466	471	477	482	
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	
832	92 012	018	023	028	033	038	044	049	054	059	
833	065	070	075	080	085	091	096	101	106	111	
834	117	122	127	132	137	143	148	153	158	163	
835	169	174	179	184	189	195	200	205	210	215	
836	221	226	231	236	241	247	252	257	262	267	
837	273	278	283	288	293	298	304	309	314	319	
838	324	330	335	340	345	350	355	361	366	371	
839	376	381	387	392	397	402	407	412	418	423	
840	428	433	438	443	449	454	459	464	469	474	
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

N.	0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	973	978	983	988
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039
852	93 044	049	054	059	064	069	075	080	085	090
853	095	100	105	110	115	120	125	131	136	141
854	146	151	156	161	166	171	176	181	186	192
855	197	202	207	212	217	222	227	232	237	242
856	247	252	258	263	268	273	278	283	288	293
857	298	303	308	313	318	323	328	334	339	344
858	349	354	359	364	369	374	379	384	389	394
859	399	404	409	414	420	425	430	435	440	445
860	450	455	460	465	470	475	480	485	490	495
861	500	505	510	515	520	526	531	536	541	546
862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	702	707	712	717	722	727	732	737	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	027	032	037	042	047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	211	216	221	226	231	236	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	325	330	335	340	345
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
881	498	503	507	512	517	522	527	532	537	542
882	547	552	557	562	567	571	576	581	586	591
883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665	670	675	680	685	689
885	694	699	704	709	714	719	724	729	734	738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905	910	915	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
891	988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95 036	041	046	051	056	061	066	071	075	080
893	085	090	095	100	105	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	182	187	192	197	202	207	211	216	221	226
896	231	236	240	245	250	255	260	265	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	424	429	434	439	444	448	453	458	463	468
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
911	952	957	961	966	971	976	980	985	990	995	
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96 047	052	057	061	066	071	076	080	085	090	
914	095	099	104	109	114	118	123	128	133	137	
915	142	147	152	156	161	166	171	175	180	185	
916	190	194	199	204	209	213	218	223	227	232	
917	237	242	246	251	256	261	265	270	275	280	
918	284	289	294	298	303	308	313	317	322	327	
919	332	336	341	346	350	355	360	365	369	374	
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	
932	942	946	951	956	960	965	970	974	979	984	
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97 035	039	044	049	053	058	063	067	072	077	
935	081	086	090	095	100	104	109	114	118	123	
936	128	132	137	142	146	151	155	160	165	169	
937	174	179	183	188	192	197	202	206	211	216	
938	220	225	230	234	239	243	248	253	257	262	
939	267	271	276	280	285	290	294	299	304	308	
940	313	317	322	327	331	336	340	345	350	354	
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

N.	0	1	2	3	4	5	6	7	8	9
950	97 772	777	782	786	791	795	800	804	809	813
951	818	823	827	832	836	841	845	850	855	859
952	864	868	873	877	882	886	891	896	900	905
953	909	914	918	923	928	932	937	941	946	950
954	955	959	964	968	973	978	982	987	991	996
955	98 000	005	009	014	019	023	028	032	037	041
956	046	050	055	059	064	068	073	078	082	087
957	091	096	100	105	109	114	118	123	127	132
958	137	141	146	150	155	159	164	168	173	177
959	182	186	191	195	200	204	209	214	218	223
960	227	232	236	241	245	250	254	259	263	268
961	272	277	281	286	290	295	299	304	308	313
962	318	322	327	331	336	340	345	349	354	358
963	363	367	372	376	381	385	390	394	399	403
964	408	412	417	421	426	430	435	439	444	448
965	453	457	462	466	471	475	480	484	489	493
966	498	502	507	511	516	520	525	529	534	538
967	543	547	552	556	561	565	570	574	579	583
968	588	592	597	601	605	610	614	619	623	628
969	632	637	641	646	650	655	659	664	668	673
970	677	682	686	691	695	700	704	709	713	717
971	722	726	731	735	740	744	749	753	758	762
972	767	771	776	780	784	789	793	798	802	807
973	811	816	820	825	829	834	838	843	847	851
974	856	860	865	869	874	878	883	887	892	896
975	900	905	909	914	918	923	927	932	936	941
976	945	949	954	958	963	967	972	976	981	985
977	989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99 034	038	043	047	052	056	061	065	069	074
979	078	083	087	092	096	100	105	109	114	118
980	123	127	131	136	140	145	149	154	158	162
981	167	171	176	180	185	189	193	198	202	207
982	211	216	220	224	229	233	238	242	247	251
983	255	260	264	269	273	277	282	286	291	295
984	300	304	308	313	317	322	326	330	335	339
985	344	348	352	357	361	366	370	374	379	383
986	388	392	396	401	405	410	414	419	423	427
987	432	436	441	445	449	454	458	463	467	471
988	476	480	484	489	493	498	502	506	511	515
989	520	524	528	533	537	542	546	550	555	559
990	564	568	572	577	581	585	590	594	599	603
991	607	612	616	621	625	629	634	638	642	647
992	651	656	660	664	669	673	677	682	686	691
993	695	699	704	708	712	717	721	726	730	734
994	739	743	747	752	756	760	765	769	774	778
995	782	787	791	795	800	804	808	813	817	822
996	826	830	835	839	843	848	852	856	861	865
997	870	874	878	883	887	891	896	900	904	909
998	913	917	922	926	930	935	939	944	948	952
999	957	961	965	970	974	978	983	987	991	996
1000	00 000	004	009	013	017	022	026	030	035	039
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Prop. Pts.

MULTIPLY	BY	TO OBTAIN
Acres	43,560	Square feet
Acres	10	Square chains
Acre-feet	43,560	Cubic feet
" "	325,851	Gallons
Arpents	0.9	Acres
Atmospheres, pressure in	29.92	Inches of mercury
" "	33.90	Feet of water
" "	14.70	Lbs./sq. in.
Barrels—oil	42	Gallons—oil
" —cement	376	Pounds—cement
Bags or sacks cement	94	" "
Board-feet	144 sq. in. × 1 in.	Cubic inches
Centimeters	.3937	Inches
Cubic feet	7.48052	Gallons
Cubic feet of water	62.4	Pounds
Cubic feet/second	.646317	Million gals./day
" " "	448.831	Gallons/min.
" " "	1.983	Acre feet/24 hours
Fathoms	6	Feet
Feet/second	.6818	Miles/hr.
Furlongs	.125	Miles
Gallons	.1337	Cubic feet
Gallons, Imperial	1.20095	U. S. gallons
" U. S.	.83267	Imperial gallons
Gallons water	8.35	Pounds of water
Hectares	2.471	Acres
Horse-power	33,000	Foot-lbs./min.
" "	.7457	Kilowatts
Inches	2.540	Centimeters
Kilograms	2.20462	Pounds
Kilometers	.6214	Miles
Labors	1,000,000	Square varas
Leagues	25,000,000	Square varas
Links	7.92	Inches
Liters	1.0567	Quarts
$\frac{\text{Width (in.)} \times \text{Thickness (in.)}}{12}$	Length (ft.)	Board feet
Meters	1.094	Yards
Miles	5280	Feet
Miles	80	Chains
Miles/hr.	1.467	Feet sec.
Miles/hr.	.8684	Knots
Miner's inches	1.5	Cubic feet/min.
Perches	.25	Gunter's chains
Poles	.25	Gunter's chains
Rods	16.5	Feet
Rods	.25	Gunter's chains
Rods	.025	Furlongs
Square miles	640	Acres
Temp. (° F) + 460	1	Abs. temp. (° F)
" " − 32	5/9	Temp (° C)
Vara (Texas)	33⅓	Inches

TABLE II

ACTUAL VALUES

OF THE

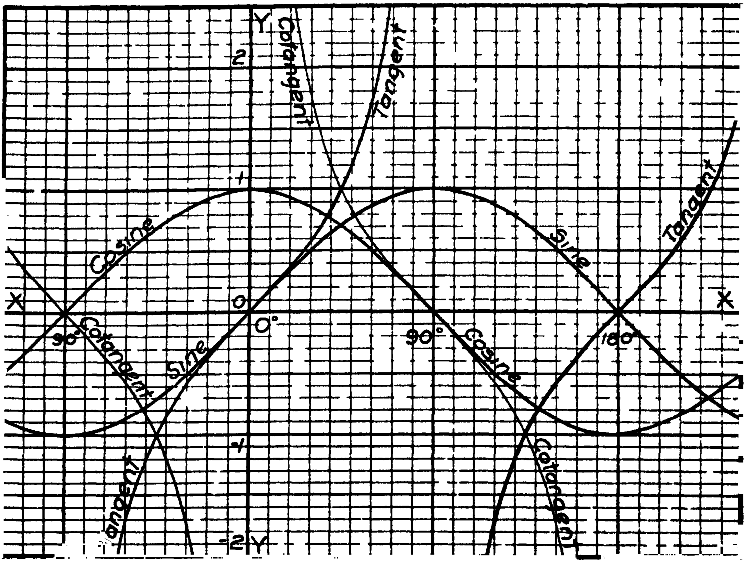
TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES



0° — Values of Trigonometric Functions — 1°

′	Sin	Tan	Ctn	Cos	
0	.00000	.00000	——	1.0000	**60**
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55	1.0000	**55**
6	175	175	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	**50**
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	**45**
16	465	465	214.86	999	44
17	495	495	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	**40**
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	**35**
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	**30**
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995	**25**
36	047	047	95.489	995	24
37	076	076	92.908	994	23
38	105	105	90.463	994	22
39	134	135	88.144	994	21
40	.01164	.01164	85.940	.99993	**20**
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	**15**
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455	68.750	.99989	**10**
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	**5**
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985	**0**
	Cos	Ctn	Tan	Sin	′

89°

′	Sin	Tan	Ctn	Cos	
0	.01745	.01746	57.290	.99985	**60**
1	774	775	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	**55**
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	**50**
11	065	066	48.412	979	49
12	094	095	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	**45**
16	211	211	45.226	976	44
17	240	240	44.639	975	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	**40**
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	**35**
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	**30**
31	647	648	37.769	965	29
32	676	677	37.358	964	28
33	705	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	**25**
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	**20**
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	**15**
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	950	11
50	.03199	.03201	31.242	.99949	**10**
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145	945	6
55	.03345	.03346	29.882	.99944	**5**
56	374	376	29.624	943	4
57	403	405	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	**0**
	Cos	Ctn	Tan	Sin	′

88°

′	Sin	Tan	Ctn	Cos	
0	.03490	.03492	28.636	.99939	**60**
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	**55**
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	**50**
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	**45**
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	**40**
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	**35**
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	**30**
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	**25**
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	**20**
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	**15**
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	**10**
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	**5**
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

87°

′	Sin	Tan	Ctn	Cos	
0	.05234	.05241	19.081	.99863	**60**
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	**55**
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	**50**
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	**45**
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	**40**
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	**35**
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	**30**
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	**25**
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	**20**
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	**15**
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	**10**
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	**5**
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

86°

4° — Values of Trigonometric Functions — 5°

′	Sin	Tan	Ctn	Cos	
0	.06976	.06993	14.301	.99756	60
1	.07005	.07022	.241	754	59
2	034	051	.182	752	58
3	063	080	.124	750	57
4	092	110	.065	748	56
5	.07121	.07139	14.008	.99746	55
6	150	168	13.951	744	54
7	179	197	.894	742	53
8	208	227	.838	740	52
9	237	256	.782	738	51
10	.07266	.07285	13.727	.99736	50
11	295	314	.672	734	49
12	324	344	.617	731	48
13	353	373	.563	729	47
14	382	402	.510	727	46
15	.07411	.07431	13.457	.99725	45
16	440	461	.404	723	44
17	469	490	.352	721	43
18	498	519	.300	719	42
19	527	548	.248	716	41
20	.07556	.07578	13.197	.99714	40
21	585	607	.146	712	39
22	614	636	.096	710	38
23	643	665	13.046	708	37
24	672	695	12.996	705	36
25	.07701	.07724	12.947	.99703	35
26	730	753	.898	701	34
27	759	782	.850	699	33
28	788	812	.801	696	32
29	817	841	.754	694	31
30	.07846	.07870	12.706	.99692	30
31	875	899	.659	689	29
32	904	929	.612	687	28
33	933	958	.566	685	27
34	962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	25
36	.08020	046	.429	678	24
37	049	075	.384	676	23
38	078	104	.339	673	22
39	107	134	.295	671	21
40	.08136	.08163	12.251	.99668	20
41	165	192	.207	666	19
42	194	221	.163	664	18
43	223	251	.120	661	17
44	252	280	.077	659	16
45	.08281	.08309	12.035	.99657	15
46	310	339	11.992	654	14
47	339	368	.950	652	13
48	368	397	.909	649	12
49	397	427	.867	647	11
50	.08426	.08456	11.826	.99644	10
51	455	485	.785	642	9
52	484	514	.745	639	8
53	513	544	.705	637	7
54	542	573	.664	635	6
55	.08571	.08602	11.625	.99632	5
56	600	632	.585	630	4
57	629	661	.546	627	3
58	658	690	.507	625	2
59	687	720	.468	622	1
60	.08716	.08749	11.430	.99619	0
	Cos	Ctn	Tan	Sin	′

85°

′	Sin	Tan	Ctn	Cos	
0	.08716	.08749	11.430	.99619	60
1	745	778	.392	617	59
2	774	807	.354	614	58
3	803	837	.316	612	57
4	831	866	.279	609	56
5	.08860	.08895	11.242	.99607	55
6	889	925	.205	604	54
7	918	954	.168	602	53
8	947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	50
11	034	071	11.024	591	49
12	063	101	10.988	588	48
13	092	130	.953	586	47
14	121	159	.918	583	46
15	.09150	.09189	10.883	.99580	45
16	179	218	.848	578	44
17	208	247	.814	575	43
18	237	277	.780	572	42
19	266	306	.746	570	41
20	.09295	.09335	10.712	.99567	40
21	324	365	.678	564	39
22	353	394	.645	562	38
23	382	423	.612	559	37
24	411	453	.579	556	36
25	.09440	.09482	10.546	.99553	35
26	469	511	.514	551	34
27	498	541	.481	548	33
28	527	570	.449	545	32
29	556	600	.417	542	31
30	.09585	.09629	10.385	.99540	30
31	614	658	.354	537	29
32	642	688	.322	534	28
33	671	717	.291	531	27
34	700	746	.260	528	26
35	.09729	.09776	10.229	.99526	25
36	758	805	.199	523	24
37	787	834	.168	520	23
38	816	864	.138	517	22
39	845	893	.108	514	21
40	.09874	.09923	10.078	.99511	20
41	903	952	.048	508	19
42	932	.09981	10.019	506	18
43	961	.10011	9.9893	503	17
44	.09990	040	.9601	500	16
45	.10019	.10069	9.9310	.99497	15
46	048	099	.9021	494	14
47	077	128	.8734	491	13
48	106	158	.8448	488	12
49	135	187	.8164	485	11
50	.10164	.10216	9.7882	.99482	10
51	192	246	.7601	479	9
52	221	275	.7322	476	8
53	250	305	.7044	473	7
54	279	334	.6768	470	6
55	.10308	.10363	9.6493	.99467	5
56	337	393	.6220	464	4
57	366	422	.5949	461	3
58	395	452	.5679	458	2
59	424	481	.5411	455	1
60	.10453	.10510	9.5144	.99452	0
	Cos	Ctn	Tan	Sin	′

84°

6° — Values of Trigonometric Functions — 7°

′	Sin	Tan	Ctn	Cos	
0	.10453	.10510	9.5144	.99452	**60**
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	**55**
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	**50**
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	**45**
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	**40**
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	**35**
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	**30**
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	**25**
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	**20**
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	**15**
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	**10**
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	**5**
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	**0**
	Cos	Ctn	Tan	Sin	′

83°

′	Sin	Tan	Ctn	Cos	
0	.12187	.12278	8.1443	.99255	**60**
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	**55**
6	360	456	.0285	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	**50**
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	**45**
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	**40**
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	**35**
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	**30**
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	**25**
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	**20**
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	**15**
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	**10**
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	**5**
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	**0**
	Cos	Ctn	Tan	Sin	′

82°

8° — Values of Trigonometric Functions — 9°

′	Sin	Tan	Ctn	Cos	
0	.13917	.14054	7.1154	.99027	**60**
1	946	084	.1004	023	59
2	.13975	113	.0855	019	58
3	.14004	143	.0706	015	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	**55**
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	**50**
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	**45**
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	**40**
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	**35**
26	666	826	.7448	919	34
27	695	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045	906	31
30	.14781	.14945	6.6912	.98902	**30**
31	810	.14975	.6779	897	29
32	838	.15005	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925	.15094	6.6252	.98880	**25**
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	**20**
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155	332	.5223	845	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	**15**
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	**10**
51	385	570	.4225	809	9
52	414	600	.4103	805	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.15500	.15689	6.3737	.98791	**5**
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

81°

′	Sin	Tan	Ctn	Cos	
0	.15643	.15838	6.3138	.98769	**60**
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	**55**
6	816	.16017	.2432	741	54
7	845	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	**50**
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	**45**
16	103	316	.1290	695	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	**40**
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555	.0405	657	36
25	.16361	.16585	6.0296	.98652	**35**
26	390	615	.0188	648	34
27	419	645	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865	633	31
30	.16505	.16734	5.9758	.98629	**30**
31	533	764	.9651	624	29
32	562	794	.9545	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	**25**
36	677	914	.9124	600	24
37	706	944	.9019	595	23
38	734	.16974	.8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	**20**
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935	.17183	5.8197	.98556	**15**
46	964	213	.8095	551	14
47	.16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	050	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	**10**
51	107	363	.7594	526	9
52	136	393	.7495	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	**5**
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365	.17633	5.6713	.98481	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

80°

10° — Values of Trigonometric Functions — 11°

′	Sin	Tan	Ctn	Cos	
0	.17365	.17633	5.6713	.98481	**60**
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	**55**
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	**50**
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	**45**
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	**40**
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	**35**
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	**30**
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	**25**
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	**20**
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	**15**
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	**10**
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	**5**
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

79°

′	Sin	Tan	Ctn	Cos	
0	.19081	.19438	5.1446	.98163	**60**
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	**55**
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	**50**
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	**45**
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	**40**
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	**35**
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	**30**
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	**25**
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	**20**
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	**15**
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	**10**
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	**5**
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

78°

12° — Values of Trigonometric Functions — 13°

′	Sin	Tan	Ctn	Cos	
0	.20791	.21256	4.7046	.97815	60
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877	347	.6845	797	57
4	905	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	55
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4.6382	.97754	50
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	45
16	246	743	.5993	717	44
17	275	773	.5928	711	43
18	303	804	.5864	705	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	40
21	388	895	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	35
26	530	047	.5357	655	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	30
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	25
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555	573	21
40	.21928	.22475	4.4494	.97566	20
41	956	505	.4434	560	19
42	.21985	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	15
46	098	658	.4134	528	14
47	126	689	.4075	521	13
48	155	719	.4015	515	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	10
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3604	.97470	5
56	382	964	.3546	463	4
57	410	.22995	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315	.97437	0
	Cos	Ctn	Tan	Sin	′

77°

′	Sin	Tan	Ctn	Cos	
0	.22495	.23087	4.3315	.97437	60
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	55
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	50
11	807	424	.2691	365	49
12	835	455	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345	46
15	.22920	.23547	4.2468	.97338	45
16	948	578	.2413	331	44
17	.22977	608	.2358	325	43
18	.23005	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	40
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	35
26	231	885	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345	.24008	4.1653	.97237	30
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	25
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285	.1178	176	21
40	.23627	.24316	4.1126	.97169	20
41	656	347	.1074	162	19
42	684	377	.1022	155	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	15
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	10
51	938	655	.0560	093	9
52	966	686	.0509	086	8
53	.23995	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065	5
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	0
	Cos	Ctn	Tan	Sin	′

76°

′	Sin	Tan	Ctn	Cos	
0	.24192	.24933	4.0108	.97030	60
1	220	964	.0058	023	59
2	249	.24995	4.0009	015	58
3	277	.25026	3.9959	008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	362	118	.9812	987	54
7	390	149	.9763	980	53
8	418	180	.9714	973	52
9	446	211	.9665	966	51
10	.24474	.25242	3.9617	.96959	50
11	503	273	.9568	952	49
12	531	304	.9520	945	48
13	559	335	.9471	937	47
14	587	366	.9423	930	46
15	.24615	.25397	3.9375	.96923	45
16	644	428	.9327	916	44
17	672	459	.9279	909	43
18	700	490	.9232	902	42
19	728	521	.9184	894	41
20	.24756	.25552	3.9136	.96887	40
21	784	583	.9089	880	39
22	813	614	.9042	873	38
23	841	645	.8995	866	37
24	869	676	.8947	858	36
25	.24897	.25707	3.8900	.96851	35
26	925	738	.8854	844	34
27	954	769	.8807	837	33
28	.24982	800	.8760	829	32
29	.25010	831	.8714	822	31
30	.25038	.25862	3.8667	.96815	30
31	066	893	.8621	807	29
32	094	924	.8575	800	28
33	122	955	.8528	793	27
34	151	.25986	.8482	786	26
35	.25179	.26017	3.8436	.96778	25
36	207	048	.8391	771	24
37	235	079	.8345	764	23
38	263	110	.8299	756	22
39	291	141	.8254	749	21
40	.25320	.26172	3.8208	.96742	20
41	348	203	.8163	734	19
42	376	235	.8118	727	18
43	404	266	.8073	719	17
44	432	297	.8028	712	16
45	.25460	.26328	3.7983	.96705	15
46	488	359	.7938	697	14
47	516	390	.7893	690	13
48	545	421	.7848	682	12
49	573	452	.7804	675	11
50	.25601	.26483	3.7760	.96667	10
51	629	515	.7715	660	9
52	657	546	.7671	653	8
53	685	577	.7627	645	7
54	713	608	.7583	638	6
55	.25741	.26639	3.7539	.96630	5
56	769	670	.7495	623	4
57	798	701	.7451	615	3
58	826	733	.7408	608	2
59	854	764	.7364	600	1
60	.25882	.26795	3.7321	.96593	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.25882	.26795	3.7321	.96593	60
1	910	826	.7277	585	59
2	938	857	.7234	578	58
3	966	888	.7191	570	57
4	.25994	920	.7148	562	56
5	.26022	.26951	3.7105	.96555	55
6	050	.26982	.7062	547	54
7	079	.27013	.7019	540	53
8	107	044	.6976	532	52
9	135	076	.6933	524	51
10	.26163	.27107	3.6891	.96517	50
11	191	138	.6848	509	49
12	219	169	.6806	502	48
13	247	201	.6764	494	47
14	275	232	.6722	486	46
15	.26303	.27263	3.6680	.96479	45
16	331	294	.6638	471	44
17	359	326	.6596	463	43
18	387	357	.6554	456	42
19	415	388	.6512	448	41
20	.26443	.27419	3.6470	.96440	40
21	471	451	.6429	433	39
22	500	482	.6387	425	38
23	528	513	.6346	417	37
24	556	545	.6305	410	36
25	.26584	.27576	3.6264	.96402	35
26	612	607	.6222	394	34
27	640	638	.6181	386	33
28	668	670	.6140	379	32
29	696	701	.6100	371	31
30	.26724	.27732	3.6059	.96363	30
31	752	764	.6018	355	29
32	780	795	.5978	347	28
33	808	826	.5937	340	27
34	836	858	.5897	332	26
35	.26864	.27889	3.5856	.96324	25
36	892	921	.5816	316	24
37	920	952	.5776	308	23
38	948	.27983	.5736	301	22
39	.26976	.28015	.5696	293	21
40	.27004	.28046	3.5656	.96285	20
41	032	077	.5616	277	19
42	060	109	.5576	269	18
43	088	140	.5536	261	17
44	116	172	.5497	253	16
45	.27144	.28203	3.5457	.96246	15
46	172	234	.5418	238	14
47	200	266	.5379	230	13
48	228	297	.5339	222	12
49	256	329	.5300	214	11
50	.27284	.28360	3.5261	.96206	10
51	312	391	.5222	198	9
52	340	423	.5183	190	8
53	368	454	.5144	182	7
54	396	486	.5105	174	6
55	.27424	.28517	3.5067	.96166	5
56	452	549	.5028	158	4
57	480	580	.4989	150	3
58	508	612	.4951	142	2
59	536	643	.4912	134	1
60	.27564	.28675	3.4874	.96126	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.27564	.28675	3.4874	.96126	60
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	55
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	50
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	45
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	40
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	35
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	30
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	25
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	20
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	15
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	10
51	.28987	287	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	5
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	0
	Cos	Ctn	Tan	Sin	′

73°

′	Sin	Tan	Ctn	Cos	
0	.29237	.30573	3.2709	.95630	60
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	55
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	50
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	45
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	40
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	35
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	30
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	25
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	20
41	376	882	.1366	275	19
42	403	914	.1334	266	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	15
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	10
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	5
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	0
	Cos	Ctn	Tan	Sin	′

72°

18° — Values of Trigonometric Functions — 19°

′	Sin	Tan	Ctn	Cos	
0	.30902	.32492	3.0777	.95106	60
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	55
6	068	685	.0595	052	54
7	095	717	.0565	043	53
8	123	749	.0535	033	52
9	151	782	.0505	024	51
10	.31178	.32814	3.0475	.95015	50
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	45
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	40
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	35
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395	.9945	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	30
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	25
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	.33783	2.9600	.94740	20
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	15
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375	665	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	10
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	5
56	447	303	.9152	590	4
57	474	335	.9125	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	0
	Cos	Ctn	Tan	Sin	′

71°

′	Sin	Tan	Ctn	Cos	
0	.32557	.34433	2.9042	.94552	60
1	584	465	.9015	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	55
6	722	628	.8878	495	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	50
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	45
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085	2.8502	.94361	40
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	35
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	30
31	408	445	.8213	254	29
32	436	477	.8187	245	28
33	463	510	.8161	235	27
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	25
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655	.35740	2.7980	.94167	20
41	682	772	.7955	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	15
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	10
51	956	101	.7700	058	9
52	.33983	134	.7675	049	8
53	.34011	167	.7650	039	7
54	038	199	.7625	029	6
55	.34065	.36232	2.7600	.94019	5
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	989	2
59	175	364	.7500	979	1
60	.34202	.36397	2.7475	.93969	0
	Cos	Ctn	Tan	Sin	′

70°

20° — Values of Trigonometric Functions — 21°

′	Sin	Tan	Ctn	Cos	
0	.34202	.36397	2.7475	.93969	**60**
1	229	430	.7450	959	59
2	257	463	.7425	949	58
3	284	496	.7400	939	57
4	311	529	.7376	929	56
5	.34339	.36562	2.7351	.93919	**55**
6	366	595	.7326	909	54
7	393	628	.7302	899	53
8	421	661	.7277	889	52
9	448	694	.7253	879	51
10	.34475	.36727	2.7228	.93869	**50**
11	503	760	.7204	859	49
12	530	793	.7179	849	48
13	557	826	.7155	839	47
14	584	859	.7130	829	46
15	.34612	.36892	2.7106	.93819	**45**
16	639	925	.7082	809	44
17	666	958	.7058	799	43
18	694	.36991	.7034	789	42
19	721	.37024	.7009	779	41
20	.34748	.37057	2.6985	.93769	**40**
21	775	090	.6961	759	39
22	803	123	.6937	748	38
23	830	157	.6913	738	37
24	857	190	.6889	728	36
25	.34884	.37223	2.6865	.93718	**35**
26	912	256	.6841	708	34
27	939	289	.6818	698	33
28	966	322	.6794	688	32
29	.34993	355	.6770	677	31
30	.35021	.37388	2.6746	.93667	**30**
31	048	422	.6723	657	29
32	075	455	.6699	647	28
33	102	488	.6675	637	27
34	130	521	.6652	626	26
35	.35157	.37554	2.6628	.93616	**25**
36	184	588	.6605	606	24
37	211	621	.6581	596	23
38	239	654	.6558	585	22
39	266	687	.6534	575	21
40	.35293	.37720	2.6511	.93565	**20**
41	320	754	.6488	555	19
42	347	787	.6464	544	18
43	375	820	.6441	531	17
44	402	853	.6418	524	16
45	.35429	.37887	2.6395	.93514	**15**
46	456	920	.6371	503	14
47	484	953	.6348	493	13
48	511	.37986	.6325	483	12
49	538	.38020	.6302	472	11
50	.35565	.38053	2.6279	.93462	**10**
51	592	086	.6256	452	9
52	619	120	.6233	441	8
53	647	153	.6210	431	7
54	674	186	.6187	420	6
55	.35701	.38220	2.6165	.93410	**5**
56	728	253	.6142	400	4
57	755	286	.6119	389	3
58	782	320	.6096	379	2
59	810	353	.6074	368	1
60	.35837	.38386	2.6051	93358	0
	Cos	**Ctn**	**Tan**	**Sin**	′

69°

′	Sin	Tan	Ctn	Cos	
0	.35837	.38386	2.6051	.93358	**60**
1	864	420	.6028	348	59
2	891	453	.6006	337	58
3	918	487	.5983	327	57
4	945	520	.5961	316	56
5	.35973	.38553	2.5938	.93306	**55**
6	.36000	587	.5916	295	54
7	027	620	.5893	285	53
8	054	654	.5871	274	52
9	081	687	.5848	264	51
10	.36108	.38721	2.5826	.93253	**50**
11	135	754	.5804	243	49
12	162	787	.5782	232	48
13	190	821	.5759	222	47
14	217	854	.5737	211	46
15	.36244	.38888	2.5715	.93201	**45**
16	271	921	.5693	190	44
17	298	955	.5671	180	43
18	325	.38988	.5649	169	42
19	352	.39022	.5627	159	41
20	.36379	.39055	2.5605	.93148	**40**
21	406	089	.5583	137	39
22	434	122	.5561	127	38
23	461	156	.5539	116	37
24	488	190	.5517	106	36
25	.36515	.39223	2.5495	.93095	**35**
26	542	257	.5473	084	34
27	569	290	.5452	074	33
28	596	324	.5430	063	32
29	623	357	.5408	052	31
30	.36650	.39391	2.5386	.93042	**30**
31	677	425	.5365	031	29
32	704	458	.5343	020	28
33	731	492	.5322	.93010	27
34	758	526	.5300	.92999	26
35	.36785	.39559	2.5279	.92988	**25**
36	812	593	.5257	978	24
37	839	626	.5236	967	23
38	867	660	.5214	956	22
39	894	694	.5193	945	21
40	.36921	.39727	2.5172	.92935	**20**
41	948	761	.5150	924	19
42	.36975	795	.5129	913	18
43	.37002	829	.5108	902	17
44	029	862	.5086	892	16
45	.37056	.39896	2.5065	.92881	**15**
46	083	930	.5044	870	14
47	110	963	.5023	859	13
48	137	.39997	.5002	849	12
49	164	.40031	.4981	838	11
50	.37191	.40065	2.4960	.92827	**10**
51	218	098	.4939	816	9
52	245	132	.4918	805	8
53	272	166	.4897	794	7
54	299	200	.4876	784	6
55	.37326	.40234	2.4855	.92773	**5**
56	353	267	.4834	762	4
57	380	301	.4813	751	3
58	407	335	.4792	740	2
59	434	369	.4772	729	1
60	.37461	.40403	2.4751	.92718	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

68°

′	Sin	Tan	Ctn	Cos	
0	.37461	.40403	2.4751	.92718	60
1	488	436	.4730	707	59
2	515	470	.4709	697	58
3	542	504	.4689	686	57
4	569	538	.4668	675	56
5	.37595	.40572	2.4648	.92664	55
6	622	606	.4627	653	54
7	649	640	.4606	642	53
8	676	674	.4586	631	52
9	703	707	.4566	620	51
10	.37730	.40741	2.4545	.92609	50
11	757	775	.4525	598	49
12	784	809	.4504	587	48
13	811	843	.4484	576	47
14	838	877	.4464	565	46
15	.37865	.40911	2.4443	.92554	45
16	892	945	.4423	543	44
17	919	.40979	.4403	532	43
18	946	.41013	.4383	521	42
19	973	047	.4362	510	41
20	.37999	.41081	2.4342	.92499	40
21	.38026	115	.4322	488	39
22	053	149	.4302	477	38
23	080	183	.4282	466	37
24	107	217	.4262	455	36
25	.38134	.41251	2.4242	.92444	35
26	161	285	.4222	432	34
27	188	319	.4202	421	33
28	215	353	.4182	410	32
29	241	387	.4162	399	31
30	.38268	.41421	2.4142	.92388	30
31	295	455	.4122	377	29
32	322	490	.4102	366	28
33	349	524	.4083	355	27
34	376	558	.4063	343	26
35	.38403	.41592	2.4043	.92332	25
36	430	626	.4023	321	24
37	456	660	.4004	310	23
38	483	694	.3984	299	22
39	510	728	.3964	287	21
40	.38537	.41763	2.3945	.92276	20
41	564	797	.3925	265	19
42	591	831	.3906	254	18
43	617	865	.3886	243	17
44	644	899	.3867	231	16
45	.38671	.41933	2.3847	.92220	15
46	698	.41968	.3828	209	14
47	725	.42002	.3808	198	13
48	752	036	.3789	186	12
49	778	070	.3770	175	11
50	.38805	.42105	2.3750	.92164	10
51	832	139	.3731	152	9
52	859	173	.3712	141	8
53	886	207	.3693	130	7
54	912	242	.3673	119	6
55	.38939	.42276	2.3654	.92107	5
56	966	310	.3635	096	4
57	.38993	345	.3616	085	3
58	.39020	379	.3597	073	2
59	046	413	.3578	062	1
60	.39073	.42447	2.3559	.92050	0
	Cos	Ctn	Tan	Sin	′

67°

′	Sin	Tan	Ctn	Cos	
0	.39073	.42447	2.3559	.92050	60
1	100	482	.3539	039	59
2	127	516	.3520	028	58
3	153	551	.3501	016	57
4	180	585	.3483	.92005	56
5	.39207	.42619	2.3464	.91994	55
6	234	654	.3445	982	54
7	260	688	.3426	971	53
8	287	722	.3407	959	52
9	314	757	.3388	948	51
10	.39341	.42791	2.3369	.91936	50
11	367	826	.3351	925	49
12	394	860	.3332	914	48
13	421	894	3313	902	47
14	448	929	.3294	891	46
15	.39474	.42963	2.3276	.91879	45
16	501	.42998	.3257	868	44
17	528	.43032	.3238	856	43
18	555	067	.3220	845	42
19	581	101	.3201	833	41
20	.39608	.43136	2.3183	.91822	40
21	635	170	.3164	810	39
22	661	205	.3146	799	38
23	688	239	.3127	787	37
24	715	274	.3109	775	36
25	.39741	.43308	2.3090	.91764	35
26	768	343	.3072	752	34
27	795	378	.3053	741	33
28	822	412	.3035	729	32
29	848	447	.3017	718	31
30	.39875	.43481	2.2998	.91706	30
31	902	516	.2980	694	29
32	928	550	.2962	683	28
33	955	585	.2944	671	27
34	.39982	620	.2925	660	26
35	.40008	.43654	2.2907	.91648	25
36	035	689	.2889	636	24
37	062	724	.2871	625	23
38	088	758	.2853	613	22
39	115	793	.2835	601	21
40	.40141	.43828	2.2817	.91590	20
41	168	862	.2799	578	19
42	195	897	.2781	566	18
43	221	932	.2763	555	17
44	248	.43966	.2745	543	16
45	.40275	.44001	2.2727	.91531	15
46	301	036	.2709	519	14
47	328	071	.2691	508	13
48	355	105	.2673	496	12
49	381	140	.2655	484	11
50	.40408	.44175	2.2637	.91472	10
51	434	210	.2620	461	9
52	461	244	.2602	449	8
53	488	279	.2584	437	7
54	514	314	.2566	425	6
55	.40541	.44349	2.2549	.91414	5
56	567	384	.2531	402	4
57	594	418	.2513	390	3
58	621	453	.2496	378	2
59	647	488	.2478	366	1
60	.40674	.44523	2.2460	.91355	0
	Cos	Ctn	Tan	Sin	′

66°

′	Sin	Tan	Ctn	Cos	
0	.40674	.44523	2.2460	.91355	60
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	55
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	50
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	45
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	40
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	35
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	25
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	20
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	15
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	5
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	0
	Cos	Ctn	Tan	Sin	′

65°

′	Sin	Tan	Ctn	Cos	
0	.42262	.46631	2.1445	.90631	60
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	55
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	50
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	45
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	40
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	35
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	30
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	25
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	20
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	15
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	10
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	5
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2.0503	.89879	0
	Cos	Ctn	Tan	Sin	′

64°

26° — Values of Trigonometric Functions — 27°

′	Sin	Tan	Ctn	Cos	
0	.43837	.48773	2.0503	.89879	**60**
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	**55**
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	**50**
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	**45**
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	**40**
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	**35**
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	**30**
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	**25**
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	**20**
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	**15**
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	**10**
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	**5**
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	**0**
	Cos	Ctn	Tan	Sin	′

63°

′	Sin	Tan	Ctn	Cos	
0	.45399	.50953	1.9626	.89101	**60**
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	099	.9570	048	56
5	.45529	.51136	1.9556	.89035	**55**
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	**50**
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	**45**
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	**40**
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	**35**
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	**30**
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	**25**
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	**20**
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	**15**
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	**10**
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	**5**
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	**0**
	Cos	Ctn	Tan	Sin	′

62°

′	Sin	Tan	Ctn	Cos	
0	.46947	.53171	1.8807	.88295	60
1	973	208	.8794	281	59
2	.46999	246	.8781	267	58
3	47024	283	.8768	254	57
4	050	320	.8755	240	56
5	.47076	.53358	1.8741	.88226	55
6	101	395	.8728	213	54
7	127	432	.8715	199	53
8	153	470	.8702	185	52
9	178	507	.8689	172	51
10	.47204	.53545	1.8676	.88158	50
11	229	582	.8663	144	49
12	255	620	.8650	130	48
13	281	657	.8637	117	47
14	306	694	.8624	103	46
15	.47332	.53732	1.8611	.88089	45
16	358	769	.8598	075	44
17	383	807	.8585	062	43
18	409	844	.8572	048	42
19	434	882	.8559	034	41
20	.47460	.53920	1.8546	.88020	40
21	486	957	.8533	.88006	39
22	511	.53995	.8520	.87993	38
23	537	.54032	.8507	979	37
24	562	070	.8495	965	36
25	.47588	.54107	1.8482	.87951	35
26	614	145	.8469	937	34
27	639	183	.8456	923	33
28	665	220	.8443	909	32
29	690	258	.8430	896	31
30	.47716	.54296	1.8418	.87882	30
31	741	333	.8405	868	29
32	767	371	.8392	854	28
33	793	409	.8379	840	27
34	818	446	.8367	826	26
35	.47844	.54484	1.8354	.87812	25
36	869	522	.8341	798	24
37	895	560	.8329	784	23
38	920	597	.8316	770	22
39	946	635	.8303	756	21
40	.47971	.54673	1.8291	.87743	20
41	.47997	711	.8278	729	19
42	.48022	748	.8265	715	18
43	048	786	.8253	701	17
44	073	824	.8240	687	16
45	.48099	.54862	1.8228	.87673	15
46	124	900	.8215	659	14
47	150	938	.8202	645	13
48	175	.54975	.8190	631	12
49	201	.55013	.8177	617	11
50	.48226	.55051	1.8165	.87603	10
51	252	089	.8152	589	9
52	277	127	.8140	575	8
53	303	165	.8127	561	7
54	328	203	.8115	546	6
55	.48354	.55241	1.8103	.87532	5
56	379	279	.8090	518	4
57	405	317	.8078	504	3
58	430	355	.8065	490	2
59	456	393	.8053	476	1
60	.48481	.55431	1.8040	.87462	0
	Cos	Ctn	Tan	Sin	′

61°

′	Sin	Tan	Ctn	Cos	
0	.48481	.55431	1.8040	.87462	60
1	506	469	.8028	448	59
2	532	507	.8016	434	58
3	557	545	.8003	420	57
4	583	583	.7991	406	56
5	.48608	.55621	1.7979	.87391	55
6	634	659	.7966	377	54
7	659	697	.7954	363	53
8	684	736	.7942	349	52
9	710	774	.7930	335	51
10	.48735	.55812	1.7917	.87321	50
11	761	850	.7905	306	49
12	786	888	.7893	292	48
13	811	926	.7881	278	47
14	837	.55964	.7868	264	46
15	.48862	.56003	1.7856	.87250	45
16	888	041	.7844	235	44
17	913	079	.7832	221	43
18	938	117	.7820	207	42
19	964	156	.7808	193	41
20	.48989	.56194	1.7796	.87178	40
21	.49014	232	.7783	164	39
22	040	270	.7771	150	38
23	065	309	.7759	136	37
24	090	347	.7747	121	36
25	.49116	.56385	1.7735	.87107	35
26	141	424	.7723	093	34
27	166	462	.7711	079	33
28	192	501	.7699	064	32
29	217	539	.7687	050	31
30	.49242	.56577	1.7675	.87036	30
31	268	616	.7663	021	29
32	293	654	.7651	.87007	28
33	318	693	.7639	.86993	27
34	344	731	.7627	978	26
35	.49369	.56769	1.7615	.86964	25
36	394	808	.7603	949	24
37	419	846	.7591	935	23
38	445	885	.7579	921	22
39	470	923	.7567	906	21
40	.49495	.56962	1.7556	.86892	20
41	521	.57000	.7544	878	19
42	546	039	.7532	863	18
43	571	078	.7520	849	17
44	596	116	.7508	834	16
45	.49622	.57155	1.7496	.86820	15
46	647	193	.7485	805	14
47	672	232	.7473	791	13
48	697	271	.7461	777	12
49	723	309	.7449	762	11
50	.49748	.57348	1.7437	.86748	10
51	773	386	.7426	733	9
52	798	425	.7414	719	8
53	824	464	.7402	704	7
54	849	503	.7391	690	6
55	.49874	.57541	1.7379	.86675	5
56	899	580	.7367	661	4
57	924	619	.7355	646	3
58	950	657	.7344	632	2
59	.49975	696	.7332	617	1
60	.50000	.57735	1.7321	.86603	0
	Cos	Ctn	Tan	Sin	′

60°

′	Sin	Tan	Ctn	Cos	
0	.50000	.57735	1.7321	.86603	**60**
1	025	774	.7309	588	59
2	050	813	.7297	573	58
3	076	851	.7286	559	57
4	101	890	.7274	544	56
5	.50126	.57929	1.7262	.86530	**55**
6	151	.57968	.7251	515	54
7	176	.58007	.7239	501	53
8	201	046	.7228	486	52
9	227	085	.7216	471	51
10	.50252	.58124	1.7205	.86457	**50**
11	277	162	.7193	442	49
12	302	201	.7182	427	48
13	327	240	.7170	413	47
14	352	279	.7159	398	46
15	.50377	.58318	1.7147	.86384	**45**
16	403	357	.7136	369	44
17	428	396	.7124	354	43
18	453	435	.7113	340	42
19	478	474	.7102	325	41
20	.50503	.58513	1.7090	.86310	**40**
21	528	552	.7079	295	39
22	553	591	.7067	281	38
23	578	631	.7056	266	37
24	603	670	.7045	251	36
25	.50628	.58709	1.7033	.86237	**35**
26	654	748	.7022	222	34
27	679	787	.7011	207	33
28	704	826	.6999	192	32
29	729	865	.6988	178	31
30	.50754	.58905	1.6977	.86163	**30**
31	779	944	.6965	148	29
32	804	.58983	.6954	133	28
33	829	.59022	.6943	119	27
34	854	061	.6932	104	26
35	.50879	.59101	1.6920	.86089	**25**
36	904	140	.6909	074	24
37	929	179	.6898	059	23
38	954	218	.6887	045	22
39	.50979	258	.6875	030	21
40	.51004	.59297	1.6864	.86015	**20**
41	029	336	.6853	.86000	19
42	054	376	.6842	.85985	18
43	079	415	.6831	970	17
44	104	454	.6820	956	16
45	.51129	.59494	1.6808	.85941	**15**
46	154	533	.6797	926	14
47	179	573	.6786	911	13
48	204	612	.6775	896	12
49	229	651	.6764	881	11
50	.51254	.59691	1.6753	.85866	**10**
51	279	730	.6742	851	9
52	304	770	.6731	836	8
53	329	809	.6720	821	7
54	354	849	.6709	806	6
55	.51379	.59888	1.6698	.85792	**5**
56	404	928	.6687	777	4
57	429	.59967	.6676	762	3
58	454	.60007	.6665	747	2
59	479	046	.6654	732	1
60	.51504	.60086	1.6643	.85717	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

59°

′	Sin	Tan	Ctn	Cos	
0	.51504	.60086	1.6643	.85717	**60**
1	529	126	.6632	702	59
2	554	165	.6621	687	58
3	579	205	.6610	672	57
4	604	245	.6599	657	56
5	.51628	.60284	1.6588	.85642	**55**
6	653	324	.6577	627	54
7	678	364	.6566	612	53
8	703	403	.6555	597	52
9	728	443	.6545	582	51
10	.51753	.60483	1.6534	.85567	**50**
11	778	522	.6523	551	49
12	803	562	.6512	536	48
13	828	602	.6501	521	47
14	852	642	.6490	506	46
15	.51877	.60681	1.6479	.85491	**45**
16	902	721	.3169	476	44
17	927	761	.6458	461	43
18	952	801	.6447	446	42
19	.51977	841	.6436	431	41
20	.52002	.60881	1.6426	.85416	**40**
21	026	921	.6415	401	39
22	051	.60960	.6404	385	38
23	076	.61000	.6393	370	37
24	101	040	.6383	355	36
25	.52126	.61080	1.6372	.85340	**35**
26	151	120	.6361	325	34
27	175	160	.6351	310	33
28	200	200	.6340	294	32
29	225	240	.6329	279	31
30	.52250	.61280	1.6319	.85264	**30**
31	275	320	.6308	249	29
32	299	360	.6297	234	28
33	324	400	.6287	218	27
34	349	440	.6276	203	26
35	.52374	.61480	1.6265	.85188	**25**
36	399	520	.6255	173	24
37	423	561	.6244	157	23
38	448	601	.6234	142	22
39	473	641	.6223	127	21
40	.52498	.61681	1.6212	.85112	**20**
41	522	721	.6202	096	19
42	547	761	.6191	081	18
43	572	801	.6181	066	17
44	597	842	.6170	051	16
45	.52621	.61882	1.6160	.85035	**15**
46	646	922	.6149	020	14
47	671	.61962	.6139	.85005	13
48	696	.62003	.6128	.84989	12
49	720	043	.6118	974	11
50	.52745	.62083	1.6107	.84959	**10**
51	770	124	.6097	943	9
52	794	164	.6087	928	8
53	819	204	.6076	913	7
54	844	245	.6066	897	6
55	.52869	.62285	1.6055	.84882	**5**
56	893	325	.6045	866	4
57	918	366	.6034	851	3
58	943	406	.6024	836	2
59	967	446	.6014	820	1
60	.52992	.62487	1.6003	.84805	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

58°

′	Sin	Tan	Ctn	Cos	
0	.52992	.62487	1.6003	.84805	60
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	55
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	50
11	263	933	.5890	635	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055	.5859	588	46
15	.53361	.63095	1.5849	.84573	45
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	40
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	35
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355	31
30	.53730	.63707	1.5697	.84339	30
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	25
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	15
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	10
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	5
56	366	775	.5438	930	4
57	391	817	.5428	915	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	0
	Cos	Ctn	Tan	Sin	′

57°

′	Sin	Tan	Ctn	Cos	
0	.54464	.64941	1.5399	.83867	60
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.5350	.83788	55
6	610	189	.5340	772	54
7	635	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	50
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805	521	.5262	645	46
15	.54829	.65563	1.5253	.83629	45
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565	41
20	.54951	.65771	1.5204	.83549	40
21	975	813	.5195	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485	36
25	.55072	.65980	1.5156	.83469	35
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105	.5127	421	32
29	169	147	.5118	405	31
30	.55194	.66189	1.5108	.83389	30
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315	.66398	1.5061	.83308	25
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	20
41	460	650	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	15
46	581	860	.4957	131	14
47	605	902	.4947	115	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	10
51	702	071	.4910	050	9
52	726	113	.4900	034	8
53	750	155	.4891	017	7
54	775	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835	920	1
60	.55919	.67451	1.4826	.82904	0
	Cos	Ctn	Tan	Sin	′

56°

′	Sin	Tan	Ctn	Cos	
0	.55919	.67451	1.4826	.82904	**60**
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	**55**
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	**50**
11	184	917	.4724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	**45**
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	**40**
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	**35**
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	**30**
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	**25**
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	**20**
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	**15**
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	**10**
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	**5**
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	**0**
	Cos	Ctn	Tan	Sin	′

55°

′	Sin	Tan	Ctn	Cos	
0	.57358	.70021	1.4281	.81915	**60**
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	**55**
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	**50**
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	**45**
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	**40**
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	**35**
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	**30**
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	**25**
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	**20**
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	**15**
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	**10**
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	**5**
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	**0**
	Cos	Ctn	Tan	Sin	′

54°

36° — Values of Trigonometric Functions — 37°

′	Sin	Tan	Ctn	Cos	
0	.58779	.72654	1.3764	.80902	**60**
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	**55**
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	**50**
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	**45**
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	**40**
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	**35**
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	**30**
31	506	.74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	**25**
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	**20**
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	**15**
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	**10**
51	972	946	.3343	021	9
52	.59995	.74991	.3335	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	**5**
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	**0**
	Cos	**Ctn.**	**Tan**	**Sin**	′

53°

′	Sin	Tan	Ctn	Cos	
0	.60182	.75355	1.3270	.79864	**60**
1	205	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	**55**
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	**50**
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	950	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	**45**
16	553	088	.3143	583	44
17	576	134	.3135	565	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	**40**
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	**35**
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	**30**
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	**25**
36	.61015	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	**20**
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	**15**
46	245	475	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	**10**
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895	1.2838	.78891	**5**
56	474	941	.2830	873	4
57	497	.77988	.2822	855	3
58	520	.78035	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

52°

38° — Values of Trigonometric Functions — 39°

′	Sin	Tan	Ctn	Cos	
0	.61566	.78129	1.2799	.78801	**60**
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	**55**
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	**50**
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	**45**
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	**40**
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	**35**
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	**30**
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	**25**
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	**20**
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	**15**
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	**10**
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	**5**
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	**0**
	Cos	Ctn	Tan	Sin	′

51°

′	Sin	Tan	Ctn	Cos	
0	.62932	.80978	1.2349	.77715	**60**
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	**55**
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	**50**
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	**45**
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	**40**
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	**35**
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	**30**
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	**25**
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	**20**
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	**15**
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	**10**
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	**5**
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	**0**
	Cos	Ctn	Tan	Sin	′

50°

′	Sin	Tan	Ctn	Cos	
0	.64279	.83910	1.1918	.76604	**60**
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	**55**
6	412	208	.1875	492	54
7	435	258	.1868	473	53
8	457	307	.1861	455	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	**50**
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	**45**
16	635	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785	248	41
20	.64723	.84906	1.1778	.76229	**40**
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.1750	154	36
25	.64834	.85157	1.1743	.76135	**35**
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	64945	.85408	1.1708	.76041	**30**
31	967	458	.1702	022	29
32	.64989	509	.1695	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965	26
35	.65055	.85660	1.1674	.75946	**25**
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	**20**
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115	.1612	775	16
45	.65276	.86166	1.1606	.75756	**15**
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	**10**
51	408	470	.1565	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	**5**
56	518	725	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	**0**
	Cos	Ctn	Tan	Sin	′

49°

′	Sin	Tan	Ctn	Cos	
0	.65606	.86929	1.1504	.75471	**60**
1	628	.86980	.1497	452	59
2	650	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395	56
5	.65716	.87184	1.1470	.75375	**55**
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.1450	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	**50**
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595	.1416	222	47
14	913	646	.1410	203	46
15	.65935	.87698	1.1403	.75184	**45**
16	956	749	.1396	165	44
17	.65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	**40**
21	066	.88007	.1363	069	39
22	088	059	.1356	050	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	**35**
26	175	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915	31
30	.66262	.88473	1.1303	.74896	**30**
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	**25**
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	**20**
41	501	.89045	.1230	683	19
42	523	097	.1224	664	18
43	545	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	**15**
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	**10**
51	718	567	.1165	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725	.1145	431	6
55	.66805	.89777	1.1139	.74412	**5**
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	**0**
	Cos	Ctn	Tan	Sin	′

48°

42° — Values of Trigonometric Functions — 43°

′	Sin	Tan	Ctn	Cos	
0	.66913	.90040	1.1106	.74314	**60**
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	**55**
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	**50**
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	**45**
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	**40**
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	**35**
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	**30**
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	**25**
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	**20**
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	**15**
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	**10**
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	**5**
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	**0**
	Cos	Ctn	Tan	Sin	′

47°

′	Sin	Tan	Ctn	Cos	
0	.68200	.93252	1.0724	.73135	**60**
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	**55**
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	**50**
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	**45**
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	**40**
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	**35**
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	**30**
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	**25**
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	**20**
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	**15**
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	**10**
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	**5**
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	**0**
	Cos	Ctn	Tan	Sin	′

46°

′	Sin	Tan	Ctn	Cos	
0	.69466	.96569	1.0355	.71934	**60**
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	**55**
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	**50**
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	**45**
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	**40**
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	**30**
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	**25**
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	**20**
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	**15**
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	**10**
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	**5**
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

45°

TABLE III

COMMON LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES

From each logarithm given, subtract 10

Table IIIa—Auxiliary Table of S and T for A in Minutes

$S = \log \sin A - \log A'$ and $T = \log \tan A - \log A'$

A'	$S+10$
0′ – 13′	6.46373
14′ – 42′	72
43′ – 58′	71
59′ – 71′	6.46370
72′ – 81′	69
82′ – 91′	68
92′ – 99′	6.46367
100′ – 107′	66
108′ – 115′	65
116′ – 121′	6.46364
122′ – 128′	63
129′ – 134′	62
135′ – 140′	6.46361
141′ – 146′	60
147′ – 151′	59
152′ – 157′	6.46358
158′ – 162′	57
163′ – 167′	56
168′ – 171′	6.46355
172′ – 176′	54
177′ – 181′	53

A'	$T+10$
0′ – 26′	6.46373
27′ – 39′	74
40′ – 48′	75
49′ – 56′	6.46376
57′ – 63′	77
64′ – 69′	78
70′ – 74′	6.46379
75′ – 80′	80
81′ – 85′	81
86′ – 89′	6.46382
90′ – 94′	83
95′ – 98′	84
99′ – 102′	6.46385
103′ – 106′	86
107′ – 110′	87
111′ – 113′	6.46388
114′ – 117′	89
118′ – 120′	90
121′ – 124′	6.46391
125′ – 127′	92
128′ – 130′	93

A'	$T+10$
131′ – 133′	6.46394
134′ – 136′	95
137′ – 139′	96
140′ – 142′	6.46397
143′ – 145′	98
146′ – 148′	99
149′ – 150′	6.46400
151′ – 153′	01
154′ – 156′	02
157′ – 158′	6.46403
159′ – 161′	04
162′ – 163′	05
164′ – 166′	6.46406
167′ – 168′	07
169′ – 171′	08
172′ – 173′	6.46409
174′ – 175′	10
176′ – 178′	11
179′ – 180′	6.46412
181′ – 182′	13
183′ – 184′	14

For small angles: $\log \sin A = \log A' + S$ **and** $\log \tan A = \log A' + T$.
For angles near 90°: $\log \cos A = \log (90° - A)' + S$, $\log \operatorname{ctn} A = \log (90° - A)' + T$ **where** A' = **number of minutes in** A, **and** $(90° - A)'$ = **number of minutes in** $90° - A$.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	——		——		——	10.00 000	60
1	6.46 373	30103	6.46 373	30103	13.53 627	10.00 000	59
2	6.76 476	17609	6.76 476	17609	13.23 524	10.00 000	58
3	6.94 085	12494	6.94 085	12494	13.05 915	10.00 000	57
4	7.06 579	9691	7.06 579	9691	12.93 421	10.00 000	56
5	7.16 270	7918	7.16 270	7918	12.83 730	10.00 000	55
6	7.24 188	6694	7.24 188	6694	12.75 812	10.00 000	54
7	7.30 882	5800	7.30 882	5800	12.69 118	10.00 000	53
8	7.36 682	5115	7.36 682	5115	12.63 318	10.00 000	52
9	7.41 797	4576	7.41 797	4576	12.58 203	10.00 000	51
10	7.46 373	4139	7.46 373	4139	12.53 627	10.00 000	50
11	7.50 512	3779	7.50 512	3779	12.49 488	10.00 000	49
12	7.54 291	3476	7.54 291	3476	12.45 709	10.00 000	48
13	7.57 767	3218	7.57 767	3219	12.42 233	10.00 000	47
14	7.60 985	2997	7.60 986	2996	12.39 014	10.00 000	46
15	7.63 982	2802	7.63 982	2803	12.36 018	10.00 000	45
16	7.66 784	2633	7.66 785	2633	12.33 215	10.00 000	44
17	7.69 417	2483	7.69 418	2482	12.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	12.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	12.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	12.23 524	9.99 999	40
21	7.78 594	2021	7.78 595	2020	12.21 405	9.99 999	39
22	7.80 615	1030	7.80 615	1931	12.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	12.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	12.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	12.13 833	9.99 999	35
26	7.87 870	1639	7.87 871	1639	12.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	12.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	12.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	12.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	12.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	12.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	12.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	12.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	12.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	11.99 219	9.99 998	25
36	8.02 002	1190	8.02 004	1190	11.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	11.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	11.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	11.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	11.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1047	11.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	11.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	11.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	11.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	11.88 304	9.99 996	15
46	8.12 647	934	8.12 651	934	11.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	11.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	11.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	11.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	11.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	11.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	11.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	11.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	11.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	11.79 587	9.99 994	5
56	8.21 189	769	8.21 195	769	11.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	11.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	11.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	11.76 538	9.99 994	1
60	8.24 186		8.24 192		11.75 808	9.99 993	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

For logarithms of sines or tangents of angles less than 3° (or logarithms of cosines or cotangents of angles greater than 87°), see Table III*a*, p. 45. When the tabular differences are large, that method is usually better. The proportional parts stated for 1° and 2° in this table are sufficient when great accuracy is not required, even if the ordinary method of interpolation is used.

89° — Logarithms of Trigonometric Functions

1° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.24 186	717	8.24 192	718	11.75 808	9.99 993	**60**
1	8.24 903	706	8.24 910	706	11.75 090	9.99 993	59
2	8.25 609	695	8.25 616	696	11.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	11.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	11.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	11.72 331	9.99 992	**55**
6	8.28 324	653	8.28 332	654	11.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	11.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	11.70 371	9.99 992	52
9	8.30 255	624	8.30 263	625	11.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	11.69 112	9.99 991	**50**
11	8.31 495	608	8.31 505	607	11.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	11.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	11.67 289	9.99 990	47
14	8.33 292	583	8.33 302	584	11.66 698	9.99 990	46
15	8.33 875	575	8.33 886	575	11.66 114	9.99 990	**45**
16	8.34 450	568	8.34 461	568	11.65 539	9.99 989	44
17	8.35 018	560	8.35 029	561	11.64 971	9.99 989	43
18	8.35 578	553	8.35 590	553	11 64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	11.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	11.63 311	9.99 988	**40**
21	8.37 217	533	8.37 229	533	11.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	11.62 238	9.99 988	38
23	8.38 276	520	8.38 289	520	11.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	11.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	11.60 677	9.99 987	**35**
26	8.39 818	502	8.39 832	502	11.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	11.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	11.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	11.58 679	9.99 985	31
30	8.41 792	480	8.41 807	480	11.58 193	9.99 985	**30**
31	8.42 272	474	8.42 287	475	11.57 713	9.99 985	29
32	8.42 746	470	8.42 762	470	11.57 238	9.99 984	28
33	8.43 216	464	8.43 232	464	11.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	11.56 304	9.99 984	26
35	8.44 139	455	8.44 156	455	11.55 844	9.99 983	**25**
36	8.44 594	450	8.44 611	450	11.55 389	9.99 983	24
37	8.45 044	445	8.45 061	446	11.54 939	9.99 983	23
38	8.45 489	441	8.45 507	441	11.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	11.54 052	9.99 982	21
40	8.46 366	433	8.46 385	432	11.53 615	9.99 982	**20**
41	8.46 799	427	8.46 817	428	11.53 183	9.99 981	19
42	8.47 226	424	8.47 245	424	11.52 755	9.99 981	18
43	8.47 650	419	8.47 669	420	11.52 331	9.99 981	17
44	8.48 069	416	8.48 089	416	11.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	11.51 495	9.99 980	**15**
46	8.48 896	408	8.48 917	408	11.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	11.50 675	9.99 979	13
48	8.49 708	400	8.49 729	401	11.50 271	9.99 979	12
49	8.50 108	396	8.50 130	397	11.49 870	9.99 978	11
50	8.50 504	393	8.50 527	393	11.49 473	9.99 978	**10**
51	8.50 897	390	8.50 920	390	11.49 080	9.99 977	9
52	8.51 287	386	8.51 310	386	11.48 690	9.99 977	8
53	8.51 673	382	8.51 696	383	11.48 304	9.99 977	7
54	8.52 055	379	8.52 079	380	11.47 921	9.99 976	6
55	8.52 434	376	8.52 459	376	11.47 541	9.99 976	**5**
56	8.52 810	373	8.52 835	373	11.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	11.46 792	9.99 975	3
58	8.53 552	367	8.53 578	367	11.46 422	9.99 974	2
59	8.53 919	363	8.53 945	363	11.46 055	9.99 974	1
60	8.54 282		8.54 308		11.45 692	9.99 974	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	710	690	670	650
2	142	138	134	130
3	213	207	201	195
4	284	276	268	260
5	355	345	335	325
6	426	414	402	390
7	497	483	469	455
8	568	552	536	520
9	639	621	603	585

	630	620	610	600
2	126	124	122	120
3	189	186	183	180
4	252	248	244	240
5	315	310	305	300
6	378	372	366	360
7	441	434	427	420
8	504	496	488	480
9	567	558	549	540

	590	580	570	560
2	118	116	114	112
3	177	174	171	168
4	236	232	228	224
5	295	290	285	280
6	354	348	342	336
7	413	406	399	392
8	472	464	456	448
9	531	522	513	504

	550	540	530	520
2	110	108	106	104
3	165	162	159	156
4	220	216	212	208
5	275	270	265	260
6	330	324	318	312
7	385	378	371	364
8	440	432	424	416
9	495	486	477	468

	510	500	490	480
2	102	100	98	96
3	153	150	147	144
4	204	200	196	192
5	255	250	245	240
6	306	300	294	288
7	357	350	343	336
8	408	400	392	384
9	459	450	441	432

	470	460	450	440
2	94	92	90	88
3	141	138	135	132
4	188	184	180	176
5	235	230	225	220
6	282	276	270	264
7	329	322	315	308
8	376	368	360	352
9	423	414	405	396

	430	420	410	400
2	86	84	82	80
3	129	126	123	120
4	172	168	164	160
5	215	210	205	200
6	258	252	246	240
7	301	294	287	280
8	344	336	328	320
9	387	378	369	360

	390	380	370	360
2	78	76	74	72
3	117	114	111	108
4	156	152	148	144
5	195	190	185	180
6	234	228	222	216
7	273	266	259	252
8	312	304	296	288
9	351	342	333	324

Prop. Pts.

88° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.54 282	360	8.54 308	361	11.45 692	9.99 974	60
1	8.54 642	357	8.54 669	358	11.45 331	9.99 973	59
2	8.54 999	355	8.55 027	355	11.44 973	9.99 973	58
3	8.55 354	351	8.55 382	352	11.44 618	9.99 972	57
4	8.55 705	349	8.55 734	349	11.44 266	9.99 972	56
5	8.56 054	346	8.56 083	346	11.43 917	9.99 971	55
6	8.56 400	343	8.56 429	344	11.43 571	9.99 971	54
7	8.56 743	341	8.56 773	341	11.43 227	9.99 970	53
8	8.57 084	337	8.57 114	338	11.42 886	9.99 970	52
9	8.57 421	336	8.57 452	336	11.42 548	9.99 969	51
10	8.57 757	332	8.57 788	333	11.42 212	9.99 969	50
11	8.58 089	330	8.58 121	330	11.41 879	9.99 968	49
12	8.58 419	328	8.58 451	328	11.41 549	9.99 968	48
13	8.58 747	325	8.58 779	326	11.41 221	9.99 967	47
14	8.59 072	323	8.59 105	323	11.40 895	9.99 967	46
15	8.59 395	320	8.59 428	321	11.40 572	9.99 967	45
16	8.59 715	318	8.59 749	319	11.40 251	9.99 966	44
17	8.60 033	316	8.60 068	316	11.39 932	9.99 966	43
18	8.60 349	313	8.60 384	314	11.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	11.39 302	9.99 964	41
20	8.60 973	309	8.61 009	310	11.38 991	9.99 964	40
21	8.61 282	307	8.61 319	307	11.38 681	9.99 963	39
22	8.61 589	305	8.61 626	305	11.38 374	9.99 963	38
23	8.61 894	302	8.61 931	303	11.38 069	9.99 962	37
24	8.62 196	301	8.62 234	301	11.37 766	9.99 962	36
25	8.62 497	298	8.62 535	299	11.37 465	9.99 961	35
26	8.62 795	296	8.62 834	297	11.37 166	9.99 961	34
27	8.63 091	294	8.63 131	295	11.36 869	9.99 960	33
28	8.63 385	293	8.63 426	292	11.36 574	9.99 960	32
29	8.63 678	290	8.63 718	291	11.36 282	9.99 959	31
30	8.63 968	288	8.64 009	289	11.35 991	9.99 959	30
31	8.64 256	287	8.64 298	287	11.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	11.35 415	9.99 958	28
33	8.64 827	283	8.64 870	284	11.35 130	9.99 957	27
34	8.65 110	281	8.65 154	281	11.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	11.34 565	9.99 956	25
36	8.65 670	277	8.65 715	278	11.34 285	9.99 955	24
37	8.65 947	276	8.65 993	276	11.34 007	9.99 955	23
38	8.66 223	274	8.66 269	274	11.33 731	9.99 954	22
39	8.66 497	272	8.66 543	273	11.33 457	9.99 954	21
40	8.66 769	270	8.66 816	271	11.33 184	9.99 953	20
41	8.67 039	269	8.67 087	269	11.32 913	9.99 952	19
42	8.67 308	267	8.67 356	268	11.32 644	9.99 952	18
43	8.67 575	266	8.67 624	266	11.32 376	9.99 951	17
44	8.67 841	263	8.67 890	264	11.32 110	9.99 951	16
45	8.68 104	263	8.68 154	263	11.31 846	9.99 950	15
46	8.68 367	260	8.68 417	261	11.31 583	9.99 949	14
47	8.68 627	259	8.68 678	260	11.31 322	9.99 949	13
48	8.68 886	258	8.68 938	258	11.31 062	9.99 948	12
49	8.69 144	256	8.69 196	257	11.30 804	9.99 948	11
50	8.69 400	254	8.69 453	255	11.30 547	9.99 947	10
51	8.69 654	253	8.69 708	254	11.30 292	9.99 946	9
52	8.69 907	252	8.69 962	252	11.30 038	9.99 946	8
53	8.70 159	250	8.70 214	251	11.29 786	9.99 945	7
54	8.70 409	249	8.70 465	249	11.29 535	9.99 944	6
55	8.70 658	247	8.70 714	248	11.29 286	9.99 944	5
56	8.70 905	246	8.70 962	246	11.29 038	9.99 943	4
57	8.71 151	244	8.71 208	245	11.28 792	9.99 942	3
58	8.71 395	243	8.71 453	244	11.28 547	9.99 942	2
59	8.71 638	242	8.71 697	243	11.28 303	9.99 941	1
60	8.71 880		8.71 940		11.28 060	9.99 940	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	360	355	350
2	72	71.0	70
3	108	106.5	105
4	144	142.0	140
5	180	177.5	175
6	216	213.0	210
7	252	248.5	245
8	288	284.0	280
9	324	319.5	315
	345	**340**	**335**
2	69.0	68	67.0
3	103.5	102	100.5
4	138.0	136	134.0
5	172.5	170	167.5
6	207.0	204	201.0
7	241.5	238	234.5
8	276.0	272	268.0
9	310.5	306	301.5
	330	**325**	**320**
2	66	65.0	64
3	99	97.5	96
4	132	130.0	128
5	165	162.5	160
6	198	195.0	192
7	231	227.5	224
8	264	260.0	256
9	297	292.5	288
	315	**310**	**305**
2	63.0	62	61.0
3	94.5	93	91.5
4	126.0	124	122.0
5	157.5	155	152.5
6	189.0	186	183.0
7	220.5	217	213.5
8	252.0	218	244.0
9	283.5	279	274.5
	300	**295**	**290**
2	60	59.0	58
3	90	88.5	87
4	120	118.0	116
5	150	147.5	145
6	180	177.0	174
7	210	206.5	203
8	210	236.0	232
9	270	265.5	261
	285	**280**	**275**
2	57.0	56	55.0
3	85.5	84	82.5
4	114.0	112	110.0
5	142.5	140	137.5
6	171.0	168	165.0
7	199.5	196	192.5
8	228.0	224	220.0
9	256.5	252	247.5
	270	**265**	**260**
2	54	53.0	52
3	81	79.5	78
4	108	106.0	104
5	135	132.5	130
6	162	159.0	156
7	189	185.5	182
8	216	212.0	208
9	243	238.5	234
	255	**250**	**245**
2	51.0	50	49.0
3	76.5	75	73.5
4	102.0	100	98.0
5	127.5	125	122.5
6	153.0	150	147.0
7	178.5	175	171.5
8	204.0	200	196.0
9	229.5	225	220.5

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.71 880	240	8.71 940	241	11.28 060	9.99 940	**60**
1	8.72 120	239	8.72 181	239	11.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	11.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	11.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	11.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	11.26 868	9.99 937	**55**
6	8.73 303	232	8.73 366	234	11.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	11.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	11.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	11.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	11.25 708	9.99 934	**50**
11	8.74 454	226	8.74 521	227	11.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	11.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	11.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	11.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	11.24 577	9.99 930	**45**
16	8.75 575	220	8.75 645	222	11.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	11.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	11.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	11.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	11.23 475	9.99 926	**40**
21	8.76 667	216	8.76 742	216	11.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	11.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	11.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	11.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	11.22 400	9.99 923	**35**
26	8.77 733	210	8.77 811	211	11.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	11.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	11.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	11.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	11.21 351	9.99 919	**30**
31	8.78 774	205	8.78 855	206	11.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	11.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	11.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	11.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	11.20 327	9.99 915	**25**
36	8.79 789	201	8.79 875	201	11.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	11.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	11.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	11.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	11.19 326	9.99 911	**20**
41	8.80 782	196	8.80 872	196	11.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	11.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	11.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	11.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	11.18 347	9.99 907	**15**
46	8.81 752	192	8.81 846	192	11.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	11.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	11.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	11.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	11.17 390	9.99 903	**10**
51	8.82 701	187	8.82 799	188	11.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	11.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	11.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	11.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	11.16 453	9.99 898	**5**
56	8.83 630	183	8.83 732	184	11.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	11.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	11.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	11.15 718	9.99 895	1
60	8.84 358		8.84 464		11.15 536	9.99 894	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	240	235	230
2	48	47.0	46
3	72	70.5	69
4	96	94.0	92
5	120	117.5	115
6	144	141.0	138
7	168	164.5	161
8	192	188.0	184
9	216	211.5	207

	225	220	215
2	45.0	44.0	43.0
3	67.5	66.0	64.5
4	90.0	88.0	86.0
5	112.5	110.0	107.5
6	135.0	132.0	129.0
7	157.5	154.0	150.5
8	180.0	176.0	172.0
9	202.5	198.0	193.5

	213	211	208
2	42.6	42.2	41.6
3	63.9	63.3	62.4
4	85.2	84.4	83.2
5	106.5	105.5	104.0
6	127.8	126.6	124.8
7	149.1	147.7	145.6
8	170.4	168.8	166.4
9	191.7	189.9	187.2

	206	203	201
2	41.2	40.6	40.2
3	61.8	60.9	60.3
4	82.4	81.2	80.4
5	103.0	101.5	100.5
6	123.6	121.8	120.6
7	144.2	142.1	140.7
8	164.8	162.4	160.8
9	185.4	182.7	180.9

	199	197	195
2	39.8	39.4	39.0
3	59.7	59.1	58.5
4	79.6	78.8	78.0
5	99.5	98.5	97.5
6	119.4	118.2	117.0
7	139.3	137.9	136.5
8	159.2	157.6	156.0
9	179.1	177.3	175.5

	193	192	190
2	38.6	38.4	38.0
3	57.9	57.6	57.0
4	77.2	76.8	76.0
5	96.5	96.0	95.0
6	115.8	115.2	114.0
7	135.1	134.4	133.0
8	154.4	153.6	152.0
9	173.7	172.8	171.0

	188	186	184
2	37.6	37.2	36.8
3	56.4	55.8	55.2
4	75.2	74.4	73.6
5	94.0	93.0	92.0
6	112.8	111.6	110.4
7	131.6	130.2	128.8
8	150.4	148.8	147.2
9	169.2	167.4	165.6

	183	182	181
2	36.6	36.4	36.2
3	54.9	54.6	54.3
4	73.2	72.8	72.4
5	91.5	91.0	90.5
6	109.8	109.2	108.6
7	128.1	127.4	126.7
8	146.4	145.6	144.8
9	164.7	163.8	162.9

Prop. Pts.

4° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.84 358	181	8.84 464	182	11.15 536	9.99 894	**60**
1	8.84 539	179	8.84 646	180	11.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	11.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	11.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	11.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	11.14 637	9.99 890	**55**
6	8.85 429	176	8.85 540	177	11.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	11.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	11.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	11.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	11.13 757	9.99 885	**50**
11	8.86 301	173	8.86 417	174	11.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	11.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	11.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	11.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	11.12 894	9.99 880	**45**
16	8.87 156	169	8.87 277	170	11.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	11.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	11.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	11.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	11.12 047	9.99 876	**40**
21	8.87 995	166	8.88 120	167	11.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	11.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	11.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	11.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	11.11 217	9.99 871	**35**
26	8.88 817	163	8.88 948	163	11.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	11.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	11.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	11.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	11.10 402	9.99 866	**30**
31	8.89 625	159	8.89 760	160	11.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	11.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	11.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	11.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	11.09 601	9.99 861	**25**
36	8.90 417	157	8.90 557	158	11.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	11.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	11.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	11.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	11.08 815	9.99 856	**20**
41	8.91 195	154	8.91 340	155	11.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	11.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	11.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	11.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	11.08 043	9.99 851	**15**
46	8.91 959	151	8.92 110	152	11.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	11.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	11.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	11.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	11.07 284	9.99 845	**10**
51	8.92 710	149	8.92 866	150	11.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	11.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	11.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	11.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	11.06 538	9.99 840	**5**
56	8.93 448	146	8.93 609	147	11.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	11.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	11.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	11.05 951	9.99 836	1
60	8.94 030		8.94 195		11.05 805	9.99 834	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	181	180	179
2	36.2	36.0	35.8
3	54.3	54.0	53.7
4	72.4	72.0	71.6
5	90.5	90.0	89.5
6	108.6	108.0	107.4
7	126.7	126.0	125.3
8	144.8	144.0	143.2
9	162.9	162.0	161.1

	177	175	173
2	35.4	35.0	34.6
3	53.1	52.5	51.9
4	70.8	70.0	69.2
5	88.5	87.5	86.5
6	106.2	105.0	103.8
7	123.9	122.5	121.1
8	141.6	140.0	138.4
9	159.3	157.5	155.7

	171	170	169
2	34.2	34.0	33.8
3	51.3	51.0	50.7
4	68.4	68.0	67.6
5	85.5	85.0	84.5
6	102.6	102.0	101.4
7	119.7	119.0	118.3
8	136.8	136.0	135.2
9	153.9	153.0	152.1

	167	165	163
2	33.4	33.0	32.6
3	50.1	49.5	48.9
4	66.8	66.0	65.2
5	83.5	82.5	81.5
6	100.2	99.0	97.8
7	116.9	115.5	114.1
8	133.6	132.0	130.4
9	150.3	148.5	146.7

	161	160	159
2	32.2	32.0	31.8
3	48.3	48.0	47.7
4	64.4	64.0	63.6
5	80.5	80.0	79.5
6	96.6	96.0	95.4
7	112.7	112.0	111.3
8	128.8	128.0	127.2
9	144.9	144.0	143.1

	157	155	153
2	31.4	31.0	30.6
3	47.1	46.5	45.9
4	62.8	62.0	61.2
5	78.5	77.5	76.5
6	94.2	93.0	91.8
7	109.9	108.5	107.1
8	125.6	124.0	122.4
9	141.3	139.5	137.7

	151	150	149
2	30.2	30.0	29.8
3	45.3	45.0	44.7
4	60.4	60.0	59.6
5	75.5	75.0	74.5
6	90.6	90.0	89.4
7	105.7	105.0	104.3
8	120.8	120.0	119.2
9	135.9	135.0	134.1

	147	145	144
2	29.4	29.0	28.8
3	44.1	43.5	43.2
4	58.8	58.0	57.6
5	73.5	72.5	72.0
6	88.2	87.0	86.4
7	102.9	101.5	100.8
8	117.6	116.0	115.2
9	132.3	130.5	129.6

Prop. Pts.

5° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.94 030	144	8.94 195	145	11.05 805	9.99 834	**60**
1	8.94 174	143	8.94 340	145	11.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	11.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	11.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	11.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	11.05 083	9.99 829	**55**
6	8.94 887	142	8.95 060	142	11.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	11.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	11.04 656	9.99 825	52
9	8.95 310	140	8.95 486	141	11.04 514	9.99 824	51
10	8.95 450	139	8.95 627	140	11.04 373	9.99 823	**50**
11	8.95 589	139	8.95 767	141	11.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	11.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	11.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	11.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	11.03 675	9.99 817	**45**
16	8.96 280	137	8.96 464	138	11.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	11.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	11.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	11.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	11.02 987	9.99 812	**40**
21	8.96 960	135	8.97 150	135	11.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	11.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	11.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	11.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	11.02 309	9.99 806	**35**
26	8.97 629	133	8.97 825	134	11.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	11.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	11.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	11.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	11.01 642	9.99 800	**30**
31	8.98 288	131	8.98 490	132	11.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	11.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	11.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	11.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	11.00 985	9.99 793	**25**
36	8.98 937	129	8.99 145	130	11.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	11.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	11.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	11.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	11.00 338	9.99 787	**20**
41	8.99 577	127	8.99 791	128	11.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	11.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	10.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	10.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	10.99 699	9.99 781	**15**
46	9.00 207	125	9.00 427	126	10.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	10.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	10.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	10.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	10.99 070	9.99 775	**10**
51	9.00 828	123	9.01 055	124	10.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	10.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	10.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	10.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	10.98 450	9.99 768	**5**
56	9.01 440	121	9.01 673	123	10.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	10.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	10.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	10.97 960	9.99 763	1
60	9.01 923		9.02 162		10.97 838	9.99 761	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	143	142	141
2	28.6	28.4	28.2
3	42.9	42.6	42.3
4	57.2	56.8	56.4
5	71.5	71.0	70.5
6	85.8	85.2	84.6
7	100.1	99.4	98.7
8	114.4	113.6	112.8
9	128.7	127.8	126.9

	140	139	138
2	28.0	27.8	27.6
3	42.0	41.7	41.4
4	56.0	55.6	55.2
5	70.0	69.5	69.0
6	84.0	83.4	82.8
7	98.0	97.3	96.6
8	112.0	111.2	110.4
9	126.0	125.1	124.2

	137	136	135
2	27.4	27.2	27.0
3	41.1	40.8	40.5
4	54.8	54.4	54.0
5	68.5	68.0	67.5
6	82.2	81.6	81.0
7	95.9	95.2	94.5
8	109.6	108.8	108.0
9	123.3	122.4	121.5

	134	133	132
2	26.8	26.6	26.4
3	40.2	39.9	39.6
4	53.6	53.2	52.8
5	67.0	66.5	66.0
6	80.4	79.8	79.2
7	93.8	93.1	92.4
8	107.2	106.4	105.6
9	120.6	119.7	118.8

	131	130	129
2	26.2	26.0	25.8
3	39.3	39.0	38.7
4	52.4	52.0	51.6
5	65.5	65.0	64.5
6	78.6	78.0	77.4
7	91.7	91.0	90.3
8	104.8	104.0	103.2
9	117.9	117.0	116.1

	128	127	126
2	25.6	25.4	25.2
3	38.4	38.1	37.8
4	51.2	50.8	50.4
5	64.0	63.5	63.0
6	76.8	76.2	75.6
7	89.6	88.9	88.2
8	102.4	101.6	100.8
9	115.2	114.3	113.4

	125	124	123
2	25.0	24.8	24.6
3	37.5	37.2	36.9
4	50.0	49.6	49.2
5	62.5	62.0	61.5
6	75.0	74.4	73.8
7	87.5	86.8	86.1
8	100.0	99.2	98.4
9	112.5	111.6	110.7

	122	121	120
2	24.4	24.2	24.0
3	36.6	36.3	36.0
4	48.8	48.4	48.0
5	61.0	60.5	60.0
6	73.2	72.6	72.0
7	85.4	84.7	84.0
8	97.6	96.8	96.0
9	109.8	108.9	108.0

Prop. Pts.

84° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.01 923	120	9.02 162	121	10.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	10.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	10.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	10.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	10.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	10.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	10.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	10.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	10.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	10.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	10.96 639	9.99 748	50
11	9.03 226	116	9.03 479	118	10.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	10.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	10.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	10.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	10.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	10.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	10.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	10.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	10.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	10.95 472	9.99 734	40
21	9.04 376	114	9.04 643	115	10.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	10.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	10.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	10.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	10.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	10.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	10.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	10.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	10.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	10.94 334	9.99 720	30
31	9.05 497	110	9.05 778	112	10.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	10.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	10.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	10.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	10.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	10.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	10.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	10.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	10.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	10.93 225	9.99 705	20
41	9.06 589	107	9.06 885	109	10.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	10.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	10.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	10.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	10.92 680	9.99 698	15
46	9.07 124	107	9.07 428	108	10.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	10.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	10.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	10.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	10.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	10.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	10.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	10.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	10.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	10.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	10.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	10.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	10.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	10.91 190	9.99 677	1
60	9.08 589		9.08 914		10.91 086	9.99 675	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	121	120	119
2	24.2	24.0	23.8
3	36.3	36.0	35.7
4	48.4	48.0	47.6
5	60.5	60.0	59.5
6	72.6	72.0	71.4
7	84.7	84.0	83.3
8	96.8	96.0	95.2
9	108.9	108.0	107.1
	118	**117**	**116**
2	23.6	23.4	23.2
3	35.4	35.1	34.8
4	47.2	46.8	46.4
5	59.0	58.5	58.0
6	70.8	70.2	69.6
7	82.6	81.9	81.2
8	94.4	93.6	92.8
9	106.2	105.3	104.4
	115	**114**	**113**
2	23.0	22.8	22.6
3	34.5	34.2	33.9
4	46.0	45.6	45.2
5	57.5	57.0	56.5
6	69.0	68.4	67.8
7	80.5	79.8	79.1
8	92.0	91.2	90.4
9	103.5	102.6	101.7
	112	**111**	**110**
2	22.4	22.2	22.0
3	33.6	33.3	33.0
4	44.8	44.4	44.0
5	56.0	55.5	55.0
6	67.2	66.6	66.0
7	78.4	77.7	77.0
8	89.6	88.8	88.0
9	100.8	99.9	99.0
	109	**108**	**107**
2	21.8	21.6	21.4
3	32.7	32.4	32.1
4	43.6	43.2	42.8
5	54.5	54.0	53.5
6	65.4	64.8	64.2
7	76.3	75.6	74.9
8	87.2	86.4	85.6
9	98.1	97.2	96.3
	106	**105**	**104**
2	21.2	21.0	20.8
3	31.8	31.5	31.2
4	42.4	42.0	41.6
5	53.0	52.5	52.0
6	63.6	63.0	62.4
7	74.2	73.5	72.8
8	84.8	84.0	83.2
9	95.4	94.5	93.6

From the top:

For **6°+** or **186°+**, read as printed; for **96°+** or **276°+**, read co-function.

From the bottom:

For **83°+** or **263°+**, read as printed; for **173°+** or **353°+**, read co-function.

Prop. Pts.

7° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.08 589	103	9.08 914	105	10.91 086	9.99 675	60
1	9.08 692	103	9.09 019	104	10.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	10.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	10.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	10.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	10.90 566	9.99 667	55
6	9.09 202	102	9.09 537	103	10.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	10.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	10.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	10.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	10.90 053	9.99 659	50
11	9.09 707	100	9.10 049	101	10.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	10.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	10.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	10.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	10.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	10.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	10.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	10.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	10.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	10.89 044	9.99 643	40
21	9.10 697	98	9.11 056	99	10.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	10.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	10.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	10.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	10.88 548	9.99 635	35
26	9.11 184	97	9.11 551	98	10.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	10.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	10.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	10.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	10.88 057	9.99 627	30
31	9.11 666	95	9.12 040	98	10.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	10.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	10.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	10.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	10.87 572	9.99 618	25
36	9.12 142	94	9.12 525	96	10.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	10.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	10.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	10.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	10.87 091	9.99 610	20
41	9.12 612	94	9.13 004	95	10.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	10.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	10.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	10.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	10.86 616	9.99 601	15
46	9.13 078	93	9.13 478	95	10.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	10.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	10.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	10.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	10.86 146	9.99 593	10
51	9.13 539	91	9.13 948	93	10.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	10.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	10.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	10.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	10.85 680	9.99 584	5
56	9.13 994	91	9.14 412	92	10.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	10.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	10.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	10.85 312	9.99 577	1
60	9.14 356		9.14 780		10.85 220	9.99 575	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	105	104	103
2	21.0	20.8	20.6
3	31.5	31.2	30.9
4	42.0	41.6	41.2
5	52.5	52.0	51.5
6	63.0	62.4	61.8
7	73.5	72.8	72.1
8	84.0	83.2	82.4
9	94.5	93.6	92.7

	102	101	99
2	20.4	20.2	19.8
3	30.6	30.3	29.7
4	40.8	40.4	39.6
5	51.0	50.5	49.5
6	61.2	60.6	59.4
7	71.4	70.7	69.3
8	81.6	80.8	79.2
9	91.8	90.9	89.1

	98	97	96
2	19.6	19.4	19.2
3	29.4	29.1	28.8
4	39.2	38.8	38.4
5	49.0	48.5	48.0
6	58.8	58.2	57.6
7	68.6	67.9	67.2
8	78.4	77.6	76.8
9	88.2	87.3	86.4

	95	94	93
2	19.0	18.8	18.6
3	28.5	28.2	27.9
4	38.0	37.6	37.2
5	47.5	47.0	46.5
6	57.0	56.4	55.8
7	66.5	65.8	65.1
8	76.0	75.2	74.4
9	85.5	84.6	83.7

	92	91	90
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

From the top:

For **7°+** or **187°+**, read as printed; for **97°+** or **277°+**, read co-function.

From the bottom:

For **82°+** or **262°+**, read as printed; for **172°+** or **352°+**, read co-function.

Prop. Pts.

82° — Logarithms of Trigonometric Functions

8° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	
0	9.14 356	89	9.14 780	92	10.85 220	9.99 575	60
1	9.14 445	90	9.14 872	91	10.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	10.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	10.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	10.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	10.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	10.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	10.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	10.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	10.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	10.84 312	9.99 557	50
11	9.15 333	88	9.15 777	90	10.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	10.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	10.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	10.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	10.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	10.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	10.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	10.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	10.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	10.83 423	9.99 539	40
21	9.16 203	86	9.16 665	88	10.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	10.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	10.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	10.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	10.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	10.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	10.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	10.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	10.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	10.82 550	9.99 520	30
31	9.17 055	84	9.17 536	86	10.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	10.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	10.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	10.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	10.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	10.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	10.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	10.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	10.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	10.81 694	9.99 501	20
41	9.17 890	83	9.18 391	84	10.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	10.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	10.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	10.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	10.81 272	9.99 492	15
46	9.18 302	81	9.18 812	84	10.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	10.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	10.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	10.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	10.80 854	9.99 482	10
51	9.18 709	81	9.19 229	83	10.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	10.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	10.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	10.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	10.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	10.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	10.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	10.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	10.80 111	9.99 464	1
60	9.19 433		9.19 971		10.80 029	9.99 462	0
	L Cos	d	L Ctn	cd	L Tan	L Sin	′

Prop. Pts.

	92	91	90
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

	89	88	87
2	17.8	17.6	17.4
3	26.7	26.4	26.1
4	35.6	35.2	34.8
5	44.5	44.0	43.5
6	53.4	52.8	52.2
7	62.3	61.6	60.9
8	71.2	70.4	69.6
9	80.1	79.2	78.3

	86	85	84
2	17.2	17.0	16.8
3	25.8	25.5	25.2
4	34.4	34.0	33.6
5	43.0	42.5	42.0
6	51.6	51.0	50.4
7	60.2	59.5	58.8
8	68.8	68.0	67.2
9	77.4	76.5	75.6

	83	82	81
2	16.6	16.4	16.2
3	24.9	24.6	24.3
4	33.2	32.8	32.4
5	41.5	41.0	40.5
6	49.8	49.2	48.6
7	58.1	57.4	56.7
8	66.4	65.6	64.8
9	74.7	73.8	72.9

From the top:

For **8°+** or **188°+**, read as printed; for **98°+** or **278°+**, read co-function.

From the bottom:

For **81°+** or **261°+**, read as printed; for **171°+** or **351°+**, read co-function.

Prop. Pts.

81° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	
0	9.19 433	80	9.19 971	82	10.80 029	9.99 462	**60**
1	9.19 513	79	9.20 053	81	10.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	10.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	10.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	10.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	10.79 622	9.99 452	**55**
6	9.19 909	79	9.20 459	81	10.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	10.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	10.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	10.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	10.79 218	9.99 442	**50**
11	9.20 302	78	9.20 862	80	10.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	10.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	10.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	10.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	10.78 818	9.99 432	**45**
16	9.20 691	77	9.21 261	80	10.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	10.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	10.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	10.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	10.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	10.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	10.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	10.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	10.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	10.78 029	9.99 411	**35**
26	9.21 458	76	9.22 049	78	10.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	10.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	10.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	10.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	10.77 639	9.99 400	**30**
31	9.21 836	76	9.22 438	78	10.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	10.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	10.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	10.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	10.77 253	9.99 390	**25**
36	9.22 211	75	9.22 824	77	10.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	10.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	10.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	10.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	10.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	77	10.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	10.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	10.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	10.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	10.76 490	9.99 368	**15**
46	9.22 952	73	9.23 586	75	10.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	10.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	10.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	10.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	10.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	10.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	10.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	10.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	10.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	10.75 739	9.99 346	**5**
56	9.23 679	73	9.24 335	75	10.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	10.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	10.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	10.75 442	9.99 337	1
60	9.23 967		9.24 632		10.75 368	9.99 335	**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	′

Prop. Pts.

	82	**81**	**80**
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

	79	**78**	**77**
2	15.8	15.6	15.4
3	23.7	23.4	23.1
4	31.6	31.2	30.8
5	39.5	39.0	38.5
6	47.4	46.8	46.2
7	55.3	54.6	53.9
8	63.2	62.4	61.6
9	71.1	70.2	69.3

	76	**75**	**74**
2	15.2	15.0	14.8
3	22.8	22.5	22.2
4	30.4	30.0	29.6
5	38.0	37.5	37.0
6	45.6	45.0	44.4
7	53.2	52.5	51.8
8	60.8	60.0	59.2
9	68.4	67.5	66.6

	73	**72**	**71**
2	14.6	14.4	14.2
3	21.9	21.6	21.3
4	29.2	28.8	28.4
5	36.5	36.0	35.5
6	43.8	43.2	42.6
7	51.1	50.4	49.7
8	58.4	57.6	56.8
9	65.7	64.8	63.9

From the top:

For **9°+**, or **189°+**, read as printed; for **99°+** or **279°+**, read co-function.

From the bottom:

For **80°+** or **260°+**, read as printed; for **170°+** or **350°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.23 967	72	9.24 632	74	10.75 368	9.99 335	2	**60**
1	9.24 039	71	9.24 706	73	10.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	10.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	10.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	10.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	10.75 000	9.99 324	2	**55**
6	9.24 395	71	9.25 073	73	10.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	10.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	10.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	10.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	10.74 635	9.99 313	3	**50**
11	9.24 748	70	9.25 437	73	10.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	10.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	10.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	10.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	10.74 273	9.99 301	2	**45**
16	9.25 098	70	9.25 799	72	10.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	10.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	10.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	10.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	10.73 914	9.99 290	2	**40**
21	9.25 445	69	9.26 158	71	10.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	10.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	10.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	10.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	10.73 557	9.99 278	2	**35**
26	9.25 790	68	9.26 514	71	10.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	10.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	10.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	10.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	10.73 203	9.99 267	3	**30**
31	9.26 131	68	9.26 867	70	10.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	10.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	10.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	10.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	10.72 852	9.99 255	3	**25**
36	9.26 470	68	9.27 218	70	10.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	10.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	10.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	10.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	10.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	69	10.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	10.72 365	9.99 238	2	18
43	9.26 940	67	9.27 704	69	10.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	10.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	10.72 158	9.99 231	2	**15**
46	9.27 140	66	9.27 911	69	10.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	10.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	10.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	10.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	10.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	69	10.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	10.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	10.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	10.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	10.71 473	9.99 207	3	**5**
56	9.27 799	63	9.28 595	67	10.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	10.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	10.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	10.71 202	9.99 197	2	1
60	9.28 060		9.28 865		10.71 135	9.99 195		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	74	**73**	**72**
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	**70**	**69**
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	**67**	**66**
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	**3**	**2**
2	13.0	0.6	0.4
3	19.5	0.9	0.6
4	26.0	1.2	0.8
5	32.5	1.5	1.0
6	39.0	1.8	1.2
7	45.5	2.1	1.4
8	52.0	2.4	1.6
9	58.5	2.7	1.8

From the top:

For **10°+** or **190°+**, read as printed; for **100°+** or **280°+**, read co-function.

From the bottom:

For **79°+** or **259°+**, read as printed; for **169°+** or **349°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.28 060	65	9.28 865	68	10.71 135	9.99 195	3	**60**
1	9.28 125	65	9.28 933	67	10.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	10.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	10.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	10.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	10.70 799	9.99 182	2	**55**
6	9.28 448	64	9.29 268	67	10.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	10.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	10.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	10.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	10.70 465	9.99 170	3	**50**
11	9.28 769	64	9.29 601	67	10.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	10.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	10.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	10.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	10.70 134	9.99 157	2	**45**
16	9.29 087	63	9.29 932	66	10.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	10.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	10.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	10.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	10.69 805	9.99 145	3	**40**
21	9.29 403	63	9.30 261	65	10.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	10.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	10.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	10.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	10.69 478	9.99 132	2	**35**
26	9.29 716	63	9.30 587	65	10.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	10.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	10.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	10.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	10.69 154	9.99 119	2	**30**
31	9.30 028	62	9.30 911	64	10.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	10.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	10.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	10.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	10.68 832	9.99 106	2	**25**
36	9.30 336	62	9.31 233	64	10.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	10.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	10.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	10.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	10.68 511	9.99 093	2	**20**
41	9.30 643	61	9.31 552	64	10.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	10.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	10.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	10.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	10.68 194	9.99 080	2	**15**
46	9.30 947	61	9.31 870	63	10.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	10.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	10.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	10.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	10.67 878	9.99 067	3	**10**
51	9.31 250	60	9.32 185	63	10.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	10.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	10.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	10.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	10.67 564	9.99 054	3	**5**
56	9.31 549	60	9.32 498	63	10.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	10.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	10.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	10.67 315	9.99 043	3	1
60	9.31 788		9.32 747		10.67 253	9.99 040		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	68	**67**	**66**
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	**64**	**63**
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7

	62	**61**	**60**
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0

	59	**3**	**2**
2	11.8	0.6	0.4
3	17.7	0.9	0.6
4	23.6	1.2	0.8
5	29.5	1.5	1.0
6	35.4	1.8	1.2
7	41.3	2.1	1.4
8	47.2	2.4	1.6
9	53.1	2.7	1.8

From the top:

For **11°+** or **191°+**, read as printed; for **101°+** or **281°+**, read co-function.

From the bottom:

For **78°+** or **258°+**, read as printed; for **168°+** or **348°+**, read co-function.

Prop. Pts.

78° — Logarithms of Trigonometric Functions

12° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.31 788	59	9.32 747	63	10.67 253	9.99 040	2	**60**
1	9.31 847	60	9.32 810	62	10.67 190	9.99 038	3	59
2	9.31 907	59	9.32 872	61	10.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	10.67 067	9.99 032	2	57
4	9.32 025	59	9.32 995	62	10.67 005	9.99 030	3	56
5	9.32 084	59	9.33 057	62	10.66 943	9.99 027	3	**55**
6	9.32 143	59	9.33 119	61	10.66 881	9.99 024	2	54
7	9.32 202	59	9.33 180	62	10.66 820	9.99 022	3	53
8	9.32 261	58	9.33 242	61	10.66 758	9.99 019	3	52
9	9.32 319	59	9.33 303	62	10.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	10.66 635	9.99 013	2	**50**
11	9.32 437	58	9.33 426	61	10.66 574	9.99 011	3	49
12	9.32 495	58	9.33 487	61	10.66 513	9.99 008	3	48
13	9.32 553	59	9.33 548	61	10.66 452	9.99 005	3	47
14	9.32 612	58	9.33 609	61	10.66 391	9.99 002	2	46
15	9.32 670	58	9.33 670	61	10.66 330	9.99 000	3	**45**
16	9.32 728	58	9.33 731	61	10.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	10.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	10.66 147	9.98 991	2	42
19	9.32 902	58	9.33 913	61	10.66 087	9.98 989	3	41
20	9.32 960	58	9.33 974	60	10.66 026	9.98 986	3	**40**
21	9.33 018	57	9.34 034	61	10.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	10.65 905	9.98 980	2	38
23	9.33 133	57	9.34 155	60	10.65 845	9.98 978	3	37
24	9.33 190	58	9.34 215	61	10.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	10.65 724	9.98 972	3	**35**
26	9.33 305	57	9.34 336	60	10.65 664	9.98 969	2	34
27	9.33 362	58	9.34 396	60	10.65 604	9.98 967	3	33
28	9.33 420	57	9.34 456	60	10.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	10.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	59	10.65 424	9.98 958	3	**30**
31	9.33 591	56	9.34 635	60	10.65 365	9.98 955	2	29
32	9.33 647	57	9.34 695	60	10.65 305	9.98 953	3	28
33	9.33 704	57	9.34 755	59	10.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	60	10.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	59	10.65 126	9.98 944	3	**25**
36	9.33 874	57	9.34 933	59	10.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	10.65 008	9.98 938	2	23
38	9.33 987	56	9.35 051	60	10.64 949	9.98 936	3	22
39	9.34 043	57	9.35 111	59	10.64 889	9.98 933	3	21
40	9.34 100	56	9.35 170	59	10.64 830	9.98 930	3	**20**
41	9.34 156	56	9.35 229	59	10.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	10.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	10.64 653	9.98 921	2	17
44	9.34 324	56	9.35 405	59	10.64 595	9.98 919	3	16
45	9.34 380	56	9.35 464	59	10.64 536	9.98 916	3	**15**
46	9.34 436	55	9.35 523	58	10.64 477	9.98 913	3	14
47	9.34 491	56	9.35 581	59	10.64 419	9.98 910	3	13
48	9.34 547	55	9.35 640	58	10.64 360	9.98 907	3	12
49	9.34 602	56	9.35 698	59	10.64 302	9.98 904	3	11
50	9.34 658	55	9.35 757	58	10.64 243	9.98 901	3	**10**
51	9.34 713	56	9.35 815	58	10.64 185	9.98 898	2	9
52	9.34 769	55	9.35 873	58	10.64 127	9.98 896	3	8
53	9.34 824	55	9.35 931	58	10.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	10.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	10.63 953	9.98 887	3	**5**
56	9.34 989	55	9.36 105	58	10.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	10.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	10.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	57	10.63 721	9.98 875	3	1
60	9.35 209		9.36 336		10.63 664	9.98 872		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	63	62	61
2	12.6	12.4	12.2
3	18.9	18.6	18.3
4	25.2	24.8	24.4
5	31.5	31.0	30.5
6	37.8	37.2	36.6
7	44.1	43.4	42.7
8	50.4	49.6	48.8
9	56.7	55.8	54.9

	60	59	58
2	12.0	11.8	11.6
3	18.0	17.7	17.4
4	24.0	23.6	23.2
5	30.0	29.5	29.0
6	36.0	35.4	34.8
7	42.0	41.3	40.6
8	48.0	47.2	46.4
9	54.0	53.1	52.2

	57	56
2	11.4	11.2
3	17.1	16.8
4	22.8	22.4
5	28.5	28.0
6	34.2	33.6
7	39.9	39.2
8	45.6	44.8
9	51.3	50.4

	55	3	2
2	11.0	0.6	0.4
3	16.5	0.9	0.6
4	22.0	1.2	0.8
5	27.5	1.5	1.0
6	33.0	1.8	1.2
7	38.5	2.1	1.4
8	44.0	2.4	1.6
9	49.5	2.7	1.8

From the top:

For **12°+** or **192°+**, read as printed; for **102°+** or **282°+**, read co-function.

From the bottom:

For **77°+** or **257°+**, read as printed; for **167°+** or **347°+**, read co-function.

Prop. Pts.

77° — Logarithms of Trigonometric Functions

13° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.35 209	54	9.36 336	58	10.63 664	9.98 872	3	**60**
1	9.35 263	55	9.36 394	58	10.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	10.63 548	9.98 867	3	58
3	9.35 373	54	9.36 509	57	10.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	10.63 434	9.98 861	3	56
5	9.35 481	55	9.36 624	57	10.63 376	9.98 858	3	**55**
6	9.35 536	54	9.36 681	57	10.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	10.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	10.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	10.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	10.63 091	9.98 843	3	**50**
11	9.35 806	54	9.36 966	57	10.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	10.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	10.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	10.62 863	9.98 831	3	46
15	9.36 022	53	9.37 193	57	10.62 807	9.98 828	3	**45**
16	9.36 075	54	9.37 250	56	10.62 750	9.98 825	3	44
17	9.36 129	53	9.37 306	57	10.62 694	9.98 822	3	43
18	9.36 182	54	9.37 363	56	10.62 637	9.98 819	3	42
19	9.36 236	53	9.37 419	57	10.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	10.62 524	9.98 813	3	**40**
21	9.36 342	53	9.37 532	56	10.62 468	9.98 810	3	39
22	9.36 395	54	9.37 588	56	10.62 412	9.98 807	3	38
23	9.36 449	53	9.37 644	56	10.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	10.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	10.62 244	9.98 798	3	**35**
26	9.36 608	52	9.37 812	56	10.62 188	9.98 795	3	34
27	9.36 660	53	9.37 868	56	10.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	10.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	55	10.62 020	9.98 786	3	31
30	9.36 819	52	9.38 035	56	10.61 965	9.98 783	3	**30**
31	9.36 871	53	9.38 091	56	10.61 909	9.98 780	3	29
32	9.36 924	52	9.38 147	55	10.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	10.61 798	9.98 774	3	27
34	9.37 028	53	9.38 257	56	10.61 743	9.98 771	3	26
35	9.37 081	52	9.38 313	55	10.61 687	9.98 768	3	**25**
36	9.37 133	52	9.38 368	55	10.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	56	10.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	55	10.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	10.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	10.61 411	9.98 753	3	**20**
41	9.37 393	52	9.38 644	55	10.61 356	9.98 750	4	19
42	9.37 445	52	9.38 699	55	10.61 301	9.98 746	3	18
43	9.37 497	52	9.38 754	54	10.61 246	9.98 743	3	17
44	9.37 549	51	9.38 808	55	10.61 192	9.98 740	3	16
45	9.37 600	52	9.38 863	55	10.61 137	9.98 737	3	**15**
46	9.37 652	51	9.38 918	54	10.61 082	9.98 734	3	14
47	9.37 703	52	9.38 972	55	10.61 028	9.98 731	3	13
48	9.37 755	51	9.39 027	55	10.60 973	9.98 728	3	12
49	9.37 806	52	9.39 082	54	10.60 918	9.98 725	3	11
50	9.37 858	51	9.39 136	54	10.60 864	9.98 722	3	**10**
51	9.37 909	51	9.39 190	55	10.60 810	9.98 719	4	9
52	9.37 960	51	9.39 245	54	10.60 755	9.98 715	3	8
53	9.38 011	51	9.39 299	54	10.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	10.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	10.60 593	9.98 706	3	**5**
56	9.38 164	51	9.39 461	54	10.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	10.60 485	9.98 700	3	3
58	9.38 206	51	9.39 569	54	10.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	10.60 377	9.98 694	4	1
60	9.38 368		9.39 677		10.60 323	9.98 690		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	58	**57**	**56**
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	**54**	**53**
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	**51**
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	**3**	**2**
2	0.8	0.6	0.4
3	1.2	0.9	0.6
4	1.6	1.2	0.8
5	2.0	1.5	1.0
6	2.4	1.8	1.2
7	2.8	2.1	1.4
8	3.2	2.4	1.6
9	3.6	2.7	1.8

From the top:

For **13°+** or **193°+**, read as printed; for **103°+** or **283°+**, read co-function.

From the bottom:

For **76°+** or **256°+**, read as printed; for **166°+** or **346°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.38 368	50	9.39 677	54	10.60 323	9.98 690	3	60
1	9.38 418	51	9.39 731	54	10.60 269	9.98 687	3	59
2	9.38 469	50	9.39 785	53	10.60 215	9.98 684	3	58
3	9.38 519	51	9.39 838	54	10.60 162	9.98 681	3	57
4	9.38 570	50	9.39 892	53	10.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	54	10.60 055	9.98 675	4	55
6	9.38 670	51	9.39 999	53	10.60 001	9.98 671	3	54
7	9.38 721	50	9.40 052	54	10.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	53	10.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	10.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	54	10.59 788	9.98 659	3	50
11	9.38 921	50	9.40 266	53	10.59 734	9.98 656	4	49
12	9.38 971	50	9.40 319	53	10.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	10.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	10.59 575	9.98 646	3	46
15	9.39 121	49	9.40 478	53	10.59 522	9.98 643	3	45
16	9.39 170	50	9.40 531	53	10.59 469	9.98 640	4	44
17	9.39 220	50	9.40 584	52	10.59 416	9.98 636	3	43
18	9.39 270	49	9.40 636	53	10.59 364	9.98 633	3	42
19	9.39 319	50	9.40 689	53	10.59 311	9.98 630	3	41
20	9.39 369	49	9.40 742	53	10.59 258	9.98 627	4	40
21	9.39 418	49	9.40 795	52	10.59 205	9.98 623	3	39
22	9.39 467	50	9.40 847	53	10.59 153	9.98 620	3	38
23	9.39 517	49	9.40 900	52	10.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	53	10.59 048	9.98 614	4	36
25	9.39 615	49	9.41 005	52	10.58 995	9.98 610	3	35
26	9.39 664	49	9.41 057	52	10.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	10.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	53	10.58 839	9.98 601	4	32
29	9.39 811	49	9.41 214	52	10.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	10.58 734	9.98 594	3	30
31	9.39 909	49	9.41 318	52	10.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	10.58 630	9.98 588	4	28
33	9.40 006	49	9.41 422	52	10.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	52	10.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	10.58 474	9.98 578	4	25
36	9.40 152	48	9.41 578	51	10.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	10.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	10.58 319	9.98 568	3	22
39	9.40 297	49	9.41 733	51	10.58 267	9.98 565	4	21
40	9.40 346	48	9.41 784	52	10.58 216	9.98 561	3	20
41	9.40 394	48	9.41 836	51	10.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	52	10.58 113	9.98 555	4	18
43	9.40 490	48	9.41 939	51	10.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	10.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	52	10.57 959	9.98 545	4	15
46	9.40 634	48	9.42 093	51	10.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	10.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	10.57 805	9.98 535	4	12
49	9.40 778	47	9.42 246	51	10.57 754	9.98 531	3	11
50	9.40 825	48	9.42 297	51	10.57 703	9.98 528	3	10
51	9.40 873	48	9.42 348	51	10.57 652	9.98 525	4	9
52	9.40 921	47	9.42 399	51	10.57 601	9.98 521	3	8
53	9.40 968	48	9.42 450	51	10.57 550	9.98 518	3	7
54	9.41 016	47	9.42 501	51	10.57 499	9.98 515	4	6
55	9.41 063	48	9.42 552	51	10.57 448	9.98 511	3	5
56	9.41 111	47	9.42 603	50	10.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	10.57 347	9.98 505	4	3
58	9.41 205	47	9.42 704	51	10.57 296	9.98 501	3	2
59	9.41 252	48	9.42 755	50	10.57 245	9.98 498	4	1
60	9.41 300		9.42 805		10.57 195	9.98 494		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	54	53	52
2	10.8	10.6	10.4
3	16.2	15.9	15.6
4	21.6	21.2	20.8
5	27.0	26.5	26.0
6	32.4	31.8	31.2
7	37.8	37.1	36.4
8	43.2	42.4	41.6
9	48.6	47.7	46.8

	51	50	49
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47
2	9.6	9.4
3	14.4	14.1
4	19.2	18.8
5	24.0	23.5
6	28.8	28.2
7	33.6	32.9
8	38.4	37.6
9	43.2	42.3

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:
For 14°+ or 194°+, read as printed; for 104°+ or 284°+, read co-function.

From the bottom:
For 75°+ or 255°+, read as printed; for 165°+ or 345°+, read co-function.

Prop. Pts.

15° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.41 300	47	9.42 805	51	10.57 195	9.98 494	3	**60**
1	9.41 347	47	9.42 856	50	10.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	51	10.57 094	9.98 488	4	58
3	9.41 441	47	9.42 957	50	10.57 043	9.98 484	3	57
4	9.41 488	47	9.43 007	50	10.56 993	9.98 481	4	56
5	9.41 535	47	9.43 057	51	10.56 943	9.98 477	3	**55**
6	9.41 582	46	9.43 108	50	10.56 892	9.98 474	3	54
7	9.41 628	47	9.43 158	50	10.56 842	9.98 471	4	53
8	9.41 675	47	9.43 208	50	10.56 792	9.98 467	3	52
9	9.41 722	46	9.43 258	50	10.56 742	9.98 464	4	51
10	9.41 768	47	9.43 308	50	10.56 692	9.98 460	3	**50**
11	9.41 815	46	9.43 358	50	10.56 642	9.98 457	4	49
12	9.41 861	47	9.43 408	50	10.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	10.56 542	9.98 450	3	47
14	9.41 954	47	9.43 508	50	10.56 492	9.98 447	4	46
15	9.42 001	46	9.43 558	49	10.56 442	9.98 443	3	**45**
16	9.42 047	46	9.43 607	50	10.56 393	9.98 440	4	44
17	9.42 093	47	9.43 657	50	10.56 343	9.98 436	3	43
18	9.42 140	46	9.43 707	49	10.56 293	9.98 433	4	42
19	9.42 186	46	9.43 756	50	10.56 244	9.98 429	3	41
20	9.42 232	46	9.43 806	49	10.56 194	9.98 426	4	**40**
21	9.42 278	46	9.43 855	50	10.56 145	9.98 422	3	39
22	9.42 324	46	9.43 905	49	10.56 095	9.98 419	4	38
23	9.42 370	46	9.43 954	50	10.56 046	9.98 415	3	37
24	9.42 416	45	9.44 004	49	10.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	10.55 947	9.98 409	4	**35**
26	9.42 507	46	9.44 102	49	10.55 898	9.98 405	3	34
27	9.42 553	46	9.44 151	50	10.55 849	9.98 402	4	33
28	9.42 599	45	9.44 201	49	10.55 799	9.98 398	3	32
29	9.42 644	46	9.44 250	49	10.55 750	9.98 395	4	31
30	9.42 690	45	9.44 299	49	10.55 701	9.98 391	3	**30**
31	9.42 735	46	9.44 348	49	10.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	10.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	10.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	10.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	10.55 456	9.98 373	3	**25**
36	9.42 962	46	9.44 592	49	10.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	10.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	10.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	10.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	10.55 213	9.98 356	4	**20**
41	9.43 188	45	9.44 836	48	10.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	10.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	10.55 067	9.98 345	3	17
44	9.43 323	44	9.44 981	48	10.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	10.54 971	9.98 338	4	**15**
46	9.43 412	45	9.45 078	48	10.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	10.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	10.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	10.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	10.54 729	9.98 320	3	**10**
51	9.43 635	45	9.45 319	48	10.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	10.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	10.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	10.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	10.54 489	9.98 302	3	**5**
56	9.43 857	44	9.45 559	47	10.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	10.54 394	9.98 295	4	3
58	9.43 946	44	9.45 654	48	10.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	10.54 298	9.98 288	4	1
60	9.44 034		9.45 750		10.54 250	9.98 284		0
	L Cos	**d**	**L Ctn**	**cd**	**L Tan**	**L Sin**	**d**	′

Prop. Pts.

	51	**50**	**49**
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	**47**	**46**
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	**44**
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	**3**
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **15°+** or **195°+**, read as printed; for **105°+** or **285°+**, read co-function.

From the bottom:

For **74°+** or **254°+**, read as printed; for **164°+** or **344°+**, read co-function.

Prop. Pts.

74° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.44 034	44	9.45 750	47	10.54 250	9.98 284	3	**60**
1	9.44 078	44	9.45 797	48	10.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	10.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	10.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	10.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	10.54 013	9.98 266	4	**55**
6	9.44 297	44	9.46 035	47	10.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	10.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	10.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	10.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	10.53 776	9.98 248	4	**50**
11	9.44 516	43	9.46 271	48	10.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	10.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	10.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	10.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	10.53 540	9.98 229	3	**45**
16	9.44 733	43	9.46 507	47	10.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	10.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	10.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	10.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	10.53 306	9.98 211	4	**40**
21	9.44 948	44	9.46 741	47	10.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	10.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	10.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	10.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	10.53 072	9.98 192	3	**35**
26	9.45 163	43	9.46 975	46	10.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	10.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	10.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	10.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	10.52 840	9.98 174	4	**30**
31	9.45 377	42	9.47 207	46	10.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	10.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	10.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	10.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	10.52 608	9.98 155	4	**25**
36	9.45 589	43	9.47 438	46	10.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	10.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	10.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	10.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	10.52 378	9.98 136	4	**20**
41	9.45 801	42	9.47 668	46	10.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	10.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	10.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	10.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	10.52 148	9.98 117	4	**15**
46	9.46 011	42	9.47 897	46	10.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	10.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	10.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	10.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	10.51 920	9.98 098	4	**10**
51	9.46 220	42	9.48 126	45	10.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	10.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	10.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	10.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	10.51 693	9.98 079	4	**5**
56	9.46 428	41	9.48 353	45	10.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	10.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	10.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	10.51 511	9.98 063	3	1
60	9.46 594		9.48 534		10.51 466	9.98 060		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	48	**47**	**46**
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	**44**	**43**
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	**41**
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	**3**
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **16°+** or **196°+**, read as printed; for **106°+** or **286°+**, read co-function.

From the bottom:

For **73°+** or **253°+**, read as printed; for **163°+** or **343°+**, read co-function.

Prop. Pts.

17° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.46 594	41	9.48 534	45	10.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	10.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	10.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	10.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	10.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	10.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	10.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	10.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	10.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	10.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	10.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	10.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	10.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	10.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	10.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	10.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	10.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	10.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	10.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	10.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	10.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	10.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	10.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	10.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	10.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	10.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	10.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	10.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	10.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	10.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	10.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	10.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	10.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	10.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	10.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	10.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	10.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	10.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	10.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	10.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	10.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	10.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	10.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	10.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	10.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	10.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	10.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	10.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	10.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	10.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	10.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	10.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	10.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	10.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	10.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	10.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	10.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	10.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	10.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	10.48 865	9.97 825	4	1
60	9.48 998		9.51 178		10.48 822	9.97 821		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5
2	7.8	1.0
3	11.7	1.5
4	15.6	2.0
5	19.5	2.5
6	23.4	3.0
7	27.3	3.5
8	31.2	4.0
9	35.1	4.5

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **17°+** or **197°+**, read as printed; for **107°+** or **287°+**, read co-function.

From the bottom:

For **72°+** or **252°+**, read as printed; for **162°+** or **342°+**, read co-function.

Prop. Pts.

72° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.48 998	39	9.51 178	43	10.48 822	9.97 821	4	**60**
1	9.49 037	39	9.51 221	43	10.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	10.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	10.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	10.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	10.48 608	9.97 800	4	**55**
6	9.49 231	38	9.51 435	43	10.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	10.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	10.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	10.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	10.48 394	9.97 779	4	**50**
11	9.49 424	38	9.51 648	43	10.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	10.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	10.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	10.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	10.48 181	9.97 759	5	**45**
16	9.49 615	39	9.51 861	42	10.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	10.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	10.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	10.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	10.47 969	9.97 738	4	**40**
21	9.49 806	38	9.52 073	42	10.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	10.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	10.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	10.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	10.47 758	9.97 717	4	**35**
26	9.49 996	38	9.52 284	42	10.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	10.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	10.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	10.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	10.47 548	9.97 696	5	**30**
31	9.50 185	38	9.52 494	42	10.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	10.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	10.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	10.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	10.47 339	9.97 674	4	**25**
36	9.50 374	37	9.52 703	42	10.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	10.47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	10.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	10.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	10.47 130	9.97 653	4	**20**
41	9.50 561	37	9.52 912	41	10.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	10.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	10.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	10.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	10.46 922	9.97 632	4	**15**
46	9.50 747	37	9.53 120	41	10.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	10.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	10.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	10.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	10.46 715	9.97 610	4	**10**
51	9.50 933	37	9.53 327	41	10.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	10.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	10.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	10.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	10.46 508	9.97 589	5	**5**
56	9.51 117	37	9.53 533	41	10.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	10.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	10.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	10.46 344	9.97 571	4	1
60	9.51 264		9.53 697		10.46 303	9.97 567		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	43	**42**	**41**
2	8.6	8.4	8.2
3	12.9	12.6	12.3
4	17.2	16.8	16.4
5	21.5	21.0	20.5
6	25.8	25.2	24.6
7	30.1	29.4	28.7
8	34.4	33.6	32.8
9	38.7	37.8	36.9

	39	**38**	**37**
2	7.8	7.6	7.4
3	11.7	11.4	11.1
4	15.6	15.2	14.8
5	19.5	19.0	18.5
6	23.4	22.8	22.2
7	27.3	26.6	25.9
8	31.2	30.4	29.6
9	35.1	34.2	33.3

	36	**5**	**4**
2	7.2	1.0	0.8
3	10.8	1.5	1.2
4	14.4	2.0	1.6
5	18.0	2.5	2.0
6	21.6	3.0	2.4
7	25.2	3.5	2.8
8	28.8	4.0	3.2
9	32.4	4.5	3.6

From the top:

For **18°+** or **198°+**, read as printed; for **108°+** or **288°+**, read co-function.

From the bottom.

For **71°+** or **251°+**, read as printed; for **161°+** or **341°+**, read co-function.

Prop. Pts.

71° — Logarithms of Trigonometric Functions

19° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.51 264	37	9.53 697	41	10.46 303	9.97 567	4	**60**
1	9.51 301	37	9.53 738	41	10.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	10.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	10.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	10.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	10.46 098	9.97 545	4	**55**
6	9.51 484	36	9.53 943	41	10.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	10.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	10.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	10.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	10.45 894	9.97 523	4	**50**
11	9.51 666	36	9.54 147	40	10.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	10.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	10.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	10.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	10.45 691	9.97 501	4	**45**
16	9.51 847	36	9.54 350	40	10.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	10.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	10.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	10.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	10.45 488	9.97 479	4	**40**
21	9.52 027	36	9.54 552	41	10.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	10.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	10.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	10.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	10.45 286	9.97 457	4	**35**
26	9.52 207	35	9.54 754	40	10.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	10.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	10.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	10.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	10.45 085	9.97 435	5	**30**
31	9.52 385	36	9.54 955	40	10.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	10.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	10.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	10.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	10.44 885	9.97 412	4	**25**
36	9.52 563	35	9.55 155	40	10.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	10.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	10.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	10.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	10.44 685	9.97 390	5	**20**
41	9.52 740	35	9.55 355	40	10.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	10.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	10.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	10.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	10.44 486	9.97 367	4	**15**
46	9.52 916	35	9.55 554	39	10.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	10.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	10.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	10.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	10.44 288	9.97 344	4	**10**
51	9.53 092	34	9.55 752	39	10.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	10.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	10.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	10.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	10.44 090	9.97 322	5	**5**
56	9.53 266	35	9.55 949	40	10.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	10.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	10.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	10.43 933	9.97 303	4	1
60	9.53 405		9.56 107		10.43 893	9.97 299		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	41	**40**	**39**
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	37	**36**	**35**
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	**5**	**4**
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

From the top:

For **19°+** or **199°+**, read as printed; for **109°+** or **289°+**, read co-function.

From the bottom:

For **70°+** or **250°+**, read as printed; for **160°+** or **340°+**, read co-function.

Prop. Pts.

70° — Logarithms of Trigonometric Functions

20° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.53 405	35	9.56 107	39	10.43 893	9.97 299	5	**60**
1	9.53 440	35	9.56 146	39	10.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	10.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	10.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	10.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	10.43 697	9.97 276	5	**55**
6	9.53 613	34	9.56 342	39	10.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	10.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	10.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	10.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	10.43 502	9.97 252	4	**50**
11	9.53 785	34	9.56 537	39	10.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	10.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	10.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	10.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	10.43 307	9.97 229	5	**45**
16	9.53 957	34	9.56 732	39	10.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	10.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	10.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	10.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	10.43 113	9.97 206	5	**40**
21	9.54 127	34	9.56 926	39	10.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	10.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	10.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	10.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	10.42 919	9.97 182	4	**35**
26	9.54 297	34	9.57 120	38	10.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	10.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	10.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	10.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	10.42 726	9.97 159	5	**30**
31	9.54 466	34	9.57 312	39	10.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	10.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	10.42 611	9.97 145	5	27
34	9.54 567	34	9.57 428	38	10.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	10.42 534	9.97 135	5	**25**
36	9.54 635	33	9.57 504	39	10.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	10.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	10.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	10.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	10.42 342	9.97 111	4	**20**
41	9.54 802	34	9.57 696	38	10.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	10.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	10.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	10.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	10.42 151	9.97 087	4	**15**
46	9.54 969	34	9.57 887	38	10.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	10.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	10.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	10.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	10.41 961	9.97 063	4	**10**
51	9.55 136	33	9.58 077	38	10.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	10.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	10.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	10.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	10.41 771	9.97 039	4	**5**
56	9.55 301	33	9.58 267	37	10.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	10.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	10.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	10.41 620	9.97 020	5	1
60	9.55 433		9.58 418		10.41 582	9.97 015		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	40	39	38
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

	37	35	34
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

	33	5	4
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

From the top:

For **20°+** or **200°+**, read as printed; for **110°+** or **290°+**, read co-function.

From the bottom:

For **69°+** or **249°+**, read as printed; for **159°+** or **339°+**, read co-function.

Prop. Pts.

69° — Logarithms of Trigonometric Functions

21° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.55 433	33	9.58 418	37	10.41 582	9.97 015	5	**60**
1	9.55 466	33	9.58 455	38	10.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	10.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	10.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	10.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	10.41 394	9.96 991	5	**55**
6	9.55 630	33	9.58 644	37	10.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	10.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	10.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	10.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	10.41 206	9.96 966	4	**50**
11	9.55 793	33	9.58 832	37	10.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	10.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	10.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	10.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	10.41 019	9.96 942	5	**45**
16	9.55 956	32	9.59 019	37	10.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	10.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	10.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	10.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	10.40 832	9.96 917	5	**40**
21	9.56 118	32	9.59 205	38	10.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	10.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	10.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	10.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	10.40 646	9.96 893	5	**35**
26	9.56 279	32	9.59 391	38	10.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	10.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	10.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	10.40 497	9.96 873	5	31
30	9.56 408	32	9.59 540	37	10.40 460	9.96 868	5	**30**
31	9.56 440	32	9.59 577	37	10.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	10.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	10.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	10.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	10.40 275	9.96 843	5	**25**
36	9.56 599	32	9.59 762	37	10.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	10.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	10.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	10.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	10.40 091	9.96 818	5	**20**
41	9.56 759	31	9.59 946	37	10.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	10.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	10.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	10.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	10.39 907	9.96 793	5	**15**
46	9.56 917	32	9.60 130	36	10.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	10.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	10.39 797	9.96 778	6	12
49	9.57 012	32	9.60 240	36	10.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	10.39 724	9.96 767	5	**10**
51	9.57 075	32	9.60 313	36	10.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	10.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	10.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	10.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	10.39 541	9.96 742	5	**5**
56	9.57 232	32	9.60 495	37	10.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	10.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	10.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	10.39 395	9.96 722	5	1
60	9.57 358		9.60 641		10.39 359	9.96 717		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	'

Prop. Pts.

	38	37	36
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	33	32	31
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	6	5	4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

From the top:

For **21°+** or **201°+**, read as printed; for **111°+** or **291°+**, read co-function.

From the bottom:

For **68°+** or **248°+**, read as printed; for **158°+** or **338°+**, read co-function.

Prop. Pts.

68° — Logarithms of Trigonometric Functions

22° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.57 358	31	9.60 641	36	10.39 359	9.96 717	6	60
1	9.57 389	31	9.60 677	37	10.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	10.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	10.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	10.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	10.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	10.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	10.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	10.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	10.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	10.38 996	9.96 665	5	50
11	9.57 700	31	9.61 040	36	10.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	10.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	10.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	10.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	10.38 816	9.96 640	6	45
16	9.57 855	30	9.61 220	36	10.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	10.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	10.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	10.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	10.38 636	9.96 614	6	40
21	9.58 008	31	9.61 400	36	10.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	10.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	10.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	10.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	10.38 456	9.96 588	6	35
26	9.58 162	30	9.61 579	36	10.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	10.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	10.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	10.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	10.38 278	9.96 562	6	30
31	9.58 314	31	9.61 758	36	10.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	10.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	10.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	10.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	10.38 099	9.96 535	5	25
36	9.58 467	30	9.61 936	36	10.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	10.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	10.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	10.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	10.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	36	10.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	10.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	10.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	10.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	10.37 744	9.96 483	6	15
46	9.58 769	30	9.62 292	35	10.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	10.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	10.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	10.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	10.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	36	10.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	10.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	10.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	10.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	10.37 391	9.96 429	5	5
56	9.59 069	29	9.62 645	35	10.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	10.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	10.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	10.37 250	9.96 408	5	1
60	9.59 188		9.62 785		10.37 215	9.96 403		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	'

Prop. Pts.

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

From the top:

For **22°+** or **202°+**, read as printed; for **112°+** or **292°+**, read co-function.

From the bottom:

For **67°+** or **247°+**, read as printed; for **157°+** or **337°+**, read co-function.

Prop. Pts.

67° — Logarithms of Trigonometric Functions

23° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.59 188	30	9.62 785	35	10.37 215	9.96 403	6	**60**
1	9.59 218	29	9.62 820	35	10.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	10.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	10.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	10.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	10.37 039	9.96 376	6	**55**
6	9.59 366	30	9.62 996	35	10.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	10.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	10.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	10.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	10.36 865	9.96 349	6	**50**
11	9.59 514	29	9.63 170	35	10.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	10.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	10.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	10.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	10.36 690	9.96 322	6	**45**
16	9.59 661	29	9.63 345	34	10.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	10.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	10.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	10.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	10.36 516	9.96 294	5	**40**
21	9.59 808	29	9.63 519	34	10.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	10.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	10.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	10.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	10.36 343	9.96 267	5	**35**
26	9.59 954	29	9.63 692	34	10.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	10.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	10.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	10.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	10.36 170	9.96 240	6	**30**
31	9.60 099	29	9.63 865	34	10.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	10.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	10.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	10.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	10.35 997	9.96 212	5	**25**
36	9.60 244	29	9.64 037	35	10.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	10.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	10.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	10.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	10.35 825	9.96 185	6	**20**
41	9.60 388	29	9.64 209	34	10.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	10.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	10.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	10.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	10.35 654	9.96 157	6	**15**
46	9.60 532	29	9.64 381	34	10.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	10.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	10.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	10.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	10.35 483	9.96 129	6	**10**
51	9.60 675	29	9.64 552	34	10.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	10.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	10.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	10.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	10.35 312	9.96 101	6	**5**
56	9.60 818	28	9.64 722	34	10.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	10.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	10.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	10.35 176	9.96 079	6	1
60	9.60 931		9.64 858		10.35 142	9.96 073		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	36	**35**
2	7.2	7.0
3	10.8	10.5
4	14.4	14.0
5	18.0	17.5
6	21.6	21.0
7	25.2	24.5
8	28.8	28.0
9	32.4	31.5

	34	**30**
2	6.8	6.0
3	10.2	9.0
4	13.6	12.0
5	17.0	15.0
6	20.4	18.0
7	23.8	21.0
8	27.2	24.0
9	30.6	27.0

	29	**28**
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	6	**5**
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:

For **23°+** or **203°+**, read as printed; for **113°+** or **293°+**, read co-function.

From the bottom:

For **66°+** or **246°+**, read as printed; for **156°+** or **336°+**, read co-function.

Prop. Pts.

66° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.60 931	29	9.64 858	34	10.35 142	9.96 073	6	**60**
1	9.60 960	28	9.64 892	34	10.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	10.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	10.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	10.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	10.34 972	9.96 045	6	**55**
6	9.61 101	28	9.65 062	34	10.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	10.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	10.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	10.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	10.34 803	9.96 017	6	**50**
11	9.61 242	28	9.65 231	34	10.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	10.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	10.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	10.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	10.34 634	9.95 988	6	**45**
16	9.61 382	29	9.65 400	34	10.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	10.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	10.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	10.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	10.34 465	9.95 960	6	**40**
21	9.61 522	28	9.65 568	34	10.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	10.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	10.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	10.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	10.34 297	9.95 931	6	**35**
26	9.61 662	27	9.65 736	34	10.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	10.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	10.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	10.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	10.34 130	9.95 902	5	**30**
31	9.61 800	28	9.65 904	33	10.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	10.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	10.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	10.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	10.33 962	9.95 873	5	**25**
36	9.61 939	27	9.66 071	33	10.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	10.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	10.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	10.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	10.33 796	9.95 844	5	**20**
41	9.62 076	28	9.66 238	33	10.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	10.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	10.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	10.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	10.33 629	9.95 815	5	**15**
46	9.62 214	27	9.66 404	33	10.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	10.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	10.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	10.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	10.33 463	9.95 786	6	**10**
51	9.62 350	27	9.66 570	33	10.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	10.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	10.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	10.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	10.33 298	9.95 757	6	**5**
56	9.62 486	27	9.66 735	33	10.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	10.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	10.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	10.33 166	9.95 733	5	1
60	9.62 595		9.66 867		10.33 133	9.95 728		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	34	33
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	6
2	5.4	1.2
3	8.1	1.8
4	10.8	2.4
5	13.5	3.0
6	16.2	3.6
7	18.9	4.2
8	21.6	4.8
9	24.3	5.4

	5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

From the top:

For **24°+** or **204°+**, read as printed; for **114°+** or **294°+**, read co-function.

From the bottom:

For **65°+** or **245°+**, read as printed; for **155°+** or **335°+**, read co-function.

Prop. Pts.

65° — Logarithms of Trigonometric Functions

25° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.62 595	27	9.66 867	33	10.33 133	9.95 728	6	60
1	9.62 622	27	9.66 900	33	10.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	10.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	10.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	10.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	10.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	10.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	10.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	10.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	10.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	10.32 804	9.95 668	5	50
11	9.62 892	26	9.67 229	33	10.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	10.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	10.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	10.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	10.32 640	9.95 639	6	45
16	9.63 026	26	9.67 393	33	10.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	10.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	10.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	10.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	10.32 476	9.95 609	6	40
21	9.63 159	27	9.67 556	33	10.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	10.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	10.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	10.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	10.32 313	9.95 579	6	35
26	9.63 292	27	9.67 719	33	10.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	10.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	10.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	10.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	10.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	33	10.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	10.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	10.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	10.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	10.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	33	10.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	10.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	10.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	10.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	10.31 826	9.95 488	6	20
41	9.63 689	26	9.68 206	33	10.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	10.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	10.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	10.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	10.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	10.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	10.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	10.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	10.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	10.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	10.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	10.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	10.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	10.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	10.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	10.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	10.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	10.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	10.31 214	9.95 372	6	1
60	9.64 184		9.68 818		10.31 182	9.95 366		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	33	32
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

From the top:

For **25°+** or **205°+**, read as printed; for **115°+** or **295°+**, read co-function.

From the bottom:

For **64°+** or **244°+**, read as printed; for **154°+** or **334°+**, read co-function.

Prop. Pts.

64° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.64 184	26	9.68 818	32	10.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	10.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	10.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	10.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	10.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	10.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	10.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	10.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	10.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	10.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	10.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	10.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	10.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	10.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	10.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	10.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	10.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	10.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	10.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	10.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	10.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	10.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	10.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	10.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	10.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	10.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	10.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	10.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	10.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	10.30 258	9.95 185	6	31
30	9.64 953	25	9.69 774	31	10.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	32	10.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	10.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	10.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	10.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	10.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	10.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995	31	10.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	10.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	10.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	10.29 911	9.95 116	6	20
41	9.65 230	25	9.70 121	31	10.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	10.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	10.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	10.29 785	9.95 090	6	16
45	9.65 331	25	9.70 247	31	10.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	10.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	10.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	10.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	10.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	10.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	10.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	10.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	10.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	10.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	10.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	10.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	10.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	10.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	10.29 315	9.94 995	7	1
60	9.65 705		9.70 717		10.29 283	9.94 988		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	32	31
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	24	7
2	4.8	1.4
3	7.2	2.1
4	9.6	2.8
5	12.0	3.5
6	14.4	4.2
7	16.8	4.9
8	19.2	5.6
9	21.6	6.3

	6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

From the top:

For **26°+** or **206°+**, read as printed; for **116°+** or **296°+**, read co-function.

From the bottom:

For **63°+** or **243°+**, read as printed; for **153°+** or **333°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.65 705	24	9.70 717	31	10.29 283	9.94 988	6	**60**
1	9.65 729	25	9.70 748	31	10.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	10.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	10.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	10.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	10.29 127	9.94 956	7	**55**
6	9.65 853	25	9.70 904	31	10.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	10.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	10.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	10.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	10.28 972	9.94 923	6	**50**
11	9.65 976	25	9.71 059	31	10.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	10.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	10.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	10.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	10.28 816	9.94 891	6	**45**
16	9.66 099	25	9.71 215	31	10.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	10.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	10.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	10.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	10.28 661	9.94 858	6	**40**
21	9.66 221	25	9.71 370	31	10.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	10.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	10.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	10.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	10.28 507	9.94 826	7	**35**
26	9.66 343	25	9.71 524	31	10.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	10.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	10.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	10.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	10.28 352	9.94 793	7	**30**
31	9.66 465	24	9.71 679	30	10.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	10.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	10.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	10.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	10.28 198	9.94 760	7	**25**
36	9.66 586	24	9.71 833	30	10.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	10.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	10.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	10.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	10.28 045	9.94 727	7	**20**
41	9.66 706	25	9.71 986	31	10.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	10.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	10.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	10.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	10.27 891	9.94 694	7	**15**
46	9.66 827	24	9.72 140	30	10.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	10.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	10.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	10.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	10.27 738	9.94 660	6	**10**
51	9.66 946	24	9.72 293	30	10.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	10.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	10.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	10.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	10.27 585	9.94 627	7	**5**
56	9.67 066	24	9.72 445	31	10.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	10.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	10.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	10.27 463	9.94 600	7	1
60	9.67 161		9.72 567		10.27 433	9.94 593		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	32	**31**
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	30	**25**
2	6.0	5.0
3	9.0	7.5
4	12.0	10.0
5	15.0	12.5
6	18.0	15.0
7	21.0	17.5
8	24.0	20.0
9	27.0	22.5

	24	**23**
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	7	**6**
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top:

For **27°+** or **207°+**, read as printed; for **117°+** or **297°+**, read co-function.

From the bottom:

For **62°+** or **242°+**, read as printed; for **152°+** or **332°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.67 161	24	9.72 567	31	10.27 433	9.94 593	6	**60**
1	9.67 185	23	9.72 598	30	10.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	10.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	10.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	10.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	10.27 280	9.94 560	7	**55**
6	9.67 303	24	9.72 750	30	10.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	10.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	10.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	10.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	10.27 128	9.94 526	7	**50**
11	9.67 421	24	9.72 902	30	10.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	10.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	10.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	10.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	10.26 977	9.94 492	7	**45**
16	9.67 539	23	9.73 054	30	10.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	10.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	10.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	10.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	10.26 825	9.94 458	7	**40**
21	9.67 656	24	9.73 205	30	10.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	10.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	10.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	10.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	10.26 674	9.94 424	7	**35**
26	9.67 773	23	9.73 356	30	10.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	10.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	10.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	10.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	10.26 524	9.94 390	7	**30**
31	9.67 890	23	9.73 507	30	10.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	10.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	10.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	10.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	10.26 373	9.94 355	6	**25**
36	9.68 006	23	9.73 657	30	10.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	10.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	10.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	10.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	10.26 223	9.94 321	7	**20**
41	9.68 121	23	9.73 807	30	10.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	10.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	10.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	10.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	10.26 073	9.94 286	7	**15**
46	9.68 237	23	9.73 957	30	10.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	10.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	10.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	10.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	10.25 923	9.94 252	7	**10**
51	9.68 351	23	9.74 107	30	10.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	10.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	10.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	10.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	10.25 774	9.94 217	7	**5**
56	9.68 466	23	9.74 256	30	10.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	10.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	10.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	10.25 655	9.94 189	7	1
60	9.68 557		9.74 375		10.25 625	9.94 182		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	31	30
2	6.2	6.0
3	9.3	9.0
4	12.4	12.0
5	15.5	15.0
6	18.6	18.0
7	21.7	21.0
8	24.8	24.0
9	27.9	27.0

	29	24
2	5.8	4.8
3	8.7	7.2
4	11.6	9.6
5	14.5	12.0
6	17.4	14.4
7	20.3	16.8
8	23.2	19.2
9	26.1	21.6

	23	22
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top:

For **28°+** or **208°+**, read as printed; for **118°+** or **298°+**, read co-function.

From the bottom:

For **61°+** or **241°+**, read as printed; for **151°+** or **331°+**, read co-function.

Prop. Pts.

29° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.68 557	23	9.74 375	30	10.25 625	9.94 182	7	**60**
1	9.68 580	23	9.74 405	30	10.25 595	9.94 175	7	59
2	9.68 603	22	9.74 435	30	10.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465	29	10.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	10.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	10.25 476	9.94 147	7	**55**
6	9.68 694	22	9.74 554	29	10.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	10.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	10.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	10.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	10.25 327	9.94 112	7	**50**
11	9.68 807	22	9.74 702	30	10.25 298	9.94 105	7	49
12	9.68 829	23	9.74 732	30	10.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	10.25 238	9.94 090	7	47
14	9.68 875	22	9.74 791	30	10.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	10.25 179	9.94 076	7	**45**
16	9.68 920	22	9.74 851	29	10.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	10.25 120	9.94 062	7	43
18	9.68 965	22	9.74 910	29	10.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	10.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	10.25 031	9.94 041	7	**40**
21	9.69 032	23	9.74 998	30	10.25 002	9.94 034	7	39
22	9.69 055	22	9.75 028	30	10.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	10.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	10.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	10.24 883	9.94 005	7	**35**
26	9.69 144	23	9.75 146	30	10.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	10.24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	10.24 795	9.93 984	7	32
29	9.69 212	22	9.75 235	29	10.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	30	10.24 736	9.93 970	7	**30**
31	9.69 256	23	9.75 294	29	10.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	10.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	10.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	10.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	10.24 589	9.93 934	7	**25**
36	9.69 368	22	9.75 441	29	10.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	10.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	10.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	10.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	10.24 442	9.93 898	7	**20**
41	9.69 479	22	9.75 588	29	10.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	10.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	10.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	10.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	10.24 295	9.93 862	7	**15**
46	9.69 589	22	9.75 735	29	10.24 265	9.93 855	8	14
47	9.69 611	22	9.75 764	29	10.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	10.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	10.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	10.24 148	9.93 826	7	**10**
51	9.69 699	22	9.75 881	29	10.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	10.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	10.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29	10.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	10.24 002	9.93 789	7	**5**
56	9.69 809	22	9.76 027	29	10.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	10.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	29	10.23 914	9.93 768	8	2
59	9.69 875	22	9.76 115	29	10.23 885	9.93 760	7	1
60	9.69 897		9.76 144		10.23 856	9.93 753		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	30	**29**
2	6.0	5.8
3	9.0	8.7
4	12.0	11.6
5	15.0	14.5
6	18.0	17.4
7	21.0	20.3
8	24.0	23.2
9	27.0	26.1

	23	**22**
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

	8	**7**
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

From the top:

For **29°+** or **209°+**, read as printed; for **119°+** or **299°+**, read co-function.

From the bottom:

For **60°+** or **240°+**, read as printed; for **150°+** or **330°+**, read co-function.

Prop. Pts.

60° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.69 897	22	9.76 144	29	10.23 856	9.93 753	7	**60**
1	9.69 919	22	9.76 173	29	10.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	10.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	10.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	10.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	10.23 710	9.93 717	8	**55**
6	9.70 028	22	9.76 319	29	10.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	10.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	10.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	10.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	10.23 565	9.93 680	7	**50**
11	9.70 137	22	9.76 464	29	10.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	10.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	10.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	10.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	10.23 420	9.93 643	7	**45**
16	9.70 245	22	9.76 609	30	10.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	10.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	10.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	10.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	10.23 275	9.93 606	7	**40**
21	9.70 353	22	9.76 754	29	10.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	10.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	10.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	10.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	10.23 130	9.93 569	7	**35**
26	9.70 461	21	9.76 899	29	10.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	10.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	10.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	10.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	10.22 985	9.93 532	7	**30**
31	9.70 568	22	9.77 044	29	10.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	10.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	10.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	10.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	10.22 841	9.93 495	8	**25**
36	9.70 675	22	9.77 188	29	10.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	10.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	10.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	10.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	10.22 697	9.93 457	7	**20**
41	9.70 782	21	9.77 332	29	10.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	10.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	10.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	10.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	10.22 553	9.93 420	8	**15**
46	9.70 888	21	9.77 476	29	10.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	10.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	10.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	10.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	10.22 409	9.93 382	7	**10**
51	9.70 994	21	9.77 619	29	10.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	10.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	10.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	10.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	10.22 266	9.93 344	7	**5**
56	9.71 100	21	9.77 763	28	10.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	10.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	10.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	10.22 151	9.93 314	7	1
60	9.71 184		9.77 877		10.22 123	9.93 307		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	30	29
2	6.0	5.8
3	9.0	8.7
4	12.0	11.6
5	15.0	14.5
6	18.0	17.4
7	21.0	20.3
8	24.0	23.2
9	27.0	26.1

	28	22
2	5.6	4.4
3	8.4	6.6
4	11.2	8.8
5	14.0	11.0
6	16.8	13.2
7	19.6	15.4
8	22.4	17.6
9	25.2	19.8

	21	8
2	4.2	1.6
3	6.3	2.4
4	8.4	3.2
5	10.5	4.0
6	12.6	4.8
7	14.7	5.6
8	16.8	6.4
9	18.9	7.2

	7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

From the top:

For **30°+** or **210°+**, read as printed; for **120°+** or **300°+**, read co-function.

From the bottom:

For **59°+** or **239°+**, read as printed; for **149°+** or **329°+**, read co-function.

Prop. Pts.

59° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.71 184	21	9.77 877	29	10.22 123	9.93 307	8	**60**
1	9.71 205	21	9.77 906	29	10.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	10.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	10.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	10.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	10.21 980	9.93 269	8	**55**
6	9.71 310	21	9.78 049	28	10.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	10.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	10.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	10.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	10.21 837	9.93 230	7	**50**
11	9.71 414	21	9.78 192	28	10.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	10.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	10.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	10.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	10.21 694	9.93 192	8	**45**
16	9.71 519	20	9.78 334	29	10.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	10.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	10.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	10.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	10.21 552	9.93 154	8	**40**
21	9.71 622	21	9.78 476	29	10.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	10.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	10.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	10.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	10.21 410	9.93 115	7	**35**
26	9.71 726	21	9.78 618	29	10.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	10.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	10.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	10.21 296	9.93 084	7	31
30	9.71 809	20	9.78 732	28	10.21 268	9.93 077	8	**30**
31	9.71 829	21	9.78 760	29	10.21 240	9.93 069	8	29
32	9.71 850	20	9.78 789	28	10.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	10.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	10.21 155	9.93 046	8	26
35	9.71 911	21	9.78 874	28	10.21 126	9.93 038	8	**25**
36	9.71 932	20	9.78 902	28	10.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	10.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	10.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	10.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	10.20 985	9.92 999	8	**20**
41	9.72 034	21	9.79 043	29	10.20 957	9.92 991	8	19
42	9.72 055	20	9.79 072	28	10.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	10.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	10.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	10.20 844	9.92 960	8	**15**
46	9.72 137	20	9.79 185	28	10.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	10.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	10.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	10.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	10.20 703	9.92 921	8	**10**
51	9.72 238	21	9.79 326	28	10.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	10.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	10.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	10.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	10.20 562	9.92 881	7	**5**
56	9.72 340	20	9.79 466	29	10.20 534	9.92 874	8	4
57	9.72 360	21	9.79 495	28	10.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	10.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	10.20 449	9.92 850	8	1
60	9.72 421		9.79 579		10.20 421	9.92 842		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	**28**
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	21	**20**
2	4.2	4.0
3	6.3	6.0
4	8.4	8.0
5	10.5	10.0
6	12.6	12.0
7	14.7	14.0
8	16.8	16.0
9	18.9	18.0

	8	**7**
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

From the top:

For **31°+** or **211°+**, read as printed; for **121°+** or **301°+**, read co-function.

From the bottom:

For **58°+** or **238°+**, read as printed; for **148°+** or **328°+**, read co-function.

Prop. Pts.

32° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.72 421	20	9.79 579	28	10.20 421	9.92 842	8	**60**
1	9.72 441	20	9.79 607	28	10.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	10.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	10.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	10.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	10.20 281	9.92 803	8	**55**
6	9.72 542	20	9.79 747	29	10.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	10.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	10.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	10.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	10.20 140	9.92 763	8	**50**
11	9.72 643	20	9.79 888	28	10.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	10.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	10.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	10.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	10.20 000	9.92 723	8	**45**
16	9.72 743	20	9.80 028	28	10.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	10.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	10.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	10.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	10.19 860	9.92 683	8	**40**
21	9.72 843	20	9.80 168	27	10.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	10.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	10.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	10.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	10.19 721	9.92 643	8	**35**
26	9.72 942	20	9.80 307	28	10.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	10.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	10.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	10.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	10.19 581	9.92 603	8	**30**
31	9.73 041	20	9.80 447	27	10.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	10.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	10.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	10.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	10.19 442	9.92 563	8	**25**
36	9.73 140	20	9.80 586	28	10.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	10.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	10.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	10.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	10.19 303	9.92 522	8	**20**
41	9.73 239	20	9.80 725	28	10.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	10.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	10.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	10.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	10.19 164	9.92 482	9	**15**
46	9.73 337	20	9.80 864	28	10.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	10.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	10.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	10.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	10.19 025	9.92 441	8	**10**
51	9.73 435	20	9.81 003	27	10.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	10.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	10.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	10.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	10.18 887	9.92 400	8	**5**
56	9.73 533	19	9.81 141	28	10.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	10.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	10.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	10.18 776	9.92 367	8	1
60	9.73 611		9.81 252		10.18 748	9.92 359		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	21
2	5.4	4.2
3	8.1	6.3
4	10.8	8.4
5	13.5	10.5
6	16.2	12.6
7	18.9	14.7
8	21.6	16.8
9	24.3	18.9

	20	19
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	9	8	7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

From the top:

For **32°+** or **212°+**, read as printed; for **122°+** or **302°+**, read co-function.

From the bottom:

For **57°+** or **237°+**, read as printed; for **147°+** or **327°+**, read co-function.

Prop. Pts.

57° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.73 611	19	9.81 252	27	10.18 748	9.92 359	8	60
1	9.73 630	20	9.81 279	28	10.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	10.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	10.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	10.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	10.18 610	9.92 318	8	55
6	9.73 727	20	9.81 418	27	10.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	10.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	10.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	10.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	10.18 472	9.92 277	8	50
11	9.73 824	19	9.81 556	27	10.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	10.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	10.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	10.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	10.18 334	9.92 235	8	45
16	9.73 921	19	9.81 693	28	10.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	10.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	10.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	10.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	10.18 197	9.92 194	8	40
21	9.74 017	19	9.81 831	27	10.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	10.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	10.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	10.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	10.18 059	9.92 152	8	35
26	9.74 113	19	9.81 968	28	10.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	10.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	10.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	10.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	10.17 922	9.92 111	9	30
31	9.74 208	19	9.82 106	27	10.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	10.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	10.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	10.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	10.17 785	9.92 069	9	25
36	9.74 303	19	9.82 243	27	10.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	10.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	10.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	10.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	10.17 648	9.92 027	9	20
41	9.74 398	19	9.82 380	27	10.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	10.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	10.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	10.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	10.17 511	9.91 985	9	15
46	9.74 493	19	9.82 517	27	10.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	10.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	10.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	10.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	10.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	28	10.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	10.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	10.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	10.17 265	9.91 908	8	6
55	9.74 662	19	9.82 762	28	10.17 238	9.91 900	9	5
56	9.74 681	19	9.82 790	27	10.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	10.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	10.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	10.17 129	9.91 866	9	1
60	9.74 756		9.82 899		10.17 101	9.91 857		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	20	19
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18	9
2	3.6	1.8
3	5.4	2.7
4	7.2	3.6
5	9.0	4.5
6	10.8	5.4
7	12.6	6.3
8	14.4	7.2
9	16.2	8.1

	8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

From the top:

For **33°+** or **213°+**, read as printed; for **123°+** or **303°+**, read co-function.

From the bottom:

For **56°+** or **236°+**, read as printed; for **146°+** or **326°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.74 756	19	9.82 899	27	10.17 101	9.91 857	8	**60**
1	9.74 775	19	9.82 926	27	10.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	10.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	10.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	10.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	10.16 965	9.91 815	9	**55**
6	9.74 868	19	9.83 062	27	10.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	10.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	10.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	10.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	10.16 829	9.91 772	9	**50**
11	9.74 961	19	9.83 198	27	10.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	10.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	10.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	10.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	10.16 693	9.91 729	9	**45**
16	9.75 054	19	9.83 334	27	10.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	10.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	10.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	10.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	10.16 558	9.91 686	9	**40**
21	9.75 147	18	9.83 470	27	10.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	10.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	10.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	10.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	10.16 422	9.91 643	9	**35**
26	9.75 239	19	9.83 605	27	10.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	10.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	10.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	10.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	10.16 287	9.91 599	8	**30**
31	9.75 331	19	9.83 740	28	10.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	10.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	10.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	10.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	10.16 151	9.91 556	9	**25**
36	9.75 423	18	9.83 876	27	10.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	10.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	10.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	10.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	10.16 016	9.91 512	8	**20**
41	9.75 514	19	9.84 011	27	10.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	10.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	10.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	10.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	10.15 881	9.91 469	9	**15**
46	9.75 605	19	9.84 146	27	10.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	10.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	10.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	10.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	10.15 746	9.91 425	9	**10**
51	9.75 696	18	9.84 280	27	10.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	10.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	10.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	10.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	10.15 612	9.91 381	9	**5**
56	9.75 787	18	9.84 415	27	10.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	10.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	10.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	10.15 504	9.91 345	9	1
60	9.75 859		9.84 523		10.15 477	9 91 336		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	26	19
2	5.2	3.8
3	7.8	5.7
4	10.4	7.6
5	13.0	9.5
6	15.6	11.4
7	18.2	13.3
8	20.8	15.2
9	23.4	17.1

	18	9
2	3.6	1.8
3	5.4	2.7
4	7.2	3.6
5	9.0	4.5
6	10.8	5.4
7	12.6	6.3
8	14.4	7.2
9	16.2	8.1

	8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

From the top:

For **34°+** or **214°+**, read as printed; for **124°+** or **304°+**, read co-function.

From the bottom:

For **55°+** or **235°+**, read as printed; for **145°+** or **325°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.75 859	18	9.84 523	27	10.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	10.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	10.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	10.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	10.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	10.15 343	9.91 292	9	**55**
6	9.75 967	18	9.84 684	27	10.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	10.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	10.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	10.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	10.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	10.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	10.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	10.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	10.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	10.15 075	9.91 203	9	**45**
16	9.76 146	18	9.84 952	27	10.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	10.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	10.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	10.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	10.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	10.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	10.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	10.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	10.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	10.14 807	9.91 114	9	**35**
26	9.76 324	18	9.85 220	27	10.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	10.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	10.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	10.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	10.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	26	10.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	10.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	10.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	10.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	10.14 540	9.91 023	9	**25**
36	9.76 501	18	9.85 487	27	10.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	10.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	10.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	10.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	10.14 406	9.90 978	9	**20**
41	9.76 590	17	9.85 620	27	10.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	10.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	10.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	10.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	10.14 273	9.90 933	9	**15**
46	9.76 677	18	9.85 754	26	10.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	10.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	10.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	10.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	10.14 140	9.90 887	9	**10**
51	9.76 765	17	9.85 887	26	10.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	10.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	10.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	10.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	10.14 007	9.90 842	10	**5**
56	9.76 852	18	9.86 020	26	10.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	10.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	10.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	10.13 900	9.90 805	9	1
60	9.76 922		9.86 126		10.13 874	9.90 796		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

From the top:

For **35°+** or **215°+**, read as printed; for **125°+** or **305°+**, read co-function.

From the bottom:

For **54°+** or **234°+**, read as printed; for **144°+** or **324°+**, read co-function.

Prop. Pts.

36° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.76 922	17	9.86 126	27	10.13 874	9.90 796	9	**60**
1	9.76 939	18	9.86 153	26	10.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	10.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	10.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	10.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	10.13 741	9.90 750	9	**55**
6	9.77 026	17	9.86 285	27	10.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	10.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	10.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	10.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	10.13 608	9.90 704	10	**50**
11	9.77 112	18	9.86 418	27	10.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	10.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	10.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	10.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	10.13 476	9.90 657	9	**45**
16	9.77 199	17	9.86 551	26	10.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	10.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	10.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	10.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	10.13 344	9.90 611	9	**40**
21	9.77 285	17	9.86 683	26	10.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	10.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	10.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	10.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	10.13 211	9.90 565	10	**35**
26	9.77 370	17	9.86 815	27	10.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	10.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	10.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	10.13 106	9.90 527	9	31
30	9.77 439	17	9 86 921	26	10.13 079	9.90 518	9	**30**
31	9.77 456	17	9.86 947	27	10.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	10.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	10.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	10.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	10.12 947	9.90 471	9	**25**
36	9.77 541	17	9.87 079	27	10.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	10.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	10.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	10.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	10.12 815	9.90 424	9	**20**
41	9.77 626	17	9.87 211	27	10.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	10.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	10.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	10.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	10.12 683	9.90 377	9	**15**
46	9.77 711	17	9.87 343	26	10.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	10.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	10.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	10.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	10.12 552	9.90 330	10	**10**
51	9.77 795	17	9.87 475	26	10.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	10.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	10.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	10.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	10.12 420	9.90 282	9	**5**
56	9.77 879	17	9.87 606	27	10.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	10.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	10.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	10.12 315	9.90 244	9	1
60	9.77 946		9.87 711		10.12 289	9.90 235		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	16	10
2	3.2	2.0
3	4.8	3.0
4	6.4	4.0
5	8.0	5.0
6	9.6	6.0
7	11.2	7.0
8	12.8	8.0
9	14.4	9.0

	9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

From the top:

For **36°+** or **216°+**, read as printed; for **126°+** or **306°+**, read co-function.

From the bottom:

For **53°+** or **233°+**, read as printed; for **143°+** or **323°+**, read co-function.

Prop. Pts.

53° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.77 946	17	9.87 711	27	10.12 289	9.90 235	10	**60**
1	9.77 963	17	9.87 738	26	10.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	10.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	10.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	10.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	10.12 157	9.90 187	9	**55**
6	9.78 047	16	9.87 869	26	10.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	10.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	10.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	10.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	10.12 026	9.90 139	9	**50**
11	9.78 130	17	9.88 000	27	10.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	10.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	10.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	10.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	10.11 895	9.90 091	9	**45**
16	9.78 213	17	9.88 131	27	10.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	10.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	10.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	10.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	10.11 764	9.90 043	9	**40**
21	9.78 296	17	9.88 262	27	10.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	10.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	10.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	10.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	10.11 633	9.89 995	10	**35**
26	9.78 379	16	9.88 393	27	10.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	10.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	10.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	10.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	10.11 502	9.89 947	10	**30**
31	9.78 461	17	9.88 524	26	10.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	10.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	10.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	10.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	10.11 371	9.89 898	10	**25**
36	9.78 543	17	9.88 655	26	10.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	10.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	10.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	10.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	10.11 241	9.89 849	9	**20**
41	9.78 625	17	9.88 786	26	10.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	10.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	10.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	10.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	10.11 110	9.89 801	10	**15**
46	9.78 707	16	9.88 916	26	10.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	10.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	10.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	10.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	10.10 980	9.89 752	10	**10**
51	9.78 788	17	9.89 046	27	10.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	10.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	10.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	10.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	10.10 849	9.89 702	9	**5**
56	9.78 869	17	9.89 177	26	10.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	10.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	10.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	10.10 745	9.89 663	10	1
60	9.78 934		9.89 281		10.10 719	9.89 653		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

52° — Logarithms of Trigonometric

Pro [illegible]

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	17	16
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

From the top:

For **37°+** or **217°+**, read as printed; for **127°+** or **307°+**, read co-function.

From the bottom:

For **52°+** or **232°+**, read as printed; for **142°+** or **322°+**, read co-function.

Pro [illegible]

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	′
0	9.78 934	16	9.89 281	26	10.10 719	9.89 653	10	**60**
1	9.78 950	17	9.89 307	26	10.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	10.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	10.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	10.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	10.10 589	9.89 604	10	**55**
6	9.79 031	16	9.89 437	26	10.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	10.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	10.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	10.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	10.10 459	9.89 554	10	**50**
11	9.79 111	17	9.89 567	26	10.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	10.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	10.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	10.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	10.10 329	9.89 504	9	**45**
16	9.79 192	16	9.89 697	26	10.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	10.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	10.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	10.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	10.10 199	9.89 455	10	**40**
21	9.79 272	16	9.89 827	26	10.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	10.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	10.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	10.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	10.10 069	9.89 405	10	**35**
26	9.79 351	16	9.89 957	26	10.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	10.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	10.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	10.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	10.09 939	9.89 354	10	**30**
31	9.79 431	16	9.90 086	26	10.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	10.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	10.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	10.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	10.09 810	9.89 304	10	**25**
36	9.79 510	16	9.90 216	26	10.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	10.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	10.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	10.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	10.09 680	9.89 254	10	**20**
41	9.79 589	16	9.90 346	25	10.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	10.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	10.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	10.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	10.09 551	9.89 203	10	**15**
46	9.79 668	16	9.90 475	26	10.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	10.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	10.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	10.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	10.09 422	9.89 152	10	**10**
51	9.79 746	16	9.90 604	26	10.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	10.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	10.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	10.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	10.09 292	9.89 101	10	**5**
56	9.79 825	15	9.90 734	25	10.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	10.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	10.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	10.09 189	9.89 060	10	1
60	9.79 887		9.90 837		10.09 163	9.89 050		**0**
	L Cos	**d**	**L Ctn**	**cd**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	26	**25**
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	17	**16**
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	15	**11**
2	3.0	2.2
3	4.5	3.3
4	6.0	4.4
5	7.5	5.5
6	9.0	6.6
7	10.5	7.7
8	12.0	8.8
9	13.5	9.9

	10	**9**
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

From the top:

For **38°+** or **218°+**, read as printed; for **128°+** or **308°+**, read co-function.

From the bottom:

For **51°+** or **231°+**, read as printed; for **141°+** or **321°+**, read co-function.

Prop. Pts.

51° — Logarithms of Trigonometric Functions

39° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.79 887	16	9.90 837	26	10.09 163	9.89 050	10	**60**
1	9.79 903	15	9.90 863	26	10.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	10.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	10.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	10.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	10.09 034	9.88 999	10	**55**
6	9.79 981	15	9.90 992	26	10.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	10.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	10.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	10.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	10.08 905	9.88 948	11	**50**
11	9.80 058	16	9.91 121	26	10.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	10.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	10.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	10.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	10.08 776	9.88 896	10	**45**
16	9.80 136	15	9.91 250	26	10.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	10.08 724	9.88 875	10	43
18	9.80 166	16	9.91 301	26	10.08 699	9.88 865	10	42
19	9.80 182	15	9.91 327	26	10.08 673	9.88 855	11	41
20	9.80 197	16	9.91 353	26	10.08 647	9.88 844	10	**40**
21	9.80 213	15	9.91 379	25	10.08 621	9.88 834	10	39
22	9.80 228	16	9.91 404	26	10.08 596	9.88 824	11	38
23	9.80 244	15	9.91 430	26	10.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	10.08 544	9.88 803	10	36
25	9.80 274	16	9.91 482	25	10.08 518	9.88 793	11	**35**
26	9.80 290	15	9.91 507	26	10.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	10.08 467	9.88 772	11	33
28	9.80 320	16	9.91 559	26	10.08 441	9.88 761	10	32
29	9.80 336	15	9.91 585	25	10.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	26	10.08 390	9.88 741	11	**30**
31	9.80 366	16	9.91 636	26	10.08 364	9.88 730	10	29
32	9.80 382	15	9.91 662	26	10.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	10.08 312	9.88 709	10	27
34	9.80 412	16	9.91 713	26	10.08 287	9.88 699	11	26
35	9.80 428	15	9.91 739	26	10.08 261	9.88 688	10	**25**
36	9.80 443	15	9.91 765	26	10.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	25	10.08 209	9.88 668	11	23
38	9.80 473	16	9.91 816	26	10.08 184	9.88 657	10	22
39	9.80 489	15	9.91 842	26	10.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	25	10.08 132	9.88 636	10	**20**
41	9.80 519	15	9.91 893	26	10.08 107	9.88 626	11	19
42	9.80 534	16	9.91 919	26	10.08 081	9.88 615	10	18
43	9.80 550	15	9.91 945	26	10.08 055	9.88 605	11	17
44	9.80 565	15	9.91 971	25	10.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	10.08 004	9.88 584	11	**15**
46	9.80 595	15	9.92 022	26	10.07 978	9.88 573	10	14
47	9.80 610	15	9.92 048	25	10.07 952	9.88 563	11	13
48	9.80 625	16	9.92 073	26	10.07 927	9.88 552	10	12
49	9.80 641	15	9.92 099	26	10.07 901	9.88 542	11	11
50	9.80 656	15	9.92 125	25	10.07 875	9.88 531	10	**10**
51	9.80 671	15	9.92 150	26	10.07 850	9.88 521	11	9
52	9.80 686	15	9.92 176	26	10.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	25	10.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	26	10.07 773	9.88 489	11	6
55	9.80 731	15	9.92 253	26	10.07 747	9.88 478	10	**5**
56	9.80 746	16	9.92 279	25	10.07 721	9.88 468	11	4
57	9.80 762	15	9.92 304	26	10.07 696	9.88 457	10	3
58	9.80 777	15	9.92 330	26	10.07 670	9.88 447	11	2
59	9.80 792	15	9.92 356	25	10.07 644	9.88 436	11	1
60	9.80 807		9.92 381		10.07 619	9.88 425		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	16	15
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	11	10
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

From the top:

For **39°**+ or **219°**+, read as printed; for **129°**+ or **309°**+, read co-function.

From the bottom:

For **50°**+ or **230°**+, read as printed; for **140°**+ or **320°**+, read co-function.

Prop. Pts.

50° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.80 807	15	9.92 381	26	10.07 619	9.88 425	10	**60**
1	9.80 822	15	9.92 407	26	10.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	10.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	10.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	10.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	10.07 490	9.88 372	10	**55**
6	9.80 897	15	9.92 535	26	10.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	10.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	10.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	10.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	10.07 362	9.88 319	11	**50**
11	9.80 972	15	9.92 663	26	10.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	10.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	10.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	10.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	10.07 234	9.88 266	11	**45**
16	9.81 047	14	9.92 792	25	10.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	10.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	10.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	10.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	10.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	25	10.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	10.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	10.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	10.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	10.06 978	9.88 158	10	**35**
26	9.81 195	15	9.93 048	25	10.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	10.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	10.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	10.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	10.06 850	9.88 105	11	**30**
31	9.81 269	15	9.93 175	26	10.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	10.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	10.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	10.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	10.06 722	9.88 051	11	**25**
36	9.81 343	15	9.93 303	26	10.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	10.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	10.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	10.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	10.06 594	9.87 996	11	**20**
41	9.81 417	14	9.93 431	26	10.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	10.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	10.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	10.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	10.06 467	9.87 942	11	**15**
46	9.81 490	15	9.93 559	25	10.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	10.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	10.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	10.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	10.06 339	9.87 887	10	**10**
51	9.81 563	15	9.93 687	25	10.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	10.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	10.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	10.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	10.06 211	9.87 833	11	**5**
56	9.81 636	15	9.93 814	26	10.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	10.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	10.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	10.06 109	9.87 789	11	1
60	9.81 694		9.93 916		10.06 084	9.87 778		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	**14**
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	11	**10**
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

From the top:

For **40°+** or **220°+**, read as printed; for **130°+** or **310°+**, read co-function.

From the bottom:

For **49°+** or **229°+**, read as printed; for **139°+** or **319°+**, read co-function.

Prop. Pts.

41° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.81 694	15	9.93 916	26	10.06 084	9.87 778	11	**60**
1	9.81 709	14	9.93 942	25	10.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	10.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	10.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	10.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	10.05 956	9.87 723	11	**55**
6	9.81 781	15	9.94 069	26	10.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	10.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	10.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	10.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	10.05 829	9.87 668	11	**50**
11	9.81 854	14	9.94 197	25	10.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	10.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	10.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	10.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	10.05 701	9.87 613	12	**45**
16	9.81 926	14	9.94 324	26	10.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	10.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	10.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	10.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	10.05 574	9.87 557	11	**40**
21	9.81 998	14	9.94 452	25	10.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	10.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	10.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	10.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	10.05 446	9.87 501	11	**35**
26	9.82 069	15	9.94 579	25	10.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	10.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	10.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	10.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	10.05 319	9.87 446	12	**30**
31	9.82 141	14	9.94 706	26	10.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	10.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	10.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	10.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	10.05 192	9.87 390	12	**25**
36	9.82 212	14	9.94 834	25	10.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	10.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	10.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	10.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	10.05 065	9.87 334	12	**20**
41	9.82 283	14	9.94 961	25	10.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	10.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	10.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	10.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	10.04 938	9.87 277	11	**15**
46	9.82 354	14	9.95 088	25	10.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	10.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	10.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	10.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	10.04 810	9.87 221	12	**10**
51	9.82 424	15	9.95 215	25	10.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	10.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	10.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	10.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	10.04 683	9.87 164	11	**5**
56	9.82 495	14	9.95 342	26	10.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	10.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	10.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	10.04 582	9.87 119	12	1
60	9.82 551		9.95 444		10.04 556	9.87 107		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	**14**
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	12	**11**
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

From the top:

For **41°+** or **221°+**, read as printed; for **131°+** or **311°+**, read co-function.

From the bottom:

For **48°+** or **228°+**, read as printed; for **138°+** or **318°+**, read co-function.

Prop. Pts.

48° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.82 551	14	9.95 444	25	10.04 556	9.87 107	11	**60**
1	9.82 565	14	9.95 469	26	10.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	10.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	10.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	10.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	10.04 429	9.87 050	11	**55**
6	9.82 635	14	9.95 596	26	10.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	10.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	10.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	10.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	10.04 302	9.86 993	11	**50**
11	9.82 705	14	9.95 723	25	10.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	10.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	10.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	10.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	10.04 175	9.86 936	12	**45**
16	9.82 775	13	9.95 850	25	10.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	10.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	10.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	10.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	10.04 048	9.86 879	12	**40**
21	9.82 844	14	9.95 977	25	10.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	10.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	10.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	10.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	10.03 922	9.86 821	12	**35**
26	9.82 913	14	9.96 104	25	10.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	10.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	10.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	10.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	10.03 795	9.86 763	11	**30**
31	9.82 982	14	9.96 231	25	10.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	10.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	10.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	10.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	10.03 668	9.86 705	11	**25**
36	9.83 051	14	9.96 357	26	10.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	10.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	10.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	10.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	10.03 541	9.86 647	12	**20**
41	9.83 120	13	9.96 484	26	10.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	10.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	10.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	10.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	10.03 414	9.86 589	12	**15**
46	9.83 188	14	9.96 611	25	10.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	10.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	10.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	10.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	10.03 288	9.86 530	12	**10**
51	9.83 256	14	9.96 738	25	10.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	10.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	10.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	10.03 186	9 86 483	11	6
55	9.83 310	14	9.96 839	25	10.03 161	9.86 472	12	**5**
56	9.83 324	14	9.96 864	26	10.03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	10.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	10.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	10.03 060	9.86 425	12	1
60	9.83 378		9.96 966		10.03 034	9.86 413		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	**13**
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	**11**
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

From the top:

For **42°+** or **222°+**, read as printed; for **132°+** or **312°+**, read co-function.

From the bottom:

For **47°+** or **227°+**, read as printed; for **137°+** or **317°+**, read co-function.

Prop. Pts.

43° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.83 378	14	9.96 966	25	10.03 034	9.86 413	12	**60**
1	9.83 392	13	9.96 991	25	10.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	10.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	10.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	10.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	10.02 908	9.86 354	12	**55**
6	9.83 459	14	9.97 118	25	10.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	10.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	10.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	26	10.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	10.02 781	9.86 295	12	**50**
11	9.83 527	13	9.97 244	25	10.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	10.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295	25	10.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	10.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	10.02 655	9.86 235	12	**45**
16	9.83 594	14	9.97 371	25	10.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	10.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	10.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	10.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	10.02 528	9.86 176	12	**40**
21	9.83 661	13	9.97 497	26	10.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	10.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	10.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	10.02 427	9.86 128	12	36
25	9.83 715	13	9.97 598	26	10.02 402	9.86 116	12	**35**
26	9.83 728	13	9.97 624	25	10.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	10.02 351	9.86 092	12	33
28	9.83 755	13	9.97 674	26	10.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	10.02 300	9.86 068	12	31
30	9.83 781	14	9.97 725	25	10.02 275	9.86 056	12	**30**
31	9.83 795	13	9.97 750	26	10.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	10.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	10.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	10.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	10.02 149	9.85 996	12	**25**
36	9.83 861	13	9.97 877	25	10.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	10.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	10.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	10.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	10.02 022	9.85 936	12	**20**
41	9.83 927	13	9.98 003	26	10.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	10.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	10.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	10.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	10.01 896	9.85 876	12	**15**
46	9.83 993	13	9.98 130	25	10.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	10.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	10.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	10.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	10.01 769	9.85 815	12	**10**
51	9.84 059	13	9.98 256	25	10.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	10.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	10.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	10.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	10.01 643	9.85 754	12	**5**
56	9.84 125	13	9.98 383	25	10.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	10.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	10.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	10.01 542	9.85 706	13	1
60	9.84 177		9.98 484		10.01 516	9.85 693		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	**13**
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	**11**
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

From the top:

For **43°+** or **223°+**, read as printed; for **133°+** or **313°+**, read co-function.

From the bottom:

For **46°+** or **226°+**, read as printed; for **136°+** or **316°+**, read co-function.

Prop. Pts.

46° — Logarithms of Trigonometric Functions

44° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.84 177	13	9.98 484	25	10.01 516	9.85 693	12	**60**
1	9.84 190	13	9.98 509	25	10.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	26	10.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	25	10.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	10.01 415	9.85 645	13	56
5	9.84 242	13	9.98 610	25	10.01 390	9.85 632	12	**55**
6	9.84 255	14	9.98 635	26	10.01 365	9.85 620	12	54
7	9.84 269	13	9.98 661	25	10.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	10.01 314	9.85 596	13	52
9	9.84 295	13	9.98 711	26	10.01 289	9.85 583	12	51
10	9.84 308	13	9.98 737	25	10.01 263	9.85 571	12	**50**
11	9.84 321	13	9.98 762	25	10.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	10.01 213	9.85 547	13	48
13	9.84 347	13	9.98 812	26	10.01 188	9.85 534	12	47
14	9.84 360	13	9.98 838	25	10.01 162	9.85 522	12	46
15	9.84 373	12	9.98 863	25	10.01 137	9.85 510	13	**45**
16	9.84 385	13	9.98 888	25	10.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	26	10.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	25	10.01 061	9.85 473	13	42
19	9.84 424	13	9.98 964	25	10.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	26	10.01 011	9.85 448	12	**40**
21	9.84 450	13	9.99 015	25	10.00 985	9.85 436	13	39
22	9.84 463	13	9.99 040	25	10.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	10.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	26	10.00 910	9.85 399	13	36
25	9.84 502	13	9.99 116	25	10.00 884	9.85 386	12	**35**
26	9.84 515	13	9.99 141	25	10.00 859	9.85 374	13	34
27	9.84 528	12	9.99 166	25	10.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	26	10.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	25	10.00 783	9.85 337	13	31
30	9.84 566	13	9.99 242	25	10.00 758	9.85 324	12	**30**
31	9.84 579	13	9.99 267	26	10.00 733	9.85 312	13	29
32	9.84 592	13	9.99 293	25	10.00 707	9.85 299	12	28
33	9.84 605	13	9.99 318	25	10.00 682	9.85 287	13	27
34	9.84 618	12	9.99 343	25	10.00 657	9.85 274	12	26
35	9.84 630	13	9.99 368	26	10.00 632	9.85 262	12	**25**
36	9.84 643	13	9.99 394	25	10.00 606	9.85 250	13	24
37	9.84 656	13	9.99 419	25	10.00 581	9.85 237	12	23
38	9.84 669	13	9.99 444	25	10.00 556	9.85 225	13	22
39	9.84 682	12	9.99 469	26	10.00 531	9.85 212	12	21
40	9.84 694	13	9.99 495	25	10.00 505	9.85 200	13	**20**
41	9.84 707	13	9.99 520	25	10.00 480	9.85 187	12	19
42	9.84 720	13	9.99 545	25	10.00 455	9.85 175	13	18
43	9.84 733	12	9.99 570	26	10.00 430	9.85 162	12	17
44	9.84 745	13	9.99 596	25	10.00 404	9.85 150	13	16
45	9.84 758	13	9.99 621	25	10.00 379	9.85 137	12	**15**
46	9.84 771	13	9.99 646	26	10.00 354	9.85 125	13	14
47	9.84 784	12	9.99 672	25	10.00 328	9.85 112	12	13
48	9.84 796	13	9.99 697	25	10.00 303	9.85 100	13	12
49	9.84 809	13	9.99 722	25	10.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	26	10.00 253	9.85 074	12	**10**
51	9.84 835	12	9.99 773	25	10.00 227	9.85 062	13	9
52	9.84 847	13	9.99 798	25	10.00 202	9.85 049	12	8
53	9.84 860	13	9.99 823	25	10.00 177	9.85 037	13	7
54	9.84 873	12	9.99 848	26	10.00 152	9.85 024	12	6
55	9.84 885	13	9.99 874	25	10.00 126	9.85 012	13	**5**
56	9.84 898	13	9.99 899	25	10.00 101	9.84 999	13	4
57	9.84 911	12	9.99 924	25	10.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	26	10.00 051	9.84 974	13	2
59	9.84 936	13	9.99 975	25	10.00 025	9.84 961	12	1
60	9.84 949		10.0000		10.00 000	9.84 949		**0**
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	13
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

From the top:

For **44°+** or **224°+**, read as printed; for **134°+** or **314°+**, read co-function.

From the bottom:

For **45°+** or **225°+**, read as printed; for **135°+** or **315°+**, read co-function.

Prop. Pts.

45° — Logarithms of Trigonometric Functions

TABLE IV

Correction in Feet from 100 Feet of Slope Taping to the Horizontal

(100) (Versine of Slope Angle)

′	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°
0	0.000	0.015	0.061	0.137	0.244	0.381	0.548	0.745	0.973	1.231	1.519	1.837	2.185	2.563	2.970	3.407	3.874	4.370	4.894
2	0.000	0.016	0.063	0.140	0.248	0.386	0.554	0.752	0.981	1.240	1.529	1.848	2.197	2.576	2.985	3.422	3.890	4.387	4.912
4	0.000	0.017	0.065	0.143	0.252	0.391	0.560	0.760	0.989	1.249	1.539	1.860	2.209	2.589	2.999	3.438	3.906	4.404	4.930
6	0.000	0.018	0.067	0.146	0.256	0.396	0.566	0.767	0.998	1.259	1.550	1.871	2.222	2.602	3.013	3.453	3.922	4.421	4.948
8	0.000	0.020	0.069	0.149	0.260	0.401	0.572	0.774	1.006	1.268	1.560	1.882	2.234	2.616	3.027	3.468	3.938	4.438	4.967
10	0.000	0.021	0.071	0.153	0.264	0.406	0.579	0.781	1.014	1.277	1.570	1.893	2.246	2.629	3.041	3.483	3.954	4.455	4.985
12	0.001	0.022	0.074	0.156	0.269	0.412	0.585	0.789	1.022	1.286	1.580	1.904	2.258	2.642	3.055	3.498	3.971	4.472	5.003
14	0.001	0.023	0.076	0.159	0.273	0.417	0.591	0.796	1.031	1.296	1.591	1.916	2.271	2.655	3.070	3.514	3.987	4.489	5.021
16	0.001	0.024	0.078	0.162	0.277	0.422	0.598	0.803	1.039	1.305	1.601	1.927	2.283	2.669	3.084	3.529	4.003	4.507	5.039
18	0.001	0.026	0.081	0.166	0.281	0.428	0.604	0.811	1.047	1.314	1.611	1.939	2.295	2.682	3.098	3.544	4.019	4.524	5.057
20	0.002	0.027	0.083	0.169	0.286	0.433	0.610	0.818	1.056	1.324	1.622	1.950	2.308	2.696	3.113	3.560	4.036	4.541	5.076
22	0.002	0.028	0.085	0.173	0.290	0.438	0.617	0.825	1.064	1.333	1.632	1.961	2.320	2.709	3.127	3.575	4.052	4.559	5.094
24	0.002	0.030	0.088	0.176	0.295	0.444	0.623	0.833	1.073	1.343	1.643	1.973	2.333	2.722	3.142	3.590	4.069	4.576	5.112
26	0.003	0.031	0.090	0.179	0.299	0.449	0.630	0.840	1.081	1.352	1.653	1.984	2.345	2.736	3.156	3.606	4.085	4.593	5.131
28	0.003	0.033	0.093	0.183	0.304	0.455	0.636	0.848	1.090	1.362	1.664	1.996	2.358	2.749	3.171	3.621	4.102	4.611	5.149
30	0.004	0.034	0.095	0.187	0.308	0.460	0.643	0.856	1.098	1.371	1.675	2.008	2.370	2.763	3.185	3.637	4.118	4.628	5.168
32	0.004	0.036	0.098	0.190	0.313	0.466	0.649	0.863	1.107	1.381	1.685	2.019	2.383	2.777	3.200	3.653	4.135	4.646	5.186
34	0.005	0.037	0.100	0.194	0.317	0.472	0.656	0.871	1.116	1.391	1.696	2.031	2.396	2.790	3.214	3.668	4.151	4.663	5.205
36	0.005	0.039	0.103	0.197	0.322	0.477	0.663	0.878	1.124	1.400	1.706	2.042	2.408	2.804	3.229	3.684	4.168	4.681	5.223
38	0.006	0.041	0.106	0.201	0.327	0.483	0.669	0.886	1.133	1.410	1.717	2.054	2.421	2.818	3.244	3.699	4.184	4.699	5.242
40	0.007	0.042	0.108	0.205	0.332	0.489	0.676	0.894	1.142	1.420	1.728	2.066	2.434	2.831	3.258	3.715	4.201	4.716	5.260
42	0.007	0.044	0.111	0.208	0.336	0.494	0.683	0.902	1.151	1.430	1.739	2.078	2.447	2.845	3.273	3.731	4.218	4.734	5.279
44	0.008	0.046	0.114	0.212	0.341	0.500	0.690	0.909	1.159	1.439	1.750	2.090	2.459	2.859	3.288	3.747	4.234	4.752	5.298
46	0.009	0.048	0.117	0.216	0.346	0.506	0.697	0.917	1.168	1.449	1.760	2.101	2.472	2.873	3.303	3.762	4.251	4.769	5.316
48	0.010	0.049	0.119	0.220	0.351	0.512	0.703	0.925	1.177	1.459	1.771	2.113	2.485	2.887	3.318	3.778	4.268	4.787	5.335
50	0.011	0.051	0.122	0.224	0.356	0.518	0.710	0.933	1.186	1.469	1.782	2.125	2.498	2.900	3.333	3.794	4.285	4.805	5.354
52	0.011	0.053	0.125	0.228	0.361	0.524	0.717	0.941	1.195	1.479	1.793	2.137	2.511	2.914	3.347	3.810	4.302	4.823	5.373
54	0.012	0.055	0.128	0.232	0.365	0.530	0.724	0.949	1.204	1.489	1.804	2.149	2.524	2.928	3.362	3.826	4.319	4.841	5.391
56	0.013	0.057	0.131	0.236	0.370	0.536	0.731	0.957	1.213	1.499	1.815	2.161	2.537	2.942	3.377	3.842	4.336	4.858	5.410
58	0.014	0.059	0.134	0.240	0.375	0.542	0.738	0.965	1.222	1.509	1.826	2.173	2.550	2.956	3.392	3.858	4.353	4.876	5.429
60	0.015	0.061	0.137	0.244	0.381	0.548	0.745	0.973	1.231	1.519	1.837	2.185	2.563	2.970	3.407	3.874	4.370	4.894	5.448

TABLE V

Stadia Coefficients, Vertical Rod

Hor. Dist. = H = $100 \cos^2 \alpha$ * Diff. Elev. = V = $100(\frac{1}{2} \sin 2\alpha)$*

	0°		1°		2°		3°	
′	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	00.00	99.97	01.74	99.88	03.49	99.73	05.23
2	100.00	00.06	99.97	01.80	99.87	03.55	99.72	05.28
4	100.00	00.12	99.97	01.86	99.87	03.60	99.71	05.34
6	100.00	00.17	99.96	01.92	99.87	03.66	99.71	05.40
8	100.00	00.23	99.96	01.98	99.86	03.72	99.70	05.46
10	100.00	00.29	99.96	02.04	99.86	03.78	99.69	05.52
12	100.00	00.35	99.96	02.09	99.85	03.84	99.69	05.57
14	100.00	00.41	99.95	02.15	99.85	03.90	99.68	05.63
16	100.00	00.47	99.95	02.21	99.84	03.95	99.68	05.69
18	100.00	00.52	99.95	02.27	99.84	04.01	99.67	05.75
20	100.00	00.58	99.95	02.33	99.83	04.07	99.66	05.80
22	100.00	00.64	99.94	02.38	99.83	04.13	99.66	05.86
24	100.00	00.70	99.94	02.44	99.82	04.18	99.65	05.92
26	99.99	00.76	99.94	02.50	99.82	04.24	99.64	05.98
28	99.99	00.81	99.93	02.56	99.81	04.30	99.63	06.04
30	99.99	00.87	99.93	02.62	99.81	04.36	99.63	06.09
32	99.99	00.93	99.93	02.67	99.80	04.42	99.62	06.15
34	99.99	00.99	99.93	02.73	99.80	04.48	99.62	06.21
36	99.99	01.05	99.92	02.79	99.79	04.53	99.61	06.27
38	99.99	01.11	99.92	02.85	99.79	04.59	99.60	06.33
40	99.99	01.16	99.92	02.91	99.78	04.65	99.59	06.38
42	99.99	01.22	99.91	02.97	99.78	04.71	99.59	06.44
44	99.98	01.28	99.91	03.02	99.77	04.76	99.58	06.50
46	99.98	01.34	99.90	03.08	99.77	04.82	99.57	06.56
48	99.98	01.40	99.90	03.14	99.76	04.88	99.56	06.61
50	99.98	01.45	99.90	03.20	99.76	04.94	99.56	06.67
52	99.98	01.51	99.89	03.26	99.75	04.99	99.55	06.73
54	99.98	01.57	99.89	03.31	99.74	05.05	99.54	06.78
56	99.97	01.63	99.89	03.37	99.74	05.11	99.53	06.84
58	99.97	01.69	99.88	03.43	99.73	05.17	99.52	06.90
60	99.97	01.74	99.88	03.49	99.73	05.23	99.51	06.96

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* See pages 163, 164, 165

	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
′								
0	99.51	06.96	99.24	08.68	98.91	10.40	98.51	12.10
2	99.51	07.02	99.23	08.74	98.90	10.45	98.50	12.15
4	99.50	07.07	99.22	08.80	98.88	10.51	98.48	12.21
6	99.49	07.13	99.21	08.85	98.87	10.57	98.47	12.26
8	99.48	07.19	99.20	08.91	98.86	10.62	98.46	12.32
10	99.47	07.25	99.19	08.97	98.85	10.68	98.44	12.38
12	99.46	07.30	99.18	09.03	98.83	10.74	98.43	12.43
14	99.46	07.36	99.17	09.08	98.82	10.79	98.41	12.49
16	99.45	07.42	99.16	09.14	98.81	10.85	98.40	12.55
18	99.44	07.48	99.15	09.20	98.80	10.91	98.39	12.60
20	99.43	07.53	99.14	09.25	98.78	10.96	98.37	12.66
22	99.42	07.59	99.13	09.31	98.77	11.02	98.36	12.72
24	99.41	07.65	99.11	09.37	98.76	11.08	98.34	12.77
26	99.40	07.71	99.10	09.43	98.74	11.13	98.33	12.83
28	99.39	07.76	99.09	09.48	98.73	11.19	98.31	12.88
30	99.38	07.82	99.08	09.54	98.72	11.25	98.29	12.94
32	99.38	07.88	99.07	09.60	98.71	11.30	98.28	13.00
34	99.37	07.94	99.06	09.65	98.69	11.36	98.27	13.05
36	99.36	07.99	99.05	09.71	98.68	11.42	98.25	13.11
38	99.35	08.05	99.04	09.77	98.67	11.47	98.24	13.17
40	99.34	08.11	99.03	09.83	98.65	11.53	98.22	13.22
42	99.33	08.17	99.01	09.88	98.64	11.59	98.20	13.28
44	99.32	08.22	99.00	09.94	98.63	11.64	98.19	13.33
46	99.31	08.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	08.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	08.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	08.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	08.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	08.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	08.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	08.68	98.91	10.40	98.51	12.10	98.06	13.78

	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
′								
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34

	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
′								
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50

	16°		17°		18°		19°	
′	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14

	20°		21°		22°		23°	
′	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16

Stadia Coefficients, Vertical Rod

	24°		25°		26°		27°	
′	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45

TABLE VI

Convergency of Meridians, Six Miles Long and Six Miles Apart, and Differences of Latitude and Longitude

LAT.	CONVERGENCY			DIFFERENCE OF LONGITUDE PER RANGE			DIFFERENCE OF LATITUDE FOR—	
	On the Parallel	Angle		In Arc		In Time	1 mi.	1 Tp.
°	*Lks.*	′	″	′	″	*Seconds*	′	
25	33.9	2	25	5	44.34	22.96	0.871	5.229
26	35.4	2	32	5	47.20	23.15		
27	37.0	2	39	5	50.22	23.35		
28	38.6	2	46	5	53.40	23.56		
29	40.2	2	53	5	56.74	23.78		
30	41.9	3	0	6	0.26	24.02	0.871	5.225
31	43.6	3	7	6	3.97	24.26		
32	45.4	3	15	6	7.87	24.52		
33	47.2	3	23	6	11.96	24.80		
34	49.1	3	30	6	16.26	25.08		
35	50.9	3	38	6	20.78	25.39	0.870	5.221
36	52.7	3	46	6	25.53	25.70		
37	54.7	3	55	6	30.52	26.03		
38	56.8	4	4	6	35.76	26.38		
39	58.8	4	13	6	41.27	26.75		
40	60.9	4	22	6	47.06	27.14	0.869	5.216
41	63.1	4	31	6	53.15	27.54		
42	65.4	4	41	6	59.56	27.97		
43	67.7	4	51	7	6.29	28.42		
44	70.1	5	1	7	13.39	28.89		
45	72.6	5	12	7	20.86	29.39	0.869	5.211
46	75.2	5	23	7	28.74	29.92		
47	77.8	5	34	7	37.04	30.47		
48	80.6	5	46	7	45.80	31.05		
49	83.5	5	59	7	55.05	31.67		
50	86.4	6	12	8	4.83	32.32	0.868	5.207
51	89.6	6	25	8	15.17	33.03		
52	92.8	6	39	8	26.13	33.74		
53	96.2	6	54	8	37.75	34.52		
54	99.8	7	9	8	50.07	35.34		
55	103.5	7	25	9	3.18	36.22	0.867	5.202
56	107.5	7	42	9	17.12	37.14		
57	111.6	8	0	9	31.97	38.13		
58	116.0	8	19	9	47.83	39.19		
59	120.6	8	38	10	4.78	40.32		
60	125.5	8	59	10	22.94	41.52	0.866	5.198
61	130.8	9	22	10	42.42	42.83		
62	136.3	9	46	11	3.38	44.22		
63	142.2	10	11	11	25.97	45.73		
64	148.6	10	38	11	50.37	47.36		
65	155.0	11	8	12	16.82	49.12	0.866	5.195
66	162.8	11	39	12	45.55	51.04		
67	170.7	12	13	13	16.88	53.12		
68	179.3	12	51	13	51.15	55.41		
69	188.7	13	31	14	28.77	57.92		
70	199.1	14	15	15	10.26	60.68	0.866	5.193

TABLE VII

Barometric Temperature Corrections

For temperatures above 50° F. the values are to be added
For temperatures below 50° F. the values are to be subtracted

Average Air Temp. °F.		**Difference of Readings in Feet**													
		0	**20**	**40**	**60**	**80**	**100**	**120**	**140**	**160**	**180**	**209**	**220**	**240**	**260**
	+ 50°	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+48°	+ 52°	0	0 1	0 2	0 2	0 3	0 4	0 5	0 5	0 6	0 7	0 8	0 9	0 9	1 0
+46°	+ 54°	0	0 2	0 3	0 5	0 6	0 8	0 9	1 1	1 3	1 4	1 6	1 7	1 9	2 0
+44°	+ 56°	0	0 2	0 5	0 7	0 9	1 2	1 4	1 6	1 9	2 1	2 4	2 6	2 8	3 1
+42°	+ 53°	0	0 3	0 6	0 9	1 3	1 6	1 9	2 2	2 5	2 8	3 1	3 5	3 8	4 1
+40°	+ 60°	0	0 4	0 8	1 2	1 6	2 0	2 4	2 7	3 1	3 5	3 9	4 3	4 7	5 1
+38°	+ 62°	0	0 5	0 9	1 4	1 9	2 4	2 8	3 3	3 8	4 2	4 7	5 2	5 7	6 1
+36°	+ 64°	0	0 5	1 1	1 6	2 2	2 7	3 3	3 8	4 4	4 9	5 5	6 0	6 6	7 1
+34°	+ 66°	0	0 6	1 3	1 9	2 5	3 1	3 8	4 4	5 0	5 7	6 3	6 9	7 5	8 2
+32°	+ 68°	0	0 7	1 4	2 1	2 8	3 5	4 2	4 9	5 7	6 4	7 1	7 8	8 5	9 2
+30°	+ 70°	0	0 8	1 6	2 4	3 1	3 9	4 7	5 5	6 3	7 1	7 9	8 6	9 4	10 2
+28°	+ 72°	0	0 9	1 7	2 6	3 5	4 3	5 2	6 0	6 9	7 8	8 6	9 5	10 4	11 2
+26°	+ 74°	0	0 9	1 9	2 8	3 8	4 7	5 7	6 6	7 5	8 5	9 4	10 4	11 3	12 3
+24°	+ 76°	0	1 0	2 0	3 1	4 1	5 1	6 1	7 1	8 2	9 2	10 2	11 2	12 2	13 3
+22°	+ 78°	0	1 1	2 2	3 3	4 4	5 5	6 6	7 7	8 8	9 9	11 0	12 1	13 2	14 3
+20°	+ 80°	0	1 2	2 4	3 5	4 7	5 9	7 1	8 2	9 4	10 6	11 8	13 0	14 1	15 3
+18°	+ 82°	0	1 3	2 5	3 8	5 0	6 3	7 5	8 8	10 0	11 3	12 6	13 8	15 1	16 3
+16°	+ 81°	0	1 3	2 7	4 0	5 3	6 7	8 0	9 4	10 7	12 0	13 4	14 7	16 0	17 4
+14°	+ 86°	0	1 4	2 8	4 2	5 7	7 1	8 5	9 9	11 3	12 7	14 1	15 6	17 0	18 4
+12°	+ 88°	0	1 5	3 0	4 5	6 0	7 5	9 0	10 4	11 9	13 4	14 9	16 4	17 9	19 4
+10°	+ 90°	0	1 6	3 1	4 7	6 3	7 9	9 4	11 0	12 6	14 1	15 7	17 3	18 9	20 4
+ 8°	+ 92°	0	1 6	3 3	4 9	6 6	8 2	9 9	11 5	13 2	14 8	16 5	18 1	19 8	21 4
+ 6°	+ 94°	0	1 7	3 5	5 2	6 9	8 6	10 4	12 1	13 8	15 6	17 3	19 0	20 7	22 5
+ 4°	+ 96°	0	1 8	3 6	5 4	7 2	9 0	10 8	12 6	14 5	16 3	18 1	19 9	21 7	23 5
+ 2°	+ 98°	0	1 9	3 8	5 7	7 5	9 4	11 3	13 2	15 1	17 0	18 9	20 7	22 6	24 5
+ 0°	+100°	0	2 0	3 9	5 9	7 9	9 8	11 8	13 7	15 7	17 7	19 6	21 6	23 6	25 5
− 2°	+102°	0	2 0	4 1	6 1	8 2	10 2	12 3	14 3	16 3	18 4	20 4	22 5	24 5	26 6
− 4°	+104°	0	2 1	4 2	6 4	8 5	10 6	12 7	14 8	17 0	19 1	21 2	23 3	25 5	27 6
− 6°	+106°	0	2 2	4 4	6 6	8 8	11 0	13 2	15 4	17 6	19 8	22 0	24 2	26 4	28 6
− 8°	+108°	0	2 3	4 6	6 8	9 1	11 4	13 7	15 9	18 2	20 5	22 8	25 1	27 3	29 6
−10°	+110°	0	2 4	4 7	7 1	9 4	11 8	14 1	16 5	18 9	21 2	23 6	25 9	28 3	30 6
−12°	+112°	0	2 4	4 9	7 3	9 7	12 2	14 6	17 0	19 5	21 9	24 4	26 8	29 2	31 7
−14°	+114°	0	2 5	5 0	7 5	10 1	12 6	15 1	17 6	20 1	22 6	25 1	27 7	30 2	32 7
−16°	+116°	0	2 6	5 2	7 8	10 4	13 0	15 6	18 1	20 7	23 3	25 9	28 5	31 1	33 7
−18°	+118°	0	2 7	5 3	8 0	10 7	13 4	16 0	18 7	21 4	24 0	26 7	29 4	32 1	34 7
−20°	+120°	0	2 7	5 5	8 2	11 0	13 7	16 5	19 2	22 0	24 7	27 5	30 2	33 0	35 7
−22°	+122°	0	2 8	5 7	8 5	11 3	14 1	17 0	19 8	22 6	25 4	28 3	31 1	33 9	36 8
−24°	+124°	0	2 9	5 8	8 7	11 6	14 5	17 4	20 3	23 3	26 2	29 1	32 0	34 9	37 8
−26°	+126°	0	3 0	6 0	9 0	11 9	14 9	17 9	20 9	23 9	26 9	29 9	32 8	35 8	38 8

For temperatures above 50° F. the values are to be added
For temperatures below 50° F. the values are to be subtracted

Average Air Temp. °F.		Difference of Readings in Feet											
		280	300	320	340	360	380	400	420	440	460	480	500
	+ 50°	0	0	0	0	0	0	0	0	0	0	0	0
+48°	+ 52°	1.1	1.2	1.3	1.3	1.4	1.5	1.6	1.6	1.7	1.8	1.9	2.0
+46°	+ 54°	2.2	2.4	2.5	2.7	2.8	3.0	3.1	3.3	3.5	3.6	3.8	3.9
+44°	+ 56°	3.3	3.5	3.8	4.0	4.2	4.5	4.7	4.9	5.2	5.4	5.7	5.9
+42°	+ 58°	4.4	4.7	5.0	5.3	5.7	6.0	6.3	6.6	6.9	7.2	7.5	7.9
+40°	+ 60°	5.5	5.9	6.3	6.7	7.1	7.5	7.9	8.2	8.6	9.0	9.4	9.8
+38°	+ 62°	6.6	7.1	7.5	8.0	8.5	9.0	9.4	9.9	10.4	10.8	11.3	11.8
+36°	+ 64°	7.7	8.2	8.8	9.3	9.9	10.4	11.0	11.5	12.1	12.6	13.2	13.7
+34°	+ 66°	8.8	9.4	10.1	10.7	11.3	11.9	12.6	13.2	13.8	14.5	15.1	15.7
+32°	+ 68°	9.9	10.6	11.3	12.0	12.7	13.4	14.1	14.8	15.6	16.3	17.0	17.7
+30°	+ 70°	11.0	11.8	12.6	13.4	14.1	14.9	15.7	16.5	17.3	18.1	18.9	19.6
+28°	+ 72°	12.1	13.0	13.8	14.7	15.6	16.4	17.3	18.2	19.0	19.9	20.7	21.6
+26°	+ 74°	13.2	14.1	15.1	16.0	17.0	17.9	18.9	19.8	20.7	21.7	22.6	23.6
+24°	+ 76°	14.3	15.3	16.3	17.4	18.4	19.4	20.4	21.4	22.5	23.5	24.5	25.5
+22°	+ 78°	15.4	16.5	17.6	18.7	19.8	20.9	22.0	23.1	24.2	25.3	26.4	27.5
+20°	+ 80°	16.5	17.7	18.8	20.0	21.2	22.4	23.6	24.7	25.9	27.1	28.3	29.5
+18°	+ 82°	17.6	18.8	20.1	21.4	22.6	23.9	25.1	26.4	27.7	28.9	30.2	31.4
+16°	+ 84°	18.7	20.0	21.4	22.7	24.0	25.4	26.7	28.0	29.4	30.7	32.1	33.4
+14°	+ 86°	19.8	21.2	22.6	24.0	25.5	26.9	28.3	29.7	31.1	32.5	33.9	35.3
+12°	+ 88°	20.9	22.4	23.9	25.4	26.9	28.4	29.9	31.3	32.8	34.3	35.8	37.3
+10°	+ 90°	22.0	23.6	25.1	26.7	28.3	29.9	31.4	33.0	34.6	36.1	37.7	39.3
+ 8°	+ 92°	23.1	24.7	26.4	28.0	29.7	31.3	33.0	34.6	36.3	37.9	39.6	41.2
+ 6°	+ 94°	24.2	25.9	27.7	29.4	31.1	32.8	34.6	36.3	38.0	39.8	41.5	43.2
+ 4°	+ 96°	25.3	27.1	28.9	30.7	32.5	34.3	36.1	37.9	39.8	41.6	43.4	45.2
+ 2°	+ 98°	26.4	28.3	30.2	32.1	33.9	35.8	37.7	39.6	41.5	43.4	45.2	47.1
+ 0°	+100°	27.5	29.5	31.4	33.4	35.4	37.3	39.3	41.2	43.2	45.2	47.1	49.1
− 2°	+102°	28.6	30.6	32.7	34.7	36.8	38.8	40.9	42.9	44.9	47.0	49.0	51.1
− 4°	+104°	29.7	31.8	33.9	36.1	38.2	40.3	42.4	44.5	46.7	48.8	50.9	53.0
− 6°	+106°	30.8	33.0	35.2	37.4	39.6	41.8	44.0	46.2	48.4	50.6	52.8	55.0
− 8°	+108°	31.9	34.2	36.5	38.7	41.0	43.3	45.6	47.8	50.1	52.4	54.7	57.0
−10°	+110°	33.0	35.4	37.7	40.1	42.4	44.8	47.1	49.5	51.8	54.2	56.6	58.9
−12°	+112°	34.1	36.5	39.0	41.4	43.8	46.3	48.7	51.1	53.6	56.0	58.4	60.9
−14°	+114°	35.2	37.7	40.2	42.7	45.3	47.8	50.3	52.8	55.3	57.8	60.3	62.8
−16°	+116°	36.3	38.9	41.5	44.1	46.7	49.3	51.9	54.4	57.0	59.6	62.2	64.8
−18°	+118°	37.4	40.1	42.7	45.4	48.1	50.8	53.4	56.1	58.8	61.4	64.1	66.8
−20°	+120°	38.5	41.2	44.0	46.7	49.5	52.2	55.0	57.7	60.5	63.2	66.0	68.7
−22°	+122°	39.6	42.4	45.2	48.1	50.9	53.7	56.6	59.4	62.2	65.1	67.9	70.7
−24°	+124°	40.7	43.6	46.5	49.4	52.3	55.2	58.1	61.0	64.0	66.9	69.8	72.7
−26°	+126°	41.8	44.8	47.8	50.8	53.7	56.7	59.7	62.7	65.7	68.7	71.7	74.6

For temperatures above 50° F. the values are to be added
For temperatures below 50° F. the values are to be subtracted

Average Air Temp. °F.		Difference of Readings in Feet													
		500	**520**	**540**	**560**	**580**	**600**	**620**	**640**	**660**	**680**	**700**	**720**	**740**	**760**
	+ 50°	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+48°	+ 52°	2.0	2.0	2.1	2.2	2.3	2.4	2.4	2.5	2.6	2.7	2.7	2.8	2.9	3.0
+46°	+ 54°	3.9	4.1	4.2	4.4	4.6	4.7	4.9	5.0	5.2	5.3	5.5	5.7	5.8	6.0
+44°	+ 56°	5.9	6.1	6.4	6.6	6.8	7.1	7.3	7.5	7.8	8.0	8.2	8.5	8.7	9.0
+42°	+ 58°	7.9	8.2	8.5	8.8	9.1	9.4	9.7	10.1	10.4	10.7	11.0	11.3	11.6	11.9
+40°	+ 60°	9.8	10.2	10.6	11.0	11.4	11.8	12.2	12.6	13.0	13.4	13.7	14.1	14.5	14.9
+38°	+ 62°	11.8	12.3	12.7	13.2	13.7	14.1	14.6	15.1	15.5	16.0	16.5	17.0	17.4	17.9
+36°	+ 64°	13.7	14.3	14.8	15.4	15.9	16.5	17.0	17.6	18.1	18.7	19.2	19.8	20.3	20.9
+34°	+ 66°	15.7	16.3	17.0	17.6	18.2	18.9	19.5	20.1	20.7	21.4	22.0	22.6	23.3	23.9
+32°	+ 68°	17.7	18.4	19.1	19.8	20.5	21.2	21.9	22.6	23.3	24.0	24.7	25.5	26.2	26.9
+30°	+ 70°	19.6	20.4	21.2	22.0	22.8	23.6	24.4	25.1	25.9	26.7	27.5	28.3	29.1	29.9
+28°	+ 72°	21.6	22.5	23.3	24.2	25.1	25.9	26.8	27.7	28.5	29.4	30.2	31.1	32.0	32.8
+26°	+ 74°	23.6	24.5	25.5	26.4	27.3	28.3	29.2	30.2	31.1	32.1	33.0	33.9	34.9	35.8
+24°	+ 76°	25.5	26.6	27.6	28.6	29.6	30.6	31.7	32.7	33.7	34.7	35.7	36.8	37.8	38.8
+22°	+ 78°	27.5	28.6	29.7	30.8	31.9	33.0	34.1	35.2	36.3	37.4	38.5	39.6	40.7	41.8
+20°	+ 80°	29.5	30.3	31.8	33.0	34.2	35.4	36.5	37.7	38.9	40.1	41.2	42.4	43.6	44.8
+18°	+ 82°	31.4	32.7	33.9	35.2	36.5	37.7	39.0	40.2	41.5	42.7	44.0	45.3	46.5	47.8
+16°	+ 84°	33.4	34.7	36.1	37.4	38.7	40.1	41.4	42.7	44.1	45.4	46.7	48.1	49.4	50.8
+14°	+ 86°	35.3	36.8	38.2	39.6	41.0	42.4	43.8	45.2	46.7	48.1	49.5	50.9	52.3	53.7
+12°	+ 88°	37.3	38.8	40.3	41.8	43.3	44.8	46.3	47.8	49.3	50.7	52.2	53.7	55.2	56.7
+10°	+ 90°	39.3	40.9	42.4	44.0	45.6	47.1	48.7	50.3	51.8	53.4	55.0	56.6	58.1	59.7
+ 8°	+ 92°	41.2	42.9	44.5	46.2	47.8	49.5	51.1	52.8	54.4	56.1	57.7	59.4	61.0	62.7
+ 6°	+ 94°	43.2	44.9	46.7	48.4	50.1	51.9	53.6	55.3	57.0	58.8	60.5	62.2	64.0	65.7
+ 4°	+ 96°	45.2	57.0	48.8	50.6	52.4	54.2	56.0	57.8	59.6	61.4	63.2	65.1	66.9	67.8
+ 2°	+ 98°	47.1	49.0	50.9	52.8	54.7	56.6	58.4	60.3	62.2	64.1	66.0	67.9	69.8	71.6
+ 0°	+100°	49.1	51.1	53.0	55.0	57.0	58.9	60.9	62.8	64.8	66.8	68.7	70.7	72.7	74.6
− 2°	+102°	51.1	53.1	55.1	57.2	59.2	61.3	63.3	65.4	67.4	69.4	71.5	73.5	75.6	77.6
− 4°	+104°	53.0	55.2	57.3	59.4	61.5	63.6	65.8	67.9	70.0	72.1	74.2	76.4	78.5	80.6
− 6°	+106°	55.0	57.2	59.4	61.6	63.8	66.0	68.2	70.4	72.6	74.8	77.0	79.2	81. 4	83.6
− 8°	+108°	57.0	59.2	61.5	63.8	66.1	68.3	70.6	72.9	75.2	77.5	79.7	82.0	84.3	86.6
−10°	+110°	58.9	61.3	63.6	66.0	68.3	70.7	73.1	75.4	77.8	80.1	82.5	84.8	87.2	89.6
−12°	+112°	60.9	63.3	65.8	68.2	70.6	73.1	75.5	77.9	80.4	82.8	85.2	87.7	90.1	92.5
−14°	+114°	62.8	65.4	67.9	70.4	72.9	75.4	77.9	80.4	83.0	85.5	88.0	90.5	93.0	95.5
−16°	+116°	64.8	67.4	70.0	72.6	75.2	77.8	80.4	83.0	85.6	88.1	90.7	93.3	95.9	98.5
−18°	+118°	66.8	69.5	72.1	74.8	77.5	80.1	82.8	85.5	88.1	90.8	93.5	96.2	98.8	101.5
−20°	+120°	68.7	71.5	74.2	77.0	79.7	82.5	85.2	88.0	90.7	93.5	96.2	99.0	101.7	104.5
−22°	+122°	70.7	73.6	76.4	79.2	82.0	84.9	87.7	90.5	93.3	96.2	99.0	101.8	104.7	107.5
−24°	+124°	72.7	75.6	78.5	81.4	84.3	87.9	90.1	93.0	95.9	98.8	101.7	104.7	107.6	110.5
−26°	+126°	74.6	77.6	80.6	83.6	86.6	89.6	92.6	95.8	98.5	101.5	104.5	017.5	110.5	113.5

For temperatures above 50° F. the values are to be added
For temperatures below 50° F. the values are to be subtracted

Average Air Temp. °F.		Difference of Readings in Feet											
		780	800	820	840	860	880	900	920	940	960	980	1000
	+ 50°	0	0	0	0	0	0	0	0	0	0	0	0
+48°	+ 52°	3.1	3.1	3.2	3.3	3.4	3.5	3.5	3.6	3.7	3.8	3.8	3.9
+46°	+ 54°	6.1	6.3	6.4	6.6	6.8	6.9	7.1	7.2	7.4	7.5	7.7	7.9
+44°	+ 56°	9.1	9.4	9.7	9.9	10.1	10.4	10.6	10.8	11.1	11.3	11.5	11.8
+42°	+ 58°	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.5	14.8	15.1	15.4	15.7
+40°	+ 60°	15.3	15.7	16.1	16.5	16.9	17.3	17.7	18.1	18.5	18.8	19.2	19.6
+38°	+ 62°	18.4	18.8	19.3	19.8	20.3	20.7	21.2	21.7	22.1	22.6	23.1	23.6
+36°	+ 64°	21.4	22.0	22.5	23.1	23.6	24.2	24.7	25.3	25.8	26.4	26.9	27.5
+34°	+ 66°	24.5	25.1	25.8	26.4	27.0	27.6	28.3	28.9	29.5	30.2	30.8	31.4
+32°	+ 68°	27.6	28.3	29.0	29.7	30.4	31.1	31.8	32.5	33.2	33.9	34.6	35.4
+30°	+ 70°	30.6	31.4	32.2	33.0	33.8	34.6	35.4	36.1	36.9	37.7	38.5	39.3
+28°	+ 72°	33.7	34.6	35.4	36.3	37.2	38.0	38.9	39.8	40.6	41.5	42.3	43.2
+26°	+ 74°	36.8	37.7	38.7	39.6	40.5	41.5	42.4	43.4	44.3	45.3	46.2	47.1
+24°	+ 76°	39.8	40.9	41.9	42.9	43.9	44.9	46.0	47.0	48.0	49.0	50.0	51.1
+22°	+ 78°	42.9	44.0	45.1	46.2	47.3	48.4	49.5	50.6	51.7	52.8	53.9	55.0
+20°	+ 80°	46.0	47.1	48.3	49.5	50.7	51.8	53.0	54.2	55.4	56.6	57.7	58.9
+18°	+ 82°	49.0	50.3	51.5	52.8	54.1	55.3	56.6	57.8	59.1	60.3	61.6	62.8
+16°	+ 84°	52.1	53.4	54.8	56.1	57.4	58.8	60.1	61.4	62.8	64.1	65.4	66.8
+14°	+ 86°	55.1	56.6	58.0	59.4	60.8	62.2	63.6	65.0	66.5	67.9	69.3	70.7
+12°	+ 88°	58.2	59.7	61.2	62.7	64.2	65.7	67.2	68.7	70.2	71.6	73.1	74.6
+10°	+ 90°	61.3	62.8	64.4	66.0	67.6	69.1	70.7	72.3	73.8	75.4	77.0	78.6
+ 8°	+ 92°	64.3	66.0	67.6	69.3	70.9	72.6	74.2	75.9	77.5	79.2	80.8	82.5
+ 6°	+ 94°	67.4	69.1	70.9	72.6	74.3	76.0	77.8	79.5	81.2	83.0	84.7	86.4
+ 4°	+ 96°	70.5	72.3	74.1	75.9	77.7	79.5	81.3	83.1	84.9	86.7	88.5	90.3
+ 2°	+ 98°	73.5	75.4	77.3	79.2	81.1	83.0	84.8	86.7	88.6	90.5	92.4	94.3
+ 0°	+100°	76.6	78.6	80.5	82.5	84.5	86.4	88.4	90.3	92.3	94.3	96.2	98.2
− 2°	+102°	79.7	81.7	83.7	85.8	87.8	89.9	91.9	94.0	96.0	98.0	100.1	102.1
− 4°	+104°	82.7	84.8	87.0	89.1	91.2	93.3	95.5	97.6	99.7	101.8	103.9	106.1
− 6°	+106°	85.8	88.0	90.2	92.4	94.6	96.8	99.0	101.2	103.4	105.6	107.8	110.0
− 8°	+108°	88.8	91.1	93.4	95.7	98.0	100.2	102.5	104.8	107.1	109.4	111.6	113.9
−10°	+110°	91.9	94.3	96.6	99.0	101.3	103.7	106.1	108.4	110.8	113.1	115.5	117.8
−12°	+112°	95.0	97.4	99.9	102.3	104.7	107.2	109.6	112.0	114.5	116.9	119.3	121.8
−14°	+114°	98.0	100.6	103.1	105.6	108.1	110.6	113.1	115.6	118.2	120.7	123.2	125.7
−16°	+116°	101.1	103.7	106.3	108.9	111.5	114.1	116.7	119.3	121.9	124.4	127.0	129.6
−18°	+118°	104.2	106.8	109.5	112.2	114.9	117.5	120.2	122.9	125.5	128.2	130.9	133.6
−20°	+120°	107.2	110.0	112.7	115.5	118.2	121.0	123.7	126.5	129.2	132.0	134.7	137.5
−22°	+122°	110.3	113.1	116.0	118.8	121.6	123.4	127.3	130.1	132.9	135.8	138.6	141.1
−24°	+124°	113.4	116.3	119.2	122.1	125.0	127.9	130.8	133.7	136.6	139.5	142.4	145.3
−26°	+126°	116.4	119.4	122.4	125.4	128.4	131.4	134.4	137.3	140.3	143.3	146.3	149.

TABLE VIII

Inches to Feet

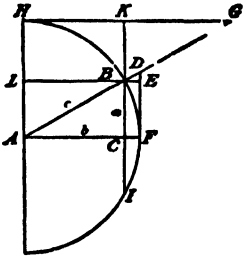
Inches		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16
0	.0000	.0052	.0104	.0156	.0208	.0260	.0313	.0365	.0417	.0469	.0521	.0573	.0625	.0677	.0729	.0781
1	.0833	.0885	.0938	.0990	.1042	.1094	.1146	.1198	.1250	.1302	.1354	.1406	.1458	.1510	.1563	.1615
2	.1667	.1719	.1771	.1823	.1875	.1927	.1979	.2031	.2083	.2135	.2188	.2240	.2292	.2344	.2396	.2448
3	.2500	.2552	.2604	.2656	.2708	.2760	.2813	.2865	.2917	.2969	.3021	.3073	.3125	.3177	.3229	.3281
4	.3333	.3385	.3438	.3490	.3542	.3594	.3646	.3698	.3750	.3802	.3854	.3906	.3958	.4010	.4063	.4115
5	.4167	.4219	.4271	.4323	.4375	.4427	.4479	.4531	.4583	.4635	.4688	.4740	.4792	.4844	.4896	.4948
6	.5000	.5052	.5104	.5156	.5208	.5260	.5313	.5365	.5417	.5469	.5521	.5573	.5625	.5677	.5729	.5781
7	.5833	.5885	.5938	.5990	.6042	.6094	.6146	.6198	.6250	.6302	.6354	.6406	.6458	.6510	.6563	.6615
8	.6667	.6719	.6771	.6823	.6875	.6927	.6979	.7031	.7083	.7135	.7188	.7240	.7292	.7344	.7396	.7448
9	.7500	.7552	.7604	.7656	.7708	.7760	.7813	.7865	.7917	.7969	.8021	.8073	.8125	.8177	.8229	.8281
10	.8333	.8385	.8438	.8490	.8542	.8594	.8646	.8698	.8750	.8802	.8854	.8906	.8958	.9010	.9063	.9115
11	.9167	.9219	.9271	.9323	.9375	.9427	.9479	.9531	.9583	.9635	.9688	.9740	.9792	.9844	.9896	.9948

TABLE IX

Trigonometric Formulas for the Solution of Plane Triangles

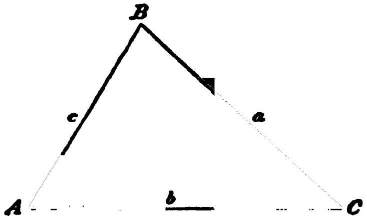
Let A = angle BAC = arc BF, and let the radius $AF = AB = AH = 1$. We then have

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\operatorname{ctn} A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = DG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2BC$



In the right-angled triangle ABC
Let $AB = c$, $AC = b$, and $BC = a$.
We then have:

1. $\sin A = \frac{a}{c} = \cos B$
2. $\cos A = \frac{b}{c} = \sin B$
3. $\tan A = \frac{a}{b} = \operatorname{ctn} B$
4. $\operatorname{ctn} A = \frac{b}{a} = \tan B$
5. $\sec A = \frac{c}{b} = \operatorname{cosec} B$
6. $\operatorname{cosec} A = \frac{c}{a} = \sec B$
7. $\operatorname{vers} A = \frac{c-b}{c} = \operatorname{covers} B$
8. $\operatorname{exsec} A = \frac{c-b}{b} = \operatorname{coexsec} B$
9. $\operatorname{covers} A = \frac{c-a}{c} = \operatorname{versin} B$
10. $\operatorname{coexsec} A = \frac{c-a}{a} = \operatorname{exsec} B$
11. $a = c \sin A = b \tan A$
12. $b = c \cos A = a \operatorname{ctn} A$
13. $c = \frac{a}{\sin A} = \frac{b}{\cos A}$
14. $a = c \cos B = b \operatorname{ctn} B$
15. $b = c \sin B = a \tan B$
16. $c = \frac{a}{\cos B} = \frac{b}{\sin B}$
17. $a = \sqrt{(c+b)(c-b)}$
18. $b = \sqrt{(c+a)(c-a)}$
19. $c = \sqrt{a^2 + b^2}$
20. $C = 90° = A + B$
21. $\text{area} = \frac{ab}{2}$



	GIVEN	SOUGHT	FORMULA
22	A, B, a	C, b, c	$C = 180° - (A + B)$, $b = \frac{a}{\sin A} \cdot \sin B$, $c = \frac{a}{\sin A} \sin (A + B)$
23	A, a, b	B, C, c	$\sin B = \frac{\sin A}{a} \cdot b$, $C = 180° - (A + B)$, $c = \frac{a}{\sin A} \cdot \sin C$.
24	C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2}C$
25		$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$
26		A, B	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$, $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$
27		c	$c = (a + b) \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)}$ $= \sqrt{a^2 + b^2 - 2ab \cos C}$
28		area	area $= \frac{1}{2}ab \sin C$.
29	a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$; $\sin \frac{1}{2}A = \sqrt{\frac{(s - b)(s - c)}{bc}}$
30			$\cos \frac{1}{2}A = \sqrt{\frac{s(s - a)}{bc}}$; $\tan \frac{1}{2}A = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}}$
31			$\sin A = \frac{2\sqrt{s(s - a)(s - b)(s - c)}}{bc}$; $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
32		area	area $= \sqrt{s(s - a)(s - b)(s - c)}$
33	A, B, C, a	area	area $= \frac{a^2 \sin B . \sin C}{2 \sin A}$

Algebraic signs of trigonometric functions in various quadrants. The radius is always +.

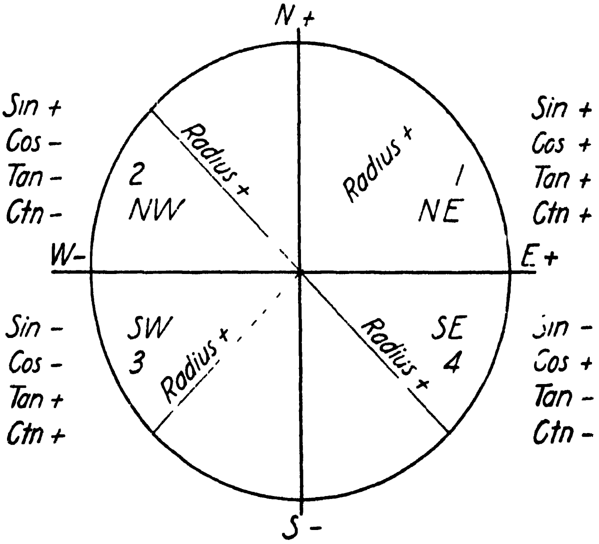


TABLE X

Practice Notes

Field notes are the records of the work done in the field and are usually kept on file for future reference Such notes must be complete, neat, legible and without ambiguous statements

In recording field notes, students should:

1 Use a 4H pencil.

2. Letter notes carefully. This page is to be used as a guide for form, style, and height.

3. Record field data in books in field, not on scratch paper

4. Designate each problem by title

5 Indicate party organization, date, weather conditions, and equipment used

6 Index the field book

7. Use plenty of space. Notes should not be crowded.

8 Draw neat sketches, roughly to scale, important details exaggerated.

9. Never erase entries in the field book. Corrections should be made by drawing a single line through the incorrect value and inserting the correct value above the erroneous one.

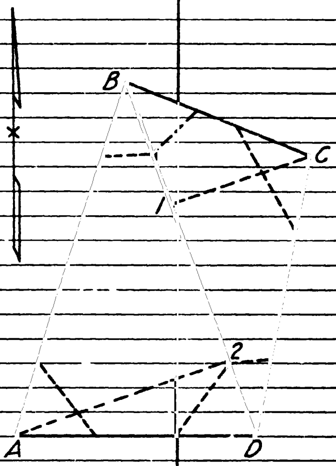
10. Remember that office engineers will use the field note book. The data must therefore be complete and admit of but one interpretation.

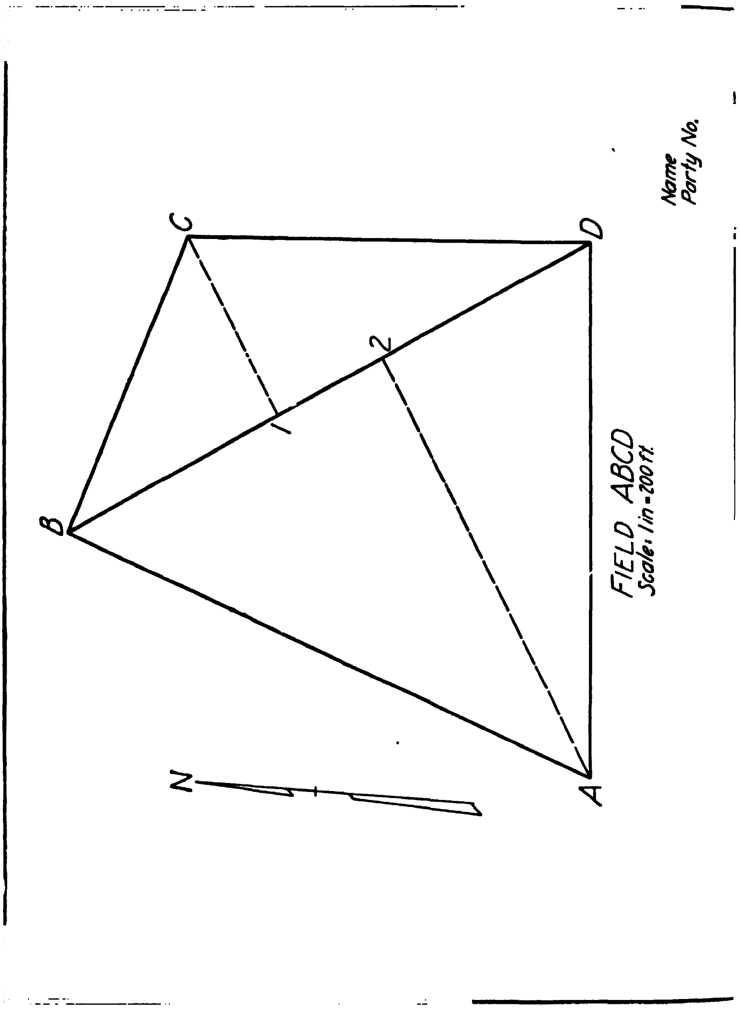
TAPE SURVEY OF FIELD ABCD

Sept. 23, 1949
Clear, cool
Party K-25
Locker 82

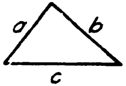
Line	Distance
	ft.
AB	608.3
BC	322.8
CD	426.8
DA	539.0
DB	627.9
A2	477.4
C1	201.9

Angle	Radius	Chord	Value	Adjst'd	Adj Value
DBC	100.0'	66.5	38°50'	-05'	38°45'
BCD	100.0'	166 9	113°08'	-05'	113°03'
CDB	100.0'	48.9	28°18'	-06'	28°12'
			180°16'		
DAB	100 0'	108.6	65°46'	+10'	65°56'
ABD	100.0'	86.9	51°30'	+11'	51°41'
BDA	100.0'	103.3	62°12'	+11'	62°23'
			179°28'		




A
B
C
D
1
2
N
FIELD ABCD
Scale: 1 in = 200 ft.
Name
Party No.

COMPUTATION OF AREA OF FIELD ABCD
3 SIDE METHOD

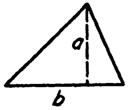
Triangle	Line	Length ft.	s $\frac{1}{2}(a+b+c)$	s-a	s-b	s-c	Logarithms	Areas sq ft.
DBC			688 75				2.83806	$\sqrt{s(s-a)(s-b)(s-c)}$
	DB = a	627.9		60.85			1 78426	
	BC = b	322.8			365.95		2.56342	
	CD = c	426.8				261 95	2.41822	
		2|1377 5					2|19 60396	
		688.75					4.80198	63384
DBA			887 50				2.94822	
	DB = a	627.9		259.70			2.41447	
	BA = b	608 3			279.30		2 44607	
	AD = c	539.0				348.60	2.54233	
		2|1775.2					2|10.35109	
		887.60					5.17555	149815
								213199 =
								4.89 Acres

Name

Party No.

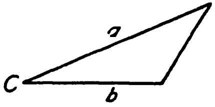
COMPUTATION OF AREA OF FIELD ABCD
BASE-ALTITUDE METHOD

Triangle	Line	Base, b ft	Alt, a	Logarithms	Double Areas sq ft	Area = $\frac{1}{2}ab$
DBC	DB	627 9		2.79789		
	CI		201.9	2.30514		
				5.10303	126774	63387
DBA	DB	627.9		2.79789		
	A2		477.4	2.67888		
				5.47677	299710	149855
					2)426484	
					213242 = 4.90 Acres	



Name
Party No.

COMPUTATION OF AREA OF FIELD ABCD
ANGLE METHOD



Triangle	Part	Value ft or °	Logarithms	Double Areas sq ft	Area = ½ab sin C
DBC	DB = a	627 9	2 79789		
	DC = b	426 8	2 63022		
	BDC = C	28°12′	9 67445		
			5 10256	126640	63320
DBA	DB = a	627 9	2 79789		
	DA = b	539 0	2 73159		
	ADB = C	62°23′	9 94747		
			5 47695	299880	149940
				2 |426520	
				213260 = 4 30 Acres	

Name
Party No

DIFFERENTIAL LEVEL CIRCUIT

Sept 20, 1949
Warm, cloudy
Party 7

Sta	B+S	HI	F−S	Elev	Description	Level 45
BM_1				615 49	U S G S BM on campus at State and	
π_1	4 73	620 22			Grant Sts	
TP_1			3 57	616 65		
π_2	2 33	618 98				
BM_2			2 14	616 84	Copper plug, E end 3rd step, S entrance	
π_3	5 75	622 59			C E Bldg	
TP_2			4 27	618 32		
π_4	2 66	620 98				
BM_3			9 33	611 65	N flange nut, water hydrant, North St,	
π_5	1 75	613 40			100 ft W of Vine St	
TP_3			4 11	609 29		
π_6	2 61	611 90				
BM_4			3 42	608 48		
	Σ=19 83		Σ=26 84	615 49	Although many surveyors use this	
			19 83	608 48	expanded form of level notes, many	
			7 01	7 01	use the concise form after the manner	
					of that shown on pg 77	

PROFILE OF CENTER LINE OF FIRST ST

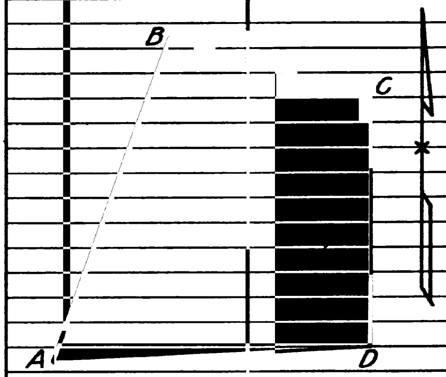
Oct 6, 1949
Cloudy, warm
Party J-7

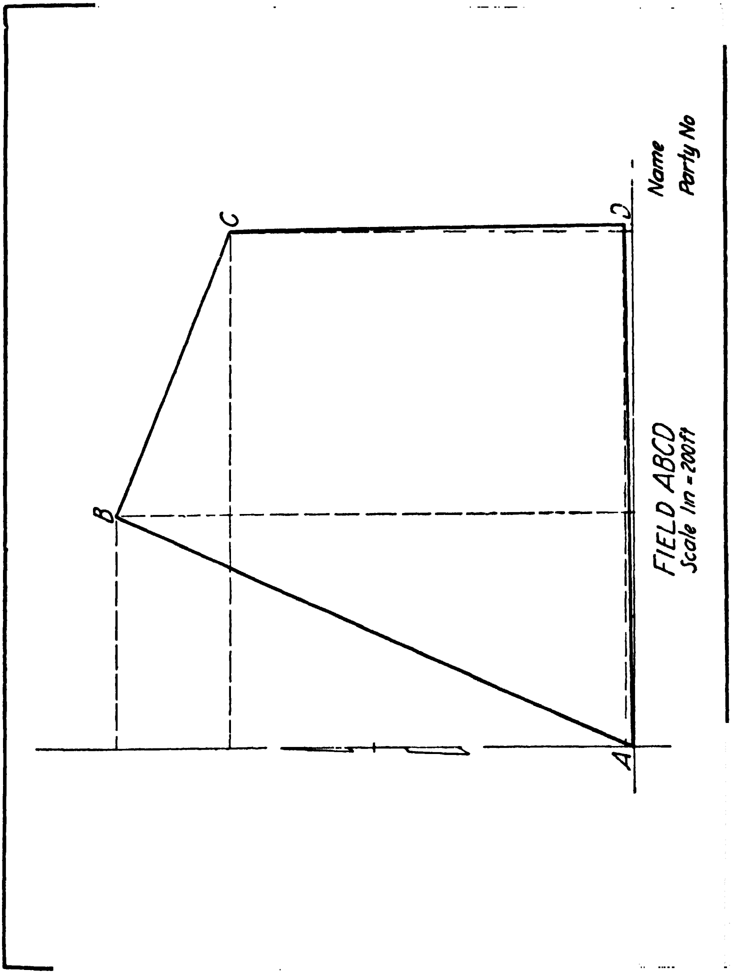
Sta	B^+S	HI	F^-S	Elevation Ground	Elevation TP&BM	Description	Level 69
BM_1					836 72	X cut in curb, NW	cor First St &
$\overline{\wedge}_1$	2.86	839 58				Donaldson Ave	
0+00			4.3	835 3			
1+00			6 1	833 5			
2+00			8 7	830 9			
TP_1			8 55		831 03		
$\overline{\wedge}_2$	4 36	835 39					
3+00			5 6	829 8			
3+35			6 0	829 4		W curb Queen St	
3+35			6 7	828 7		W gutter " "	
3+50			6 1	829.5		℄ " "	
3+65			6 8	828.6		E gutter " "	
3+65			6 1	829 3		E curb " "	
4+00			7 2	828 2			
4+60			8 3	827 1		Break in grade	
5+00			7 0	828.4			
TP_2			6 72		828 67		
$\overline{\wedge}_3$	8 25	836 92					
6+00			6 2	830 7			
		etc					

COMPASS TRAVERSE OF FIELD ABCD

Oct. 2, 1949
Clear, cool
Party K-25
Locker 17

Station Occupied	Station Sighted	Distance ft.	Needle Reading	Interior Angle
A	D		N89°10'E	65°50'
	B	608.3	N23°20'E	
B	A		S23°30'W	90°45'
	C	322.8	S67°15'E	
C	B		N67°00'W	113°00'
	D	426.8	S 0°00'	
D	C		N 0°10'E	90°50'
	A	539.0	S89°20'W	
				360°25'




A
B
C
D
FIELD ABCD
Scale 1in = 200ft
Name
Party No

FIELD DATA & CALCULATION OF ADJUSTED BEARINGS

Station Occ'd	Station Sg't'd	Distance Ft	Needle Reading	Interior Angle	Correction	Corrected Angle	Adjusted Bearing
A	D		N89°10'E	65°50'	-06'	65°44'	
	B	608 3	N23°20 E				N 23°20 E
B	A		S 23°30 W	90°45'	-06'	90°39'	
	C	322 8	S 67°15'E				S 67°19 E
C	B		N67°00'W	113°00'	-06'	112°54'	
	D	426 8	S 0°00'W				S 0°13'E
D	C		N 0°10 E	90°50	-07'	90°43'	
	A	539 0	S 89°20 W				S 89°04 W
	Σ =	1896 9		360°25'		360°00'	

Name

Party No

CALCULATION OF LATITUDES & DEPARTURES

Line	AB	BC	CD	DA
Lat	558 5	124 5	426 8	8 8
Log Lat	2 74706	2 09511	2 63022	0.94348
Log Cos Brg	9 96294	9 58618	10 00000	8 21189
Log Dist	2 78412	2 50893	2 63022	2 73159
Log Sin Brg	9 59778	9 96504	7 57767	9 99994
Log Dep	2 38190	2 47397	0 20789	2.73153
Dep	240 9	297 8	1 6	538 9

Name
Party No

ERROR OF CLOSURE, D.M D, AREA

Line	Latitude		Departure		Adjusted		D M D	Double Area	
	N	S	E	W	Lat	Dep		N	S
AB	558.5		240.9		+559 0	+240.4	240 4	134380	
BC		124 5	297.8		-124 3	+297 6	778 4		96754
CD		426 8	1.6		-426 4	+1 3	1077 3		459367
DA		8.8		538 9	-8.3	-539.3	539 3		4476
	558 5	560.1	540 3	538 9				134380	560597
	16		14						

$2\,\lfloor 426217$

213109 sq ft =

4.90 Acres

Error of Closure $= \frac{1}{d} = \frac{\sqrt{16^2+14^2}}{1896\ 9} = \frac{1}{895}$

Latitudes and Departures Adjusted by Compass Rule

Line	AB	BC	CD	DA
Log Lat	2.74741	2 09447	2.62982	0 91908
Log DMD	2 38093	2 89120	3 03234	2 73183
Log Double Area	5 12834	4 98567	5 66216	3 65091
Double Area	134380	96754	459367	4476

Name

Party No

RECTANGULAR COORDINATES

TOTAL LATITUDES & DEPARTURES FOR PLOTTING

Line	Cor. Lat.	Cor. Dep.	Point	Total Lat.	Total Dep.
AB	+559.0	+240.4	A	0.0	0.0
BC	-124.3	+297.6	B	+559.0	+240.4
CD	-426.4	+1.3	C	+434.7	+538.0
DA	-8.3	-539.3	D	+8.3	+539.3
			A	0.0	0.0

Name

Party No.

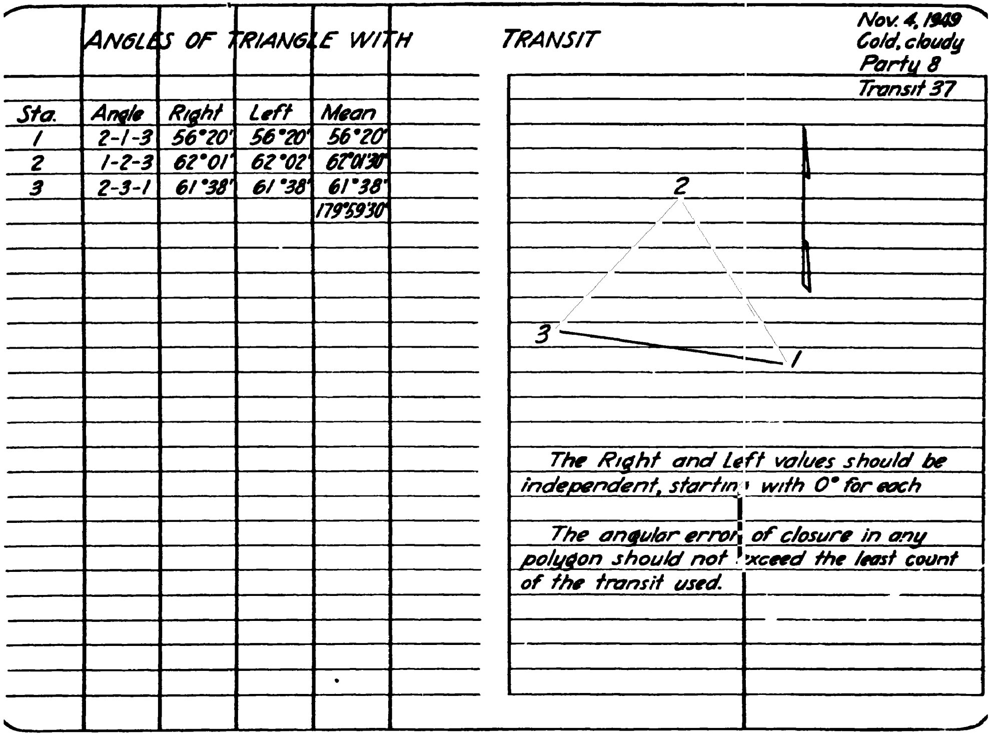
AREA BY RECTANGULAR COORDINATES

	N(y)	E(x)	Diff in Adjacent x Values	Double Areas +	Double Areas −
A	0 0	0 0	-298.9	0 0	
B	559.0	240 4	+538.0	300742 0	
C	434 7	538 0	+298 9	129931 8	
D	8 3	539 3	-538.0		4465 4
			Σ = 0 0	430673 8	4465 4
				- 4465 4	
			2	426208.4	
				213104 2 sq ft =	
				4 90 Acres	

Direct multiplication or logarithms may be used

Name

Party No



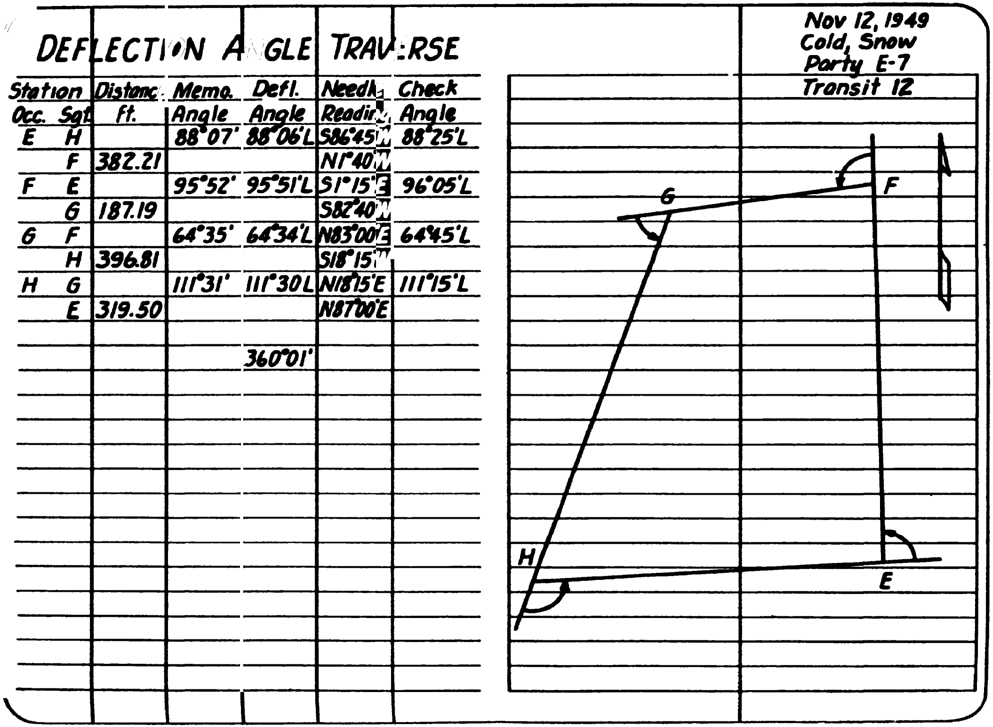
ANGLES OF TRIANGLE WITH TRANSIT

Nov. 4, 1949
Cold, cloudy
Party 8
Transit 37

Sta.	Angle	Right	Left	Mean
1	2-1-3	56°20′	56°20′	56°20′
2	1-2-3	62°01′	62°02′	62°01′30″
3	2-3-1	61°38′	61°38′	61°38′
				179°59′30″

The Right and Left values should be independent, starting with 0° for each

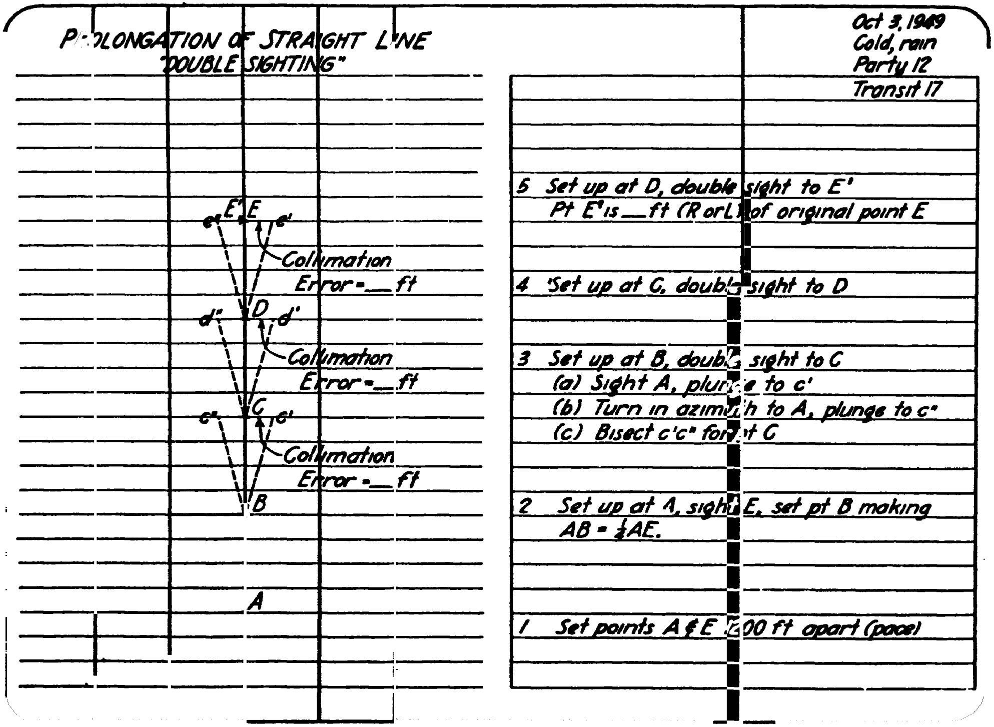
The angular error of closure in any polygon should not exceed the least count of the transit used.



DEFLECTION ANGLE TRAVERSE

Station		Distanc	Memo.	Defl.	Needl	Check
Occ.	Sgt	ft.	Angle	Angle	Readin	Angle
E	H		88°07'	88°06'L	S86°45'W	88°25'L
	F	382.21			N1°40'W	
F	E		95°52'	95°51'L	S1°15'E	96°05'L
	G	187.19			S82°40'W	
G	F		64°35'	64°34'L	N83°00'E	64°45'L
	H	396.81			S18°15'W	
H	G		111°31'	111°30L	N18°15'E	111°15'L
	E	319.50			N87°00'E	
				360°01'		

Nov 12, 1949
Cold, Snow
Party E-7
Transit 12



PROLONGATION OF STRAIGHT LINE
"DOUBLE SIGHTING"

Oct 3, 1949
Cold, rain
Party 12
Transit 17

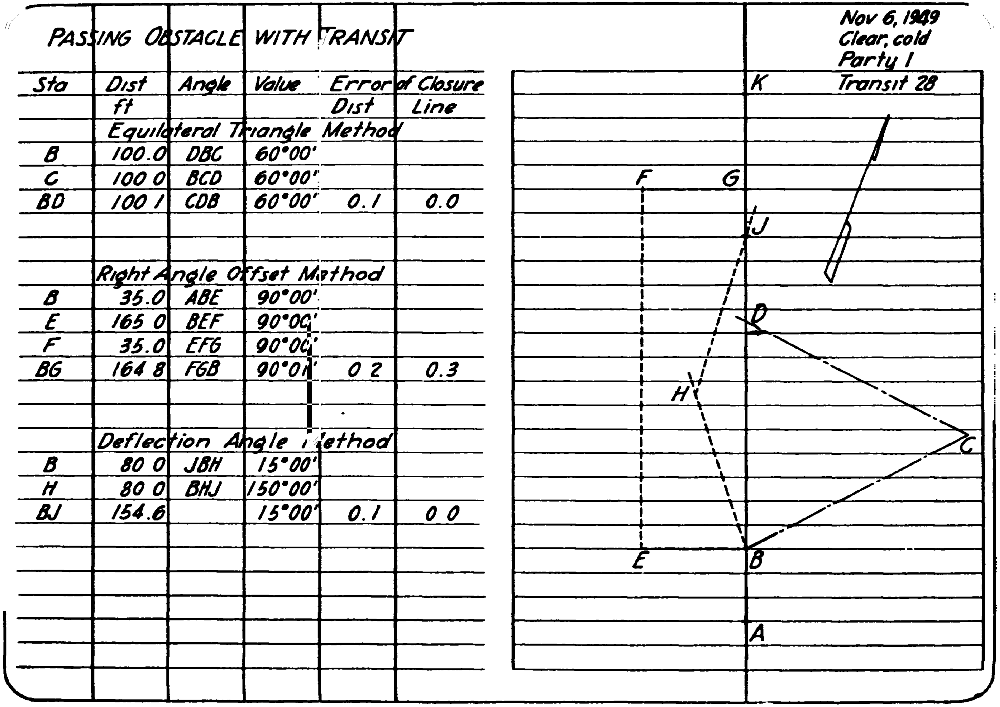
5 Set up at D, double sight to E'
Pt E' is __ ft (R or L) of original point E

4 Set up at C, double sight to D

3 Set up at B, double sight to C
(a) Sight A, plunge to c'
(b) Turn in azimuth to A, plunge to c"
(c) Bisect c'c" for pt C

2 Set up at A, sight E, set pt B making
AB = ½AE.

1 Set points A & E 200 ft apart (pace)



PASSING OBSTACLE WITH TRANSIT

Nov 6, 1949
Clear, cold
Party 1
Transit 28

Sta	Dist ft	Angle	Value	Error of Closure Dist	Error of Closure Line
	Equilateral Triangle Method				
B	100.0	DBC	60°00'		
C	100.0	BCD	60°00'		
BD	100.1	CDB	60°00'	0.1	0.0
	Right Angle Offset Method				
B	35.0	ABE	90°00'		
E	165.0	BEF	90°00'		
F	35.0	EFG	90°00'		
BG	164.8	FGB	90°01'	0.2	0.3
	Deflection Angle Method				
B	80.0	JBH	15°00'		
H	80.0	BHJ	150°00'		
BJ	154.6		15°00'	0.1	0.0

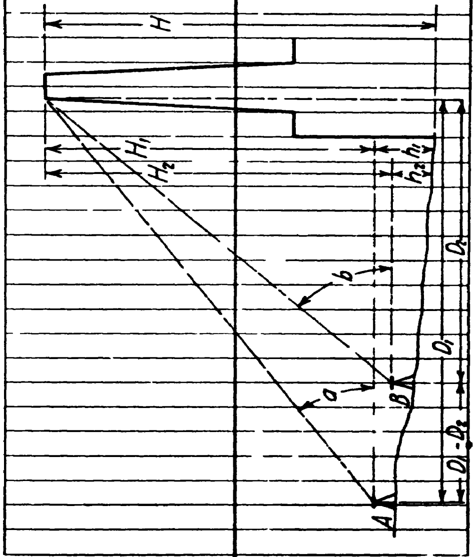
HEIGHT OF PURDUE STACK

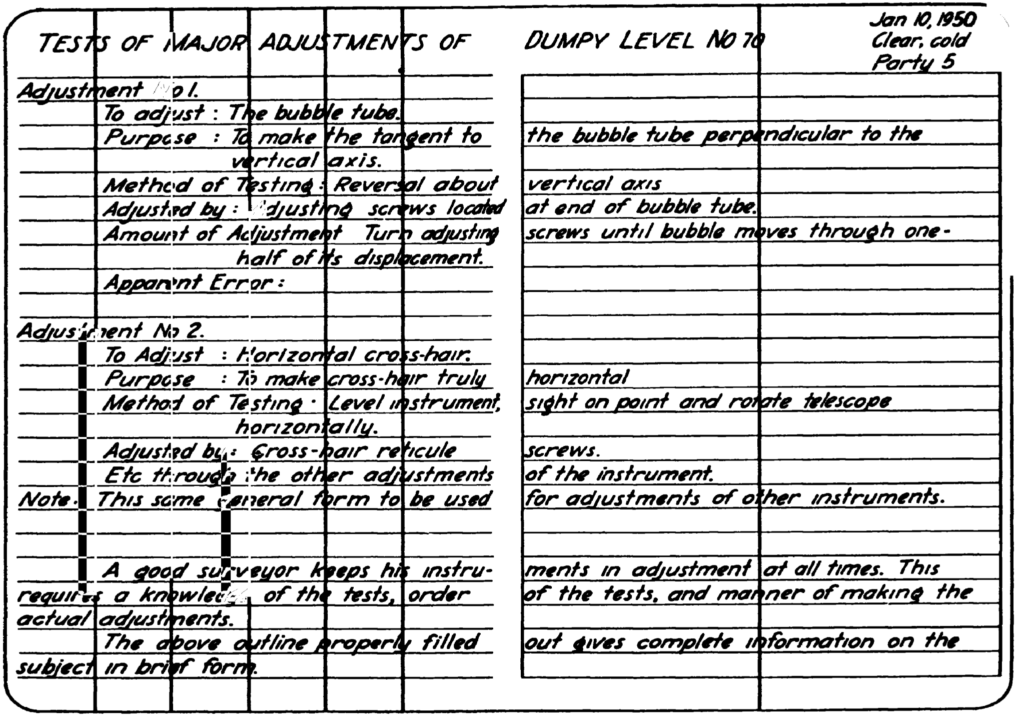
Nov 20, 1949
Cold, windy
Party 3
Transit 15

Sta	Vertical Angle	D_1-D_2 ft	-S
A	26°45′30″	166 8	4 550 = h_1
B	37°25′30″		4 158 = h_2

Equations

$$D_1 = H_1 \cot a$$
$$D_2 = H_2 \cot b$$
$$H = H_1 + h_1 = H_2 + h_2$$
$$H_1 = H_2 - (h_1 - h_2)$$
$$D_1 = H_2 \cot a - (h_1 - h_2) \cot a$$
$$D_1 - D_2 = H_2(\cot a - \cot b) - (h_1 - h_2) \cot a$$
$$H_2 = \frac{D_1 - D_2 + (h_1 - h_2) \cot a}{\cot a - \cot b}$$
$$H = H_2 - h_2$$





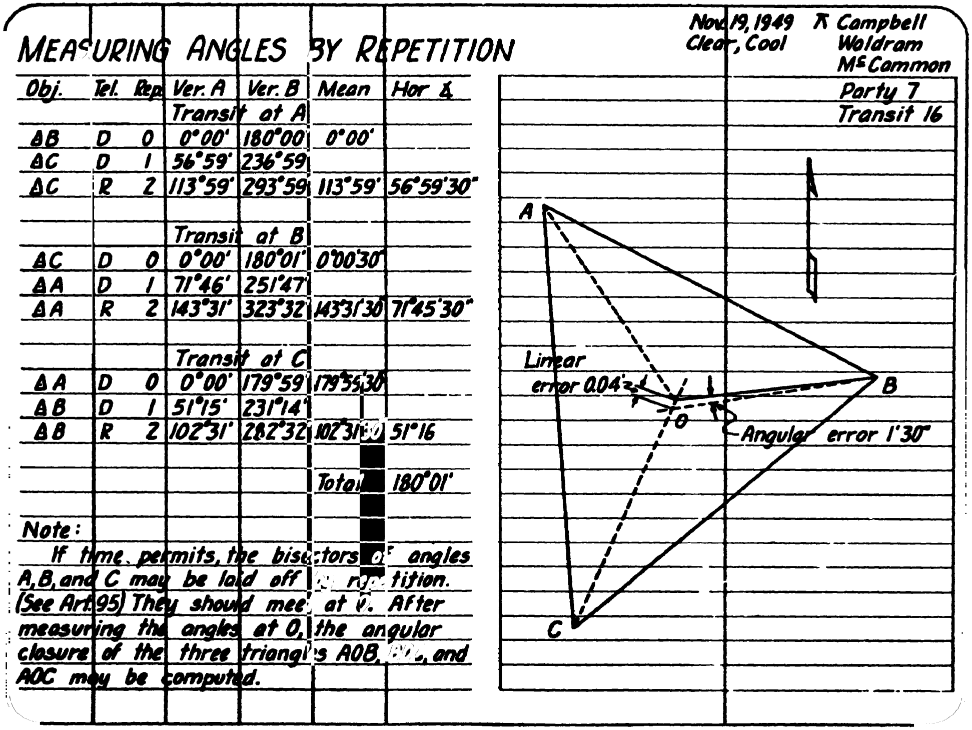
TESTS OF MAJOR ADJUSTMENTS OF DUMPY LEVEL No 70

Jan 10, 1950
Clear, cold
Party 5

Adjustment No 1.
To adjust : The bubble tube.
Purpose : To make the tangent to the bubble tube perpendicular to the
vertical axis.
Method of Testing : Reversal about vertical axis
Adjusted by : Adjusting screws located at end of bubble tube.
Amount of Adjustment Turn adjusting screws until bubble moves through one-
half of its displacement.
Apparent Error :

Adjustment No 2.
To Adjust : Horizontal cross-hair.
Purpose : To make cross-hair truly horizontal
Method of Testing : Level instrument, sight on point and rotate telescope
horizontally.
Adjusted by : Cross-hair reticule screws.
Etc through the other adjustments of the instrument.
Note. This same general form to be used for adjustments of other instruments.

A good surveyor keeps his instru- ments in adjustment at all times. This
requires a knowledge of the tests, order of the tests, and manner of making the
actual adjustments.
The above outline properly filled out gives complete information on the
subject in brief form.



MEASURING ANGLES BY REPETITION

Nov. 19, 1949 ⊼ Campbell
Clear, Cool Woldram
McCommon
Party 7
Transit 16

Obj.	Tel.	Rep.	Ver. A	Ver. B	Mean	Hor ∠
			Transit at A			
ΔB	D	0	0°00'	180°00'	0°00'	
ΔC	D	1	56°59'	236°59'		
ΔC	R	2	113°59'	293°59'	113°59'	56°59'30"
			Transit at B			
ΔC	D	0	0°00'	180°01'	0°00'30"	
ΔA	D	1	71°46'	251°47'		
ΔA	R	2	143°31'	323°32'	143°31'30"	71°45'30"
			Transit at C			
ΔA	D	0	0°00'	179°59'	179°59'30"	
ΔB	D	1	51°15'	231°14'		
ΔB	R	2	102°31'	282°32'	102°31'30"	51°16'
					Total [redacted]	180°01'

Note:
If time permits, the bisectors [redacted] angles A, B, and C may be laid off [illegible] repetition. (See Art. 95) They should meet at O. After measuring the angles at O, the angular closure of the three triangles AOB, [illegible], and AOC may be computed.

HEIGHT OF SMOKE STACK ABOVE BENCH MARK

April 2, 1950 Cloudy, Cool
⊼ Campbell
McCammon
Waldman
Party 7
Transit 15

Horizontal Angles

Sta.	Single ∠	Double ∠	½ Double	H.I.
B	60°14'	120°28'	60°14'	4.15
A	72°41'	145°20'	72°40'	8.55

C is point at top of stack. B.M. is at bottom stack

Vertical Angles

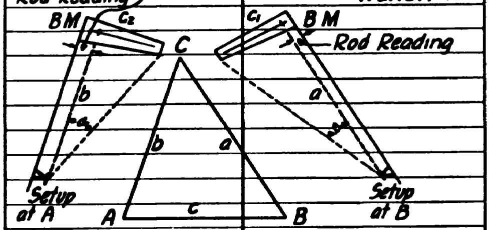
	1st	2nd	Mean
β	+20°28'	+20°28'	+20°28'
α	+21°51'	+21°51'	+21°51'

Rod Readings

From setup at B = 4.15
From setup at A = 8.55
Taped length AB 401.50
∠ C = 180° − (60°14' + 72°40') = 47°06'

Using Sine Law – In Triangle ABC

(1) Log c (401.50) = 2.60369
(2) Colog C (47°06') = 0.13517
(3) Log sin A (72°40') = 9.97982
(4) Log sin B (60°14') = 9.93855
Log b = (1)+(2)+(4) = 2.67741
b = 475.79
Log a = (1)+(2)+(3) = 2.71868
a = 523.21



Log a = 2.71868		Log b	= 2.67741
Log tan β = 9.57197		Log tan α	= 9.60313
Log cc₁ = 2.29065		Log cc₂	= 2.28054
cc₁ = 195.28'		cc₂	= 190.78'
Rod = 4.15'		Rod	= 8.55'
Height = 199.43'		Height	= 199.33

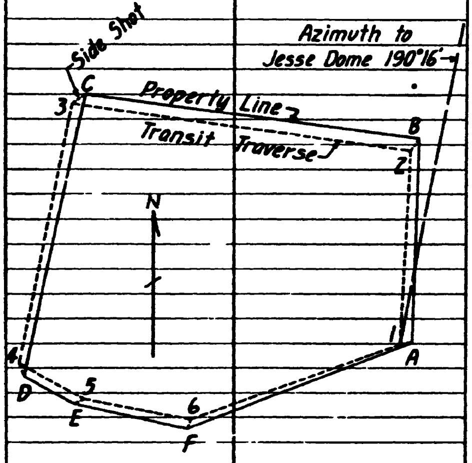
Mean Height above B.M. = 199.38'

See pages 183 & 184

OFF-LINE TRAVERSE OF PROPERTY LINES

April 9,1950 ⊼ Marcus
Clear, Warm Nomikos
Kay
Party 13
Transit 7

Obj.	Ver. A	Ver. B	Taped Distance	Compass Reading	Calculated Bearing
		Transit	at Station 1		
A	252°16'	72°16'	15.4	N72½°E	N72°16'E
2	182°36'	2°36'	584.2	N2½°E	N2°36'E
6	68°22'	248°22'	617.2	S68½°W	S68°22'W
		Transit	at Station 2		
1	2°36'	182°36'	584.2	S2½°W	S2°36'W
B	201°07'	21°07'	14.9	N21°E	N21°07'E
3	98°21'	278°21'	945.3	N81½°W	N81°39'W
		Transit	at Station 3		
2	278°21'	98°21'	945.3	S81½°E	S81°39'E
C	225°34'	45°34'	16.4	N45½°E	N45°34'E
4	10°14'	190°14'	800.4	S10½°W	S10°14'W
		Transit	at Station 4		
3	190°14'	10°14'	800.4	N10½°E	N10°14'E
D	339°27'	159°27'	23.4	S20½°E	S20°33'E
5	299°20'	119°20'	207.5	S60½°E	S60°40'E
		Transit	at Station 5		
4	119°20'	299°20'	207.5	N60½°W	N60°40'W
E	76°05'	256°05'	9.7	S76°W	S76°05'W
6	281°20'	101°20'	302.8	S78½°E	S78°40'E
		Transit	at Station 6		
5	101°20'	281°20'	302.8	N78½°W	N78°40'W
F	15°30'	195°30'	9.4	S15½°W	S15°30'W
1	248°22'	68°22'	617.2	N68½°E	N68°22'E



Note: Description, computations, and plat of this survey are given on pages 203, 307 & 308

TABLE XI

Natural Sines and Cosines

′	0°		1°		2°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0 0000000	1 0000000	0 0174524	0 9998477	0 0348995	0 9993908	60
1	0002909	1 0000000	0177432	9998426	0351902	9993806	59
2	0005818	9999998	0180341	9998374	0354809	9993704	58
3	0008727	9999996	0183249	9998321	0357716	9993600	57
4	0011636	9999993	0186158	9998267	0360623	9993495	56
5	0014544	9999989	0189066	9998213	0363530	9993390	55
6	0017453	9999985	0191974	9998157	0366437	9993284	54
7	0020362	9999979	0194883	9998101	0369344	9993177	53
8	0023271	9999973	0197791	9998044	0372251	9993069	52
9	0026180	9999966	0200699	9997986	0375158	9992960	51
10	0 0029089	0 9999958	0 0203608	0 9997927	0 0378065	0 9992851	50
11	0031998	9999949	0206516	9997867	0380971	9992740	49
12	0034907	9999939	0209424	9997807	0383878	9992629	48
13	0037815	9999928	0212332	9997745	0386785	9992517	47
14	0040724	9999917	0215241	9997683	0389692	9992404	46
15	0043633	9999905	0218149	9997620	0392598	9992290	45
16	0046542	9999892	0221057	9997556	0395505	9992176	44
17	0049451	9999878	0223965	9997492	0398411	9992060	43
18	0052360	9999863	0226873	9997426	0401318	9991944	42
19	0055268	9999847	0229781	9997360	0404224	9991827	41
20	0 0058177	0 9999831	0 0232690	0 9997292	0 0407131	0 9991709	40
21	0061086	9999813	0235598	9997224	0410037	9991590	39
22	0063995	9999795	0238506	9997155	0412944	9991470	38
23	0066904	9999776	0241414	9997086	0415850	9991350	37
24	0069813	9999756	0244322	9997015	0418757	9991228	36
25	0072721	9999736	0247230	9996943	0421663	9991106	35
26	0075630	9999714	0250138	9996871	0424569	9990983	34
27	0078539	9999692	0253046	9996798	0427475	9990859	33
28	0081448	9999668	0255954	9996724	0430382	9990734	32
29	0084357	9999644	0258862	9996649	0433288	9990609	31
30	0 0087265	0 9999619	0 0261769	0 9996573	0 0436194	0 9990482	30
31	0090174	9999593	0264677	9996497	0439100	9990355	29
32	0093083	9999567	0267585	9996419	0442006	9990227	28
33	0095992	9999539	0270493	9996341	0444912	9990098	27
34	0098900	9999511	0273401	9996262	0447818	9989968	26
35	0101809	9999482	0276309	9996182	0450724	9989837	25
36	0104718	9999452	0279216	9996101	0453630	9989706	24
37	0107627	9999421	0282124	9996020	0456536	9989573	23
38	0110535	9999389	0285032	9995937	0459442	9989440	22
39	0113444	9999357	0287940	9995854	0462347	9989306	21
40	0 0116353	0 9999323	0 0290847	0 9995770	0 0465253	0 9989171	20
41	0119261	9999289	0293755	9995684	0468159	9989035	19
42	0122170	9999254	0296662	9995599	0471065	9988899	18
43	0125079	9999218	0299570	9995512	0473970	9988761	17
44	0127987	9999181	0302478	9995424	0476876	9988623	16
45	0130896	9999143	0305385	9995336	0479781	9988484	15
46	0133805	9999105	0308293	9995247	0482687	9988344	14
47	0136713	9999065	0311200	9995157	0485592	9988203	13
48	0139622	9999025	0314108	9995066	0488498	9988061	12
49	0142530	9998984	0317015	9994974	0491403	9987919	11
50	0 0145439	0 9998942	0 0319922	0 9994881	0 0494308	0 9987775	10
51	0148348	9998900	0322830	9994788	0497214	9987631	9
52	0151256	9998856	0325737	9994693	0500119	9987486	8
53	0154165	9998812	0328644	9994598	0503024	9987340	7
54	0157073	9998766	0331552	9994502	0505929	9987194	6
55	0159982	9998720	0334459	9994405	0508835	9987046	5
56	0162890	9998673	0337366	9994308	0511740	9986898	4
57	0165799	9998625	0340274	9994209	0514645	9986748	3
58	0168707	9998577	0343181	9994110	0517550	9986598	2
59	0171616	9998527	0346088	9994009	0520455	9986447	1
60	0 0174524	0 9998477	0 0348995	0 9993908	0 0523360	0 9986295	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		

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′	3°		4°		5°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.0523360	0.9986295	0.0697565	0 9975641	0.0871557	0.9961947	60
1	.0526264	.9986143	.0700467	.9975437	.0874455	.9961693	59
2	.0529169	.9985989	.0703368	.9975233	.0877353	.9961438	58
3	.0532074	.9985835	.0706270	.9975028	.0880251	.9961183	57
4	.0534979	.9985680	.0709171	.9974822	.0883148	.9960926	56
5	.0537883	.9985524	.0712073	.9974615	.0886046	.9960669	55
6	.0540788	.9985367	.0714974	.9974408	.0888943	9960411	54
7	.0543693	.9985209	.0717876	.9974199	.0891840	.9960152	53
8	.0546597	.9985050	.0720777	.9973990	.0894738	.9959892	52
9	.0549502	.9984891	.0723678	.9973780	.0897635	.9959631	51
10	0.0552406	0.9984731	0.0726580	0.9973569	0.0900532	0.9959370	50
11	.0555311	.9984570	.0729481	.9973357	.0903429	.9959107	49
12	.0558215	.9984408	.0732382	.9973145	.0906326	.9958844	48
13	.0561119	.9984245	.0735283	.9972931	.0909223	.9958580	47
14	.0564024	.9984081	.0738184	.9972717	.0912119	.9958315	46
15	.0566928	.9983917	.0741085	.9972502	.0915016	.9958049	45
16	.0569832	.9983751	.0743986	.9972286	.0917913	.9957783	44
17	.0572736	.9983585	.0746887	.9972069	.0920809	.9957515	43
18	.0575640	.9983418	.0749787	.9971851	.0923706	.9957247	42
19	.0578544	.9983250	.0752688	.9971633	.0926602	.9956978	41
20	0.0581448	0.9983082	0.0755589	0.9971413	0.0929499	0.9956708	40
21	.0584352	.9982912	.0758489	.9971193	.0932395	.9956437	39
22	.0587256	.9982742	.0761390	.9970972	.0935291	.9956165	38
23	.0590160	.9982570	.0764290	.9970750	.0938187	.9955893	37
24	.0593064	.9982398	.0767190	.9970528	.0941083	.9955620	36
25	.0595967	.9982225	.0770091	.9970304	.0943979	.9955345	35
26	.0598871	.9982052	.0772991	.9970080	.0946875	.9955070	34
27	.0601775	.9981877	.0775891	.9969854	.0949771	.9954795	33
28	.0604678	.9981701	.0778791	.9969628	.0952666	.9954518	32
29	.0607582	.9981525	.0781691	.9969401	.0955562	.9954240	31
30	0.0610485	0.9981348	0.0784591	0.9969173	0.0958458	0.9953962	30
31	.0613389	.9981170	.0787491	.9968945	.0961353	.9953683	29
32	.0616292	.9980991	.0790391	.9968715	.0964248	.9953403	28
33	.0619196	.9980811	.0793290	.9968485	.0967144	.9953122	27
34	.0622099	.9980631	.0796190	.9968254	.0970039	.9952840	26
35	.0625002	.9980450	.0799090	.9968022	.0972934	.9952557	25
36	.0627905	.9980267	.0801989	.9967789	.0975829	.9952274	24
37	.0630808	.9980084	.0804889	.9967555	.0978724	.9951990	23
38	.0633711	.9979900	.0807788	.9967321	.0981619	.9951705	22
39	.0636614	.9979716	.0810687	.9967085	.0984514	.9951419	21
40	0.0639517	0.9979530	0.0813587	0.9966849	0.0987408	0.9951132	20
41	.0642420	.9979343	.0816486	.9966612	.0990303	.9950844	19
42	.0645323	.9979156	.0819385	.9966374	.0993197	.9950556	18
43	.0648226	.9978968	.0822284	.9966135	.0996092	.9950266	17
44	.0651129	.9978779	.0825183	.9965895	.0998986	.9949976	16
45	.0654031	.9978589	.0828082	.9965655	.1001881	.9949685	15
46	.0656934	.9978399	.0830981	.9965414	.1004775	.9949393	14
47	.0659836	.9978207	.0833880	.9965172	.1007669	.9949101	13
48	.0662739	.9978015	.0836778	.9964929	.1010563	.9948807	12
49	.0665641	.9977821	.0839677	.9964685	.1013457	.9948513	11
50	0.0668544	0.9977627	0.0842576	0.9964440	0.1016351	0.9948217	10
51	.0671446	.9977433	.0845474	.9964195	.1019245	.9947921	9
52	.0674349	.9977237	.0848373	.9963948	.1022138	.9947625	8
53	.0677251	.9977040	.0851271	.9963701	.1025032	.9947327	7
54	.0680153	.9976843	.0854169	.9963453	.1027925	.9947028	6
55	.0683055	.9976645	.0857067	.9963204	.1030819	.9946729	5
56	.0685957	.9976445	.0859966	.9962954	.1033712	.9946428	4
57	.0688859	.9976245	.0862864	.9962704	.1036605	.9946127	3
58	.0691761	.9976045	.0865762	.9962452	.1039499	.9945825	2
59	.0694663	.9975843	.0868660	.9962200	.1042392	.9945523	1
60	0.0697565	0.9975641	0.0871557	0.9961947	0.1045285	0.9945219	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	86°		85°		84°		′

′	6°		7°		8°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.1045285	0.9945219	0.1218693	0.9925462	0.1391731	0.9902681	60
1	.1048178	.9944914	.1221581	.9925107	.1394612	.9902275	59
2	.1051070	.9944609	.1224468	.9924751	.1397492	.9901869	58
3	.1053963	.9944303	.1227355	.9924394	.1400372	.9901462	57
4	.1056856	.9943996	.1230241	.9924037	.1403252	.9901055	56
5	.1059748	.9943688	.1233128	.9923679	.1406132	.9900646	55
6	.1062641	.9943379	.1236015	.9923319	.1409012	.9900237	54
7	.1065533	.9943070	.1238901	.9922959	.1411892	.9899826	53
8	.1068425	.9942760	.1241788	.9922599	.1414772	.9899415	52
9	.1071318	.9942448	.1244674	.9922237	.1417651	.9899003	51
10	0.1074210	0.9942136	0.1247560	0.9921874	0.1420531	0.9898590	50
11	.1077102	.9941823	.1250446	.9921511	.1423410	.9898177	49
12	.1079994	.9941510	.1253332	.9921147	.1426289	.9897762	48
13	.1082885	.9941195	.1256218	.9920782	.1429168	.9897347	47
14	.1085777	.9940880	.1259104	.9920416	.1432047	.9896931	46
15	.1088669	.9940563	.1261990	.9920049	.1434926	.9896514	45
16	.1091560	.9940246	.1264875	.9919682	.1437805	.9896096	44
17	.1094452	.9939928	.1267761	.9919314	.1440684	.9895677	43
18	.1097343	.9939610	.1270646	.9918944	.1443562	.9895258	42
19	.1100234	.9939290	.1273531	.9918574	.1446440	.9894838	41
20	0.1103126	0.9938969	0.1276416	0.9918204	0.1449319	0.9894416	40
21	.1106017	.9938648	.1279302	.9917832	.1452197	.9893994	39
22	.1108908	.9938326	.1282186	.9917459	.1455075	.9893572	38
23	.1111799	.9938003	.1285071	.9917086	.1457953	.9893148	37
24	.1114689	.9937679	.1287956	.9916712	.1460830	.9892723	36
25	1117580	.9937355	.1290841	.9916337	.1463708	.9892298	35
26	.1120471	.9937029	.1293725	.9915961	.1466585	.9891872	34
27	.1123361	.9936703	.1296609	.9915584	.1469463	.9891445	33
28	.1126252	.9936375	.1299494	.9915206	.1472340	.9891017	32
29	.1129142	.9936047	.1302378	.9914828	.1475217	.9890588	31
30	0.1132032	0.9935719	0.1305262	0.9914449	0.1478094	0.9890159	30
31	.1134922	.9935389	.1308146	.9914069	.1480971	.9889728	29
32	.1137812	.9935058	.1311030	.9913688	.1483848	.9889297	28
33	.1140702	.9934727	.1313913	.9913306	.1486724	.9888865	27
34	.1143592	.9934395	.1316797	.9912923	.1489601	.9888432	26
35	.1146482	.9934062	.1319681	.9912540	.1492477	.9887998	25
36	.1149372	.9933728	.1322564	.9912155	.1495353	.9887564	24
37	.1152261	.9933393	.1325447	.9911770	.1498230	.9887128	23
38	1155151	.9933057	.1328330	.9911384	.1501106	.9886692	22
39	.1158040	.9932721	.1331213	.9910997	.1503981	.9886255	21
40	0.1160929	0.9932384	0.1334096	0.9910610	0.1506857	0.9885817	20
41	.1163818	.9932045	.1336979	.9910221	.1509733	.9885378	19
42	.1166707	.9931706	.1339862	.9909832	.1512608	.9884939	18
43	.1169596	.9931367	.1342744	.9909442	.1515484	9884498	17
44	.1172485	.9931026	.1345627	.9909051	.1518359	.9884057	16
45	.1175374	.9930685	.1348509	.9908659	.1521234	.9883615	15
46	.1178263	.9930342	.1351392	.9908266	.1524109	.9883172	14
47	.1181151	.9929999	.1354274	.9907873	.1526984	.9882728	13
48	1184040	.9929655	.1357156	.9907478	.1529858	.9882284	12
49	.1186928	.9929310	.1360038	.9907083	.1532733	9881838	11
50	0.1189816	0.9928965	0.1362919	0.9906687	0.1535607	0.9881392	10
51	.1192704	.9928618	.1365801	.9906290	.1538482	.9880945	9
52	.1195593	.9928271	.1368683	.9905893	.1541356	.9880497	8
53	.1198481	9927922	.1371564	.9905494	.1544230	.9880048	7
54	.1201368	.9927573	.1374445	.9905095	.1547104	.9879599	6
55	.1204256	.9927224	.1377327	.9904694	.1549978	.9879148	5
56	.1207144	.9926873	.1380208	.9904293	.1552851	.9878697	4
57	.1210031	.9926521	.1383089	.9903891	.1555725	.9878245	3
58	.1212919	.9926169	.1385970	.9903489	.1558598	.9877792	2
59	.1215806	.9925816	.1388850	.9903085	.1561472	.9877338	1
60	0 1218693	0.9925462	0.1391731	0 9902681	0.1564345	0 9876883	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	83°		82°		81°		

Natural Sines and Cosines

′	9°		10°		11°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.1564345	0.9876883	0.1736482	0.9848078	0.1908090	0.9816272	60
1	.1567218	.9876428	.1739346	.9847572	.1910945	.9815716	59
2	.1570091	.9875972	.1742211	.9847066	.1913801	.9815160	58
3	.1572963	.9875514	.1745075	.9846558	.1916656	.9814603	57
4	.1575836	.9875057	.1747939	.9846050	.1919510	.9814045	56
5	.1578708	.9874598	.1750803	.9845542	.1922365	.9813486	55
6	.1581581	.9874138	.1753667	.9845032	.1925220	.9812927	54
7	.1584453	.9873678	.1756531	.9844521	.1928074	.9812366	53
8	.1587325	.9873216	.1759395	.9844010	.1930928	.9811805	52
9	.1590197	.9872754	.1762258	.9843498	.1933782	.9811243	51
10	0.1593069	0.9872291	0.1765121	0.9842985	0.1936636	0.9810680	50
11	.1595940	.9871827	.1767984	.9842471	.1939490	.9810116	49
12	.1598812	.9871363	.1770847	.9841956	.1942344	.9809552	48
13	.1601683	.9870897	.1773710	.9841441	.1945197	.9808986	47
14	.1604555	.9870431	.1776573	.9840924	.1948050	.9808420	46
15	.1607426	.9869964	.1779435	.9840407	.1950903	.9807853	45
16	.1610297	.9869496	.1782298	.9839889	.1953756	.9807285	44
17	.1613167	.9869027	.1785160	.9839370	.1956609	.9806716	43
18	.1616038	.9868557	.1788022	.9838850	.1959461	.9806147	42
19	.1618909	.9868087	.1790884	.9838330	.1962314	.9805576	41
20	0.1621779	0.9867615	0.1793746	0.9837808	0.1965166	0.9805005	40
21	.1624650	.9867143	.1796607	.9837286	.1968018	.9804433	39
22	.1627520	.9866670	.1799469	.9836763	.1970870	.9803860	38
23	.1630390	.9866196	.1802330	.9836239	.1973722	.9803286	37
24	.1633260	.9865722	.1805191	.9835715	.1976573	.9802712	36
25	.1636129	.9865246	.1808052	.9835189	.1979425	.9802136	35
26	.1638999	.9864770	.1810913	.9834663	.1982276	.9801560	34
27	.1641868	.9864293	.1813774	.9834136	.1985127	.9800983	33
28	.1644738	.9863815	.1816635	.9833608	.1987978	.9800405	32
29	.1647607	.9863336	.1819495	.9833079	.1990829	.9799827	31
30	0.1650476	0.9862856	0.1822355	0.9832549	0.1993679	0.9799247	30
31	.1653345	.9862375	.1825215	.9832019	.1996530	.9798667	29
32	.1656214	.9861894	.1828075	.9831487	.1999380	.9798086	28
33	.1659082	.9861412	.1830935	.9830955	.2002230	.9797504	27
34	.1661951	.9860929	.1833795	.9830422	.2005080	.9796921	26
35	.1664819	.9860445	.1836654	.9829888	.2007930	.9796337	25
36	.1667687	.9859960	.1839514	.9829353	.2010779	.9795752	24
37	.1670556	.9859475	.1842373	.9828818	.2013629	.9795167	23
38	.1673423	.9858988	.1845232	.9828282	.2016478	.9794581	22
39	.1676291	.9858501	.1848091	.9827744	.2019327	.9793994	21
40	0.1679159	0.9858013	0.1850949	0.9827206	0.2022176	0.9793406	20
41	.1682026	.9857524	.1853808	.9826668	.2025024	.9792818	19
42	.1684894	.9857035	.1856666	.9826128	.2027873	.9792228	18
43	.1687761	.9856544	.1859524	.9825587	.2030721	.9791638	17
44	.1690628	.9856053	.1862382	.9825046	.2033569	.9791047	16
45	.1693495	.9855561	.1865240	.9824504	.2036418	.9790455	15
46	.1696362	.9855068	.1868098	.9823961	.2039265	.9789862	14
47	.1699228	.9854574	.1870956	.9823417	.2042113	.9789268	13
48	.1702095	.9854079	.1873813	.9822873	.2044961	.9788674	12
49	.1704961	.9853583	.1876670	.9822327	.2047808	.9788079	11
50	0.1707828	0.9853087	0.1879528	0.9821781	0.2050655	0.9787483	10
51	.1710694	.9852590	.1882385	.9821234	.2053502	.9786886	9
52	.1713560	.9852092	.1885241	.9820686	.2056349	.9786288	8
53	.1716425	.9851593	.1888098	.9820137	.2059195	.9785689	7
54	.1719291	.9851093	.1890954	.9819587	.2062042	.9785090	6
55	.1722156	.9850593	.1893811	.9819037	.2064888	.9784490	5
56	.1725022	.9850091	.1896667	.9818485	.2067734	.9783889	4
57	.1727887	.9849589	.1899523	.9817933	.2070580	.9783287	3
58	.1730752	.9849086	.1902379	.9817380	.2073426	.9782684	2
59	.1733617	.9848582	.1905234	.9816826	.2076272	.9782080	1
60	0.1736482	0.9848078	0.1908090	0.9816272	0.2079117	0.9781476	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		

′	12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.2079117	0.9781476	0.2249511	0.9743701	0.2419219	0.9702957	60
1	.2081962	.9780871	.2252345	.9743046	.2422041	.9702253	59
2	.2084807	.9780265	.2255179	.9742390	.2424863	.9701548	58
3	.2087652	.9779658	.2258013	.9741734	.2427685	.9700842	57
4	.2090497	.9779050	.2260846	.9741077	.2430507	.9700136	56
5	.2093341	.9778442	.2263680	.9740419	.2433329	.9699428	55
6	.2096186	.9777832	.2266513	.9739760	.2436150	.9698720	54
7	.2099030	.9777222	.2269346	.9739100	.2438971	.9698011	53
8	.2101874	.9776611	.2272179	.9738439	.2441792	.9697301	52
9	.2104718	.9775999	.2275012	.9737778	.2444613	.9696591	51
10	0.2107561	0.9775387	0.2277844	0.9737116	0.2447433	0.9695879	50
11	.2110405	.9774773	.2280677	.9736453	.2450254	.9695167	49
12	2113248	.9774159	.2283509	.9735789	.2453074	.9694453	48
13	.2116091	.9773544	.2286341	.9735124	.2455894	.9693740	47
14	.2118934	.9772928	.2289172	.9734459	.2458713	.9693025	46
15	.2121777	.9772311	.2292004	.9733793	.2461533	.9692309	45
16	.2124619	.9771693	.2294835	.9733125	.2464352	.9691593	44
17	.2127462	.9771075	.2297666	.9732458	.2467171	.9690875	43
18	.2130304	.9770456	.2300497	.9731789	.2469990	.9690157	42
19	.2133146	.9769836	.2303328	.9731119	.2472809	.9689438	41
20	0.2135988	0.9769215	0.2306159	0.9730449	0.2475627	0.9688719	40
21	.2138829	.9768593	.2308989	.9729777	.2478445	.9687998	39
22	.2141671	.9767970	.2311819	.9729105	.2481263	.9687277	38
23	.2144512	.9767347	.2314649	.9728432	.2484081	.9686555	37
24	.2147353	.9766723	.2317479	.9727759	.2486899	.9685832	36
25	.2150194	.9766098	.2320309	.9727084	.2489716	.9685108	35
26	.2153035	.9765472	.2323138	.9726409	.2492533	.9684383	34
27	.2155876	.9764845	.2325967	.9725733	.2495350	.9683658	33
28	.2158716	.9764218	.2328796	.9725056	.2498167	.9682931	32
29	.2161556	.9763589	.2331625	.9724378	.2500984	.9682204	31
30	0.2164396	0.9762960	0.2334454	0.9723699	0.2503800	0.9681476	30
31	.2167236	.9762330	.2337282	.9723020	.2506616	.9680748	29
32	.2170076	.9761699	.2340110	.9722339	.2509432	.9680018	28
33	.2172915	.9761068	.2342938	.9721658	.2512248	.9679288	27
34	.2175754	.9760435	.2345766	.9720976	.2515063	.9678557	26
35	.2178593	.9759802	.2348594	.9720294	.2517879	.9677825	25
36	.2181432	.9759168	.2351421	.9719610	.2520694	.9677092	24
37	.2184271	.9758533	.2354248	.9718926	.2523508	.9676358	23
38	.2187110	.9757897	.2357075	.9718240	.2526323	.9675624	22
39	.2189948	.9757260	.2359902	.9717554	.2529137	.9674888	21
40	0.2192786	0.9756623	0.2362729	0.9716867	0.2531952	0.9674152	20
41	.2195624	.9755985	.2365555	.9716180	.2534766	.9673415	19
42	.2198462	.9755345	.2368381	.9715491	.2537579	.9672678	18
43	.2201300	.9754706	.2371207	.9714802	.2540393	.9671939	17
44	.2204137	.9754065	.2374033	.9714112	.2543206	.9671200	16
45	.2206974	.9753423	.2376859	.9713421	.2546019	.9670459	15
46	.2209811	.9752781	.2379684	.9712729	.2548832	.9669718	14
47	.2212648	.9752138	.2382510	.9712036	.2551645	.9668977	13
48	.2215485	.9751494	.2385335	.9711343	.2554458	.9668234	12
49	.2218321	.9750849	.2388159	.9710649	.2557270	.9667490	11
50	0.2221158	0.9750203	0.2390984	0.9709953	0.2560082	0.9666746	10
51	.2223994	.9749556	.2393808	.9709258	.2562894	.9666001	9
52	.2226830	.9748909	.2396633	.9708561	.2565705	.9665255	8
53	.2229666	.9748261	.2399457	.9707863	.2568517	.9664508	7
54	.2232501	.9747612	.2402280	.9707165	.2571328	.9663761	6
55	.2235337	.9746962	.2405104	.9706466	.2574139	.9663012	5
56	.2238172	.9746311	.2407927	.9705766	.2576950	.9662263	4
57	.2241007	.9745660	.2410751	.9705065	.2579760	.9661513	3
58	.2243842	.9745008	.2413574	.9704363	.2582570	.9660762	2
59	.2246676	.9744355	.2416396	.9703661	.2585381	.9660011	1
60	0.2249511	0.9743701	0.2419219	0.9702957	0.2588190	0.9659258	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	77°		76°		75°		

′	15°		16°		17°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.2588190	0.9659258	0.2756374	0.9612617	0.2923717	0.9563048	60
1	.2591000	.9658505	.2759170	.9611815	.2926499	.9562197	59
2	.2593810	.9657751	.2761965	.9611012	.2929280	.9561345	58
3	.2596619	.9656996	.2764761	.9610208	.2932061	.9560492	57
4	.2599428	.9656240	.2767556	.9609403	.2934842	.9559639	56
5	.2602237	.9655484	.2770352	.9608598	.2937623	.9558785	55
6	.2605045	.9654728	.2773147	.9607792	.2940403	.9557930	54
7	.2607853	.9653968	.2775941	.9606984	.2943183	.9557074	53
8	.2610662	.9653209	.2778736	.9606177	.2945963	.9556218	52
9	.2613469	.9652449	.2781530	.9605368	.2948743	.9555361	51
10	0.2616277	0.9651689	0 2784324	0.9604558	0.2951522	0.9554502	50
11	.2619085	.9650927	.2787118	.9603748	.2954302	.9553643	49
12	.2621892	.9650165	2789911	.9602937	.2957081	.9552784	48
13	.2624699	.9649402	.2792704	.9602125	.2959859	.9551923	47
14	.2627506	.9648638	.2795497	.9601312	.2962638	.9551062	46
15	.2630312	.9647873	.2798290	.9600499	.2965416	.9550199	45
16	.2633118	.9647108	.2801083	.9599684	.2968194	.9549336	44
17	.2635925	.9646341	.2803875	9598869	.2970971	.9548473	43
18	.2638730	.9645574	.2806667	.9598053	.2973749	.9547608	42
19	.2641536	.9644806	.2809459	.9597236	.2976526	.9546743	41
20	0.2644342	0.9644037	0.2812251	0.9596418	0.2979303	0.9545876	40
21	.2647147	.9643268	.2815042	.9595600	.2982079	.9545009	39
22	.2649952	.9642497	.2817833	.9594781	.2984856	.9544141	38
23	.2652757	.9641726	.2820624	.9593961	.2987632	.9543273	37
24	.2655561	.9640954	.2823415	.9593140	.2990408	.9542403	36
25	.2658366	.9640181	.2826205	.9592318	.2993184	.9541533	35
26	.2661170	.9639407	.2828995	.9591496	.2995959	.9540662	34
27	.2663973	.9638633	.2831785	.9590672	.2998734	.9539790	33
28	.2666777	.9637858	.2834575	.9589848	.3001509	.9538917	32
29	.2669581	.9637081	.2837364	.9589023	.3004284	.9538044	31
30	0.2672384	0.9636305	0.2840153	0.9588197	0.3007058	0.9537170	30
31	.2675187	.9635527	.2842942	.9587371	.3009832	.9536294	29
32	.2677989	.9634748	.2845731	.9586543	.3012606	.9535418	28
33	.2680792	.9633969	.2848520	.9585715	.3015380	.9534542	27
34	.2683594	.9633189	.2851308	.9584886	.3018153	.9533664	26
35	.2686396	.9632408	.2854096	.9584056	.3020926	.9532786	25
36	.2689198	.9631626	.2856884	.9583226	.3023699	.9531907	24
37	.2692000	.9630843	.2859671	.9582394	.3026471	.9531027	23
38	.2694801	.9630060	.2862458	.9581562	.3029244	.9530146	22
39	.2697602	.9629275	.2865246	.9580729	.3032016	.9529264	21
40	0.2700403	0.9628490	0.2868032	0.9579895	0.3034788	0.9528382	20
41	.2703204	.9627704	.2870819	.9579060	.3037559	.9527499	19
42	.2706004	.9626917	.2873605	.9578225	.3040331	.9526615	18
43	.2708805	.9626130	.2876391	.9577389	.3043102	.9525730	17
44	.2711605	.9625342	.2879177	.9576552	.3045872	.9524844	16
45	.2714404	.9624552	.2881963	.9575714	.3048643	.9523958	15
46	.2717204	.9623762	.2884748	.9574875	.3051413	.9523071	14
47	2720003	.9622972	.2887533	.9574035	.3054183	.9522183	13
48	.2722802	.9622180	.2890318	.9573195	.3056953	.9521294	12
49	.2725601	.9621387	.2893103	.9572354	.3059723	.9520404	11
50	0 2728400	0.9620594	0.2895887	0.9571512	0.3062492	0.9519514	10
51	.2731198	.9619800	.2898671	.9570669	.3065261	.9518623	9
52	.2733997	.9619005	.2901455	.9569825	.3068030	.9517731	8
53	.2736794	.9618210	.2904239	.9568981	.3070798	.9516838	7
54	.2739592	.9617413	.2907022	.9568136	.3073566	.9515944	6
55	.2742390	.9616616	.2909805	.9567290	.3076334	.9515050	5
56	.2745187	.9615818	.2912588	.9566443	.3079102	.9514154	4
57	.2747984	.9615019	.2915371	.9565595	.3081869	.9513258	3
58	.2750781	.9614219	.2918153	.9564747	.3084636	.9512361	2
59	.2753577	.9613418	.2920935	.9563898	.3087403	.9511464	1
60	0.2756374	0.9612617	0.2923717	0.9563048	0.3090170	0.9510565	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	74°		73°		72°		′

′	18°		19°		20°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.3090170	0.9510565	0.3255682	0.9455186	0.3420201	0.9396926	60
1	.3092936	.9509666	.3258432	.9454238	.3422935	.9395931	59
2	.3095702	.9508766	.3261182	.9453290	.3425668	.9394935	58
3	.3098468	.9507865	.3263932	.9452341	.3428400	.9393938	57
4	.3101234	.9506963	.3266681	.9451391	.3431133	.9392940	56
5	.3103999	.9506061	.3269430	.9450441	.3433865	.9391942	55
6	.3106764	.9505157	.3272179	.9449489	.3436597	.9390943	54
7	.3109529	.9504253	.3274928	.9448537	.3439329	.9389942	53
8	.3112294	.9503348	.3277676	.9447584	.3442060	.9388942	52
9	.3115058	.9502443	.3280424	.9446630	.3444791	.9387940	51
10	0.3117822	0.9501536	0.3283172	0.9445675	0.3447521	0.9386938	50
11	.3120586	.9500629	.3285919	.9444720	.3450252	.9385934	49
12	.3123349	.9499721	.3288666	.9443764	.3452982	.9384930	48
13	.3126112	.9498812	.3291413	.9442807	.3455712	.9383925	47
14	.3128875	.9497902	.3294160	.9441849	.3458441	.9382920	46
15	.3131638	.9496991	.3296906	.9440890	.3461171	.9381913	45
16	.3134400	.9496080	.3299653	.9439931	.3463900	.9380906	44
17	.3137163	.9495168	.3302398	.9438971	.3466628	.9379898	43
18	.3139925	.9494255	.3305144	.9438010	.3469357	.9378889	42
19	.3142686	.9493341	.3307889	.9437048	.3472085	.9377880	41
20	0.3145448	0.9492426	0.3310634	0.9436085	0.3474812	0.9376869	40
21	.3148209	.9491511	.3313379	.9435122	.3477540	.9375858	39
22	.3150969	.9490595	.3316123	.9434157	.3480267	.9374846	38
23	.3153730	.9489678	.3318867	.9433192	.3482994	.9373833	37
24	.3156490	.9488760	.3321611	.9432227	.3485720	.9372820	36
25	.3159250	.9487842	.3324355	.9431260	.3488447	.9371806	35
26	.3162010	.9486922	.3327098	.9430293	.3491173	.9370790	34
27	.3164770	.9486002	.3329841	.9429324	.3493898	.9369774	33
28	.3167529	.9485081	.3332584	.9428355	.3496624	.9368758	32
29	.3170288	.9484159	.3335326	.9427386	.3499349	.9367740	31
30	0.3173047	0.9483237	0.3338069	0.9426415	0.3502074	0.9366722	30
31	.3175805	.9482313	.3340810	.9425444	.3504798	.9365703	29
32	.3178563	.9481389	.3343552	.9424471	.3507523	.9364683	28
33	.3181321	.9480464	.3346293	.9423498	.3510246	.9363662	27
34	.3184079	.9479538	.3349034	.9422525	.3512970	.9362641	26
35	.3186836	.9478612	.3351775	.9421550	.3515693	.9361618	25
36	.3189593	.9477684	.3354516	.9420575	.3518416	.9360595	24
37	.3192350	.9476756	.3357256	.9419598	.3521139	.9359571	23
38	.3195106	.9475827	.3359996	.9418621	.3523862	.9358547	22
39	.3197863	.9474897	.3362735	.9417644	.3526584	.9357521	21
40	0.3200619	0.9473966	0.3365475	0.9416665	0.3529306	0.9356495	20
41	.3203374	.9473035	.3368214	.9415686	.3532027	.9355468	19
42	.3206130	.9472103	.3370953	.9414705	.3534748	.9354440	18
43	.3208885	.9471170	.3373691	.9413724	.3537469	.9353412	17
44	.3211640	.9470236	.3376429	.9412743	.3540190	.9352382	16
45	.3214395	.9469301	.3379167	.9411760	.3542910	.9351352	15
46	.3217149	.9468366	.3381905	.9410777	.3545630	.9350321	14
47	.3219903	.9467430	.3384642	.9409793	.3548350	.9349289	13
48	.3222657	.9466493	.3387379	.9408808	.3551070	.9348257	12
49	.3225411	.9465555	.3390116	.9407822	.3553789	.9347223	11
50	0.3228164	0.9464616	0.3392852	0.9406835	0.3556508	0.9346189	10
51	.3230917	.9463677	.3395589	.9405848	.3559226	.9345154	9
52	.3233670	.9462736	.3398325	.9404860	.3561944	.9344119	8
53	.3236422	.9461795	.3401060	.9403871	.3564662	.9343082	7
54	.3239174	.9460854	.3403796	.9402881	.3567380	.9342045	6
55	.3241926	.9459911	.3406531	.9401891	.3570097	.9341007	5
56	.3244678	.9458968	.3409265	.9400899	.3572814	.9339968	4
57	.3247429	.9458023	.3412000	.9399907	.3575531	.9338928	3
58	.3250180	.9457078	.3414734	.9398914	.3578248	.9337888	2
59	.3252931	.9456132	.3417468	.9397921	.3580964	.9336846	1
60	0.3255682	0.9455186	0.3420201	0.9396926	0.3583679	0.9335804	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	71°		70°		69°		

Natural Sines and Cosines

′	21°		22°		23°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.3583679	0.9335804	0.3746066	0.9271839	0.3907311	0.9205049	60
1	.3586395	.9334761	.3748763	.9270748	.3909980	.9203912	59
2	.3589110	.9333718	.3751459	.9269658	.3912066	.9202774	58
3	.3591825	.9332673	.3754156	.9268566	.3915343	.9201635	57
4	.3594540	.9331628	.3756852	.9267474	.3918019	.9200496	56
5	.3597254	.9330582	.3759547	.9266380	.3920695	.9199356	55
6	.3599968	.9329535	.3762243	.9265286	.3923371	.9198215	54
7	.3602682	.9328488	.3764938	.9264192	.3926047	.9197073	53
8	.3605395	.9327439	.3767632	.9263096	.3928722	.9195931	52
9	.3008108	.9326390	.3770327	.9262000	.3931397	.9194788	51
10	0.3610821	0 9325340	0.3773021	0.9260902	0.3934071	0.9193644	50
11	.3613534	.9324290	.3775714	.9259805	.3936745	.9192499	49
12	.3616246	.9323238	.3778408	.9258706	.3939419	.9191353	48
13	.3618958	.9322186	.3781101	.9257606	.3942093	.9190207	47
14	.3621669	.9321133	.3783794	.9256506	.3944766	.9189060	46
15	.3624380	.9320079	.3786486	.9255405	.3947439	.9187912	45
16	.3627091	.9319024	3789178	.9254303	.3950111	.9186763	44
17	.3629802	.9317969	.3791870	.9253201	.3952783	9185614	43
18	.3632512	.9316912	.3794562	9252097	.3955455	.9184464	42
19	.3635222	.9315855	.3797253	9250993	.3958127	.9183313	41
20	0.3637932	0.9314797	0.3799944	0.9249888	0.3960798	0.9182161	40
21	.3640641	.9313739	.3802634	.9248782	.3963468	.9181009	39
22	.3643351	.9312679	.3805324	.9247676	.3966139	.9179855	38
23	.3646059	.9311619	.3808014	.9246568	.3968809	.9178701	37
24	.3648768	.9310558	.3810704	.9245460	.3971479	.9177546	36
25	.3651476	.9309496	.3813393	.9244351	.3974148	.9176391	35
26	.3654184	.9308434	.3816082	.9243242	.3976818	.9175234	34
27	.3656891	.9307370	.3818770	.9242131	.3979486	.9174077	33
28	.3659599	.9306303	.3821459	.9241020	.3982155	.9172919	32
29	3662306	.9305241	.3824147	.9239908	.3984823	.9171760	31
30	0.3665012	0.9304176	0.3826834	0.9238795	0.3987491	0.9170601	30
31	.3667719	.9303109	.3829522	.9237682	.3990158	.9169440	29
32	.3670425	.9302042	.3832209	.9236567	.3992825	.9168279	28
33	.3673130	.9300974	.3834895	9235452	.3995492	.9167118	27
34	.3675836	.9299905	.3837582	.9234336	.3998158	.9165955	26
35	.3678541	.9298835	.3840268	.9233220	.4000825	.9164791	25
36	.3681246	.9297765	.3842953	.9232102	.4003490	.9163627	24
37	.3683950	.9296694	.3845639	.9230984	.4006156	.9162462	23
38	.3686654	.9295622	.3848324	9229865	.4008821	.9161297	22
39	.3689358	9294549	3851008	.9228745	.4011486	.9160130	21
40	0.3692061	0 9293475	0.3853693	0.9227624	0.4014150	0.9158963	20
41	.3694765	.9292401	.3856377	.9226503	.4016814	.9157795	19
42	.3697468	.9291326	.3859060	.9225381	.4019478	.9156626	18
43	.3700170	.9290250	.3861744	.9224258	.4022141	.9155456	17
44	.3702872	.9289173	.3864427	.9223134	.4024804	.9154286	16
45	.3705574	.9288096	.3867110	.9222010	.4027467	.9153115	15
46	.3708276	.9287017	.3869792	.9220884	.4030129	.9151943	14
47	.3710977	.9285938	.3872474	.9219758	.4032791	.9150770	13
48	.3713678	.9284858	.3875156	.9218632	.4035453	.9149597	12
49	.3716379	.9283778	.3877837	9217504	.4038114	.9148422	11
50	0.3719079	0.9282696	0.3880518	0.9216375	0.4040775	0.9147247	10
51	.3721780	.9281614	.3883199	.9215246	.4043436	.9146072	9
52	.3724479	.9280531	.3885880	.9214116	.4046096	.9144895	8
53	.3727179	.9279447	.3888560	.9212986	.4048756	.9143718	7
54	.3729878	.9278363	.3891240	.9211854	.4051416	.9142540	6
55	.3732577	.9277277	.3893919	.9210722	.4054075	.9141361	5
56	.3735275	.9276191	.3896598	.9209589	.4056734	.9140181	4
57	.3737973	.9275104	.3899277	.9208455	.4059393	.9139001	3
58	.3740671	.9274016	.3901955	.9207320	.4062051	.9137819	2
59	.3743369	.9272928	.3904633	.9206185	.4064709	.9136637	1
60	0.3746066	0 9271839	0 3907311	0.9205049	0.4067366	0.9135455	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	68°		67°		66°		

′	24°		25°		26°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0 4067366	0 9135455	0 4226183	0 9063078	0 4383711	0 8987940	60
1	4070024	9134271	4228819	9061848	4386326	8986665	59
2	4072681	9133087	4231455	9060618	4388940	8985389	58
3	4075337	9131902	4234090	9059386	4391553	8984112	57
4	4077993	9130716	4236725	9058154	4394166	8982834	56
5	4080649	9129529	4239360	9056922	4396779	8981555	55
6	4083305	9128342	4241994	9055688	4399392	8980276	54
7	4085960	9127154	4244628	9054454	4402004	8978996	53
8	4088615	9125965	4247262	9053219	4404615	8977715	52
9	4091269	9124775	4249895	9051983	4407227	8976433	51
10	0 4093923	0 9123584	0 4252528	0 9050746	0 4409838	0 8975151	50
11	4096577	9122393	4255161	9049509	4412448	8973868	49
12	4099230	9121201	4257793	9048271	4415059	8972584	48
13	4101883	9120008	4260425	9047032	4417668	8971290	47
14	4104536	9118815	4263056	9045792	4420278	8970014	46
15	4107189	9117620	4265687	9044551	4422887	8968727	45
16	4109841	9116425	4268318	9043310	4425496	8967440	44
17	4112492	9115229	4270949	9042068	4428104	8966153	43
18	4115144	9114033	4273579	9040825	4430712	8964864	42
19	4117795	9112835	4276208	9039582	4433319	8963575	41
20	0 4120445	0 9111637	0 4278838	0 9038338	0 4435927	0 8962285	40
21	4123096	9110438	4281467	9037093	4438534	8960994	39
22	4125745	9109238	4284095	9035847	4441140	8959703	38
23	4128395	9108038	4286723	9034600	4443746	8958411	37
24	4131044	9106837	4289351	9033353	4446352	8957118	36
25	4133693	9105635	4291979	9032105	4448957	8955824	35
26	4136342	9104432	4294606	9030856	4451562	8954529	34
27	4138990	9103228	4297233	9029606	4454167	8953234	33
28	4141638	9102024	4299859	9028356	4456771	8951938	32
29	4144285	9100819	4302485	9027105	4459375	8950641	31
30	0 4146932	0 9099613	0 4305111	0 9025853	0 4461978	0 8949344	30
31	4149579	9098406	4307736	9024600	4464581	8948045	29
32	4152226	9097199	4310361	9023347	4467184	8946746	28
33	4154872	9095990	4312986	9022092	4469786	8945446	27
34	4157517	9094781	4315610	9020838	4472388	8944146	26
35	4160163	9093572	4318234	9019582	4474990	8942844	25
36	4162808	9092361	4320857	9018325	4477591	8941542	24
37	4165453	9091150	4323481	9017068	4480192	8940240	23
38	4168097	9089938	4326103	9015810	4482792	8938936	22
39	4170741	9088725	4328726	9014551	4485392	8937632	21
40	0 4173385	0 9087511	0 4331348	0 9013292	0 4487992	0 8936326	20
41	4176028	9086297	4333970	9012031	4490591	8935021	19
42	4178671	9085082	4336591	9010770	4493190	8933714	18
43	4181313	9083866	4339212	9009508	4495789	8932406	17
44	4183956	9082649	4341832	9008246	4498387	8931098	16
45	4186597	9081432	4344453	9006982	4500984	8929789	15
46	4189239	9080214	4347072	9005718	4503582	8928480	14
47	4191880	9078995	4349692	9004453	4506179	8927169	13
48	4194521	9077775	4352311	9003188	4508775	8925858	12
49	4197161	9076554	4354930	9001921	4511372	8924546	11
50	0 4199801	0 9075333	0 4357548	0 9000654	0 4513967	0 8923234	10
51	4202441	9074111	4360166	8999386	4516563	8921920	9
52	4205080	9072888	4362784	8998117	4519158	8920606	8
53	4207719	9071665	4365401	8996848	4521753	8919291	7
54	4210358	9070440	4368018	8995578	4524347	8917975	6
55	4212996	9069215	4370634	8994307	4526941	8916659	5
56	4215634	9067989	4373251	8993036	4529535	8915342	4
57	4218272	9066762	4375866	8991763	4532128	8914024	3
58	4220909	9065535	4378482	8990489	4534721	8912705	2
59	4223546	9064307	4381097	8989215	4537313	8911385	1
60	0 4226183	0 9063078	0 4383711	0 8987940	0 4539905	0 8910065	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	65°		64°		63°		′

′	27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.4539905	0.8910065	0.4694716	0.8829476	0.4848096	0.8746197	60
1	.4542497	.8908744	.4697284	.8828110	.4850640	.8744786	59
2	.4545088	.8907423	.4699852	.8826743	.4853184	.8743375	58
3	.4547679	.8906100	.4702419	.8825376	.4855727	.8741963	57
4	.4550269	.8904777	.4704986	.8824007	.4858270	.8740550	56
5	.4552839	.8903453	.4707553	.8822638	.4860812	.8739137	55
6	.4555449	.8902128	.4710119	.8821269	.4863354	.8737722	54
7	.4558038	.8900803	.4712685	.8819898	.4865895	.8736307	53
8	.4560627	.8899476	.4715250	.8818527	.4868436	.8734891	52
9	.4563216	.8898149	.4717815	.8817155	.4870977	.8733475	51
10	0.4565804	0.8896822	0.4720380	0.8815782	0.4873517	0.8732058	50
11	.4568392	.8895493	.4722944	.8814409	.4876057	.8730640	49
12	.4570979	.8894164	.4725508	.8813035	.4878597	.8729221	48
13	.4573566	.8892834	.4728071	.8811660	.4881136	.8727801	47
14	.4576153	.8891503	.4730634	.8810284	.4883674	.8726381	46
15	.4578739	.8890171	.4733197	.8808907	.4886212	.8724960	45
16	.4581325	.8888839	.4735759	.8807530	.4888750	.8723538	44
17	.4583910	.8887506	.4738321	.8806152	.4891288	.8722116	43
18	.4586496	.8886172	.4740882	.8804774	.4893825	.8720693	42
19	.4589080	.8884838	.4743443	.8803394	.4896361	.8719269	41
20	0.4591665	0.8883503	0.4746004	0.8802014	0.4898897	0.8717844	40
21	.4594248	.8882166	.4748564	.8800633	.4901433	.8716419	39
22	.4596832	.8880830	.4751124	.8799251	.4903968	.8714993	38
23	.4599415	.8879492	.4753683	.8797869	.4906503	.8713566	37
24	.4601998	.8878154	.4756242	.8796486	.4909038	.8712138	36
25	.4604580	.8876815	.4758801	.8795102	.4911572	.8710710	35
26	.4607162	.8875475	.4761359	.8793717	.4914105	.8709281	34
27	.4609744	.8874134	.4763917	.8792332	.4916638	.8707851	33
28	.4612325	.8872793	.4766474	.8790946	.4919171	.8706420	32
29	.4614906	.8871451	.4769031	.8789559	.4921704	.8704989	31
30	0.4617486	0.8870108	0.4771588	0.8788171	0.4924236	0.8703557	30
31	.4620066	.8868765	.4774144	.8786783	.4926767	.8702124	29
32	.4622646	.8867420	.4776700	.8785394	.4929298	.8700691	28
33	.4625225	.8866075	.4779255	.8784004	.4931829	.8699256	27
34	.4627804	.8864730	.4781810	.8782613	.4934359	.8697821	26
35	.4630382	.8863383	.4784364	.8781222	.4936889	.8696386	25
36	.4632960	.8862036	.4786919	.8779830	.4939419	.8694949	24
37	.4635538	.8860688	.4789472	.8778437	.4941948	.8693512	23
38	.4638115	.8859339	.4792026	.8777043	.4944476	.8692074	22
39	.4640692	.8857989	.4794579	.8775649	.4947005	.8690636	21
40	0.4643269	0.8856639	0.4797131	0.8774254	0.4949532	0.8689196	20
41	.4645845	.8855288	.4799683	.8772858	.4952060	.8687756	19
42	.4648420	.8853936	.4802235	.8771462	.4954587	.8686315	18
43	.4650996	.8852584	.4804786	.8770064	.4957113	.8684874	17
44	.4653571	.8851230	.4807337	.8768666	.4959639	.8683431	16
45	.4656145	.8849876	.4809888	.8767268	.4962165	.8681988	15
46	.4658719	.8848522	.4812438	.8765868	.4964690	.8680544	14
47	.4661293	.8847166	.4814987	.8764468	.4967215	.8679100	13
48	.4663866	.8845810	.4817537	.8763067	.4969740	.8677655	12
49	.4666439	.8844453	.4820086	.8761665	.4972264	.8676209	11
50	0.4669012	0.8843095	0.4822634	0.8760263	0.4974787	0.8674762	10
51	.4671584	.8841736	.4825182	.8758859	.4977310	.8673314	9
52	.4674156	.8840377	.4827730	.8757455	.4979833	.8671866	8
53	.4676727	.8839017	.4830277	.8756051	.4982355	.8670417	7
54	.4679298	.8837656	.4832824	.8754645	.4984877	.8668967	6
55	.4681869	.8836295	.4835370	.8753239	.4987399	.8667517	5
56	.4684439	.8834933	.4837916	.8751832	.4989920	.8666066	4
57	.4687009	.8833569	.4840462	.8750425	.4992441	.8664614	3
58	.4689578	.8832206	.4843007	.8749016	.4994961	.8663161	2
59	.4692147	.8830841	.4845552	.8747607	.4997481	.8661708	1
60	0.4694716	0.8829476	0.4848096	0.8746197	0.5000000	0.8660254	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	62°		61°		60°		′

′	30°		31°		32°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.5000000	0.8660254	0.5150381	0.8571673	0.5299193	0.8480481	60
1	.5002519	.8658799	.5152874	.8570174	.5301659	.8478939	59
2	.5005037	.8657344	.5155367	.8568675	.5304125	.8477397	58
3	.5007556	.8655887	.5157859	.8567175	.5306591	.8475853	57
4	.5010073	.8654430	.5160351	.8565674	.5309057	.8474309	56
5	.5012591	.8652973	.5162842	.8564173	.5311521	.8472765	55
6	.5015107	.8651514	.5165333	.8562671	.5313986	.8471219	54
7	.5017624	.8650055	.5167824	.8561168	.5316450	.8469673	53
8	.5020140	.8648595	.5170314	.8559664	.5318913	.8468126	52
9	.5022655	.8647134	.5172804	.8558160	.5321376	.8466579	51
10	0.5025170	0.8645673	0.5175293	0.8556655	0.5323839	0.8465030	50
11	.5027685	.8644211	.5177782	.8555149	.5326301	.8463481	49
12	.5030199	.8642748	.5180270	.8553643	.5328763	.8461932	48
13	.5032713	.8641284	.5182758	.8552135	.5331224	.8460381	47
14	.5035227	.8639820	.5185246	.8550627	.5333685	.8458830	46
15	.5037740	.8638355	.5187733	.8549119	.5336145	.8457278	45
16	.5040252	.8636889	.5190219	.8547609	.5338605	.8455726	44
17	.5042765	.8635423	.5192705	.8546099	.5341065	.8454172	43
18	.5045276	.8633956	.5195191	.8544588	.5343523	.8452618	42
19	.5047788	.8632488	.5197676	.8543077	.5345982	.8451064	41
20	0.5050298	0.8631019	0.5200161	0.8541564	0.5348440	0.8449508	40
21	.5052809	.8629549	.5202646	.8540051	.5350898	.8447952	39
22	.5055319	.8628079	.5205130	.8538538	.5353355	.8446395	38
23	.5057828	.8626608	.5207613	.8537023	.5355812	.8444838	37
24	.5060338	.8625137	.5210096	.8535508	.5258268	.8443279	36
25	.5062846	.8623664	.5212579	.8533992	.5360724	.8441720	35
26	.5065355	.8622191	.5215061	.8532475	.5363179	.8440161	34
27	.5067863	.8620717	.5217543	.8530958	.5365634	.8438600	33
28	.5070370	.8619243	.5220024	.8529440	.5368089	.8437039	32
29	.5072877	.8617768	.5222505	.8527921	.5370543	.8435477	31
30	0.5075384	0.8616292	0.5224986	0.8526402	0.5372996	0.8433914	30
31	.5077890	.8614815	.5227466	.8524881	.5375449	.8432351	29
32	.5080396	.8613337	.5229945	.8523360	.5377902	.8430787	28
33	.5082901	.8611859	.5232424	.8521839	.5380354	.8429222	27
34	.5085406	.8610380	.5234903	.8520316	.5382806	.8427657	26
35	.5087910	.8608901	.5237381	.8518793	.5385257	.8426091	25
36	.5090414	.8607420	.5239859	.8517269	.5387708	.8424524	24
37	.5092918	.8605939	.5242336	.8515745	.5390158	.8422956	23
38	.5095421	.8604457	.5244813	.8514219	.5392608	.8421388	22
39	.5097924	.8602975	.5247290	.8512693	.5395058	.8419818	21
40	0.5100426	0.8601491	0.5249766	0.8511167	0.5397507	0.8418249	20
41	.5102928	.8600007	.5252241	.8509639	.5399955	.8416679	19
42	.5105429	.8598523	.5254717	.8508111	.5402403	.8415108	18
43	.5107930	.8597037	.5257191	.8506582	.5404851	.8413536	17
44	.5110431	.8595551	.5259665	.8505053	.5407298	.8411965	16
45	.5112931	.8594064	.5262139	.8503522	.5409745	.8410390	15
46	.5115431	.8592576	.5264613	.8501991	.5412191	.8408816	14
47	.5117930	.8591088	.5267085	.8500459	.5414637	.8407241	13
48	.5120429	.8589599	.5269558	.8498927	.5417082	.8405666	12
49	.5122927	.8588109	.5272030	.8497394	.5419527	.8404090	11
50	0.5125425	0.8586619	0.5274502	0.8495860	0.5421971	0.8402513	10
51	.5127923	.8585127	.5276973	.8494325	.5424415	.8400936	9
52	.5130420	.8583635	.5279443	.8492790	.5426859	.8399357	8
53	.5132916	.8582143	.5281914	.8491254	.5429302	.8397778	7
54	.5135413	.8580649	.5284383	.8489717	.5431744	.8396199	6
55	.5137908	.8579155	.5286853	.8488179	.5434187	.8394618	5
56	.5140404	.8577660	.5289322	.8486641	.5436628	.8393037	4
57	.5142899	.8576164	.5291790	.8485102	.5439069	.8391455	3
58	.5145393	.8574668	.5294258	.8483562	.5441510	.8389873	2
59	.5147887	.8573171	.5296726	.8482022	.5443951	.8388290	1
60	0.5150381	0.8571673	0.5299193	0.8480481	0.5446390	0.8386706	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		

′	33°		34°		35°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.5446390	0.8386706	0.5591929	0.8290376	0.5735764	0.8191520	60
1	.5448830	.8385121	.5594340	.8288749	.5738147	.8189852	59
2	.5451269	.8383536	.5596751	.8287121	.5740529	.8188182	58
3	.5453707	.8381950	.5599162	.8285493	.5742911	.8186512	57
4	.5456145	.8380363	.5601572	.8283864	.5745292	.8184841	56
5	.5458583	.8378775	.5603981	.8282234	.5747672	.8183169	55
6	.5461020	.8377187	.5606390	.8280603	.5750053	.8181497	54
7	.5463456	.8375598	.5608798	.8278972	.5752432	.8179824	53
8	.5465892	.8374009	.5611206	.8277340	.5754811	.8178151	52
9	.5468328	.8372418	.5613614	.8275708	.5757190	.8176476	51
10	0.5470763	0.8370827	0.5616021	0.8274074	0.5759568	0.8174801	50
11	.5473198	.8369236	.5618428	.8272440	.5761946	.8173125	49
12	.5475632	.8367643	5620834	.8270806	.5764323	.8171449	48
13	.5478066	8366050	.5623239	.8269170	.5766700	.8169772	47
14	.5480499	.8364456	.5625645	.8267534	.5769076	.8168094	46
15	.5482932	.8362862	.5628049	.8265897	.5771452	.8166416	45
16	.5485365	.8361266	.5630453	.8264260	.5773827	.8164736	44
17	.5487797	.8359670	.5632857	.8262622	.5776202	.8163056	43
18	.5490228	.8358074	.5635260	.8260983	.5778576	.8161376	42
19	.5492659	.8356476	.5637663	.8259343	.5780950	.8159695	41
20	0.5495090	0.8354878	0.5640066	0.8257703	0.5783323	0.8158013	40
21	.5497520	.8353279	.5642467	.8256062	.5785696	.8156330	39
22	.5499950	.8351680	.5644869	.8254420	.5788069	.8154647	38
23	.5502379	.8350080	.5647270	.8252778	.5790440	.8152963	37
24	.5504807	.8348479	.5649670	.8251135	.5792812	.8151278	36
25	.5507236	.8346877	.5652070	.8249491	.5795183	.8149593	35
26	.5509663	.8345275	.5654469	.8247847	.5797553	.8147906	34
27	.5512091	.8343672	.5656868	.8246202	.5799923	.8146220	33
28	.5514518	.8342068	.5659267	.8244556	.5802292	.8144532	32
29	.5516944	.8340463	.5661665	.8242909	.5804661	.8142844	31
30	0.5519370	0.8338858	0.5664062	0.8241262	0.5807030	0.8141155	30
31	.5521795	.8337252	.5666459	.8239614	.5809397	.8139466	29
32	.5524220	.8335646	.5668856	.8237965	.5811765	.8137775	28
33	.5526645	.8334038	.5671252	.8236316	.5814132	.8136084	27
34	.5529069	.8332430	.5673648	.8234666	.5816498	.8134393	26
35	.5531492	.8330822	.5676043	.8233015	.5818864	.8132701	25
36	.5533915	.8329212	.5678437	.8231364	.5821230	.8131008	24
37	.5536338	.8327602	.5680832	.8229712	.5823595	.8129314	23
38	.5538760	.8325991	.5683225	.8228059	.5825959	.8127620	22
39	.5541182	.8324380	.5685619	.8226405	.5828323	.8125925	21
40	0.5543603	0.8322768	0.5688011	0.8224751	0.5830687	0.8124229	20
41	.5546024	.8321155	.5690403	.8223096	.5833050	.8122532	19
42	.5548441	.8319541	.5692795	.8221440	.5835412	.8120835	18
43	.5550864	.8317927	.5695187	.8219784	.5837774	.8119137	17
44	.5553283	.8316312	.5697577	.8218127	.5840136	.8117439	16
45	.5555702	.8314696	.5699968	.8216469	.5842497	.8115740	15
46	.5558121	.8313080	.5702357	.8214811	.5844857	.8114040	14
47	.5560539	.8311463	.5704747	.8213152	.5847217	.8112339	13
48	.5562956	.8309845	.5707136	.8211492	.5849577	.8110638	12
49	.5565373	.8308226	.5709524	.8209832	.5851936	.8108936	11
50	0.5567790	0.8306607	0.5711912	0.8208170	0.5854294	0.8107234	10
51	.5570206	.8304987	.5714299	.8206509	.5856652	.8105530	9
52	.5572621	.8303366	.5716686	.8204846	.5859010	.8103826	8
53	.5575036	.8301745	.5719073	.8203183	.5861367	.8102122	7
54	.5577451	.8300123	.5721459	.8201519	.5863724	.8100416	6
55	.5579865	.8298500	.5723844	.8199854	.5866080	.8098710	5
56	.5582279	.8296877	.5726229	.8198189	.5868435	.8097004	4
57	.5584692	.8295252	.5728614	.8196523	.5870790	.8095296	3
58	.5587105	.8293628	.5730998	.8194856	.5873145	.8093588	2
59	.5589517	.8292002	.5733381	.8193189	.5875499	.8091879	1
60	0.5591929	0.8290376	0.5735764	0.8191520	0.5877853	0.8090170	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	56°		55°		54°		′

′	36°		37°		38°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.5877853	0.8090170	0.6018150	0.7986355	0.6156615	0.7880108	60
1	.5880206	.8088460	.6020473	.7984604	.6158907	.7878316	59
2	.5882558	.8086749	.6022795	.7982853	.6161198	.7876524	58
3	.5884910	.8085037	.6025117	.7981100	.6163489	.7874732	57
4	.5887262	.8083325	.6027439	.7979347	.6165780	.7872939	56
5	.5889613	.8081612	.6029760	.7977594	.6168069	.7871145	55
6	.5891964	.8079899	.6032080	.7975839	.6170359	.7869350	54
7	.5894314	.8078185	.6034400	.7974084	.6172648	.7867555	53
8	.5896663	.8076470	.6036719	.7972329	.6174936	.7865759	52
9	.5899012	.8074754	.6039038	.7970572	.6177224	.7863963	51
10	0.5901361	0.8073038	0.6041356	0.7968815	0.6179511	0.7862165	50
11	.5903709	.8071321	.6043674	.7967058	.6181798	.7860367	49
12	.5906057	.8069603	.6045991	.7965299	.6184084	.7858569	48
13	.5908404	.8067885	.6048303	.7963540	.6186370	.7856770	47
14	.5910750	.8066166	.6050624	.7961780	.6188655	.7854970	46
15	.5913096	.8064446	.6052940	.7960020	.6190939	.7853169	45
16	.5915442	.8062726	.6055255	.7958259	.6193224	.7851368	44
17	.5917787	.8061005	.6057570	.7956497	.6195507	.7849566	43
18	.5920132	.8059283	.6059884	.7954735	.6197790	.7847764	42
19	.5922476	.8057560	.6062198	.7952972	.6200073	.7845961	41
20	0.5924819	0.8055837	0.6064511	0.7951208	0.6202355	0.7844157	40
21	.5927163	.8054113	.6066824	.7949444	.6204636	.7842352	39
22	.5929505	.8052389	.6069136	.7947678	.6206917	.7840547	38
23	.5931847	.8050661	.6071447	.7945913	.6209198	.7838741	37
24	.5934189	.8048938	.6073758	.7944146	.6211478	.7836935	36
25	.5936530	.8047211	.6076069	.7942379	.6213757	.7835127	35
26	.5938871	.8045484	.6078379	.7940611	.6216036	.7833320	34
27	.5941211	.8043756	.6080689	.7938843	.6218314	.7831511	33
28	.5943550	.8042028	.6082998	.7937074	.6220592	.7829702	32
29	.5945889	.8040299	.6085306	.7935304	.6222870	.7827892	31
30	0.5948228	0.8038569	0.6087614	0.7933533	0.6225146	0.7826082	30
31	.5950566	8036838	.6089922	.7931762	.6227423	.7824270	29
32	.5952904	8035107	.6092229	.7929990	.6229698	7822459	28
33	.5955241	.8033375	.6094535	.7928218	.6231974	7820646	27
34	.5957577	.8031642	.6096841	.7926445	.6234248	7818833	26
35	.5959913	8029909	.6099147	.7924671	.6236522	7817019	25
36	.5962249	.8028175	.6101452	.7922896	.6238796	7815205	24
37	.5964584	.8026440	.6103756	.7921121	.6241069	7813390	23
38	.5966918	.8024705	.6106060	.7919345	.6243342	7811574	22
39	.5969252	8022969	6108363	.7917569	6245614	7809757	21
40	0.5971586	0.8021232	0.6110666	0.7915792	0.6247885	0.7807940	20
41	.5973919	.8019495	.6112969	.7914014	.6250156	.7806123	19
42	.5976251	.8017756	.6115270	.7912235	.6252427	7804304	18
43	.5978583	.8016018	.6117572	.7910456	.6254696	7802485	17
44	.5980915	8014278	.6119873	.7908676	.6256966	7800665	16
45	.5983246	8012538	.6122173	.7906896	.6259235	.7798845	15
46	.5985577	.8010797	.6124473	.7905115	.6261503	7797024	14
47	.5987906	.8009056	.6126772	.7903333	.6263771	.7795202	13
48	.5990236	.8007314	.6129071	.7901550	.6266038	.7793380	12
49	.5992565	8005571	.6131369	7899767	6268305	7791557	11
50	0.5994893	0.8003827	0.6133666	0.7897983	0.6270571	0.7789733	10
51	.5997221	.8002083	.6135964	.7896198	.6272837	.7787909	9
52	.5999549	.8000338	.6138260	.7894413	.6275102	.7786084	8
53	.6001876	.7998593	.6140556	.7892627	.6277366	.7784258	7
54	.6004202	.7996847	.6142852	.7890841	.6279631	.7782431	6
55	.6006528	.7995100	.6145147	.7889054	.6281894	.7780604	5
56	.6008854	.7993352	.6147442	.7887266	.6284157	.7778777	4
57	.6011179	.7991604	.6149736	.7885477	.6286420	.7776949	3
58	.6013503	.7989855	.6152029	.7883688	.6288682	.7775120	2
59	.6015827	.7988105	.6154322	.7881898	.6290943	.7773290	1
60	0 6018150	0.7986355	0.6156615	0 7880108	0 6293204	0.7771460	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	53°		52°		51°		′

′	39°		40°		41°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.6293204	0.7771460	0.6427876	0.7660444	0.6560590	0.7547096	60
1	.6295464	.7769629	.6430104	.7658574	.6562785	.7545187	59
2	.6297724	.7767797	.6432332	.7656704	.6564980	.7543278	58
3	.6299983	.7765965	.6434559	.7654832	.6567174	.7541368	57
4	.6302242	.7764132	.6436785	.7652960	.6569367	.7539457	56
5	.6304500	.7762298	.6439011	.7651087	.6571560	.7537546	55
6	.6306758	.7760464	.6441236	.7649214	.6573752	.7535634	54
7	.6309015	.7758629	.6443461	.7647340	.6575944	.7533721	53
8	.6311272	.7756794	.6445685	.7645465	.6578135	.7531808	52
9	.6313528	.7754957	.6447909	.7643590	.6580326	.7529894	51
10	0.6315784	0.7753121	0.6450132	0.7641714	0.6582516	0.7527980	50
11	.6318039	.7751283	.6452355	.7639838	.6584706	.7526065	49
12	.6320293	.7749445	.6454577	.7637960	.6586895	.7524149	48
13	.6322547	.7747606	.6456798	.7636082	.6589083	.7522233	47
14	.6324800	.7745767	.6459019	.7634204	.6591271	.7520316	46
15	.6327053	.7743926	.6461240	.7632325	.6593458	.7518398	45
16	.6329306	.7742086	.6463460	.7630445	.6595645	.7516480	44
17	.6331557	.7740244	.6465679	.7628564	.6597831	.7514561	43
18	.6333809	.7738402	.6467898	.7626683	.6600017	.7512641	42
19	.6336059	.7736559	.6470116	.7624802	6602202	.7510721	41
20	0.6338310	0.7734716	0.6472334	0.7622919	0.6604386	0.7508800	40
21	.6340559	.7732872	.6474551	.7621036	.6606570	.7506879	39
22	.6342808	.7731027	.6476767	.7619152	.6608751	.7504957	38
23	.6345057	.7729182	.6478984	.7617268	6610936	.7503034	37
24	.6347305	.7727336	.6481199	.7615383	.6613119	.7501111	36
25	.6349553	.7725489	.6483414	.7613497	.6615300	.7499187	35
26	.6351800	.7723642	.6485628	.7611611	.6617482	.7497262	34
27	.6354046	.7721794	.6487842	.7609724	.6619662	.7495337	33
28	.6356292	.7719945	.6490056	.7607837	.6621842	.7493411	32
29	.6358537	.7718096	.6492268	.7605949	.6624022	.7491484	31
30	0.6360782	0.7716246	0.6494480	0.7604060	0.6626200	0.7489557	30
31	.6363026	.7714395	.6496692	.7602170	.6628379	.7487629	29
32	.6365270	.7712544	.6498903	.7600280	.6630557	.7485701	28
33	.6367513	.7710692	.6501114	.7598389	.6632734	.7483772	27
34	.6369756	.7708840	.6503324	.7596498	.6634910	.7481842	26
35	.6371998	.7706986	.6505533	.7594606	.6637087	.7479912	25
36	.6374240	.7705132	.6507742	.7592713	.6639262	.7477981	24
37	.6376481	.7703278	.6509951	.7590820	.6641437	.7476049	23
38	.6378721	.7701423	.6512158	.7588926	.6643612	.7474117	22
39	.6380961	.7699567	.6514366	.7587031	.6645785	.7472184	21
40	0.6383201	0.7697710	0.6516572	0.7585136	0.6647959	0.7470251	20
41	.6385440	.7695853	.6518778	.7583240	.6650131	.7468317	19
42	.6387678	.7693996	.6520984	.7581343	.6652304	.7466382	18
43	.6389916	.7692137	.6523189	.7579446	.6654475	.7464446	17
44	.6392153	.7690278	.6525394	.7577548	.6656646	.7462510	16
45	.6394390	.7688418	.6527598	.7575650	.6658817	.7460574	15
46	.6396626	.7686558	.6529801	.7573751	.6660987	.7458636	14
47	.6398862	.7684697	.6532004	.7571851	.6663156	.7456699	13
48	.6401097	.7682835	.6534206	.7569951	.6665325	.7454760	12
49	.6403332	.7680973	.6536408	.7568050	.6667493	.7452821	11
50	0.6405566	0.7679110	0.6538609	0.7566148	0.6669661	0.7450881	10
51	.6407799	.7677246	.6540810	.7564246	.6671828	.7448941	9
52	.6410032	.7675382	.6543010	.7562343	.6673994	.7446999	8
53	.6412264	.7673517	.6545209	.7560439	.6676160	.7445058	7
54	.6414496	.7671652	.6547408	.7558535	.6678326	.7443115	6
55	.6416728	.7669785	.6549607	.7556630	.6680490	.7441173	5
56	.6418958	.7667918	.6551804	.7554724	.6682655	.7439229	4
57	.6421189	.7666051	.6554002	.7552818	.6684818	.7437285	3
58	.6423418	.7664183	.6556198	.7550911	.6686981	.7435340	2
59	.6425647	.7662314	.6558395	.7549004	.6689144	.7433394	1
60	0.6427876	0.7660444	0.6560590	0.7547096	0.6691306	0.7431448	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	50°		49°		48°		′

′	42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.6691306	0.7431448	0.6819984	0.7313537	0.6946584	0.7193398	60
1	.6693468	.7429502	.6822111	.7311553	.6948676	.7191377	59
2	.6695628	.7427554	.6824237	.7309568	.6950767	.7189355	58
3	.6697789	.7425606	.6826363	.7307583	.6952858	.7187333	57
4	.6699948	.7423658	.6828489	.7305597	.6954949	.7185310	56
5	.6702108	.7421708	.6830613	.7303610	.6957039	.7183287	55
6	.6704266	.7419758	.6832738	.7301623	.6959128	.7181263	54
7	.6706424	.7417808	.6834861	.7299635	.6961217	7179238	53
8	.6708582	.7415857	.6836984	.7297646	.6963305	.7177213	52
9	.6710739	.7413905	.6839107	.7295657	.6965392	7175187	51
10	0.6712895	0.7411953	0.6841229	0.7293668	0.6967479	0.7173161	50
11	.6715051	.7410000	.6843350	.7291677	.6969565	.7171134	49
12	.6717206	.7408046	.6845471	.7289686	.6971651	.7169106	48
13	.6719361	.7406092	.6847591	.7287695	.6973736	.7167078	47
14	.6721515	.7404137	.6849711	.7285703	.6975821	.7165049	46
15	.6723668	.7402181	.6851830	.7283710	.6977905	.7163019	45
16	.6725821	.7400225	.6853948	.7281716	.6979988	.7160989	44
17	.6727973	.7398268	.6856066	.7279722	.6982071	.7158959	43
18	.6730125	.7396311	.6858184	.7277728	.6984153	.7156927	42
19	.6732276	.7394353	.6860300	.7275732	.6986234	.7154895	41
20	0.6734427	0.7392394	0.6862416	0.7273736	0.6988315	0.7152863	40
21	.6736577	.7390435	.6864532	.7271740	.6990396	.7150830	39
22	.6738727	.7388475	.6866647	.7269743	.6992476	.7148796	38
23	.6740876	.7386515	.6868761	.7267745	.6994555	.7146762	37
24	.6743024	.7384553	.6870875	.7265747	.6996633	.7144727	36
25	.6745172	.7382592	.6872988	.7263748	.6998711	.7142691	35
26	.6747319	.7380629	.6875101	.7261748	.7000789	.7140655	34
27	.6749466	.7378666	.6877213	.7259748	.7002866	.7138618	33
28	.6751612	.7376703	.6879325	.7257747	.7004942	.7136581	32
29	.6753757	.7374738	.6881435	.7255746	7007018	.7134543	31
30	0.6755902	0.7372773	0.6883546	0.7253744	0.7009093	0.7132504	30
31	.6758046	.7370808	.6885655	.7251741	.7011167	.7130465	29
32	.6760190	.7368842	6887765	.7249738	.7013241	.7128426	28
33	.6762333	.7366875	.6889873	.7247734	.7015314	.7126385	27
34	.6764476	.7364908	.6891981	.7245729	.7017387	.7124344	26
35	.6766618	.7362940	.6894089	.7243724	.7019459	.7122303	25
36	6768760	.7360971	.6896195	.7241719	.7021531	.7120260	24
37	.6770901	.7359002	.6898302	.7239712	.7023601	.7118218	23
38	6773041	.7357032	6900407	.7237705	.7025672	7116174	22
39	.6775181	.7355061	.6902512	.7235698	7027741	7114130	21
40	0.6777320	0.7353090	0.6904617	0.7233690	0.7029811	0.7112086	20
41	.6779459	.7351118	.6906721	.7231681	.7031879	7110041	19
42	6781597	.7349146	.6908824	.7229671	.7033947	.7107995	18
43	.6783734	.7347173	.6910927	.7227661	.7036014	.7105948	17
44	6785871	.7345199	.6913029	.7225651	.7038081	.7103901	16
45	.6788007	.7343225	.6915131	.7223640	.7040147	7101854	15
46	.6790143	.7341250	.6917232	.7221628	.7042213	7099806	14
47	.6792278	.7339275	.6919332	.7219615	.7044278	7097757	13
48	6794413	.7337299	.6921432	.7217602	.7046342	.7095707	12
49	6796547	.7335322	6923531	.7215589	.7048406	7093657	11
50	0.6798681	0.7333345	0 6925630	0.7213574	0.7050469	0.7091607	10
51	.6800813	.7331367	.6927728	.7211559	.7052532	.7089556	9
52	.6802946	.7329388	.6929825	.7209544	.7054594	.7087504	8
53	.6805078	.7327409	.6931922	.7207528	.7056655	.7085451	7
54	.6807209	.7325429	.6934018	.7205511	.7058716	.7083398	6
55	.6809339	.7323449	.6936114	.7203494	.7060776	.7081345	5
56	.6811469	.7321467	.6938209	.7201476	.7062835	.7079291	4
57	.6813599	.7319486	.6940304	.7199457	.7064894	.7077236	3
58	.6815728	.7317503	6942398	.7197438	.7066953	.7075180	2
59	.6817856	.7315521	.6944491	.7195418	.7069011	.7073124	1
60	0.6819984	0.7313537	0 6946584	0.7193398	0 7071068	0 7071068	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	47°		46°		45°		

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